ACCRETIONAL HEATING OF THE SATELLITES OF SATURN AND URANUS

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Voyager images of the satellites of Saturn and Uranus have shown that these bodies are characterized by remarkable diversity and surprisingly complex geologic histories. Despite their small sizes, a number of the satellites show unambiguous evidence for resurfacing. Satellites that have clearly undergone at least one episode of resurfacing include Enceladus, Dione, Miranda, Ariel, and Titania [1,2]. Less compelling evidence has also been presented for resurfacing on Mimas and Rhea. In the case of Enceladus and perhaps Ariel, tidal dissipation may have been largely responsible for heating the satellites. The energy source that apparently led to melting in the other satellites, however, is less clear. For all the satellites, long-lived radiogenic heating is entirely inadequate to produce melting. We have therefore turned to investigation of accretional heating as a possible energy source. Numerous models of accretional heating of planetary bodies have been developed previously, especially for the terrestrial planets. All such models suffer from a need to select preferred values of critical model parameters in order to obtain meaningful results. Our goal here has been to develop a detailed model for the heating of these small satellites, and then to explore the consequences of variations in the free parameters in the model. Specifically, we attempt to determine for what range of conditions melting will occur in these satellites. Along with varying a number of model parameters, we also consider the important effects of inclusion of small amounts of ammonia and (in the case of the uranus satellites) methane in the system.

We begin with a swarm of bodies from which the satellite will accrete. We allow accretion to take place gravitationally, calculating the amount of heat deposited, the depth over which it is deposited, the subsequent transport of heat within the satellite, the rate of growth of the satellite, and the satellite's evolving internal thermal profile. We consider two reservoirs of particles from which a satellite may accrete: bodies in orbit about the planet (cis-planetary debris) and bodies in orbit about the sun (trans-planetary debris). The initial surface density $\sigma_c$ of the cis-planetary particle disk is obtained by distributing the present mass of the satellite over an annular region centered on the planet, with inner and outer radii at the distances at which the gravitational attraction of the present satellite is balanced by the attractions of its inner and outer nearest neighbors, respectively. The surface density $\sigma_t$ of the trans-planetary disk is determined by similarly distributing a small fraction $\beta$ of the planet's mass over a sun-centered annular region. The impacting particles are assumed to have a density equal to that of the present satellite, and to have a size distribution of the form:

$$\frac{\partial \hat{N}(r_p)}{\partial r_p} \propto r_p^{-\alpha}$$

(1)

where $\hat{N}(r_p)$ is the number flux of particles of radius $r_p$, and $\alpha$ is a constant.

In calculating the accretion rate and the impact velocity of the impactors from both particle disks, we follow the formulation of Safronov [3]. We use the Safronov parameter $\theta_c$ for the cis-planetary disk, defined by

$$\theta_c = GM_s/R_s u_c^2$$

(2)

where $G$ is the gravitational constant, $M_s$ is the mass of the satellite, $R_s$ is the radius of the satellite, and $u_c$ is the velocity of an impactor relative to the circular orbit. The Safronov parameter is related to the eccentricity of the particles in the disk, and is a free parameter in the model. It then follows that the impact velocity $v_c$ is given by

$$v_c = \sqrt{u_c^2 + 2GM_s/R_s}$$

(3)
and that the mass flux \( \dot{M}_c \) is given by

\[
\dot{M}_c = \frac{4\pi(1 + 2\theta_c)}{P_s} \sigma_c R_c^2
\]

where \( P_s \) is the satellite's orbital period. Following Kaula [4], we also define similar expressions for \( v_t \) and \( \dot{M}_t \), the impact velocity and mass flux for debris from the trans-planetary disk.

Having developed expressions for the size distribution, impact velocity, and mass flux of particles from both sources, we then calculate deposition of thermal energy by the impact process. We assume that some fraction \( \eta \) of an impactor's kinetic energy is deposited beneath the satellite's surface; \( \eta \) is another free parameter in the model. When an impact takes place, a roughly hemispheric shock wave centered on the impact point is generated. As it passes through the material of the satellite, energy is deposited in that material. The specific energy deposited is largest near the impact, and decreases monotonically with distance from the impact point.

We have solved the Rankine-Hugoniot equations for the deposition of specific energy by shock passage, using published experimental data on shock propagation through \( \text{H}_2\text{O} \) ice. The specific energy \( \epsilon \) deposited as a function of distance \( r \) from the impact point is taken to be

\[
\epsilon = \epsilon_0 \left(\frac{r_i}{r}\right)^\gamma
\]

where \( r_i \) is the impactor radius. Solution of the Rankine-Hugoniot equations for ice show that an expression of this form is appropriate, and give a value of \( \gamma = 3.1 \). Energy deposition is described by eq. (5) out to the point at which the shock pressure drops below the Hugoniot elastic limit for ice [5], and beyond that point is assumed to be zero. Impacts are assumed to be uniformly distributed over the satellite. Energy deposition is integrated over depth and over the particle size distribution for both impactor sources, so that the total energy deposition as a function of depth is obtained.

The satellite is allowed to grow, increasing in radius as it depletes the particle disks from which it is growing. Simultaneously, we solve the heat conduction equation for the growing satellite, allowing heat to be transported in response to internal thermal gradients. The thermal parameters used are those appropriate for \( \text{H}_2\text{O} \) ice. Near the surface, heat transport will also be significantly influenced by the physical mixing that takes place due to impacts, introducing an additional effective thermal diffusivity. The effective diffusivity due to impact mixing at a depth \( z \) is given by

\[
K(z) = \frac{1}{2} n_x z^2
\]

where \( n_x \) is the number of times a layer of depth \( z \) is turned over by impacts per unit time. We have developed an expression for \( K(z) \) based on the regolith mixing model of Gault et al. [6].

Growth of the satellite takes place until the source of impactors is exhausted. The free parameters in the model are then \( \beta \) (the fraction of the planet's mass in the trans-planetary disk), \( \theta_c \) and \( \theta_t \) (Safronov parameters for impactors in both disks), \( \alpha \) (the particle size distribution exponent), and \( \eta \) (the impactor kinetic energy partitioning coefficient).

We have performed two sets of calculations. In the first, we have done a large number of calculations of accretional heating of Saturn's satellite Rhea, allowing for large variations in all the parameters of interest. A sample result is given in Figure 1, for \( \beta = 0.001, \theta_c = 4, \theta_t = 4, \alpha = 3.0, \eta = 0.2, \) and \( \gamma = 3.1 \). Reasonable variations in \( \beta \) have a negligible effect on the results. Even placing 10% of the mass of Saturn in the trans-planetary nebula has little effect; the reason is that the timescale for accretion from the cis-planetary disk is very short in comparison to that for the trans-planetary disk. Variations in \( \theta_c \) are important. For large values of \( \theta_c \) (i.e.,
for low impactor eccentricities), accretion times are extremely short; for example, \( \sim 200 \) yr for \( \theta_e = 50 \). Impact velocities are also low, so energy is deposited only very near the surface. In contrast, low values of \( \theta_e \) lead to longer accretion times (\( \sim 5000 \) yr for \( \theta_e = 2 \)), and deeper deposition of energy. Variations in \( \alpha \) are also important; smaller values of \( \alpha \) mean that more of the mass flux is in large impactors and increases the amount of energy deposited at depth. As one would expect, variations in \( \eta \) have a significant effect on the peak internal temperature achieved.

In the second set of calculations, we have calculated accretional temperature profiles for the inner major satellites of Saturn (Mimas – Rhea) and for all the major satellites of Uranus. The maximum temperatures achieved are: Mimas 83 K, Enceladus 93 K, Tethys 145 K, Dione 158 K, Rhea 195 K, Miranda 82 K, Ariel 164 K, Umbriel 160 K, Titania 214 K, Oberon 199 K. Given the uncertainties in the input parameters, these temperatures may be uncertain by as much as a few tens of degrees. It is clear that temperatures will never exceed 273 K, so melting would not be expected if the satellites were made of pure H\(_2\)O ice. However, it is probable that the saturnian satellites contain some NH\(_3\), and it is possible that the uranian satellites also contain some CH\(_4\). A peritectic melt will form in the H\(_2\)O – NH\(_3\) system at a temperature of 173 K, and it has been suggested that ice containing CH\(_4\) could be substantially mobilized at temperatures as low as 100 K [7]. Given the uncertainty in the calculations and the likelihood of NH\(_3\) in the saturnian satellites and CH\(_4\) in the uranian satellites, these calculations suggest that, among these satellites, only Mimas and Enceladus cannot have undergone resurfacing as a result of accretional heating.


![Figure 1 — Calculated thermal profiles within Rhea during accretion. Curves are plotted at equal satellite mass intervals. Total accretion time is \( \sim 2000 \) yr.](image-url)