A NEW HYDRODYNAMIC ANALYSIS OF DOUBLE LAYERS

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ABSTRACT

A genuine two-fluid model of plasmas with collisions permits the calculation of dynamic (not necessarily static) electric fields and double layers inside of plasmas including oscillations and damping. For the first time a macroscopic model for coupling of electromagnetic and Langmuir waves was achieved with realistic damping. Starting points were laser-produced plasmas showing very high dynamic electric fields in nonlinear force-produced cavitory and inverted double layers in agreement with experiments. Applications for any inhomogeneous plasma as in laboratory or in astrophysical plasmas can then be followed up by a transparent hydrodynamic description. Results are the rotation of plasmas in magnetic fields and a new second harmonics resonance, explanation of the measured inverted double layers, explanation of the observed density-independent, second harmonics emission from laser-produced plasmas, and a laser acceleration scheme by the very high fields of the double layers.

I. INTRODUCTION

There is a close similarity between the double layers in the surface of laser-produced plasmas and a wide class of astrophysical plasmas (Hora, 1975). In both cases a high temperature plasma is produced which expands into vacuum or into gases of much less density. During this dynamic process, a separation of space charges will happen at the plasma surfaces when the equithermal electrons with their much higher velocity than that of the ions will expand much faster generating first a negatively charged cloud followed by a positively charged cloud of the ions. Then the more or less homogeneous and space charge quasi-neutral plasma follows. The separation of the electrons and ions with a net neutral charge is a double layer (DL) in which electric fields persist within these plasma areas (Fig. 1). These fields were suggested for explaining phenomena in extraterrestrial plasmas by Alfvén (1958) not without hefty opposition of other plasma theoreticians. Even the more advanced presentation (Alfvén, 1981) was commented by Kulsrud (1983) as “Alfvén’s electric fields whose origin is intuitively not clear.” These fields and double layers were also suggested to be involved with the solar atmosphere (Alfvén and Carlqvist, 1967; Carlqvist, 1979, 1982; Torvén et al., 1985), in the ionosphere and magnetosphere of the Earth and the magnetosphere of Jupiter (Shawhan, 1976), and with the striated structure of the barium clouds when expanding in the ionosphere (Haerendel et al., 1976).

In laser-produced plasmas, these double layers in the surface of the expanding plasma were thought to be involved with the measured speeding up of the ions to multi-kiloelectronvolt energies as measured by Linlor (1963) while the particle temperatures were 100 eV or less. However, the analysis of the double layer (Hora et al., 1967) with a derivation of its thickness being of a Debye length (Hora, 1975) arrived at a number of accelerated ions which was $10^5$ times less than measured. A completely different acceleration mechanism had to be derived by nonlinear forces (as a generalization of the ponderomotive forces) of nonthermal electrodynamic interaction of the laser radiation with the plasma (Hora, 1969).
The direct experimental proof of the double layer and high electric fields in laser-produced plasmas was given by Mendel and Olson (1975) where the bending of an ion beam passing the double layer led to the measurement of electric fields of 10 kV/cm. The generation of electrostatic potentials was measured by Pearlman and Dahlbacka (1977), and a more detailed study using Rogowski coils (Eliezer and Ludmirsky, 1983), Figure 2, with a high temporal resolution to 50 psec or less (Ludmirsky et al., 1985; Eliezer et al., 1986) arrived at the discovery of the inverted double layers and spatially oscillating behavior of the double layers (Hora et al., 1984).

The first measurement of the double layers in space plasmas was not before 1977 (Mozer et al., 1977; Temerin et al., 1982; Temerin and Mozer, 1984) using S3-3 satellite data. These results, together with the laboratory experiments on double layers as reviewed by Hershkowitz (1985), emerged from the initial theory on plasma double layers by Langmuir (1929), Bohm (1949), Bernstein et al. (1957), and Knorr and Goertz (1974), and from computer simulations by DeGroot et al. (1977) and Sato and Okuda (1980). Turbulence theory by Yabe et al. (1981) arrived at electric fields inside of these turbulence areas and may be interpreted as some sort of a double layer behavior. The laboratory experiments showed double layers in mercury discharges (Torvén, 1981; Stangeby and Allen, 1973), Q machines (Sato et al., 1976), and triple devices where two plasmas at different electric potential are connected through grids by a plasma which has then a double layer according to the difference of the voltages plus the difference of the voltages between the two outer plasmas (Coakley and Hershkowitz, 1981; Quon and Wong, 1976; Leung et al., 1980). The geometry can be one-dimensional (Hershkowitz et al., 1981), two-dimensional (Baker et al., 1981), or three-dimensional (Merlino et al., 1984).

A special motivation for studying the double layer in laser-produced plasma was given from the theory of the nonlinear force (Hora, 1969, 1974, 1981; Lindl and Kaw, 1971; Peratt and Watterson, 1977; Peratt, 1979). The electrodynamic, dielectrically caused acceleration of plasma by laser radiation is based on the force acting on the high density electron gas in the plasma being pushed or pulled and the ion gas has to follow then by the electric fields generated between the two fluids. When the essential properties of the nonlinear force were derived from the space charge quasi-neutral plasma model (Hora, 1969), the mentioned fields were disguised by the presumptions of the model. However, the existence of the fields of the description of the single electron motion (Hora, 1971) was evident, and a search was overdue since the beginning of the work on the noninear force in 1965.

While the treatment of the double layers and the high electric fields is essentially no problem on the basis of the kinetic theory with the Vlasov equation (Knorr and Goertz, 1974), the inclusion of collisions for the conditions of the high density laser-produced plasmas would have been necessary for which the complications of the collision processes for the kinetic theory would be a problem. How important the collision processes are in laser-produced plasma can be seen from several examples. Simply, the classical optical constants (Hora, 1981) can be evaluated only by carefully watching the numerical problems close to a pole of the related functions where the change of the real part or the imaginary part of the optical constant can be by a factor $10^3$ or much more for a change of the plasma temperature or the plasma density by less than 1 percent. Another drastic example is the theory of Denisov's resonance absorption (to distinguish from a new resonance found by Hora and Ghatak, 1985) where the derivation based on the electric field by White and Chen (1974) arrived at a negative infinite pole of the function for the effective dielectric function of the plasma was collisionless. Introducing a tiny little bit of absorption (collisions), however, caused a swap of the pole from minus infinity to nearly plus infinity (Hora, 1979). Collisions are therefore essential in laser-produced plasmas.

The use of N-particle simulation of the plasma (with N = $10^6$) by computers could again not be used as the physics of the collisions could be covered yet only in a limited way and only first attempts have been done to correctly treat Coulomb collisions now in a simplified way by using supercomputers (Yabe, 1985). The difficulties in this macroscopic theory, however, are in the presumptions of space charge quasi-neutrality that could not at all be used to treat the electric fields or double layers in plasmas. It even could not describe the coupling of the longitudinal ("electrostatic") Langmuir waves with transversal electromagnetic waves in plasma (Schamel, 1979).
II. THE GENUINE TWO-FLUID MODEL

The macroscopic hydrodynamic theory for the consequent description of the double layers and the generated electric fields required the use of the complete two fluids for electrons and ions including collisions, viscosity, equipartition of temperatures between ions and electrons, optical constants with the correct nonlinear dependence on the laser intensity (about an incorrect formula, see e.g., Duderstadt and Moses, 1983), and including the general expression of the nonlinear force apart from the thermokinetic force given by the gas dynamic pressure (Hora, 1969, 1981, 1985a). In one spatial dimension, the problem was then to solve the following seven quantities depending on the spatial coordinate x and the time t for given initial and boundary values: the density, temperature and velocity (in the x-direction) for electrons, the same for ions \(n_e, T_e, v_e, n_i, T_i, v_i\) and the electric field \(E\) (in the x-direction) differing from the external electric and magnetic fields \(E_L\) and \(H_L\) of the incident laser radiation. For the seven functions, seven differential equations are available: the equations of continuity for electrons and ions, the equations of motions for electrons and ions, the equations of energy conservation for electrons and ions, and the Poisson (or better Gaussian) equation (Lalousis and Hora, 1983). For the whole three-dimensional description there have to be added the two variables for the other components of the electron velocity and the same for the ion velocities for which the four further velocity components of the equation of motion are accounted. Instead of the longitudinal electric field component \(E\) of the one-dimensional case, all three components of \(E\) and that of the magnetic field \(H\) generated in the plasma during the complex dynamics have to be included for which instead of the Gaussian law in one dimension, the six components of the Maxwellian equations have to be used. All together, there are 16 equations for the 16 quantities to be determined in space and time, automatically also reproducing the complete development of the so-called spontaneous magnetic fields in the laser-produced plasmas.

The solution of the one-dimensional problem allowed for numerics is very complicated in this general property of the plasma because the time steps have to be very much shorter than the shortest plasma oscillation time. For the plasmas at irradiation with neodymium glass laser radiation, the time steps have then to be shorter than 0.1 fs. In order to arrive at physically detectable results in the picosecond scale, long computer runs have to go on, where for each time step the Maxwellian equations also have to be solved for the incident laser radiation with the correct conditions for the reflected wave. For the treatment of the reflection field, a very quick computation by a matrix procedure was invented (Lalousis, 1983). The whole computation had to be using a very unusual Eulerian code instead of the usual Lagrangian codes because of the appropriate inclusion of the description of the electric fields produced inside the plasma. The basic problem of the boundary conditions in this case ran into instabilities, and a special new method for a stable solution had to be discovered as derived by numerical experiments (Lalousis, 1983; Lalousis and Hora, 1984).

The results described in the following were attained by using a CD 7600 computer and a Cray 1. The stability of the computation and the correctness of the output was confirmed after the runs up to the picosecond range by checks of the conservation of energy. Also the fact that the gain or loss of energy of relativistic electrons, fired through the then not longer static and conservative electric fields with potentials, but having the dynamic electric fields \(E\) where

\[
\oint E \cdot dx \neq 0
\]

resulted in reasonable numbers of the gain or loss of electron energies (Green et al., 1986), was a proof of the correct computations.
When using the genuine two-fluid code, the appearance of strong electric oscillations was marked. For a plasma without laser irradiation, the following initial condition was chosen: a fully ionized hydrogen plasma slab of 10 μm thickness with a linear increase of the electron density from \(5 \times 10^{20} \text{cm}^{-3}\) at \(x = 0\) to \(10^{21} \text{cm}^{-3}\) at \(x = 10 \mu\text{m}\) was taken at time \(t = 0\) with same ion density and an electron and ion temperature \(T_e = T_i = 10^3 \text{eV}\) at \(t = 0\). The initial velocities were \(v_e = v_i = 0\) everywhere at \(t = 0\) and, consequently, the electric field \(E = 0\) at \(t = 0\). Working with time steps of \(1.5 \times 10^{-16} \text{s}\) (1/30 of the shortest plasma period of \(5 \times 10^{-15} \text{s}\)) at \(x = 10 \mu\text{m}\), expanding plasma showed a very strong oscillation of the electric field displayed by electrons moving down the ramp and being returned. The field was always negative, never positive, because the electron cloud went back to the initial position within the ions or less. At later times an "ambipolar" oscillation field was noted (Figs. 3 to 5) which decayed faster when the initial plasma temperature was lower (higher collision frequency). The oscillations were damped out and a bent profile of the electric field resulted, nearly unchanged along the whole expanding plasma profile. The field had the highest negative values at \(x = 0\) of \(2.6 \times 10^6 \text{V/cm}\). This value was interpreted for a temperature of \(10^3 \text{eV}\) and a length of \(10^{-3} \text{cm}\), reaching a value of \(3 \times 10^6 \text{V/cm}\), of a "potential" of \(10^3 \text{kT}\) was assumed. As we have a time-dependent evaluation of the field \(E\) due to the plasma dynamics, we have no longer a conservative field and therefore no potential. These fields are then, strictly speaking, no longer electrostatic fields, and the generated double layer is, strictly speaking, not an electrostatic double layer, though the result is close to the picture of one.

An analytical description of the numerically very general result is possible with some approximations: The Poisson equation, which was formulated for a potential as an inhomogeneous differential equation to the homogeneous Laplace differential equation, is then only an approximation as the fields are, strictly speaking, no longer conservative. The following Gauss law was used where \(n_e\) and \(n_i\) have to be considered as time-dependent. The non-conservative character of these fields, equation (1), can be used to produce an acceleration or a stopping of charged particles by manipulating the time dependence of \(n_e\) and \(n_i\). From the time-dependent electric field, we get the Gauss law by time differentiation, substitution of the equations of continuity, and integration over the spatial coordinate (without discussing the integration constant),

\[
\frac{\partial}{\partial t} E = 4\pi e (n_e v_e - Zv_i n_i) .
\]

Further time differentiation, substitution by the equations of motion and re-arrangement of the terms with the collision frequency \(\nu\) results in

\[
\frac{\partial^2 E}{\partial t^2} + \nu \frac{\partial E}{\partial t} + \omega_{po}^2 E = E_0 \omega_{po}^2 + \frac{4\pi e}{m_e} \frac{\partial}{\partial x} \left( E_L^2 + H_L^2 \right)/8\pi + 4\pi e \nu (n_e v_i - Z n_i v_e)
\]

where

\[
E_0 = \frac{4\pi e}{\omega_{po}^2} \left[ \frac{\partial}{\partial x} \left( \frac{3n_i k T_i}{m_i} + 2n_i v_i^2 \right) - \frac{\partial}{\partial x} \right] \frac{3n_i k T_e}{m_e} + n_e v_e^2
\]
and
\[ \omega_{po}^2 = 4\pi e^2 \frac{n_e}{m_e} + \frac{Z^2 n_i}{m_i} \] (5)

The driving laser field \( E_L \) and \( H_L \) were used for the following section. Neglecting (3) for \( \nu \ll \omega_p \) and assuming a vanishing laser field \( (E_L = H_L = 0) \), the local solution of (3) results in an electric field,
\[ E = E_0 \left\{ 1 - \exp \left( -\frac{\nu^2}{2} t \right) \right\} \left[ \cos \left( \sqrt{\omega_{po}^2 - \nu^2} t \right) + \frac{\nu}{2\sqrt{\omega_{po}^2 - \nu^2}} \sin \left( \sqrt{\omega_{po}^2 - \nu^2} t \right) \right] \] (6)

which oscillates with a frequency close to the plasma frequency. These oscillations, however, are damped (exponentially decaying) by the collision frequency such that after a time \( t >> 2/\nu \) a nearly constant electric field \( E \) remains, as seen numerically (Fig. 5). This field \( E \) is determined by the spatial gradients of the enthalpy of the ions and electrons given in the brackets within the square bracket of equation (4) divided by the particle masses.

The (nearly static) electric field has an understandable order of magnitude at least for the early time of the damping processes of an initially stationary inhomogeneous plasma where any electron and ion velocity is small and from the big ratio of the ion to the electron mass. It follows,
\[ E \approx \frac{4\pi e}{\omega_{po}^2 m_e} \frac{\partial}{\partial x} 3n_e kT_e \] (7)

or
\[ eE \approx \frac{1}{n_e} \frac{d}{dx} 3n_e kT_e \] (8)

We see that the electric field \( E \) is simply caused by the gradients of the electron density and/or the temperature temporally changing. Therefore the expression “inhomogeneity field” or “dynamic electric field” has been used. In the stationary approximation (8) the inhomogeneity field corresponds to the (thermionic) work function for the electrons that moved from the plasma interior to the vacuum (or an electrode) outside corresponding to the spread Debye sheath (Hora, 1983).

This result of a quasi-potential value \( eE, \lambda = eV_e = 3kT_e \) corresponds to the measured 600 volts in a tokamak of a maximum temperature of 200 eV where the missing factor 3 was mentioned as an unexplained result (Razumova, 1983). If there are experimental conditions where, instead of a factor 3, a factor 10 (Eliezer and Ludmirsky, 1983) has been measured from the electric fields in laser-produced plasmas with (spread) Debye lengths over 10 to 100 times of its usual value, this may be explained for the more general conditions of the time developing enthalpy in (4) which was simplified in (8). Higher values than a factor 3 were also measured in cases of double layer experiments.
**IV. ELECTRIC FIELDS AND DL'S WITH LASER INTERACTIONS**

For the case of incident laser radiation, the computer output of the following cases will be discussed (Hora et al., 1984). A 25 μm thick plasma slab of initial $10^3$ eV temperature and zero velocity with an ion and electron density of symmetric parabolic shape very close to the value in Figure 6 for $t = 0.5$ ps is given. No laser interaction occurs during the first 0.5 ps such that the minor thermal expansion does not change much of the initial density profiles while this time is long enough to damp down the fast electric oscillations. At $t = 0.5$ ps, a neodymium glass laser field incident from the left-hand side is switched on with a vacuum amplitude of $10^{16}$ W/cm². The resulting electric field density $E_L/\sqrt{2\pi}$ averaged over a laser period is given in Figure 7 showing an exponential decay for $x > 8$ μm because of superdense plasma there. At several time steps up to 1.5 ps, the resulting densities (Fig. 6) and ion velocity (Fig. 8) are given. The density (Fig. 6) shows a strong minimum (caviton) at $x = 5$ μm indicating the predominance of the nonlinear force-driven ponderomotion. Plasma blocks with ion velocities up to $10^7$ cm/s are created in agreement with simplified estimates of the strong acceleration densities.

The resulting differences of the ion and electron densities are given in Figure 9. They cause fast changing electric fields $E$ given in Figure 10 reaching values beyond $10^8$ V/cm. This value corresponds to the expected numbers: the dielectrically swollen laser field $E_L$ in the plasma can be up to $10^{11}$ V/cm decaying to zero within $10^{-3}$ cm.

Using similar simplifying approximations as in equation (6), including the oscillating laser field, the longitudinal (dynamic electric) field $E$ from (3) is given by

$$E = 4\pi e \left[ \frac{\partial}{\partial x} \left( \frac{3n_i kT_i}{m_i} + Zn_i v_i^2 \right) - \frac{\partial}{\partial x} \left( \frac{3n_e kT_e}{m_e} + n_e v_e^2 \right) + \frac{1}{m_e} \frac{\partial}{\partial x} (E_L^2 + H_L^2) \right] + \frac{\omega_p^2 - 4\omega^2}{(\omega_p^2 - 4\omega^2)^2 + \nu^2 \omega^2}$$

$$+ \frac{4\pi e}{m_e} \frac{\partial}{\partial x} \left( \frac{E_L^2 + H_L^2}{2\omega t} \right) \cos 2\omega t + \frac{2\nu \omega}{(\omega_p^2 - 4\omega^2)^2 + \nu^2 \omega^2}$$

where the first term represents the former quasi-static field $E$, (4) with its damped-fast oscillations but modified by the amplitude of the fast time-averaged laser field density $E_L^2 + H_L^2$ which is dominant before the gas dynamic pressure $n_e kT_e$ acts. As $E_L^2 + H_L^2$ changes fast (still very slow compared to the laser oscillation time), a quite complicated result for $E_x$ can be seen in Figure 12, in which the exact result is given without the simplification of equation (9). Considering the complicated time dependence of $n_i, n_e, T_i, T_e, E_{L,i}$ and $H_{L,i}$, the term "potential" is no longer applicable and $E$ is a dynamic electric field following equation (1). Only at stationary conditions, the pressure may be a potential or one may consider a ponderomotive potential.

The second and last terms in equation (9) oscillate quickly with twice the laser frequency. As $E_x$ is directed to the x-direction, i.e., perpendicular to the $E_L$ of the laser field, we have — obviously for the very first time — the coupling of the transverse electromagnetic wave with the longitudinal plasma waves which is made possibly only by overcoming the restriction of the quasi-neutrality of the earlier two-fluid theory, and without the artificial inclusions of microscopic model assumptions. The last term in equation (9) has a resonance denominator, causing a very steep increase of the oscillation amplitude at $2\omega = \omega_p$. As we consider a case of purely perpendicular incidence without
any surface rippling and no self-focusing, we have here a new type of resonance mechanism acting in the evanescent part of the wave in a depth of 4 times the critical density, if there is still sufficient laser intensity. This resonance is basically different from Denisov's resonance absorption which works at oblique incidence for p-polarization only (Denisov, 1957). The new type of perpendicular incidence resonance can be significant (Hora and Ghatak, 1985) as will be discussed in Section V with other phenomena.

The numerical result of Figure 9 can explain the inverted double layers in laser-produced plasmas if cavitons are produced by the nonlinear forces. The existence of the electric fields in plasma surfaces had been shown directly by electron beam probes and from electrostatic acceleration of a small number of the nonlinear force-accelerated ions. A more systematic experiment was done by Eliezer and Ludmirsky (1983), Ludmirsky et al. (1985), and Eliezer et al. (1986) where the temporal dependence of charge of the expanding plasma and the temporal change of the target potential were measured. A very unexpected observation was that the plasma leaving the target was first positively charged and then negatively charged. This was in contrast to the general expectation that an electron cloud should first leave the plasma. The picture changes, however, if we look at all fields at the surface and in the interior of the plasma in the genuine two-fluid model if a nonlinear force-driven caviton is generated. Figure 9 shows, near x = 25 μm, where no laser light acts, that a negatively charged plasma expands before the positively charged plasma follows. Near x = 0, one sees that first a strong positively charged plasma is emitted and then a negatively charged plasma before a nearly neutral plasma follows. This is the result of the caviton generation. Though the experiment (Eliezer and Ludmirsky, 1983) was on the nanosecond time scale, the comparison with the picosecond processes should be justified not only by the correct polarity of the plasma charges but also from other experiments that showed the picosecond buildup of the cavitons (Briand et al., 1985). The experiment of Eliezer and Ludmirsky (1983) is an indirect proof that they had also generated cavitons.

A further experiment which can be explained is the energy upshift of alpha particles from laser fusion pellets. It was observed (Gazit et al., 1979) that the DT-alpha particles from laser fusion pellets had not the expected maximum energy of 3.56 MeV but showed an upshift by Δε of up to 0.5 MeV. The exact description of the interaction of the alphas with the spatially and temporally varying electric field E(x,t) in the (one-dimensional) plasma corona is very complicated as the field is non-conservative. The velocity of the alpha particle, v, with an initial velocity, v₀ and mass, mα is given by the complex integral equation,

\[ v(x) = v_0 + \frac{2e}{mα} \int_{t1}^{t2} E[x(t),t] dt \quad x = v(t)dx. \]  

For a very simplified estimate we use,

\[ d \left( \frac{mα}{2} v^2 \right) = 2eE[x(t),t] dx, \]  

with an average value \( \bar{E} \) of E to give the increase of the alpha energy,

\[ Δε = 2\bar{E}E Δx, \]  

after acceleration along a length Δx of the plasma corona. In order to reach Δε = 0.5 MeV for Δx = 10 μm, we find \( \bar{E} = 2.7 \times 10^3 \text{ V/cm} \). Such fields for Nd glass laser pulses of \( 10^{16} \text{ W/cm}^2 \) are possible only if the nonlinear force-produced cavitons (Fig. 10) are present, since lengths very much larger than 10 μm are not realistic. Thermally produced fields of up to \( 10^9 \text{ V/cm} \) could not produce the measured upshifts of 0.5 MeV. Our results, therefore, are
not only a rough explanation of the alpha upshift by the large electric fields in the cavitons but are also a clear
indication that no thermal electric field can cause the measured upshifts.

We have preliminary results on the exact numerical solution of equation (10) from E-values derived from
laser plasma dynamics (Green et al., 1985). It was discovered that broad E-maxima move within 0.3 to 0.9 of the
speed of light (Fig. 10). The correct phasing of the charged particles in the field does lead to an acceleration by
multiples of the estimate of equation (12). It can be shown how today available CO$_2$ lasers (Antares) with 80 TW
short laser pulses and a sequence of several pulses can shift electron clouds of GeV energy to TeV electron energy.
The caviton (nonlinear force) fields of the type in Figure 12 of $10^{11}$ V/cm act like the (non-conservative) pump fields
in the microwave cavities of an accelerator. The phasing of the nonlinear force field electron acceleration is an
extension of the concept based on many years of work on the nonlinear force and the then recent results on high
electric fields in plasmas (Clark et al., 1985).

**V. DISCUSSION AND FURTHER RESULTS**

Against all prior assumptions of space charge quasi-neutrality of plasmas, our analysis of genuine two-fluid
hydrodynamics has shown very high electric fields inside of plasmas. These are simply given by gradients of density
and/or temperature (inhomogeneity fields) modified by plasma oscillations due to changes in mechanical motion for
free expansion or due to the nonlinear force-produced block motion or cavitons. A consequence for laser fusion of
the resonance at perpendicular incidence may be significant, but it is only one of numerous anomalous and nonlinear
phenomena known. A more important consequence, however, is the fact that the electric fields in the double layers
change the thermal conductivity drastically. In order to fit experiments with too low temperatures of the interior of
the plasma-irradiated pellet and the low fusion neutron emission with the computations, fitting factors $f$ for reduc-
tion of the thermal conduction were used since 1979 (Ding et al., 1983; Richardson et al., 1986) which were around
1/100. The results of the double layers offer a quantitative theory for this reduction. This and further consequences
of the reviewed results will be discussed in this section (Hora, 1985b).

**A. Double Layers and Reduction of Thermal Conduction**

The generation of electric fields and double layers inside of plasmas at gradients of density and/or tempera-
ture can cause the inhibition (reduction) of thermal conductivity below the Spitzer-value for the plasma electrons.
This inhibition was detected indirectly from laser fusion experiments when the interior of the compressed pellet did
not reach the temperatures expected from electronic thermal conduction (Cicchitelli et al., 1984), expressed by a
reduction factor $f$. This can be understood simply from Figure 11 where a double layer is produced between a hot
laser-irradiated corona and the cold pellet interior.

The energetic electrons have left the positive area (causing a mostly negligible preheat), and the following
electrons are returned by the positive charges. If a total disconnection of the electron transport through the double
layer is considered because of the return current of the electrons, only the ions can transport the heat. The thermal
conductivity $\kappa$ is then that of the ions, $\kappa_i$, given by that of the electrons $\kappa_e$,

$$\kappa = \kappa_i = \kappa_e (m_i/m_e)^{1/2} , \quad (13)$$

where $m$ is the electron mass and $m_i$ is the ion mass. This gives the factor $\kappa/\kappa_e = 1/70$ for the ion mass of deuterium
and tritium used in the experiments where a computation fit with a factor 1/100 was shown (Ding et al., 1983;
Richardson et al., 1986).
This explanation of the reduced thermal conduction by the double layer does not take into account that the electrons in the hot plasma may have a Maxwellian equilibrium distribution of their energy with a small number of very fast electrons penetrating the double layer. The factor $f$ of the thermal conduction by the fast electrons through the double layer is given by the ratio of the energy flux density of the electrons (of temperature $T$) in the $x$-direction $E_{\text{out}}$ at $x = x_2$ in Figure 11 over the energy flux density $E_{\text{in}}$ of the electrons incident from the left-hand side at $x = x_1$.

$$f = \frac{E_{\text{out}}}{E_{\text{in}}}$$  \hspace{1cm} (14)

Based on an equilibrium distribution $n$ of the electrons with the velocity $v = (v_x; v_y; v_z)$

$$n(v_x,v_y,v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} n_o \exp\left(-\frac{mv^2}{2kT}\right),$$

where $n_o$ is the (spatial) electron density, we find,

$$E_{\text{in}} = \int_{-\infty}^{+\infty} v_x \frac{m}{2} v^2 n dv_x dv_y dv_z = 4\pi n_o m(kT/2\pi)^{3/2}. \tag{16}$$

The flux density $E_{\text{out}}$ must take into account the fact that the energy of the electrons beyond the double layer is reduced by the electric potential $\varepsilon V_o$ of the layer and only electrons with a velocity component in the $x$-direction $v_x > v_{xo} = (2\varepsilon V_o/m)^{1/2}$ will be transmitted. This results in,

$$E_{\text{out}} = \int_{+\infty}^{+\infty} \int_{+\infty}^{+\infty} \int_{+\infty}^{+\infty} v_x (mv^2/2 - \varepsilon V_o) n dv_y dv_z \tag{17}$$

$$= 4\pi n_o m(kT/2 m)^{3/2} \exp(-\alpha), \tag{18}$$

where

$$\alpha = \varepsilon V_o/(kT). \tag{19}$$

The final result

$$f = \exp(-\varepsilon V_o/kT) \tag{20}$$

is then a simple Boltzmann factor.

From the experiments (Eliezer et al., 1985) there may be good reasons that $\varepsilon V_o$ is more than 5 kT up to at least 10 kT. In this case $f$ is less than 1/70 given from the thermal ion conduction for D-T plasma. If we, however, work with the simple (one-dimensional) adiabatic relation $\varepsilon V_o = 3kT$, the factor $f$ is 1/20, showing a well reduced but electronically dominated thermal conduction.
We conclude that the reduction of the thermal conductivity by the electrostatic double layer between hot and cold plasma does not necessarily drop down to the low value of the ion conductivity, and a reduced electronic thermal conduction by the energetic tail of the electron energy distribution may remain. For the analysis of future experiments, these variables of thermal conductivity factors have to be taken into account if no further competitive mechanisms (e.g., turbulence, classical thermal conductivity in inhomogeneous media) are taken into account. With respect to the energetic (so-called “hot”) electrons in laser-produced plasmas, it has been found that there does not exist a fast Maxwellian tail of the energy distribution (McCall, 1983) proving that these energies are due to nonthermal quiver motion. These electrons would not be able to contribute to the thermal conduction mechanism discussed here. Another indication that these energetic electrons are not of a thermal nature (very probably representing the coherent quiver motion) is the very anistropic “butterfly” directivity of the x-ray emission.

The reduced thermal conductivity in the double layers at steep thermal or density gradients, as given by the dynamic electric field strength $E$ (inhomogeneity field), equation (9), is an important consideration in pellet ablation-compression computations whether the driving is by particle beams or by lasers. As long as no nonlinear forces, nonlinear optical response (absorption), and parametric effects are involved, there is a lot of similarity to the laser driving where the computer evaluation of the hydrodynamics automatically results in a compression of the plasma below the driver heated ablating corona. As a sufficient temperature is needed for the compressed plasma in the pellet core, the heat transport between corona and core is essential. If the classical electronic conductivity is used (without change by the inhomogeneity fields or the space charges of the double layers), it is no surprise that the laser ablation resulted in high core densities well after the mechanical recoil, but the temperatures were too low (Yaakobi et al., 1984) and the neutron gains from fusion were $10^4$ times less than expected at this ablation mode (Hora, 1981).

It should be noted that the inhibition of electron transport by the double layer (Fig. 11) is valid also for the energetic (erroneously called “hot”) electrons. Even if their energy is some 100 keV as in CO$_2$ laser-irradiated fusion pellets, the number of electrons to produce a Debye layer only can move to the pellet interior to preheat the plasma. The following electrons, especially if they have no fast Maxwellian tail of a distribution, cannot pass the 100 keV DL. The usual electron preheat in pellets is then only a few mJ at some 100 J absorption of laser radiation.

B. New Resonance at Perpendicular Incidence

The only resonance phenomenon (to be distinguished from parametric instabilities) at laser-plasma interaction is Denisov’s (1957) resonance absorption which only may work at oblique incidence of laser radiation for p-polarization. White and Chen (1974) published the first derivation with the electric field description for a collisionless plasma, showing a resonance maximum of the electric field component of the laser field in the direction perpendicular to the surface at the critical density for laser light which is obliquely incident and p-polarized. The resonance in this case is in the evanescent field region below the reflection point of the propagating radiation. When generalizing this derivation (Hora, 1979) to the case with collisions, the pole of the effective dielectric constant suddenly changes from minus infinity to a high positive value and the width of Denisov’s resonance maximum can be directly calculated in a transparent way (Hora, 1981).

In difference to this, a resonance was found (Hora and Ghatak, 1985) at perpendicular incidence of the (laser driven) longitudinal dynamic plasma field $E$ (not the laser field) of such magnitudes that some phenomena at perpendicular incidence may be explained now where Denisov resonance was mentioned hoping that density ripple provides the necessary oblique incidence. This was questionable with respect to the low angle of incidence.

While the results on the numerical theory of the genuine two-fluid model were most general, the simplified analytical evaluation of the equations was possible by neglecting terms because of the electrons to ion mass ratios,
dropping discussions of integration constants and reducing to local differentiations and by coupling with Maxwell’s
equations. In a laser-irradiated plasma for perpendicular incidence, an inhomogeneous oscillation equation is then
derived (with driving terms) for the (longitudinally oscillating dynamic) electric field \( E \) which is perpendicular to
the driving laser field \( E_L \) (and \( H_L \)). The solution of the differential equation resulted in equation (9).

The last term in equation (9) significantly indicated a resonance of \( \omega_p = 2\omega \) (4 times the critical density).
This was noted approximately before and evaluated roughly numerically (Hora et al., 1984; Hora and Ghatak,
1985). The more precise evaluation was performed by Goldsworthy et al. (1986). It is stressed again that in evalu-
ating the last term in equation (9) before time averaging, the whole nonlinear force needs to be strong enough such
that the term proportional to \( \sin(2\omega t) \) resonantly dominates. The coefficient of this term is

\[
E_R = \frac{2v\omega}{(\omega_p^2 - 4\omega^2)^2 + 16v^2\omega^2} \frac{e}{2m} \frac{\partial}{\partial x} (E_L^2 + H_L^2) .
\]  

(21)

In order to get the solutions \( E_L \) and \( H_L \) from the inhomogeneous plasma we especially select the condition that the
electron density is increasing linearly in the region of the evanescent field. In this case, the wave equation can be
solved by Airy functions (Lindl and Kaw, 1971; Goldsworthy et al., 1986). The full resonance amplitude given in
equation (21) can now be evaluated numerically for any slope of the linear density profile and a constant temperature
(collision frequency) by numerically solving \( E_L \), deriving \( H_L \) from Maxwell’s equations and calculation \( n \), and
using these values to compute the resonance amplitude \( E_R \).

Numerical evaluation of the resonance phenomenon described in the previous sections was carried out for a
plasma irradiated by neodymium glass laser light.

In Figure 12 the value of \( E_R \) of the resonant field amplitude is plotted as a function of depth \( x \) where the zero
of the depth axis represents the critical layer. Noting that the resonant field depends linearly on the incident laser
intensity, only the results of the realistic case, an initial intensity of \( 10^{16} \) W/cm², are discussed.

The electron collision frequency \( v \) is density dependent and is given by

\[
v = 2.72 \times 10^{-5} \frac{n_e}{T^{3/2}} \ln \Lambda
\]  

(22)

where \( n_e \) is the electron density per cubic centimeter, \( T_e \) is the electron temperature in electron volts (eV), and \( \ln \Lambda \)
is the Coulomb logarithm.

Results have been obtained for several different plasma temperatures, of which the case for 1 keV is given in
Figure 12. The gradient of the density profile was varied as a parameter of the curves. The gradient is determined by
\( \alpha \),

\[
\alpha^2 = (\partial n_e/\partial x)^{-1}\omega/c ,
\]  

(23)

where the maximum of each curve is at such depth \( x \) where the density has reached 4 times the critical density.
Figure 12 shows the results for the conditions \( T_e = 1 \) keV for different depths of the maxima. The density gradients
\( \alpha^2 \) range from 140 to 240. \( T_e \) is the effective temperature (chaotic plus coherent motion of the electrons) which
can well have the values of \( 10^4 \) eV at high laser intensities. Figure 13 evaluates the maximum field \( E_{max} \) of \( E_R \) as \( E_{max}/E_L \)
related to the amplitude of the laser field in vacuum for various plasma temperatures.
Figure 13 shows that any strong resonance effect can be expected only when the profile has a very high steepening such that 4 times the density is reached at one wavelength or less below the critical density. This high steepening, however, is not unusual in cases where the nonlinear force is dominating the plasma dynamics (Ahlstrom, 1982; and Montes and Willi, 1982).

For laser-plasma interaction at perpendicular incidence a resonance is analyzed which produces high electric fields oscillating with the second harmonic perpendicular to the plasma surface (longitudinal oscillations). These fields are found in the application of a new genuine two-fluid hydrodynamic theory which is not restricted by space charge quasi-neutrality. For linear density profiles beyond the critical density, the resonance maxima are evaluated on the basis of the Airy functions and reach considerably high values for such profiles which can be generated by nonlinear force driving of the laser-plasma dynamics. Even the necessary high temperatures (appearing then as quiver energy as in the theory of the optical constants) seem to be reasonable. This perpendicular resonance mechanism may possibly be distinguished from the ordinary nonlinear force acceleration by the appearance of electron bursts.

**C. Density Independent Second Harmonics Emission**

A rather surprising phenomenon was reported by Mayer et al. (1982). Irradiating a plane target in vacuum by a neodymium glass laser, a side-on time-integrated picture in the second harmonic frequency showed the large plasma plume in nearly constant 2\(\omega\) intensity though the plasma density has been lower by orders of magnitudes in the outermost parts of the plasma than in the focus. A similar observation was detected more precisely (Aleksandrova et al., 1985) from a 400 \(\mu\)m diameter pellet irradiated by a 2 ns rectangular neodymium glass laser pulse (Delfin), where a nearly constant 2\(\omega\) radiation from a sphere of 2 mm diameter (to which the pellet corona had expanded during the laser irradiation) was detected. The fact that the very low peripheric plasma density emits the same 2\(\omega\) radiation as the inner part of the cut-off density can be explained by the middle term of equation (9). The factor is nearly density-independent at low \(\omega_p (\omega_p << \omega)\), and the standing wave pattern may result in a constant nonlinear force factor; therefore, this term of equation (9) should produce a spatially constant term of the dynamic electric field \(E\), as long as the laser is shining.

While this gives a qualitative explanation of the observation, a quantitative evaluation of the transfer of the dipole oscillation of \(E\) into emission of electromagnetic radiation results in an emission power of about \(10^6\) watts (Goldsworthy et al., 1986). The experimental evaluation of the calibration of the experimental results in a 2\(\omega\)-power of about \(10^5\) watts (Fedotov et al., 1985).

**D. E x B Rotation of Plasmas**

Since the dynamic electric fields, e.g., (9), in plasmas are (apart from the oscillations, damping, and transient effects of internal and/or external plasma dynamics) in a simplified way due to gradients of electron density and/or temperature, their \(E \times B\) interaction with external magnetic fields \(B\) may cause drift motion or rotation of plasmas. We shall first discuss this as examples with plasmas without laser irradiation, e.g., with tokamaks and stellarators, and then consider the extremely high \(E\)-fields by the nonlinear forces in laser-produced plasmas that describe fast block acceleration of plasma. There is a similarity to the simple ambipolar field effects.

The consequences for dynamic inhomogeneity electric fields in tokamaks are not only the modification of the thermal conduction but also the resulting basic change in the dynamics. The radial decay of density and temperature in any plasma column produces an inhomogeneity field in the radial direction which under stationary conditions is given by equation (9)
This field combines with the toroidal magnetic field \( B \) and causes a drift with the velocity of the poloidal plasma rotation in meters/second (Fig. 14)

\[
v_{\text{rot}} = \frac{3T}{rB},
\]

where the electron temperature is in electronvolts, the radius \( r \) of the plasma column is in meters, and \( B \) is in Tesla.

Measurements from tokamaks fully agree with the result of equation (25). Bell (1979) measured rotation velocity \( v = 2 \times 10^3 \text{ m/s} \) for \( r = 2 \times 10^{-2} \text{ m} \), \( B = 0.5 \text{T} \), \( T_e = 50 \text{ eV} \) for which case equation (25) results in \( v = 2.4 \times 10^3 \text{ m/s} \). These plasma rotations were detected from the Doppler shift of \( H\alpha \)-lines, with similar agreement with equation (25), by Sigmar et al. (1974) who did not interpret them as plasma rotation, but as an anomaly of hot protons in the banana and plateau regimes. The agreement with equation (25), however, favors an interpretation of a simple rotation.

The same is with the experiment at the stellarator W7, where the result of 1980 agrees with a rotation according to equation (25). As this experiment was with tangential neutral beam injection, one would have had to exclude the rotation of these neutrals, which is difficult. Recent measurements at W7 without neutral beam injection but with plasma production by intensive microwave irradiation and heating (Thumm, 1985) result in exactly the same rotation given by equation (25).

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Figure 1. Between the vacuum range A and the space charge neutral interior of homogeneous plasma C, the plasma surface sheath is depleted by the escape of fast electrons until such a strong space charge is built up that the following fast electrons from the plasma C are electrostatically returned into C. The electric field $E(x)$, due to the space charge density $\rho(x)$, and the resulting potential $V$ are given schematically (Hora, 1975).
Figure 2. Experiment for a laser-irradiated pellet whose potential and the field [by the Rogowski coil $I(t)$] are measured.
Figure 3. Time-dependent development of the longitudinal dynamic electric field $E_x$ along the density with an initial ramp of linear plasma of initial temperature $10^7$ K of $5 \times 10^{20}$ cm$^{-3}$ at $x = 0$ and $10^{21}$ cm$^3$ at $x = 10 \mu$m (Lalousis and Hora, 1983).
$T = 10^7 \, \text{K}$

$n_{e,\text{max}} = 10^{21} \, \text{cm}^{-3}$

Figure 4. Same as Figure 3 for times of 10 and 12 plasma oscillation period (Lalousis nad Hora, 1983).
Figure 5. Same as Figure 3 for times of 40 to 42 periods (Lalouat and Hora, 1983).

Electric field: \( E = 2.58 \times 10^6 \text{ V/cm} \)

Temperature: \( T = 10^7 \text{ K} \)
Figure 6. Ion density of a 25 μm thick hydrogen plasma slab initially at rest and 1 keV temperature irradiated from the left-hand side by a $10^{16}$ W/cm² Nd glass laser. At $t = 0.6$ ps the density is very similar to its initial value. The energy maximum near $x = 4$ μm produces a caviton by nonlinear forces.
Figure 7. Density of the electric field energy of the laser (without the electrostatic fields generated within the plasma).
Figure 8. Ion velocity \( v_o \) at several time steps for a plasma, as in Figure 3. A block of plasma is generated with a velocity up to \( 10^7 \) cm/s.
Figure 9. The genuine two-fluid model shows the difference between ion density $n_i$ and electron density $n_e$ with the surprising result of a positive difference (space charge) before the caviton and a negative region behind the caviton (inverted double layer as observed by Eliezer and Ludmirsky, 1983). Near $x = 25 \mu m$ the laserless plasma expands normally with a negative periphery.
Figure 10. Electric field $E = E_s$ inside the plasma of Figure 3 dynamically evolving with absolute values beyond $10^8$ V/cm near the caviton produced by the nonlinear laser forces at times (in picoseconds) 0.6--; 0.8--; 1.0--; 1.1--.
Figure 11. The positive charge of the double layer between the hot and the cold plasma causes a return of the electrons to the hot plasma with the exception of the electrons from the energetic tail of the energy distribution (Cicchitelli et al., 1984).
Figure 12. Resonance amplitude $E_R$ of the longitudinal electric field, as a function of the depth $x$ below the critical density, for neodymium glass laser irradiation of $10^{16}$ W/cm$^2$ into plasma with a temperature of 1 keV. The parameter for profile steepening $\alpha^2$ ranges from 100 to 240. The critical density $n_e$ corresponds to the axis $x = 0$. 
Figure 13. Combining the resonace maxima of Figure 12 and for other temperatures given as ratio to the incident laser amplitude for various plasma temperatures in electron volts depending on the depth $x$ below the critical density for neodymium glass laser radiation.
Figure 14. Poloidal rotation of tokamak plasma by $E \times B$ forces where the electric field $E$ is the inhomogeneity field.