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EXAMPLE ON HOW TO MODEL AND SIMULATE TURBULENCE FOR FLIGHT SIMULATORS

John C. Houbolt
NASA Langley Research Center
Hampton, Virginia

Theré has been a lot of analytical development on gust response in the past several years, but evidently the material has not been disseminated very well.

Therefore, I would like to first discuss length scale, L; using the spectrum differently; how σ and L form a combined parameter; why L is not important; and the exceedance number N_0.

Consider Figure 1 which deals with the scale of turbulence. Note that sometimes it is improper to derive an artificial or apparent value for turbulence length scale and then label it as the integral scale of turbulence. Suppose we have some data, as depicted in Figure 1, and then we curve fit an analytical function to the data. We do this specifically to deduce a value of L that makes the function fit the data. We should be very careful and not call this deduced value the integral scale of turbulence. Keep in mind that what we are doing is not only measuring the turbulence but also measuring the phemonenon that is causing the turbulence. The value of L may thus be misleading.

Figure 2 shows the power spectrum as obtained from measurements of turbulence and winds for very different intervals of sampling times ranging from 1 second to 1 minute, to an hour, to a day, to a week, to a month, to a year, and to five years. Just about all wavelengths of turbulence are possible in this representation of the turbulence spectra. If we fit a chosen function to the data, say a von Karman function, we might deduce a scale of turbulence on the order of 1000 miles. Thus, be very careful how you describe the scale of turbulence because it depends on the phenomenon and on the time interval of sampling. In the case of sampling over years, we are working with wavelengths that may be several thousand miles long.

For a number of years I have advocated that spectral functions should be looked at in a different way; that is, use the same spectrum function or functions that we have used before but interpret them differently. For example, we can rearrange the von Karman spectrum function so that is appears as shown in Figure 3. There is only a single line at the high frequencies. We combine both the severity and scale of turbulence to form a new parameter, designated as σ_1 in the figure. Non-dimensionalizing the spectrum with this parameter results in all the curves condensing to the elegant form shown. Working with this modified form of the analytical function greatly simplifies the rest of the analysis.

For example, suppose we have made measurements in a patch of turbulent air and have deduced the power spectrum shown schematically in Figure 4. If you make use of the function depicted in Figure 3, you can calculate automatically the combined parameter $\sigma_w/L^{1/3}$ by the equation:

$$\frac{\sigma_w}{L^{1/3}} = [1.919\ \Omega_1^{5/3}\ \phi_w(\Omega_1)]^{1/2} \tag{1}$$

This equation is obtained by simply going to the straight line portion of the curve, any place along it, and inserting the values of the abscissa, Ω_1, and ordinate, $\phi_w(\Omega_1)$, into the equation. Do not try to separate the severity from the scale in any more detail. They are combined in the parameter, σ_1, and they should be used that way.

Let's also make an inference from this observation. For a given set of data, $\sigma_w/L^{1/3}$ is a constant value. What does that infer? It infers the results shown in Figure 5.

From this figure we can, if indeed we want to, split it out and write σ_w as a function of L; specifically, $\sigma_w = CL^{1/3}$. It is not surprising then that the British have come up with the notion that the turbulence severity tends to vary according to the third power of the gust gradient distance. Spectral theory predicts this behavior if L is equated to gust gradient distance H as is often supposed. But again, I remind you, although this behavior can be inferred, it is not necessary to separate σ_w from L; σ_1 should be used as a combined parameter.

When we use the combined parameter, σ_1, we find the output spectrum of the vertical acceleration for an airplane as a function of the reduced frequency appears as shown in Figure 6. The influence of scale shows up only in a minor way at the lefthand tails of the curves; the influence is inconsequential with respect to the overall acceleration that the airplane feels because the primary airplane response takes place out in the region of frequency where scale is completely out of the picture. This observation is true for all the airplanes I have examined so far. As an aside, we should keep in mind that at the very low frequencies where scale does have a minor effect, we are dealing with wavelengths where the pilot, the autopilot, or the navigation system is controlling the airplane. The question of turbulence scale is thus a moot point.

Some questions have arisen about the number of zero crossing values, N_0, particularly with regard to certain pertinent integrals which do not converge. However, if it is done right, there is no problem getting a meaningful value of N_0. The N_0 integral will converge to a realistic value if the proper ingredients are included in the analysis. These are specifically the two functions shown in the middle of the equation on Figure 7. This equation depicts in simplified form the spectrum for the vertical acceleration of the center of gravity (c.g.). The first function on the right-hand side of the equation is a simplified form of the airplane transfer function. The last function represents the gust spectrum in simplified form. The second function takes into account gust penetration effects; notice the k^2 falloff at high frequency. The third term takes into account the effects of spanwise variation in turbulence. This avoids the usual assumption that the gusts are uniform in the spanwise direction. Observe how the effect of spanwise variation falls off inversely with k at the high frequency. Notice that the spanwise effects function also contains the aspect ratio A. When the two middle functions shown in Figure 7 are included, no problem is involved in determining the value of N_0.

Some simplified results for N_0 that have been obtained will now be discussed. To start the discussion, it is noted that the study of a number of airplanes indicates that the reduced frequency k_0 is related to the reduced short-period frequency by:

$$k_0 = 1.29\, k_s^{0.6} \tag{2}$$

where

$$k_s = \omega_s \frac{c}{2V} \tag{3}$$

In turn, the zero crossing value follows:

$$N_0 = \frac{V}{\pi c} k_0 \tag{4}$$

Consider now the history of the gust loads analysis. If we consider the load on an airplane when it enters a sharp-edged gust such as shown in Figure 8, the load or lift on the airplane is given by:

$$L = \frac{a}{2} \rho S V^2 \frac{U}{V} \tag{5}$$

Equating this lift to an equivalent incremental acceleration gives:

$$\Delta n = \frac{L}{W} = \frac{a \rho S V}{2W} U \tag{6}$$

Note that the basic parameter which involves the combination of the variables a, ρ, s, V, and W is an equation we have seen and used for years. Its continued use, however, has led us into a trap. Later I will show that by rearranging the form of the basic parameter, our results will be greatly simplified. This equation is a first cut at establishing the vertical acceleration the airplane will feel when entering a sharp edge gust. We recognize, however, that gust penetration effects, non-steady lift effects, and the vertical motion of the aircraft tend to alleviate the load. In the early years--the 1940's--we introduced an alleviation factor (K) in the equation:

$$\Delta n = \frac{a \rho S V}{2W} K_g U \tag{7}$$

The factor was arbitrarily derived and was plotted as a function of the wing loading on the airplane as illustrated in Figure 9. We recognized, however, that the wing loading was not the right parameter to use when we started analyzing the acceleration in a more rational way, that is, when we began to include penetration effects, non-steady lift effects, and airplane motion effects.

When these various effects were taken into account the results shown in Figure 10 were obtained. The fundamental assumption leading to this figure is that the airplane is a point mass which moves in the vertical direction only; the gust was assumed uniform across the span. The incremental acceleration is noted to be of the same form as obtained for a sharp edge gust, except that a rationally derived alleviation factor, K_g, is introduced. K_g was found to be a function of the mass parameter μ. The gust shape assumed was a one minus cosine with a gust gradient distance H of 10 to 12 chords. U was taken to be on the order of 50 fps. Actually, there is nothing magic in the choice of the one-cosine gust; it is arbitrary. A triangle or half sine wave would have served equally well.

Progressing historically, the power spectral techniques for analyzing the response of aircraft in turbulence began to be introduced. Some basic results obtained are shown in Figure 11. The equation for vertical acceleration:

$$\sigma_{\Delta n} = \frac{a\rho SV}{2W} K_\phi \, \sigma_w$$

$$= \frac{V}{cg} \frac{K_\phi}{\mu} \sigma_w \tag{8}$$

is found to be analogous to the discrete gust equation, except that the gust severity and acceleration values are now expressed in rms units. The alleviation factor K_ϕ is also found to be a function of the mass parameter μ, and in addition is found to depend on 2L/c. This ratio L/c is analogous to the gust gradient distance in the discrete gust formulation. We should note that if the gust spectrum had been introduced as depicted in Figure 3 (i.e., as a function of σ_1), then the various curves in Figure 11 would collapse to nearly a single curve.

When everything is put together in a simple rational way, the gust response equation for acceleration can be shown to collapse to the very simple result:

$$\Delta n = 1.5 \sqrt{\frac{0.13}{\alpha}} \tag{9}$$

However, Equation 9 is the complete equation for designing an airplane for gust penetrations; α is the angle of attack of the airplane necessary to maintain level flight, where α has the value at which $C_L = 0$. That is all there is. The equation automatically takes into account the altitude of the airplane, the speed of the airplane, the weight, all the alleviation factors, everything. I believe this to be a profound equation. People should be aware of it and it should be introduced into the regulations. We must note, however, that we have not been able to change the regulations for 40 years so the chances of getting this equation into the regulations appear slim.

Note the inferences from the equation. If you run into turbulence, one of the first things you want to do is slow down a little. To slow down but maintain altitude you've got to increase α. Increasing α gives you smaller incremental accelerations. As I mentioned, this is a fascinating equation, and I hope we can make the aviation community aware that it exists.

I also have derived generalized equations for N_0. If we had started with the von Karman expression, the N_0 value is simply given by:

$$N_0 = \frac{1.084}{\sqrt{c\alpha}} \tag{10}$$

Again, all flight conditions are taken into account in this equation. The only item determining N_0 is α. If we had started with the Dryden spectrum, the same form of the result is found but the constant is different:

$$N_0 = \frac{0.858}{\sqrt{c\alpha}} \tag{11}$$

Now consider the aspects of turbulence for simulator applications. There has been trouble in the past with the simulations of turbulence in flight simulators. This is primarily because only one component was used. There has been some attempt to alleviate this situation with added sophistication but overall this has not been realistic. Specifically, attempts have been made to include non-stationary turbulence such as a modulation times a stationary kind of random turbulence. But invariably when pilots fly the simulator they comment that "It does not seem realistic." It is not surprising that it does not seem realistic because the simulation is not very realistic. As I have mentioned on previous occasions, turbulence is three-dimensional in nature, and this must be taken into account.

For example, as shown in Figure 12 there are, in general, three forces and three moments due to turbulence. Not all these forces are important, not all the moments are important. There are three, in particular, that are significant. They are: (1) vertical force, (2) pitch moment, (3) rolling moment. In many cases, pitching and rolling moment have not been taken into account. We must look at the turbulence situation in a little more realistic fashion. We cannot have a rolling moment if we make the assumption that the turbulence is uniform in the spanwise direction. There is a spanwise gradient in the turbulence just like there is a variation in longitudinal direction of flight. When we take into account the spanwise gradient you will have rolling moments on an airplane. All pilots know this fact. During approach an airplane can suddenly be thrown into a 20 degree roll condition. So in simulation studies we should at least include the vertical force, pitch moment, and roll moment because these are the important ones. In general we have not done so. The question is, how do we do that? The remainder of the presentation gives a quick insight as to how we can introduce the vertical force and the two important moments into simulation studies in a very realistic way.

Figure 13 introduces the notion of cross spectra. Along paths W_1 and W_2 we have different turbulence time histories. We have, in turn, differing cross spectra according to the separation distances that are involved. Let's take this into account in deriving the equations that produce the vertical force and the rolling moment.

Consider the vertical force as an example. We can simulate this very rationally in a simulator. The lift is given by:

$$L = \frac{a}{2} \rho S V^2 \frac{W}{V} \tag{12}$$

or

$$L = \frac{1}{2} \rho V^2 S c_L \tag{13}$$

where

$$c_L = a \frac{W}{V} \tag{14}$$

The actual form of the equation for L is much more complicated than shown, but if we considered the equation in complete form and took the Fourier transform of the lift coefficient you would arrive at the F_{c_L} function:

$$F_{c_L}(\omega) = \frac{a}{V} (P + iQ)(R + iS) F_w(\omega) \tag{15}$$

Because we have non-steady lift effects, we work with complex numbers in the frequency plane; (P + iQ) gives the in-phase and out-of-phase lift components that are due to gust penetration effects; (R + iS) is a similar kind of function but it occurs due to the spanwise variation in turbulence. It would take a week of lectures to present the complete derivation of (R + iS) but I'll indicate its basic nature as a final result. Finally, in Equation 15 we have $F_w(\omega)$ the Fourier transform of the turbulence itself. From the Fourier transform we can readily deduce the power spectrum of the lift coefficient as:

$$\phi_{c_L} = \frac{a^2}{V^2} (P^2 + Q^2)(R^2 + S^2)\phi_w \tag{16}$$

An indication of the nature of some of these functions is given in Figure 14. If we penetrated a sharp-edged gust, the lift would grow as sketched in the upper part of the figure. Converting to the frequency plane, the $(P^2 + Q^2)$ function as shown is obtained.

The function (R + iS) is the one term that comes about because of the explicit consideration of the spanwise variation in turbulence. It involves evaluating the integral:

$$R^2 + S^2 = 2 \int_0^2 \int_{-1}^{1-S} \frac{c(S+\eta)}{C_o} \frac{c(\eta)}{C_o} \phi_{12}(|S|,\omega) d\eta ds \tag{17}$$

where c is wing chord and ϕ_{12} is the cross spectra. Evaluating the integral gives the function: 1/(1 + 0.55AK). A good approximation to this function is sketched in Figure 15.

For purposes of illustration, I have adapted:

$$\phi_W = \frac{\sigma_1{}^2 \left(\frac{2L}{c}\right)^2}{1 + \left(\frac{2L}{c} K\right)^2} \tag{18}$$

as the power spectrum of the input gust. I have introduced σ_1, the combined severity and scale parameter, and this makes all the spectra for c_L fall at the same points at high frequency.

Figure 16 shows the power spectrum of the lift coefficient as a function of reduced frequency. When all the functions are put together the equation:

$$\phi_{c_L} = \frac{a^2}{V^2} \sigma_1{}^2 \frac{2500}{1 + 4743k^2 + 45357k^4} \tag{19}$$

represents a quite accurate curve fit of the spectrum result. We now ask the question: Is there a differential equation which when considered could lead to this function? The answer is yes, and the equation is:

$$213\left(\frac{c}{2V}\right)^2 c_L + c_L = 50 \frac{a}{V} \sigma_1 W_n \tag{20}$$

This is a differential equation that would yield the spectrum given by Equation 19. If we wish, we can have coefficients in the equation vary during an approach according to the way the speed of the airplane is varying. The W_n on the right-hand side of the equation is white noise as obtained from a white noise generator; the equation automatically shapes the white noise to an appropriate turbulence spectrum. The approach for simulation is illustrated in Figure 17. Utilizing a white noise generator, feed the white noise into the analog of this differential equation. A time-varying c_L is generated which you input into the simulator, specifically to the equation for vertical motion. A realistic simulation of vertical force on the airplane is thus obtained.

For rolling moment (see Figure 18), it is essential to take into account the spanwise variation in turbulence. The general equation for the spectrum of the rolling moment coefficient is:

$$\phi_{CM} = \frac{a^2}{16V^2} \frac{A^2}{A_o{}^2} \, I \, (P^2 + Q^2)\phi_w \tag{21}$$

where

$$A = \frac{b^2}{S} \quad \text{and} \quad A_o = \frac{b}{C_o}$$

The nature of the integral I of Equation 21 is:

$$I = 2 \int_0^2 \int_{-1}^{1-S} \frac{c(S + \eta)}{C_o} \frac{c(\eta)}{C_o} (S + \eta)\eta \; \phi_{12}(151,\omega) d\eta ds \tag{22}$$

is shown in Figure 19.

The rolling moment integral is a little more complicated than the vertical force integral because we have to take moment arms into account. The very definite pronounced peak in Figure 19 is associated with wavelengths near the span of the aircraft. Indeed a very good approximation to the value of k at which this peak occurs at π/A. A very useful and simple approximation to I is:

$$I = \frac{5.57\nu}{7.84 + \nu^2} \times \frac{0.32 - 0.26\varepsilon}{1 + 0.8\varepsilon} \tag{23}$$

where

$$\nu = \frac{c}{2L} A \sqrt{1 + \left[\frac{2L}{c} k\right]^2} \tag{24}$$

Note that a different frequency argument than k alone is found.

Figure 20 shows the spectrum for rolling moment coefficient as a function of a reduced frequency. The equation:

$$\phi_{CM} = \frac{a^2}{16V^2} \frac{A^2}{A_o{}^2} \sigma_1{}^2 \frac{33.8}{1 + 685k^2 + 1473k^4} \tag{25}$$

fits that curve exceptionally well. There is a differential equation that can lead to this spectrum which we will discuss later.

Figure 21 is added here to show again the non-importance of L. The spectrum of the rolling moment coefficient is at the top of the figure. When we include the transfer function for the airplane, $|H|^2$, that is associated with roll dynamic behavior, you get the output spectrum for roll angle as shown at the bottom of the figure. The scale of turbulence is not important in the consideration because the predominant response is in the frequency range that is not influenced by the scale of turbulence.

The differential equation for the rolling moment coefficient is:

$$38\left[\frac{c}{2V}\right]^2 c_M + 28\,\frac{c}{2V}\,\dot{c}_M + c_M = \frac{a}{4V}\,\frac{A}{A_0}\,5.81\,W_n \qquad (26)$$

Again, as in the case of the vertical force (see Figure 22), you have a white noise generator, you feed its output into the analog of the differential equation (Equation 26), and out comes the time varying moment coefficient; you input this to your simulator, specifically to the rolling equation of motion. The simulation of the rolling moment due to a turbulence encounter will then automatically be taken into account.

QUESTION: Hal Murrow (NASA Langley). Two points I would like to make. On the spectrum correction factor, I agree that there needs to be a correction. The point that is unclear is the magnitude of the correction and probably the biggest reason for this is the fact that in our instrumentation system for the B-57B we have some anti-aliasing filters. Their effect has to also be taken into account to determine the magnitude of the correction to apply. The second point I wanted to make is that we are talking about hypersonic airplanes nowadays, the Orient Express, that sort of thing. If you think of the primary response of the airplane as being in the short-period mode and calculate what that would be, it would go down to the very low frequencies or wavelengths. In these regions it would make a difference as to what is the value of L. I'm not convinced that L and σ are directly related in all cases.

ANSWER: That is something we will argue about in the future. You will not get down to those low frequencies with any airplane.

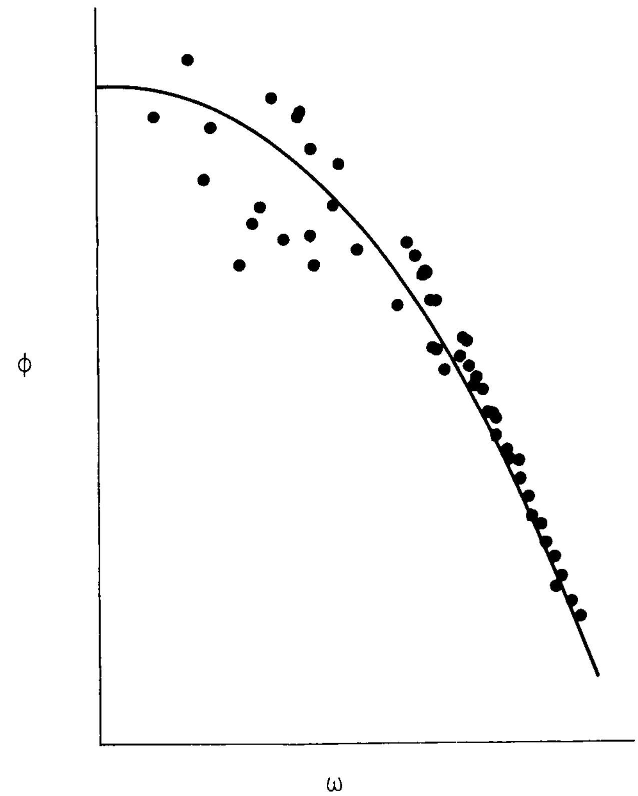


Figure 1. Curve fit of an analytical function to deduce a value for L.

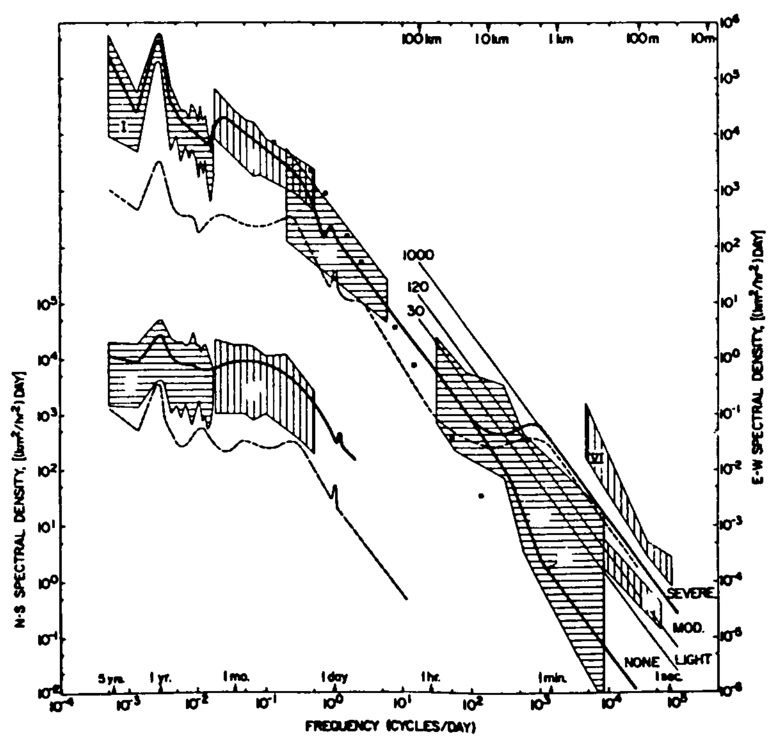


Figure 2. Power spectrum.

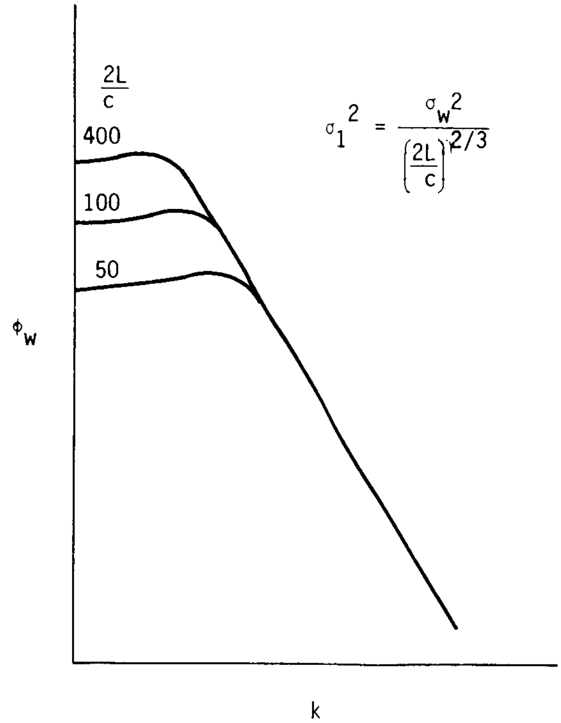


Figure 3. A von Karman spectrum.

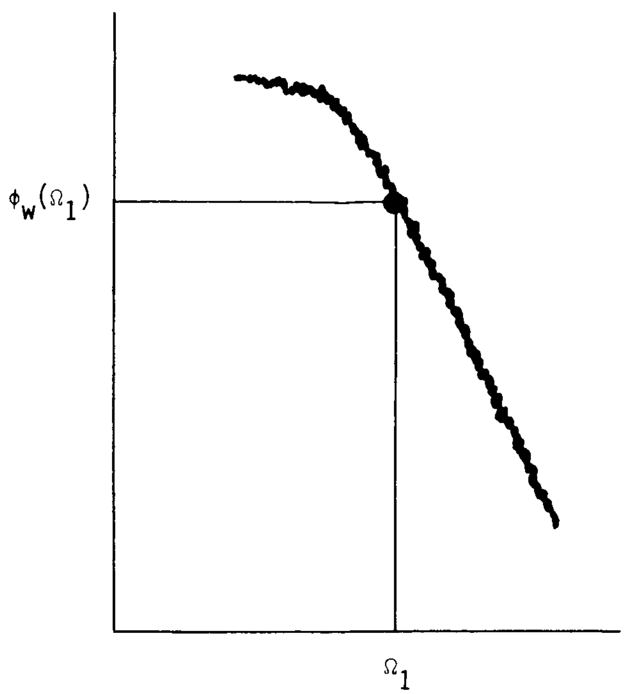


Figure 4. Example of a power spectrum for a patch of turbulent air.

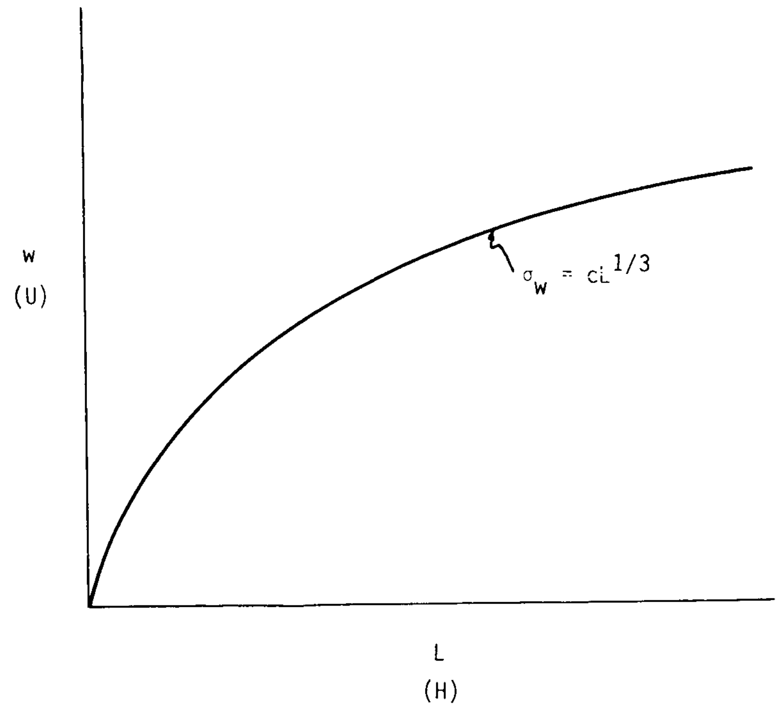


Figure 5. Influence from the relationship of σ_w, L, and C; namely, $C = \sigma/L^{1/3}$.

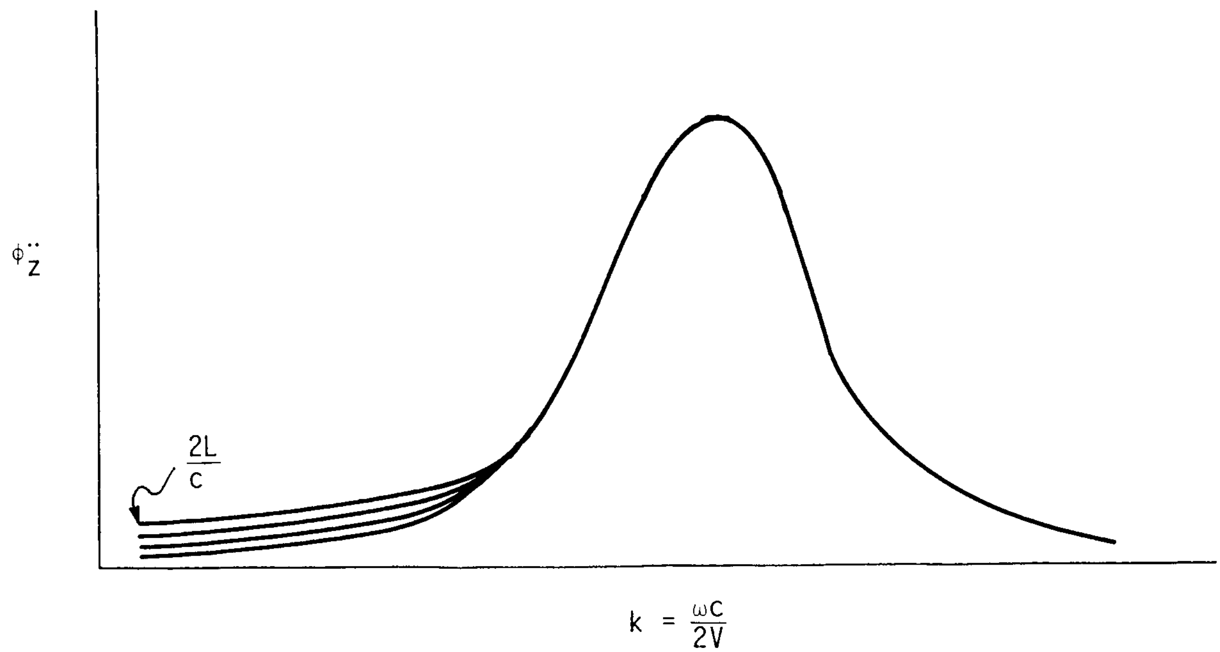


Figure 6. Example of an output spectrum for vertical acceleration of an airplane which illustrates the influence of the scale.

$$\phi_{\ddot{z}} = \frac{a_1 k^2}{1 + a_1 k^2} \times \frac{\beta^2}{\beta^2 + 1.5\pi k + \pi^2 M k^2} \times \frac{1}{1 + 0.55AK} \times \frac{\sigma_w^2}{1 + \left(\frac{2L}{c}\, k\right)^{5/3}}$$

Gust Spectrum

Spanwise Effects

Gust Penetration Effects

Transfer Function

Figure 7. Simplified form of the spectrum of vertical acceleration of the center of gravity.

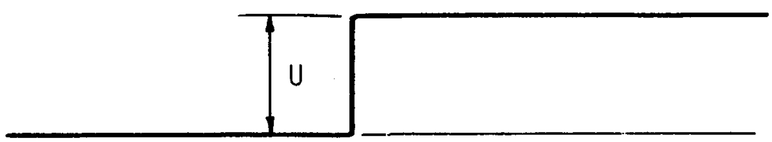


Figure 8. Sharp edge gust.

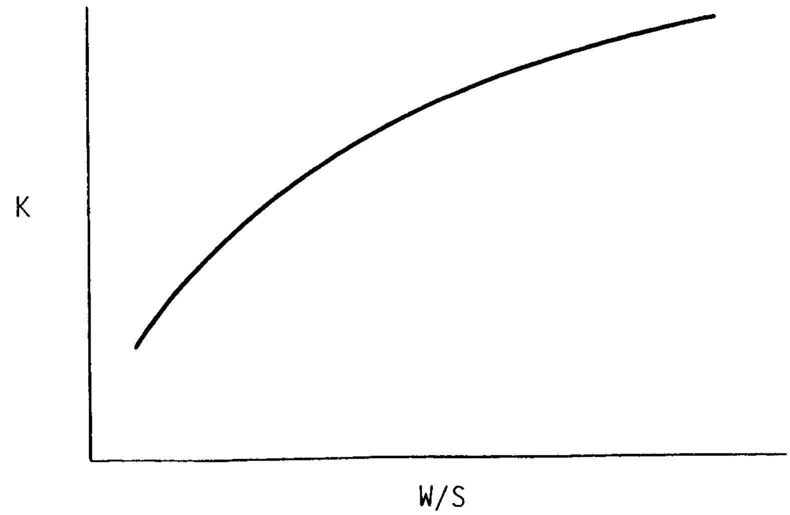


Figure 9. A 1940's version of the gust alleviation factor as a function of wing loading.

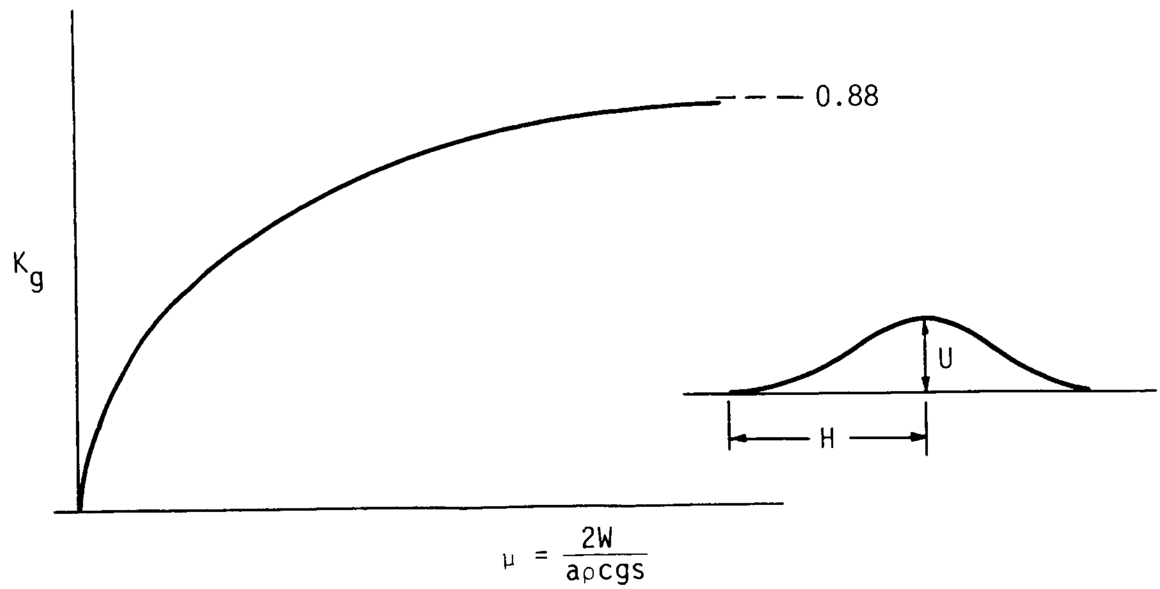


Figure 10. The gust alleviation factor as a function of mass parameter.

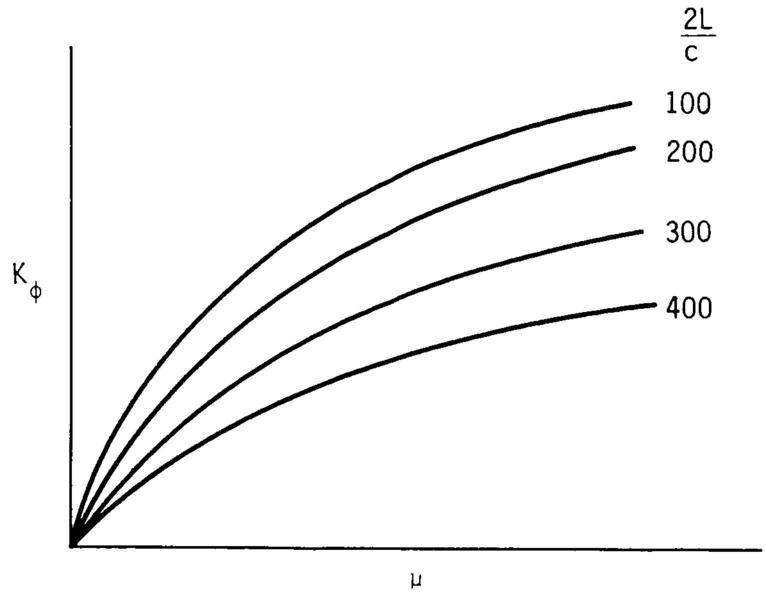


Figure 11. The gust alleviation factor as a function of the mass parameter as well as showing its dependence on gust gradient distance.

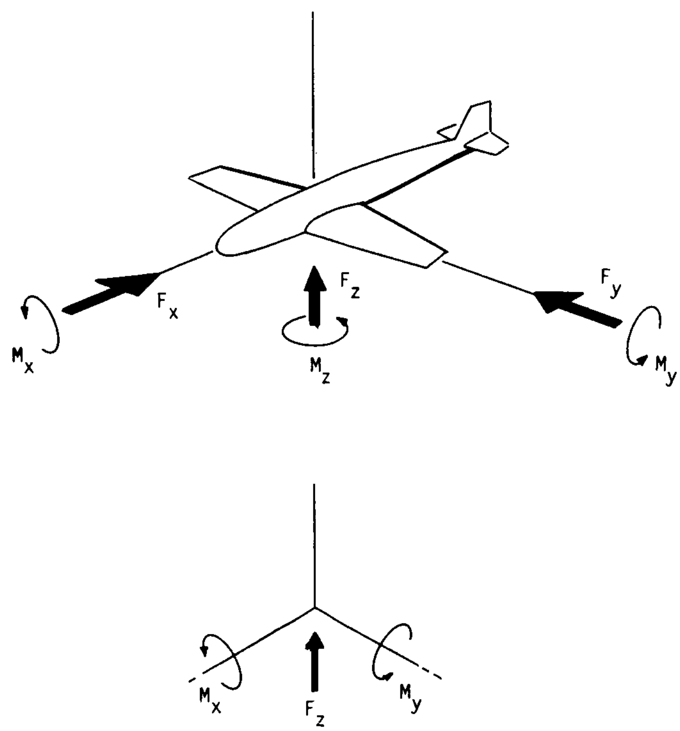


Figure 12. The forces and moments due to turbulence.

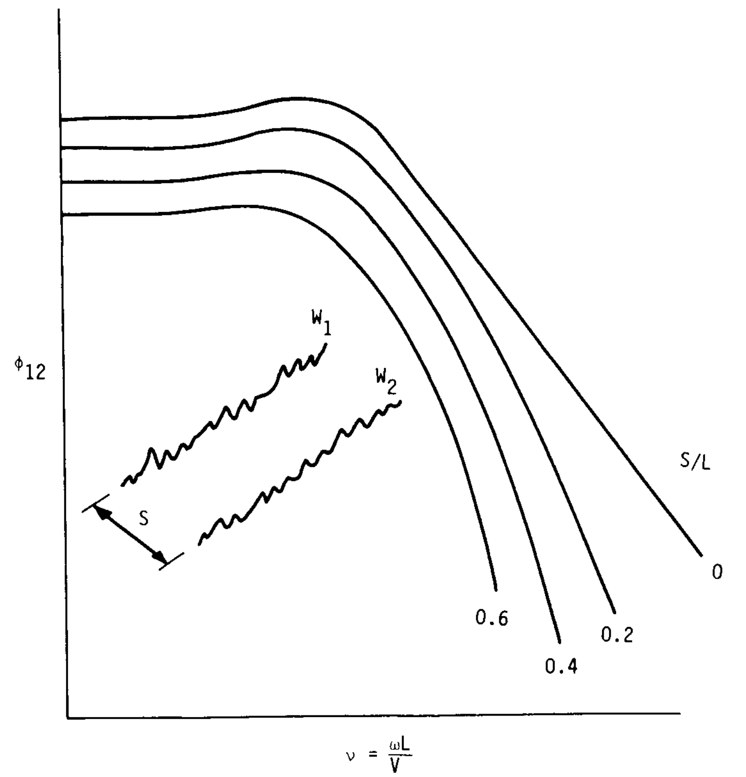
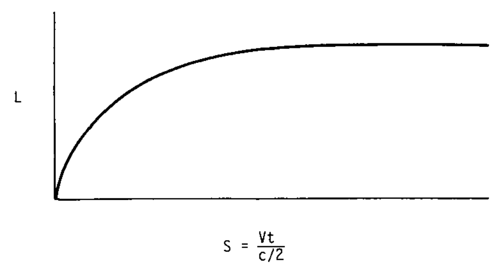
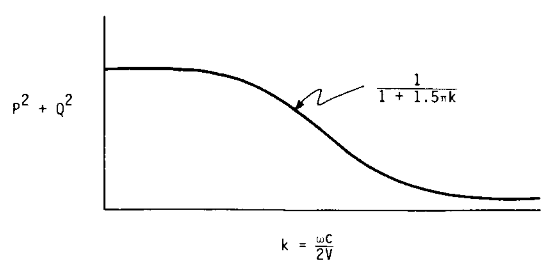


Figure 13. Illustrating the effect of separation distance on cross spectra.



(a) Time plane



(b) Frequency plane

Figure 14. Lift relationships as a function of time and frequency components.

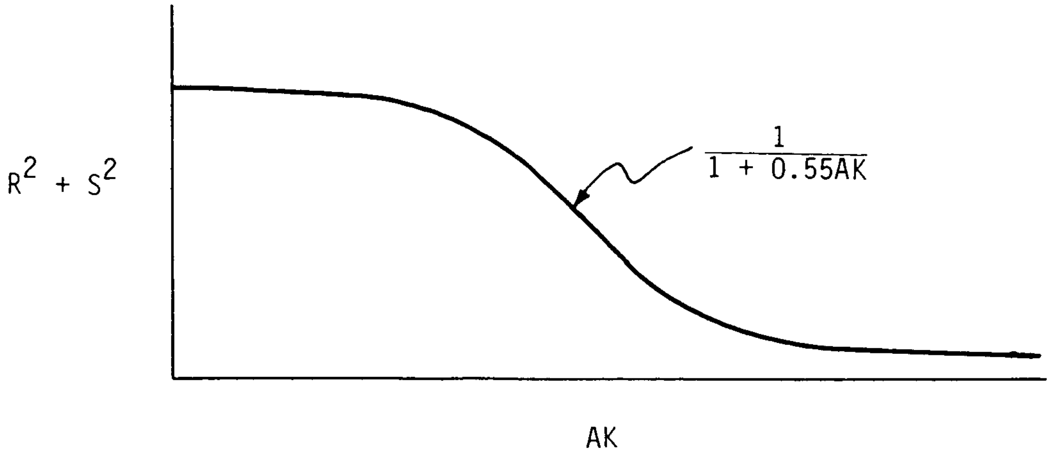


Figure 15. Spanwise variation effects with a consideration of cross spectra.

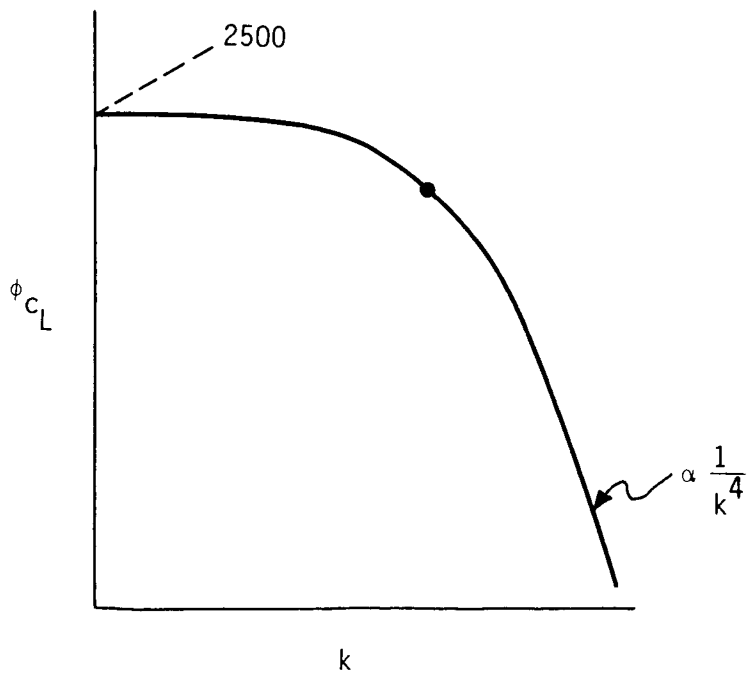


Figure 16. Power spectrum of the lift coefficient as a function of reduced frequency.

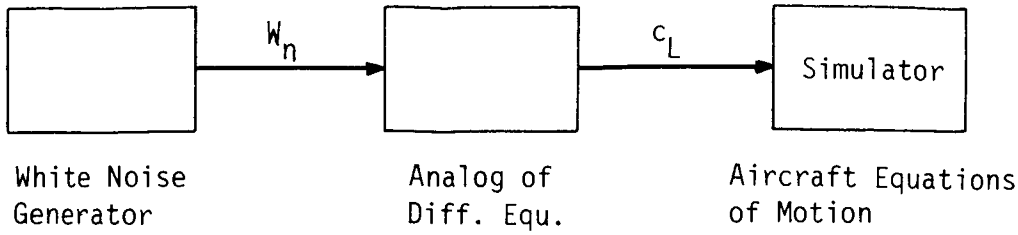


Figure 17. A realistic simulation approach.

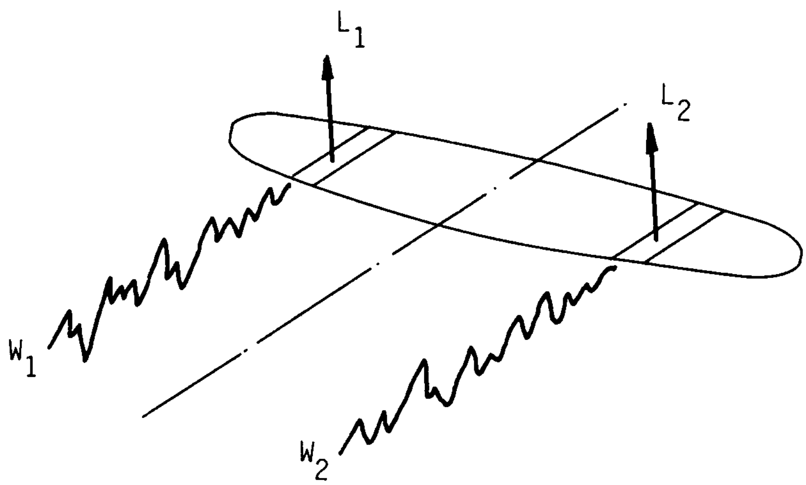


Figure 18. Spanwise variation in turbulence illustration.

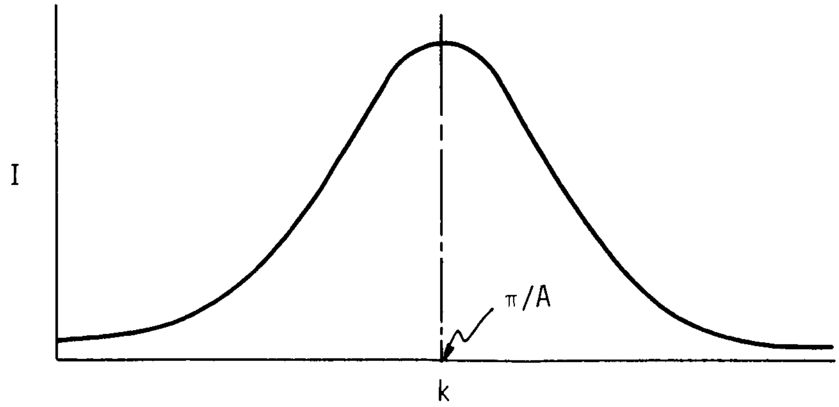


Figure 19. Rolling moment integral relationship to reduced frequency.

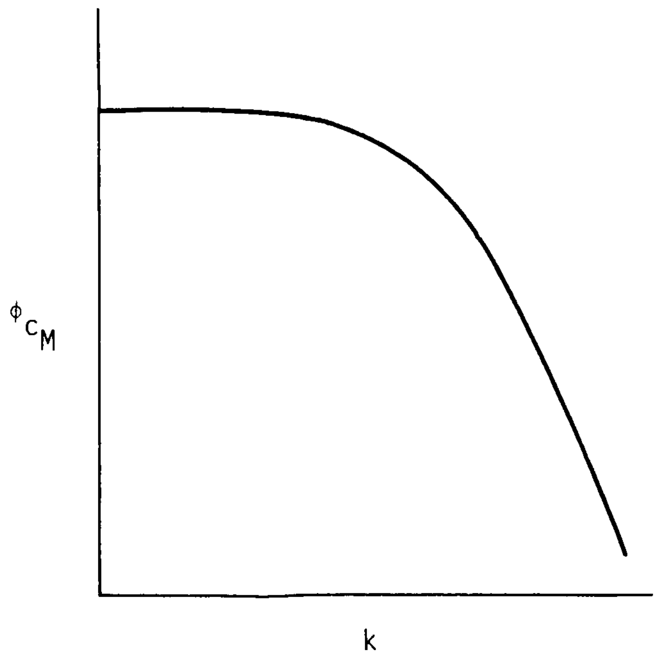


Figure 20. Rolling moment coefficient as a function of reduced frequency.

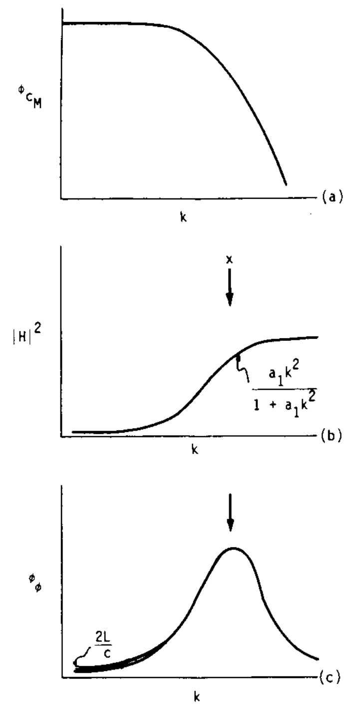


Figure 21. Output spectrum (ϕ_ϕ) obtained from spectrum of rolling moment coefficient (ϕ_{CM}) by a consideration of the transfer function $|H|^2$, i.e., $\phi_\phi = |H|^2\phi_{CM}$.

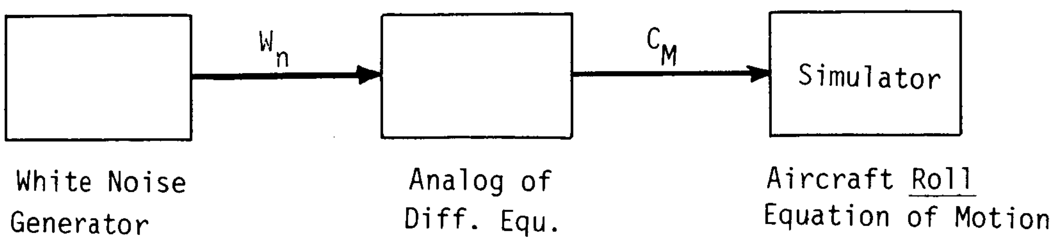


Figure 22. Roll behavior simulation.