FREQUENCY STABILIZATION IN INJECTION CONTROLLED PULSED CO₂ LASERS WITH UNSTABLE RESONATOR CAVITIES

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SUMMARY

Longitudinal mode selection by injection has been demonstrated as a viable technique for tailoring a TEA-CO₂ laser with pulse energies of a Joule or greater to fit the requirements of a coherent lidar transmitter. Once reliable generation of single-longitudinal-mode (SLM) pulses is obtained, one can study the intrapulse frequency variation and attempt to determine the sources of frequency sweeping, or chirp. These sources include the effect of the decaying plasma, the thermal gradient due to the energy dissipation associated with the laser mechanism itself, and the pressure shift of the center frequency of the laser transition. The use of the positive-branch unstable resonator as an efficient means of coupling a discharge with transverse spatial dimensions of the order of centimeters to an optical cavity mode introduces another concern: namely, what can be done to emphasize transverse mode discrimination in an unstable resonator cavity while maintaining high coupling efficiency. These issues are briefly discussed in the paper, and representative experimental examples are included.

DESCRIPTION OF EXPERIMENTAL APPARATUS

Mode selection in a TEA-CO₂ laser cavity through the use of injection techniques provides high peak-power SLM pulses with relatively low intensities of injected radiation [refs. 1,2]. Provided the selected mode is near the center of the pressure-broadened TEA-CO₂ gain transition and the frequency proximity requirements are met, as little as 10 µW of injected power is sufficient to ensure a single-frequency pulse with peak power of several Megawatts.

An experimental coherent CO₂ lidar has been developed at JPL which uses an injection-controlled TEA-CO₂ transmitter. The injection control technique has been refined to the level at which this lidar has been used to measure vertical profiles of atmospheric backscatter and atmospheric wind velocity on several occasions, with particular emphasis on collecting vertical aerosol backscatter profile with enough frequency during the past four years to permit a statistical analysis of seasonal differences, trends, etc. The studies of injection control of the laser have been reported in the literature [refs. 1-4]. The lidar optical block diagram, indicating the front-end injection, is displayed in Figure 1. A brief description follows.

The TEA-CO₂ laser is a modified Lumonics model 102-2 normally operated at pressures around 760 torr with a flowing gas mixture of He:CO₂:N₂ (85:9:6). The gain section is 80 cm and the total cavity length is 2.4 m, corresponding to a longitudinal mode spacing of 62.5 MHz. The optical cavity is formed from a Littrow mounted reflection grating (150 lines/mm) and a germanium meniscus convex output coupler. The inside
and outside radii of curvature of the Ge coupler are 15 m and 41.5 m, respectively, and both surfaces are AR coated for 10.6 μm to a diameter of 40 mm. The central 14.6 mm square area of the inside surface is uncoated yielding an intensity reflection coefficient for normal incidence of approximately 40%. This positive branch unstable resonator cavity has a magnification of approximately 2.2 (fractional power loss per roundtrip = 80%) and provides a near-diffraction limited far-field pattern. The injection oscillator is a CW CO₂ waveguide laser operated at a pressure near 200 torr. The waveguide laser energy which is reflected by the TEA laser optical cavity is focused onto a cooled HgCdTe photodetector. This detector signal is used as a diagnostic for automatic (feedback loop) control of the relationship between the waveguide laser frequency and the longitudinal mode frequencies (mc/2L) of the TEA cavity.

The frequency stability of the injection-controlled TEA-CO₂ laser was examined by a heterodyne technique, schematically illustrated in Fig. 2. The signal was recorded at 30 MHz intermediate frequency corresponding to the center frequency of the filter/amplifier. In order not to use the transient digitizer at a speed higher than 50 MHz, where the A/D conversion was not accurate enough for our purpose, the 30 MHz IF signal was converted to 10 MHz through an RF mixer followed by a 20 MHz filter. The time evolution of the period of the IF signal was calculated with a temporal resolution equivalent to 2 periods (200 ns) in order to improve the accuracy of the frequency measurement (300 kHz). Representative examples of results will be shown following a brief discussion of sources of frequency sweeping.

**INTRAPULSE FREQUENCY SWEEPING**

The frequency sweeping of the laser follows any changes in the cavity refractive index, n, at the selected mode frequency. The three major potential sources of phase shifting due to refractive index temporal variations are briefly discussed in the following paragraphs.

The refractive index variation associated with the time dependent electron density in the decaying discharge plasma occurs mainly during the initial part of the pulse build-up time but may still represent a substantial refractive index perturbation at the time of the peak of the optical pulse [ref. 5]. This effect also causes the mode selection zone to be offset slightly with respect to the frequency of the selected mode of the pre-discharge TEA laser cavity [ref. 3]. Assuming for our laser an average electron density of 10¹³ cm⁻³ during a 0.5μs excitation pulse, the plasma frequency νₚ is 2.10¹¹ Hz. The frequency change, in the laser cavity, due to this electron density is calculated to be about 5 MHz. (The observed frequency deviation near the time of the appearance of the gainswitched-spike-maximum (GSSM) is approximately 2 MHz).

One should also consider the mode pulling effect arising from the frequency difference between the TEA molecular gain curve maximum νₒ and the injected radiation frequency ν₄ coincident with the resonant mode of the empty resonator. The equivalent frequency pulling is:

\[
\Delta ν₄ = \frac{c}{4π} \frac{νₒ - ν₄}{Δνₒ} \frac{gₖ}{L}
\]
where \( c \) is the light velocity, \( \Delta v_0 \) is the half width half maximum of the TEA molecular gain curve, \( g \) is the gain of the medium in the cavity, and \( t/L \) is the filling factor. The injected laser frequency normally corresponds to the center frequency of a low pressure (10 torr) CO\(_2\) laser tuned to the same line as that of the TEA laser. A careful study of the pressure shift of CO\(_2\) transition frequencies was done by SooHoo et al. [ref. 6] with the pressure ranging from 10\(^{-2}\) torr to 1 torr, but little is known about the pressure shift between 10 torr and 1 atm. The problem is further complicated by the dependence on the gas mixing composition. Referring to Agalakov's work [ref. 7], one may calculate a shift of +240 MHz for a 10% CO\(_2\) 90% He gas mix and lines in the P branch. Hollins and Jordan [ref. 8] measured a very small shift of -5 MHz for a 40% CO\(_2\), 20% N\(_2\), 40% He gas mix. This small shift appears to be consistent with Agalakov’s work, because it can be attributed to a cancellation of shifts of opposite sign caused by CO\(_2\) - CO\(_2\) (or CO\(_2\) - N\(_2\)) and CO\(_2\) - He collisions. A pressure shift of +240 MHz is large enough to induce a 2 MHz mode pulling when the gain coefficient \( g \) is changing from 0 to 2.1\times10\(^{-2}\) cm\(^{-1}\). The mode pulling due to the saturation of the CO\(_2\) molecules in the non-active region of the cavity is an order of magnitude smaller and can be neglected.

Finally, laser-induced medium perturbation (LIMP) has been found to be a major cause of changes in frequency \( \nu \) during the optical pulse and is related in the following way to the output energy \( E \), the beam radius \( \sigma \), and the cavity length \( L \) [9]:

\[
\Delta \nu = \frac{2KRv}{\pi C_v} \frac{E}{\sigma^4 L} t^2 = \alpha t^2
\]

where \( K \) is the Gladstone-Dale constant and \( R \), \( C_v \) the usual thermodynamic constants. For a cavity with transverse dimension of its spatial modes larger than 1 cm, and output energy of 1 J (assuming 100% loss), \( \alpha \) is smaller than 50 kHz/ps.

The salient features of this qualitative analysis of frequency sweeping mechanisms are depicted in Figure 3. Although frequency variations appearing before the gain switch spike maximum (GSSM) are presented, only those occurring near or after the GSSM contribute to the spectral broadening of the laser pulse.

**TRANSVERSE MODE DISCRIMINATION**

It is important to analyze the properties of the lowest order transverse modes in an unstable resonator cavity which is appropriate for a TEA-CO\(_2\) laser. Often the mode loss separation of the first 2-3 modes is small enough such that if the lowest-loss mode suffers incrementally due to spatial hole burning or some other spatial nonuniformity, the appearance of another transverse mode during a portion of the pulse duration may result. It has been pointed out by Weiner [ref. 10] that the unstable resonator mode loss discrimination is sensitive to tilt, such that the optical axis, which is determined by the line through the centers of curvative of the cavity defining mirrors, intersects the output coupler at an off center position. The degree of tilt can be described by the normalized off-axis distances \( \xi_x, y = h_x, y/\sigma \), as depicted in Figure 4. (In Figure 4 the reflective portion of the output coupling optic is represented by the shaded square, which is offset with respect to the laser output intensity pattern.) The optical axis intersects the output coupler at the origin of coordinates. Weiner analyzed the effects of
off-axis tilts on mode-loss discrimination for a high equivalent-Fresnel-number example \( (N_{eq}=9.6) \), using calculations based on the asymptotic technique.

To analyze the transverse mode properties of the TEA laser positive branch unstable resonator in a more general fashion, we use the Huygens-Fresnel integral equation for a one-dimensional (strip resonator) case [ref. 11] to define eigenfunctions \( u_m(x) \) and eigenvalues \( \gamma_m = \gamma_m \exp(i\phi_m) \), corresponding to the mode amplitude distributions and the energy losses and phase shifts per round trip in the cavity. Applying symmetry arguments to the orthogonal dimension, the two-dimensional transverse mode shape, fractional power loss and mode frequency are given by:

\[
u_{mnq} = \frac{c q + c (\phi_m + \phi_n - 2kL)}{2\pi L}
\]

In order to solve the integral equation, an iterative method (the Prony method) was used. This was first applied by Siegman and Miller [ref. 12] to the case of a symmetric resonator with circular mirrors. An iterative method is preferable to the asymptotic method for low Fresnel number.

To illustrate an example of the effects of a slight off-axis tilt on mode-loss separation, we have tabulated in Table I the results of the mode loss calculations for the three lowest order modes, comparing the on-axis case with the case for which \( \epsilon = 0.3 \). The \( N_{eq} = 1.3 \) and the magnification factor \( M = 2.2 \) for this case. It is noteworthy that the lowest order mode loss separation is quite small for the on-axis case but changes dramatically for \( \epsilon = 0.3 \). In fact, our analysis indicates that experimental alignment tolerances which result in a small but non-zero value for \( \epsilon \), even when onaxis symmetry is the goal, will result in a much larger mode-loss separation than is indicated in the ideal \( \epsilon = 0 \) case. However, an intentional tilt in order to achieve a value for \( \epsilon \) in the neighborhood of \( 0.25 - 0.5 \) should definitely help to ensure single transverse mode oscillation throughout the laser pulse.

### Table I. Unstable Resonator Mode Losses (%)

<table>
<thead>
<tr>
<th>Mode index, ((m,n))</th>
<th>On-axis (even symmetric)</th>
<th>Off-axis ((\epsilon=0.3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>80.0</td>
<td>65.2</td>
</tr>
<tr>
<td>(0,1)</td>
<td>80.9</td>
<td>80.0</td>
</tr>
<tr>
<td>(1,1)</td>
<td>81.8</td>
<td>88.5</td>
</tr>
</tbody>
</table>

EXPERIMENT RESULTS AND DISCUSSION

Two examples which are typical displays of the TEA-CO₂ laser pulse shape and corresponding chirp record are shown in Figures 5 and 6. The decreasing frequency characteristic at the beginning of each pulse can be attributed to the plasma
decay. The dotted line on the figure represents the expected plasma decay effect. As expected, the chirp due to the LIMP mechanism is less than 100 kHz. The same results were obtained when we shifted the injection frequency by 60 MHz with respect to the transition center frequency of the atmospheric pressure broadened TEA line. In other words, any effect of mode pulling towards the line-center frequency was not discernible. The record shown in Figure 5 reveals a large frequency shift (7-8 MHz) which appears just after the gain-switched-spike and disappears synchronously with the sharp amplitude spike at 1.5 μs. The occurrence of such sudden frequency shifts was found to be strongly dependent on the alignment of the laser cavity in the region near the "on-axis" configuration. These frequency shifts were not observed for a slightly misaligned "off-axis" configuration, which was a surprise to us until the transverse mode studies were accomplished. The numerical analysis of the transverse modes of the unstable resonator with characteristics matching our TEA-CO$_2$ laser provides a plausible interpretation of these rather large frequency shifts. The significant difference in spatial intensity distributions of the two most dominant transverse modes provides a reasonable mechanism for the sudden appearance and disappearance of the next higher mode just after the GSS. The medium is highly saturated just after the GSS, and spatial variation in the degree of saturation might allow the second mode to gain the advantage for a period of time. This would be most probable when the mode-loss separation is small.

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REFERENCES


Figure 1. Optical block diagram of coherent lidar system using an injection-controlled TEA-CO\textsubscript{2} laser transmitter with an unstable resonator cavity.

Figure 2. Block diagram of heterodyne detection apparatus for frequency chirp measurement of TEA-CO\textsubscript{2} laser pulse.
Figure 3. Semi-quantitative depiction of the temporal dependence of the pulse output frequency, assuming a selected cavity mode near the low-pressure laser transition center frequency.

Figure 4. Geometry of the off-axis case in which the output intensity pattern (with approximately square exterior dimensions) is displaced from the central reflective patch (also square) of the output coupler.
- LARGE PLASMA DECAY EFFECT
- INADEQUATE MODE LOSS DISCRIMINATION

Figure 5. Example of the intrapulse frequency variation when a sudden jump to a second transverse mode occurred just after the GSSM.

- LARGE PLASMA DECAY EFFECT
- ADEQUATE MODE LOSS DISCRIMINATION

Figure 6. Example of the intrapulse frequency variation when oscillation was maintained on a single transverse mode as well as single longitudinal mode.