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THREE-DIMENSIONAL MAGNETOSTATIC MODELS OF THE LARGE-SCALE CORONA

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INTRODUCTION

There is a relatively stable component of the large-scale corona that persists for periods of the order of a solar rotation. This component is captured in the synoptic charts constructed from the daily coronagraph observation of the density structures visible in Thomson-scattered light from the photosphere (e.g., Fisher and Sime 1984). The Thomson-scattered light is optically thin and the coronagraph images correspond to electron densities integrated along the line of sight. A long standing need therefore exists for the development of theoretical models of the corona capable of relating the observed integrated density to the actual three-dimensional coronal structures. It is the magnetic field and electric current in the highly conducting medium of the corona that order the density into various structures. Unfortunately, the coronal magnetic field cannot be measured to any useful spatial resolution. Only the longitudinal component of the magnetic field in the photosphere can be measured routinely with a magnetograph. An obvious approach for a theoretical model to adopt is to use the measured photospheric magnetic field as an input and extrapolate for both the magnetic field and density in the corona. Until recently, this approach has been feasible only with the potential field model (e.g., Newkirk and Altschuler 1970). The disadvantage, as is well known, is that a potential magnetic field does not interact with the plasma and the model predicts no density structure. The idea to use linear force-free magnetic fields with a constant α, proposed by Nakagawa (1973), has difficulties of nonuniqueness and infinite total magnetic energy when applied to coronal models in spherical geometry (Seehafer 1978). Non-constant α force-free magnetic fields and general non-potential magnetic fields in static equilibrium have so far proved to be mathematically intractable in three-dimensional geometry. In this paper, we describe a special class of magnetostatic equilibria, which are mathematically simple and yet sufficiently versatile so as to fit any arbitrary normal magnetic flux prescribed at the photosphere. With these solutions, the corona can be modeled with precisely the same mathematical procedure as has previously been done with potential fields. The magnetostatic model predicts, in addition to the coronal magnetic field, the three-dimensional coronal density which can be compared with coronagraph observations. This is an ongoing project being conducted at the High Altitude Observatory (HAO), using data from the Kitt Peak Magnetograph and several coronagraphs operated by HAO.

THREE-DIMENSIONAL MAGNETOSTATIC MODELS

The corona is in a state of expansion (Parker 1963, Hundhausen 1972). The expansion is believed to be negligible in the low corona, and, as a first approximation, the low corona is taken to be in static equilibrium and described by

$$\frac{1}{4\pi}\left(\vec{\nabla}\times\vec{B}\right)\times\vec{B} - \vec{\nabla}p - \frac{\rho G M_\odot}{r^2}\hat{r} = 0\,, \qquad [1]$$

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$$\vec{\nabla} \cdot \vec{B} = 0 , \qquad [2]$$

where $\vec{B}$, p and ρ are, respectively, the magnetic field, pressure and density, G is Newton's gravitational constant, $M_\odot$ the solar mass and r the heliocentric distance. To close the equations, we may relate the temperature to p and ρ through the ideal gas law and specify an equation for steady energy balance. The balance of energy in the corona plays an important role in determining coronal structures. The detailed processes are complicated and our current knowledge about them is very incomplete. In any case, to account for these processes would render the magntostatic problem quite intractable. It therefore seems reasonable, as a first step, to ignore the energy equation and seek solutions to equations (1) and (2) with no further constraints. Even within such an artificial consideration, the magnetostatic problem is not readily tractable (see the review by Low (1985a)). Equations (1) and (2) have not been treated in fully three-dimensional geometry except for the cases of the potential magnetic field and the constant-α force-free magnetic field.

In a recent development, Low (1985b) showed that equations (1) and (2) can be reduced to a simpler problem without assuming any special geometry if the electric current density $\vec{J}$ is assumed to flow everywhere perpendicular to the solar gravity. In this case, employing the usual spherical coordinates, we can express the magnetic field and electric current density as

$$\vec{B} = \Psi \hat{r} + \frac{1}{r}\frac{\partial \Phi}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial \Phi}{\partial \phi}\hat{\phi} , \qquad [3]$$

$$\vec{J} = \frac{c}{4\pi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\Psi - \frac{\partial \Phi}{\partial r}\right)\hat{\theta} - \frac{c}{4\pi}\frac{1}{r}\frac{\partial}{\partial \theta}\left(\Psi - \frac{\partial \Phi}{\partial r}\right)\hat{\phi} , \qquad [4]$$

where Ψ is an arbitrary function of $\dfrac{\partial \Phi}{\partial r}$ and r. The magnetostatic problem reduces to solving

$$\frac{\partial}{\partial r}\left(r^2 \Psi(\frac{\partial \Phi}{\partial r}, r)\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial \Phi}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 \Phi}{\partial \phi^2} = 0 , \qquad [5]$$

for Φ as a boundary value problem given $\Psi(\dfrac{\partial \Phi}{\partial r}, r)$. The freedom to choose the form of Ψ arises from our not imposing an energy equation. From the solution Φ, the magnetic field is given by equation (3) and the equilibrium pressure and density are given by

$$p = p_0(r) - \frac{1}{8\pi}\Psi(\frac{\partial \Phi}{\partial r}, r) + \frac{1}{4\pi}\int^{\frac{\partial \Phi}{\partial r}} dv\, \Psi(v, r) , \qquad [6]$$

$$\rho = \rho_0(r) +$$

$$\frac{r^2}{GM_\odot}\left(\frac{1}{4\pi}\frac{1}{r^2\sin^2\theta}\frac{\partial \Phi}{\partial \phi}\frac{\partial \Psi}{\partial \phi} + \frac{1}{4\pi r^2}\frac{\partial \Phi}{\partial \theta}\frac{\partial \Psi}{\partial \theta} - \frac{1}{8\pi}\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial r}(\frac{\partial \Phi}{\partial \phi})^2 - \frac{1}{8\pi r^2}\frac{\partial}{\partial r}(\frac{\partial \Phi}{\partial \theta})^2\right) , \qquad [7]$$

where p_0 and ρ_0 are the pressure and density of an arbitrary spherically symmetric hydrostatic atmosphere. The familiar case of a potential magnetic field is recovered if we set $\Psi = \dfrac{\partial \Phi}{\partial r}$, and then only the hydrostatic components in equations (6) and (7) remain. In general, equation (5) is nonlinear and suitable numerical methods need to be developed to solve it. If we set

$$\Psi = \eta(r)\frac{\partial \Phi}{\partial r}, \qquad [8]$$

where η is given, equation (7) is linear in Φ and standard numerical methods for elliptic equations can be employed to solve the boundary value problem. Bogdan and Low (1986) considered several cases of given η for which the boundary value problem is soluble analytically in terms of classical functions. In particular if

$$\eta(r) = (1 + \frac{a}{r})^2, \qquad [9]$$

where a is a constant, equation (7) admits the series solution

$$\Phi(r,\theta,\phi) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left(A_{mn}(r + a)^{-n-1} + B_{mn}(r + a)^{n}\right) Y_n^m(\theta,\phi), \qquad [10]$$

where A_{mn} and B_{mn} are constant coefficients and Y_n^m are the usual spherical harmonics. As to be expected, the classical spherical-harmonic expansion for the potential field is obtained if $a = 0$. In fact, equation (9) shows that $\Psi \rightarrow \Phi$ in the limit of $r \rightarrow \infty$ so that in that limit, the magnetic field is potential. In other words, the electric current is confined to the low corona within a heliocentric distance of the order of a. For a given non-zero a, equation (10) can be fitted with a prescribed radial magnetic field at an inner spherical surface identified with the solar surface, using $B_r = \Psi$. An outer boundary condition is needed to determine the coefficients A_{mn} and B_{mn} uniquely. This outer condition may take the form of requiring the magnetic field to vanish as $r \rightarrow \infty$ or that the magnetic field becomes purely radial at some outer spherical surface (e.g., Newkirk and Altschuler 1970). The procedure follows essentially the same steps as that of the classical potential field model. However, the magnetic field is not potential for a non-zero with the equilibrium pressure and density is given by equations (6) and (7).

DISCUSSION

We plan to generate three-dimensional magnetospheric models of the corona based on equations (9) and (10), with an input of the photospheric normal magnetic flux from the Kitt Peak Magnetograph. This is a first opportunity for constructing a three-dimensional magnetostatic corona which is based on photospheric magnetograms and can be tested against coronagraph observations. By varying the free parameter a, it will be interesting to see if we can achieve a reasonable agreement between the density predicted by the model and the observed line-of-sight integrated density. The model is based on several artificial assumptions and we do not have *apriori* reasons to expect excellent agreement. The assumption of the electric current flowing perpendicular to gravity is not as severe as it may appear. Such a distribution of the electric current may result from stretching a magnetic field radially outward, starting from a potential state, as demonstrated by the solutions described by equation (10) for $a > 0$ (Bogdan and Low 1986). The outward stretch of the magnetic field may result from the buoyancy of magnetic structures. The assumption of equation (9) is more restrictive. It requires that the presence of electric current can be characterized by a single parameter, namely, a. The illustrative examples in Bogdan and Low (1986) look promising, but it remains to be seen from actual modelling with data if this is a workable hypothesis.

REFERENCES

Bogdan, T. J., and Low, B. C. 1986, "Three Dimensional Structures of Magnetostatic

Atmospheres. II. Modelling the Large-Scale Corona" , *Ap.J.* , in press.

Fisher, R., and Sime, D. G. 1984, "Rotational Characteristics of the White-Light Corona: 1965 - 1983" , *Ap.J.* **287,** 959.

Hundhausen, A. J. 1972, "Coronal Expansion and the Solar Wind" (Springer-Verlag).

Low, B. C. 1985a, "Some Recent Developments in the Theoretical Dynamics of Magnetic Fields" , *SolarPhys.* **100,** 309.

Low, B. C. 1985b, "Three-Dimensional Structures of Magnetostatic Atmospheres. I. Theory" , *Ap.J.* **293,** 31.

Nakagawa, Y. 1973, "A Practical Representation of the Solar Magnetic Field" , *Astron.Astrophys* **27,** 95.

Newkirk, Jr., G., and Altschuler, M. D. 1970, "Magnetic Fields and the Solar Corona. III. The Observed Connection between Magnetic Fields and the Density Structures of the Corona" , *Solar-Phys.* **13,** 131.

Parker, E. N. 1963, "Interplanetary Dynamical Processes" (Interscience).

Seehafer, N. 1978, "Determination of Constant α Force-Free Solar Magnetic Fields from Magnetograph Data" , *SolarPhys.* **58,**215.