ALTIMETER MEASUREMENTS FOR THE DETERMINATION
OF THE EARTH'S GRAVITY FIELD

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INTRODUCTION

The University of Texas Center for Space Research (UT/CSR) research efforts under NASA Grant NAG5-746 during the time period from March 18, 1986, to March 14, 1987, have concentrated on the following areas:

- Refining altimeter and altimeter crossover measurement models for precise orbit determination and for the solution of the earth's gravity field,
- Performing experiments using altimeter data for the improvement of precise satellite ephemerides and the determination of the earth's gravity field,
- Analyzing an optimal relative data weighting algorithm to combine various data types in the solution of the gravity field.
- Examining observability of geopotential coefficients using altimeter crossover measurements from two satellites with different inclinations and altitudes.

Copies of abstracts for some of the publications and conference presentations related to the research are attached in the appendices.

ALTIMETER MODEL REFINEMENT

A model of the quasi-stationary sea surface topography (SST) was implemented in the University of Texas Orbit Processor (UTOPIA). The model uses a surface spherical harmonics representation of the Levitus model of the SST, and it includes the capability to estimate the SST model in terms of spherical harmonics coefficients using altimeter data. The model of SST is part of the dynamically consistent altimeter measurement model which includes the modeling of the dynamical effect of the
global ocean surface on the satellite orbit.

A generalized theory for efficient computation of altimeter crossover measurements for both single and dual satellites has been developed. Software refinement is underway to implement dual-satellite crossover computation capability. Future work also includes the modeling of dual-satellite crossover measurement types in UTOPIA.

ALTIMETER CROSSOVER EXPERIMENTS

Orbit determination experiments using altimeter crossover observations were performed. Altimeter crossover and ground-based tracking data were used to determine SEASAT 12-day arc with orbit epoch beginning at July 7, 1978. When resonant coefficients and 10\times10 gravity field coefficients were solved using crossovers and tracking data, the SEASAT orbit was computed with a better accuracy. A simulation experiment was then performed to confirm the above results. In conclusion, the use of crossover measurements significantly improves the radial component of the estimated orbit. The improvements in the geopotential gained from processing crossover measurements will reduce the geographically correlated orbit error.

Observability of zonal geopotential coefficients using dual-satellite crossover measurements was examined using linear theory. A variation of inclination and altitude of altimetric satellites was used to compute observability indexes which reflect the strength of zonal variation using altimeter crossover data from two different satellites. Preliminary results indicated that zonals are observable using dual-satellite crossovers.

ALTIMETER EXPERIMENT

Techniques used to generate the altimeter information equations for the TOPEX gravity field solution have been developed, and a numerical experiment to determine the gravity field using SEASAT altimeter data was performed. Information equations for the SEASAT altimeter data were
created for two arcs (12 and 18 days) during the 17-day repeat orbit and one arc (24 days) during the
3-day repeat orbit. The altimeter data were edited at 10 m and then time sampled at the rate of once
every 20 seconds. The resulting information equations included the geopotential coefficient
complete to degree and order 42 and the quasi-stationary sea surface topography (SST) complete to
degree and order 10. The reference orbits were computed using PTGF1 with altimeter residuals
greater than 10 meters completely edited. The numerical simulation used 12 days of laser, S-band,
crossover and altimeter data computed at the same times as the real data for SEASAT
7/7/78-7/18/78. Only when the altimeter data were included with the other observations was it
possible to solve for a 20×20 geopotential (no model error). No accurate SST solutions were
obtained, but this was probably due to a lack of other satellite data to provide separation for the SST
and the geopotential. Future work will concentrate on filtering the altimeter data and creating
altimeter normal points.

APPLICATION OF ALTIMETER DATA

A new technique was developed to interpolate unevenly spaced GEOS-3 and SEASAT altimeter
data to generate the mean sea surface, or marine geoid, with high resolution. Green functions of the
biharmonic operator, in any number of dimensions, are used for minimum curvature interpolation of
irregularly spaced data points. The interpolating curve (or surface) is a linear combination of Green
functions centered at each data point. In one (or two) dimensions, this technique is equivalent to
cubic spline (or bicubic spline) interpolation, while in three dimensions, it corresponds to
multiquadratic interpolation. Although this new technique is relatively slow, it is more flexible than
the spline method since both slopes and values can be used to find a surface. Moreover, noisy data
can be fit in a least-squares sense by reducing the number of model parameters. These properties are
well suited for interpolating irregularly spaced satellite altimeter profiles. The long wavelength
radial orbit error is suppressed by differentiating each profile. The shorter wavelength noise is
reduced by the least-squares fit to nearby profiles. Using this technique with 0.5 million GEOS-3 and SEASAT data points, it was found that the marine geoid of the areas covering the Gulf of Mexico and the Caribbean are highly correlated with the sea floor topography. This suggests that similar applications in more remote areas may reveal new features of the sea floor. Future plans include application of altimeter data to produce a high resolution global ocean surface map. GEOSAT altimeter data, if available, will be included.

**RELATIVE DATA WEIGHTING**

An optimal relative data weighting algorithm was developed to combine various data types for the solution of the earth's gravity field. The algorithm minimizes the error introduced by the observation noise and scales the *a priori* variances for an optimal solution. Evaluations and experiments are underway to examine the ability of the data weighting algorithm for the optimal solution of the earth's gravity field.

**APPLICATION OF ALTIMETER DATA TO THE SOLUTION OF THE GRAVITY FIELD**

SEASAT altimeter data were processed and included in the solution for the TOPEX gravity field, PTGF2A. The experimental solution of the PTGF2A gravity field is part of the continuing collaborative effort between UT/CSR and NASA/GSFC to develop the TOPEX gravity model. The purpose, in part, is to assess the roles of both the altimeter crossover and the direct altimeter data in their contribution to the solution of the earth's gravity field. Preliminary gravity field information equations were generated for both the SEASAT altimeter crossover and direct altimeter data. The altimeter information equations were included in the gravity field solution. Assessment of their respective contribution to the gravity field is underway. The technique for the generation of normal points for direct altimeter data is in development.
PUBLICATIONS/CONFERENCES

The following is a list of publications and conference presentations pertinent to the research activities under NASA Grant NAG5-746:

- **Altimeter Crossover Experiments**, R. S. Nerem, C. K. Shum, and J. C. Ries. Presented at the Sixth UT/GSFC TOPEX Gravity Model Development Meeting held at The University of Texas at Austin, April 8–10, 1986.


- **Biharmonic Spline Interpolation of GEOS-3 and SEASAT Altimeter Data**, D. T. Sandwell. Submitted to EOS for publication. Presented at the Fall Meeting of the American Geophysical Union, San Francisco, California, December 1986. A copy of the abstract is included in Appendix B.


ALTIMETER CROSSOVER METHODS FOR PRECISION ORBIT DETERMINATION

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The ability of satellite-borne radar altimeter data to measure the global ocean surface with sub-centimeter accuracy provides a unique data set to aid precision orbit determination as well as to map the earth's gravity field and its geoid. Although satellite altimetry has a distinct advantage in its global distribution of data, consideration of possible error sources when altimeter data are directly used for orbit determination and geophysical mapping suggests several disadvantages. In particular, long wavelength oceanographic features and non-temporal ocean topography can be absorbed into the orbit when altimeter data are directly used. Altimeter crossover data, which eliminate the dependence on the non-temporal ocean topography, provide a valuable data set in its global coverage. In this investigation, techniques to employ altimeter crossover data to perform precision orbit determination and to recover geophysical parameters have been developed. New techniques of using dual or multiple satellite crossover data for precision orbit determination and geodetic parameter mapping have been investigated.

Acknowledgment: This research was supported by NASA under Contract No. NAG 5-746.
APPENDIX B
BIHARMONIC SPLINE INTERPOLATION OF GEOS-3
AND SEASAT ALTIMETER DATA

ABSTRACT

Green functions of the biharmonic operator, in any number of dimensions, are used for minimum curvature interpolation of irregularly spaced data points. The interpolating curve (or surface) is a linear combination of Green functions centered at each data point. In one (or two) dimensions, this technique is equivalent to cubic spline (or bicubic spline) interpolation, while in three dimensions, it corresponds to multiquadratic interpolation. Although this new technique is relatively slow, it is more flexible than the spline method since both slopes and values can be used to find a surface. Moreover, noisy data can be fit in a least-squares sense by reducing the number of model parameters. These properties are well suited for interpolating irregularly spaced satellite altimeter profiles. The long wavelength radial orbit error is suppressed by differentiating each profile. The shorter wavelength noise is reduced by the least-squares fit to nearby profiles. Using this technique with 0.5 million GEOS-3 and SEASAT data points, it was found that the marine geoid of the Caribbean area is highly correlated with the sea floor topography. This suggests that similar applications in more remote areas may reveal new features of the sea floor.
APPENDIX C
Altimeter Crossover Methods for Precision Orbit Determination

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The ability of satellite-borne radar altimeter data to measure the global ocean surface with sub-centimeter accuracy provides a unique data set to aid precision orbit determination as well as to map the earth's gravity field and its geoid. Although satellite altimetry has a distinct advantage in its global distribution of data, consideration of possible error sources when altimeter data are used directly for orbit determination and geophysical mapping suggests several disadvantages. In particular, long wavelength oceanographic features and non-temporal ocean topography can be absorbed into orbit and geodetic parameters when altimeter data are used directly. The non-temporal ocean topography, mostly due to errors in the marine geoid, currently has uncertainty of several meters and is significant when decimeter radial orbit accuracy of certain satellite orbits, such as that of TOPEX, is desired.

A technique which eliminates the altimeter's dependence on the non-temporal ocean topography is the use of altimeter measurements at the points where the orbit ground tracks intersect. These points are referred to as "crossover points." A crossover measurement is defined as the difference in the altimeter measurements at the points where the satellite ground tracks cross. Since the constant part of each altimeter measurement will be due to the marine geoid and permanent sea surface topography, the crossover or differenced altimeter measurements will be a function of the orbit error and the time-varying ocean surface topography. The orbit error will be the major contributor to the crossover measurement. The orbit error is primarily a long wavelength feature with once/revolution (40,000 km) and twice/revolution (20,000 km) components dominating. Although the non-temporal portion of the ocean topography can be eliminated at the crossover point, the remaining temporal changes, such as ocean tide, unmodeled orbit error, short wavelength phenomena as well as altimeter time tag error, can still be aliased into the radial orbit error on a global basis. With the exception of the geographically correlated error due to inaccuracy in the earth's gravity field, global computation and analysis of single-satellite crossover residuals can provide valuable information about radial orbit error sources and the reduction of interesting geophysical parameters. A new technique has been developed to use dual-satellite crossover measurements, or crossovers from two satellites carrying altimeter instruments, for precision orbit determination. The dual-satellite crossover technique is found to be capable of observing variation of the earth's zonal harmonics which constitutes part of the geographically correlated gravity error.

The TOPEX/POSEIDON mission, which will be conducted jointly by the National Aeronautics and Space Administration (NASA) and France's Centre Nationale d'Etudes
Spatiale (CNES), will carry precise radar altimeter instruments in an attempt to measure oceanic topography including the mean and variable surface geostrophic currents and tides of the world's oceans, and to lay the foundation for a continuing program to provide long-term observations of the ocean's circulation and its variability. To meet the objectives of the TOPEX/POSEIDON mission, the satellite altitude relative to the center of mass of the earth must be determined to an accuracy of 14 centimeters or better. In the same time span, the European Space Agency (ESA) will launch another altimetric satellite, ERS-1. The application of the dual-satellite crossover technique combining ERS-1 and TOPEX/POSEIDON altimeter crossover measurements will improve the ERS-1 orbit. Since ERS-1 will cover most of the ocean surface with its 98° inclination, the use of dual-satellite crossover measurements between ERS-1 and TOPEX/POSEIDON will provide a significant means for the mapping of geophysical and oceanographic phenomena of interest.

In this investigation, single- and dual-satellite altimeter crossover techniques are developed for precision orbit determination and the reduction of geophysical parameters. Observability of zonal harmonics using dual-satellite crossover measurement was studied. Numerical experiments were carried out to examine the role of both single- and dual-satellite crossover techniques to the contribution of precision orbit determination and reduction of geophysical parameters.

Acknowledgment. Funding for this research is provided by NASA, Contract No. NAG5-746.
APPENDIX D
ALTIMETER METHODS FOR THE DETERMINATION OF THE EARTH’S GRAVITY FIELD

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The ability of satellite-borne radar altimeter data to measure the global ocean surface with high precision and dense spatial coverage provides a unique tool for the mapping of the earth’s gravity field and its geoid. The altimeter crossover measurements, created by differencing direct altimeter measurements at the subsatellite points where the orbit ground tracks intersect, have the distinct advantage of eliminating geoid error and other nontemporal or long period oceanographic features. In the 1990’s, the joint U.S./French TOPEX/POSEIDON mission and the European Space Agency’s ERS-1 mission will carry radar altimeter instruments capable of global ocean mapping with high precision. This investigation aims at the development and application of dynamically consistent direct altimeter and altimeter crossover measurement models to the simultaneous mapping of the earth’s gravity field and its geoid, the ocean tides and the quasi-stationary component of the dynamic sea surface topography. Altimeter data collected by SEASAT, GEOS-3 and GEOSAT are used for the investigation.

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GSc. Analysis of Satellite Altimetry (oral presentation)

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