CLOUD FLUID MODELS OF GAS DYNAMICS AND STAR FORMATION IN GALAXIES

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ABSTRACT

The large dynamic range of star formation in galaxies, and the apparently complex environmental influences involved in triggering or suppressing star formation, challenge our understanding. The key to this understanding may be the detailed study of simple physical models for the dominant nonlinear interactions in interstellar cloud systems. We describe one such model, a generalized Oort model cloud fluid, and explore two simple applications of it. The first of these is the relaxation of an isolated volume of cloud fluid following a disturbance. Though very idealized, this closed box study suggests a physical mechanism for starbursts, which is based on the approximate commensurability of massive cloud lifetimes and cloud collisional growth times. The second application is to the modeling of colliding ring galaxies. In this case, the driving processes operating on a dynamical timescale interact with the local cloud processes operating on the above timescales. The result is a variety of interesting nonequilibrium behaviors, including spatial variations of star formation that do not depend monotonically on gas density.
I. INTRODUCTION

A. The Dynamic Range of Star Formation in Galaxies

The discovery of Sargent and Searle (1970) that several Zwicky irregular galaxies are essentially "extragalactic HII regions" provided one of the earliest indications that current star formation rates (SFRs) in other galaxies could be very high compared to the SFR in our galaxy. It was clear to these authors that the relatively high SFRs in these galaxies could not persist for any significant fraction of a Hubble time, if only because the implied gas consumption time was short. Later, Gerola and Seiden showed that relatively strong (i.e. enhancement factors of an order of a few) bursts of star formation, separated by relatively long quiescent periods, could be a consequence of self-propagating star formation in relatively small and essentially three-dimensional systems like the dwarf irregulars (see e.g. the review of Seiden and Gerola 1982). At the same time, the discovery of Larson and Tinsley (1978) that bursts of star formation were apparently ubiquitous in Arp interacting galaxies showed that high SFRs were not unique to dwarf galaxies. More recently, the observation that merger remnants are undergoing very extended starbursts (e.g. Joseph and Wright 1985, and further references in the review of Schweizer 1986), provides very dramatic evidence that large galaxies are capable of very large net SFRs.

Moreover, these examples of high SFRs in galaxies do not show the full dynamic range of star formation in galaxies, since galactic SFRs extend to very low values as well. One example, is given by the class of anemic spirals of van den Bergh (e.g. 1977). However, these galaxies have low gas densities as well as small SFRs. A better example is provided by the gas-rich, low-surface-brightness (LSB) galaxies discovered by Thuan and Seitzer (1979), and further studied by Romanishin et al. (1982). As Schommer and Bothun (1983) point out, these galaxies may provide evidence for suppression of star formation in otherwise normal disk galaxies.

Many questions are raised by the existence of variations in star formation between galaxies as large as those between the LSB galaxies and the starburst galaxies. What are the mechanisms responsible? What are the circumstances required to induce bursts or the suppression of star formation in galaxies? What is the precise role of interactions? Is the star formation process in starburst galaxies an extreme extension of the normal mechanisms of star formation in disk galaxies or is it a different process entirely? How can we tell observationally?

With respect to the latter two questions, there is evidence that the star formation efficiencies per unit mass in starburst galaxies can range up to about two orders of magnitude higher than in normal spirals (Rieke et al. 1980, Sanders and Mirabel 1985, Young et al. 1986, Sanders et al. 1986). In itself this result does not answer the question of whether starbursts represent a nonlinear continuation of a normal (e.g. Schmidt-law) mode of star formation, or whether once some threshold is exceeded a qualitatively different mode appears. If there are two distinct modes of star formation, then the dispersion in optical and near-infrared colors in color-color diagrams gives an indication of the relative importance of burst versus continuous star formation.
in galaxies (Larson and Tinsley 1978, Struck-Marcell and Tinsley 1978, and Telesco 1983). However, these models do not directly address the question of mechanisms.

At the present time, with radio continuum and optical line surveys, and with IRAS data and near-infrared mapping of individual galaxies, there is a wealth of new information available on star formation in galaxies. To decipher the systematics and mechanisms of star formation in galaxies, it has been and will continue to be especially important to have statistical studies of large sets of data, collected in a consistent way (e.g. the IRAS data (Lonsdale et al. 1985), or optical line work like that of Balzano 1983 and Keel et al. 1985). For the present, we leave the intriguing questions posed above, but will return in the concluding section to consider a possible explanation for the large dynamic range of star formation in galaxies, which is suggested by a simple physical model.

Following some discussion of the general role of simple models, a specific model is presented in section II. In sections III and IV two applications of the model are considered — relaxation in a closed box and the evolution of ring galaxies.

B. The Heuristic Role of Simple Physical Models

In order to model and understand the wide range of phenomena associated with star formation in galaxies, theorists have used a variety of tools and techniques, including continuum kinetic or fluid approaches, discrete N-body models with approximations to include interstellar gas cloud and cloud-star interactions, and modeling with a stochastic component of the star formation. This range parallels that in studies of fluid turbulence, and other nonlinear phenomena. The various approaches have different advantages and disadvantages, both from the point of view of faithfully representing the phenomena, and the practical point of view of being able to perform the calculations and interpret them, analytically or on existing computers. In the end, to derive a consistent interpretation of the many aspects of star formation in galaxies (and hopefully some predictions!), a variety of increasingly sophisticated approaches will be needed. Because of the extreme complexity of the numerical models, the various approaches must be carefully compared to each other and to observation.

On the other hand, to achieve a consistent physical understanding of a complex problem it is very helpful to begin with the study of a relatively simple phenomenological model which captures the essence of the phenomenon (e.g. the Ising model of ferromagnetism or the Kolmogorov model of turbulence). Of course, in using such a simple model one must remember that while it may succeed in mocking-up the dominant physical processes in some relevant range of parameter space, it may miss the full interplay of these processes in some other parameter range, or it may miss the emergence of new processes. For the problems of galaxy-scale star formation and gas dynamics the ideas behind the classic Oort cycle — i.e. that clouds are built up by collisional coalescence and massive clouds are broken up as a result of internal star formation activity — have provided the basis for such a simple model for some time. Examples of the usefulness of the Oort picture include the work of Field and Saslaw (1965), who showed that a kinetic (coalescence) equation for the evolution of the cloud spectrum with Oort cycle interactions yielded power-law
solutions like the observed solar neighborhood cloud spectrum, and implied a
dependence of the star formation rate on gas density in accordance with the
empirical Schmidt law. Another example of a cloud collisional model is
provided by Larson's (1969, 1974, 1975, 1976) numerical models of collapsing
protogalaxies, which included density-dependent star formation and energy
dissipation in cloud collisions, and which yielded structures matching many
observations of elliptical galaxies. More recently, a variety of numerical
calculations (e.g. Casoli and Combes 1982, Combes and Gerin 1985, Kwan and
Valdez 1983, Hausman and Roberts, 1984, Roberts and Hausman 1984 and Tomisaka
1984), which include Oort-type interactions, have shown that a spiral density
wave can drive collisional processes, leading to the buildup of giant clouds or
cloud complexes, and presumably, enhanced star formation.

These examples share in common the feature that the interstellar medium is
assumed to consist of an ensemble of distinct clouds (with perhaps a large
range of sizes and masses). The cumulative interactions among the clouds are
used as a link between the large-scale disturbance and the gas dynamics and
star formation on small-scales. This approach, like many turbulence theories
(e.g. eddy viscosity or mixing length theories), is an essentially
phenomenological treatment of intermediate-scale interactions (like the
"inertial" range in incompressible turbulence). It attempts to model the
cumulative effects of small-scale interactions, without incorporating the
details of those interactions (e.g. the physics of the formation of an indi-
nual massive star). At the same time, the intermediate-scales (cloud
ensembles) can be driven by large-scale disturbances or instabilities (e.g.
density waves, gravitational or Parker-Jeans instabilities), and their non-
linear feedbacks can, in turn, effect the development of the large-scale
instability. Thus, a model of the intermediate-scale interactions serves as an
essential tool for applying knowledge of the physics of the interstellar medium
in our Galaxy and other nearby galaxies, to a variety of galaxy-scale problems.

In the original Oort model, and many of the later versions (e.g. Field and
Saslaw 1965), a number of physical processes of possible importance in the ISM
were not considered. Several of these can be incorporated quite readily into a
continuum formalism derived from a kinetic equation (see Scalo and Struck-
Marcell 1984), such as the fact that cloud collisions at high relative velocity
probably lead to cloud disruption, rather than coalescence (Hausman 1982,
Gilden 1984), or that clouds can be formed from the more or less random local
compression of the intercloud material by runaway O, B stars (Bania and Lyon
1980). The possible effects of an intercloud medium on the cloud ensemble
(e.g. drag) could also be included; however, we have argued elsewhere (Scalo
and Struck-Marcell 1984) that such effects are probably not dominant in most
situations.

The interchange of material between cloud and intercloud media, or more
generally, between multiple phases in the ISM, is an intricate subject in its
own right. The view of the ISM as a gas in several co-existing thermal phases,
with a dynamic, time-dependent balance (McKee and Ostriker 1977, Ikeuchi et al.
1984, Bodifee and de Loore 1985), is fundamentally orthogonal (though not
contradictory) to the Oort picture. However, since most of the interstellar
gas is located in relatively dense clouds, the multiple phase picture is
probably more relevant to the study of the intercloud gas.
Another area of substantial ignorance is the role of magnetic fields on all scales. Most workers have concentrated on the effects of magnetic fields on small-scales (e.g. angular momentum transport, flux loss, etc.), but they may also contribute an effective viscosity and additional dissipation on intermediate and possibly large scales (e.g. Clifford and Elmegreen 1983, Clifford 1984, 1985, Elmegreen 1985). The effects of magnetic fields may be crudely included in a hydrodynamic model as additions to the cross sections or rates of viscous transport and dissipation.

Perhaps the most serious questions about the relevance of an Oort-type cloud model to gas in galaxies are raised by the conclusion of Scalo (1985, and references therein) that a hierarchical structure of clouds within clouds is implied by an analysis of observations in a variety of wave-bands of the ISM in our Galaxy on a wide range of scales. In Scalo (1985) it is suggested that about five levels of this hierarchy have been observed, with several clumps contained within each clump of the next higher level. The hierarchecal picture does not necessarily call into question the importance of cloud collisional interactions, and cloud-star interactions, which are the basics of the Oort model. However, at the least, it muddies the simple physical picture of the original Oort model, e.g. the definition of a "cloud" and the proper treatment of collisions in a hierarchecal structure, and in fact, makes the definition of statistical averages ambiguous.

Nonetheless, despite the difficulties and ambiguities a cloud ensemble or generalized Oort picture still provides a viable basis for building deterministic and physically understandable models of large scale gas dynamics and star formation in galaxies (see e.g. Chiang and Prendergast 1985). As a starting point for such a model, we have suggested (Scalo and Struck-Marcell 1984) a general kinetic equation for the joint coordinate position, velocity, and mass distribution of an interstellar cloud ensemble as a function of time. Unfortunately, this kinetic equation is too complex to serve as a very practical tool in itself (although N-cloud type calculations can simulate it). Thus, again following the tradition in the study of fluids and turbulence, we can take velocity and mass moments to derive hydrodynamic equations.

To proceed from this point one must adopt specific models for the cloud interactions, which act like sources and sinks in the hydrodynamic equations. A variety of approximate forms were suggested in Scalo and Struck-Marcell 1984. In the next section we will discuss a relatively simple example of an Oort-type model. In later sections we will apply it to the ring galaxy problem in particular, and explore the insights it offers on the starburst problem in general.

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1This procedure is of course quite idealized since we do not know the mass and velocity distributions or how they may vary with time. However, within the range of reasonable functional forms for these (where what is reasonable is based in part on what we know of the cloud distributions in the solar neighborhood), this amounts to uncertainty in coefficients or order unity.
II. AN OORT MODEL EXAMPLE

Given our great ignorance about the nature of the interactions in the interstellar medium in galaxies, the choice of what methods to use in a simple model, let alone the functional dependence of the interactions, will not be unique and must be partially based on considerations of practicality or feasibility. We have chosen to study, as a general-purpose model for a variety of applications, a hydrodynamic model, averaged over the mass and velocity distributions of clouds, with equations for the number density of clouds $n$, the mean mass of clouds $m$, and the cloud velocity dispersion $c$ (or equivalently the internal energy). The equations are very similar to the normal hydrodynamic equations except for the presence of extra source terms. We include source terms for the usual Oort-cycle interactions, including: cloud collisional coalescence, collisional energy dissipation, the breakup of massive clouds and the acceleration of the fragments due to the stellar winds, expanding HII regions, and supernovae that result from massive star formation. We have also included the process of cloud disruption in high relative velocity collisions. Figure 1 gives a schematic overview of these processes.

Among the key 'philosophical' choices we have made in setting up this model are: 1) We assume that the characterization of the problem by the mean values of cloud number density, cloud mass, mean flow velocity, and velocity dispersion (i.e. the random motions of clouds), preserves much of the essential physics of the problem (more on this later), 2) we do not assume constant collisional cross sections and direct gas density-dependent star formation, and 3) we do not assume the cloud fluid is isothermal (motivated in part by the possible observational comparisons since velocity dispersions can be measured across the face of galaxies). In the remainder of this section we consider in a little more detail a couple of the key aspects of the adopted model (henceforth the Oort model), including the parametrizations of the collisional cross sections and the rate of cloud disruption by internal star formation activity. We also comment briefly on the role of time delays due to the finite lifetime of massive clouds, a role we have found to be crucial in controlling the qualitative and quantitative evolution of the models.

A. Collisional Rates

The form adopted for the rate of change of the number of clouds per unit volume due to coalescence or disruption in collisions is

$$\frac{dn}{dt}_{\text{collision}} = -\alpha_n f_c(m,c)m^{2/3}n^2c,$$  \hspace{1cm} (1)

where the $n^2c$ factor gives the usual quadratic dependence on cloud number density and linear dependence on velocity dispersion for binary collisions. The factor $m^{2/3}$ represents the mass dependence of the geometric cross section, assuming that the internal density of clouds is roughly constant, and the factor $\alpha_n$ contains the generalized cross section amplitude (i.e. the ratio of effective to geometrical cross section). The additional collisional nonlinearities are contained in the function $f_c$, for which we have adopted the simple parametrized form

$$f_c = \left[ \frac{1-(c/c_r)}{1+(c/c_r)} \right],$$  \hspace{1cm} (2)
Figure 1. Schematic overview of the important processes in an Oort model of an interacting cloud system (from Struck-Marcell and Scalo 1986). The first panel shows cloud collisional coalescence (disruption) in low (high) relative velocity collisions and adopted parametrization for the rate this process. The second panel illustrates the process of massive cloud break-up as a result of internal star formation activity, and again the model parametrization is given. The third panel illustrates the 'coalescence overshoot', which occurs when the timescale for massive cloud collisional build-up is less than the cloud lifetime to internal star formation.
where \( c_{\text{cr}} \) is the characteristic velocity dispersion such that the average relative kinetic energy in collisions equals the mean gravitational binding of a cloud. Thus, for example, when \( c \) increases past the value \( c = c_{\text{cr}} \), the sign of the collisional cross section changes, giving the assumed change from coalescence to disruption. The exponent \( r \) determines the abruptness or nonlinearity of this change, and is treated as a parameter of the model. The form of the function is such that it saturates at a maximum (minimum), where we have essentially 100% efficient coalescence (disruption) at \( c < c_{\text{cr}} \) (\( c > c_{\text{cr}} \)). Examples of the function, with different values of \( r \) are shown in Fig. 2, taken from Scalo and Struck-Marcell 1986.

The simple binding energy approximation contained in eq. (2) does not by any means fully represent the complex physics involved in cloud collisions, see, for example, the recent numerical hydrodynamical calculations of Gilden (1984), Lattanzio et al. (1985) and Hunter et al. (1986). Moreover, even the numerical hydrodynamic calculations are highly idealized compared to real interactions between clouds containing internal hierarchecal clumping. Thus, it is presently hard to see how to substantially improve upon the physically plausible, if crude, approximation above.

B. Internal Star Formation

In this simple Oort model we also assume that at any time some fraction of the clouds are sufficiently massive to form stars efficiently and then suffer disruption as a result. (For a recent observational reference see Leisawitz 1985.) We have chosen the following parametrization for this fraction as a function of mean cloud mass compared to a critical cloud mass for star formation \( m_{\text{SF}} \),

\[
    f_{\text{SF}}(m) = \frac{(m/m_{\text{SF}})^s}{1+(m/m_{\text{SF}})^s}, \tag{3}
\]

where the exponent \( s \), like the exponent \( r \) in eq. (2), is a parameter of the model, which characterizes the nonlinearity, or rapid turn-on of star formation, at \( m = m_{\text{SF}} \) (see Fig. 2). The ratio \( f_{\text{SF}}(m)/f_{\text{SF}}(m_0) \), for some characteristic (e.g. equilibrium) cloud mass \( m_0 \), gives the relative rate of cloud disruption by internal star formation, and also the relative fraction of clouds that are forming massive stars. If the mean gas density is constant this ratio can also be taken as an indicator of relative star formation rate. (Strictly speaking, this interpretation also requires particular assumptions about the evolution of the cloud spectrum, see Scalo and Struck-Marcell 1986.)

The particular form for \( f_{\text{SF}} \) in eq. (3) is somewhat arbitrary, but it embodies several physically relevant features quite naturally. First of all, the fraction of clouds massive enough to form stars (or at least the probability of massive cloud formation) is a monotonically increasing function
Figure 2. Several examples of the parametrized coalescence – disruption function $f_c$ and star formation function $f_{SF}$ are shown (from Struck-Marcell and Scalo 1986). Depending on the values of the parameters (especially the exponents $r, s$), the form of these functions can range from nearly linear to an almost step function form. The latter implies a strong threshold behavior.

of mean cloud mass. Secondly, since $f_{SF}$ is defined as the fraction of clouds forming massive stars, it must have a maximum saturation value of $f_{SF, \text{max}} < 1$. On the basis of the first point, we might simply choose to approximate $f_{SF}$ by a power-law in $m$. The saturation effect, however, suggests the form of the denominator on the right hand side of eq. (3). The form of the denominator
also reflects an attempt to constrain \( f_{SF}(m) \) based on the possible evolution of simple cloud mass spectra (see Appendix A in Struck-Marcell and Scalo 1986).

Note that a steep increase in \( f_{SF} \) occurs at \( m = m_{SF} \) if \( s > 1 \). On the other hand, it is apparent from Fig. 2 that if \( s < 2 \), \( f_{SF} \) is sufficiently smooth that there is only a modest increase at 'threshold'.

As in the case of the collision function \( f_c \), it is difficult to go beyond these qualitative considerations. Basically, the function \( f_{SF} \) is dependent on two highly uncertain quantities: 1) The efficiency of massive star formation as a function of cloud mass, and 2) the most probably time-dependent mass distribution of the cloud ensemble. Ongoing observations of star-forming regions in our Galaxy should provide helpful constraints on the first factor.

We might hope that N-body simulations of interacting cloud ensembles would provide useful constraints on the second quantity, in the same way that molecular dynamics experiments yield information on net chemical reaction cross sections. The analogy is imperfect of course, because we don't have a first-principles understanding of cloud interactions, and the detailed results of N-cloud calculations may depend sensitively on the assumptions made about these interactions. Nonetheless, we can illustrate the usefulness of these calculations with one example from the work of Kwan and Valdes (1983). The evolution of the mass spectrum \( g(m) \) of a cloud ensemble following passage through a spiral density wave, on a timescale which is assumed to be shorter than the massive cloud lifetime, is shown in Fig. 2 of their paper. The form of the mass spectrum does change significantly with time, but qualitatively, it is characterized by a single dominant peak at all times, supporting the basic consistency of a model based on the mean mass. Moreover, a narrowing of the mass distribution as the mean mass increases and a steepening at the high-mass end are apparent in their figure (for this essentially pure coalescence case). These features indicate a fairly rapid increase of the mass fraction greater than some (relatively large) critical mass as a function of the mean mass. If this is also convolved with a threshold behavior in the star formation efficiency, then it is quite plausible that the exponent \( s > 2 \) in eq. (3).

C. The Rate Equations with Time Delays

With the expressions above for the collision and star formation terms, the equations for the rate of change of the cloud number density and velocity dispersion squared due to these interactions in an isolated fluid element can be written

\[
\frac{dn}{dt} = \alpha \left( m c^2 \right)^{2/3} n_c^{2/3} + \beta \int_{n_{SF}(m(t-T_d))}^{n(t-T_d)} n(t-T_d) \\
\frac{d(n c^2)}{dt} = \alpha m^{2/3} n c^2 + \beta c_{SF}(m(t-T_d)) n(t-T_d)
\]

(see Scalo and Struck-Marcell 1984, Scalo and Struck-Marcell 1986 for more)
details). With an additional assumption for the rate of conversion of gas into stellar remnants, $dm/dt$, the equation set is closed. The full hydrodynamical equations, with pressure terms and other spatial gradients, are derived and discussed in detail in Scalo and Struck-Marcell 1984.

As will be shown in the following section, it is essential to include the finite cloud lifetime in the cloud disruption terms in eqs. (4). (The importance of the cloud lifetime is readily apparent in several $N$-cloud studies of density waves, e.g. Hausman and Roberts (1984), Tomisaka (1984).) Specifically, massive clouds are assumed to have a fixed lifetime $\tau_d$, and thus, the cloud disruption rate at time $t$ is proportional to the number of massive star-forming clouds at time $t-\tau_d$, i.e. $n(t-\tau_d)\nu_{SF}(m(t-\tau_d))$. With the inclusion of the time delay effect our simple Oort model is complete.

To summarize, we have attempted to model, in a very general way, the principle nonlinear effects associated with cloud collisions and the feedbacks of massive stars on the cloud ensemble. To keep the model simple a number of processes noted in the introduction have not been included. Eventually, it would be interesting to extend the model to include some of these. In general, there is no reason that the hydrodynamic formalism must be restricted to Oort-type interactions. Many potentially interesting kinetic effects are excluded in a mean fluid model, but in this case some idea of what has been lost can be obtained by comparing to $N$-cloud calculations. (Multiple-fluid hydrodynamic models are also possible, and even two-scale models have proven of great use in atmospheric physics.)

Even with the many omissions this 'simple' model appears at first sight to be very complex, and dependent on many parameters. However, as far as the qualitative evolutionary behavior is concerned, this turns out not to be the case. First of all, the equation set (4) has a single equilibrium (with $n_0$, $m_0$, $c_0$ real and $>0$) at constant gas mass density, and if the equations are nondimensionalized in units of this equilibrium six dimensionless parameters remain. These include: the exponents $r,s$, the critical mass for star formation $m_{SF}/m_0$, (in units of $m_0$ for convenience) the critical velocity dispersion for disruption in collisions $c_{cr}/c_0$, (in units of $c_0$) a dissipation efficiency factor, and $T$ the ratio of the cloud lifetime to the equilibrium cloud collision time. The extensive parameter study in Struck-Marcell and Scalo 1986, shows that, as long a $S>2$, only the parameter $T$ effects the qualitative behavior of the model, the quantitative effects of the other parameters decouple to the extent that their individual effects are fairly understandable physically (see the following section). Thus, we believe that this deterministic Oort model can serve as a useful tool for studying a variety of problems in galactic gas dynamics, and as an aid in the interpretation of other more complex calculations.

III. APPLICATIONS I: A MECHANISM FOR STARBURSTS

A. Evolution in a Closed Box

The simplest application of the model eqs. (4) is to the study of the
evolution of a cloud system in an isolated closed box with constant gas density \( p = mn \), following a disturbance. Some of the essential results of such calculations were given in Scalo and Struck-Marcell 1986, and an extensive parameter study is reported in Struck-Marcell and Scalo 1986, so we limit the discussion in this section to a brief summary.

We have found, on the basis of an extensive grid of numerical integrations of eqs. (4) and linear stability analysis, that following an arbitrary disturbance, the closed box cloud system relaxes within a few cloud collision times to one of two generic behaviors. If \( T \), the cloud lifetime to collision time parameter (henceforth simply the 'time delay' parameter \( T \)) is assumed to be less than some critical value \( T_{cr} \), the system relaxes rapidly to the single stable equilibrium state \( (n_0, m_0, c_0) \). On the other hand, if \( T > T_{cr} \) the system relaxes to a stable closed curve, rather than a point, in the \( (n, m, c) \) phase space. In this case the system undergoes nonlinear, self-excited oscillations. At \( T = T_{cr} \) the system undergoes a so-called Hopf bifurcation, so that for \( T \) slightly greater than \( T_{cr} \) the attracting set is a single-period limit cycle. The bifurcation is characterized not only by the appearance of the limit cycle, but also by the fact that the original equilibrium state becomes unstable. Thus, even if the system is in a state near equilibrium, if \( T > T_{cr} \), it will evolve out to the limit cycle.

The value of \( T_{cr} \) is found to be of order unity for virtually the whole range of astronomically interesting values of the other parameters. Thus, since cloud lifetimes are probably of the same order as cloud collision times, the bifurcation phenomenon is relevant to galactic cloud systems. Moreover, since \( T_{cr} \) is insensitive to the other parameters, it is not expected to be a singular or unusual phenomenon. Even more generally, the bifurcation is not restricted to the precise form of the Oort model terms of eq. (4), see Scalo and Struck-Marcell 1986.

If \( T \) is increased far enough beyond \( T_{cr} \), a second bifurcation occurs, this time to a double-looped limit cycle with two bursts of different amplitude per cycle. Indeed, further increases of \( T \) lead to a series of bifurcations and eventually to deterministic chaos in the phase space. In this limit the SFR vs. time looks essentially stochastic, with frequent bursts (Scalo and Struck-Marcell 1986).

This result leads us to consider the physical meaning of high \( T \). If the cloud lifetime consists essentially of the protostellar collapse time plus the main-sequence lifetime of massive stars, it should be more or less constant universally (at a given stellar metallicity). On the other hand, the equilibrium cloud collision time depends inversely on the cloud density and cross section, which turns out to be the product of the mean gas density of the cloud fluid and a slowly varying function. Thus, increased gas density implies increased \( T \) (\( T \propto \rho \) roughly). Note, however, that \( m \) and \( f_{SF} \) are highly nonlinear functions of \( T \). Thus, this is not a Schmidt law density dependence.

B. A Starburst Mechanism and Systematics

With the essentially mathematical questions of the generic existence and stability of the limit cycle bifurcation resolved, we can proceed to the
questions of the physical mechanism of the bifurcation and its consequences for galactic gas dynamics. First the mechanism: In Scalo and Struck-Marcell 1986 we attributed the bifurcation to a "coalescence overshoot", where, as a result of the relatively long cloud lifetime, a massive star-forming cloud can continue to grow for some time before suffering disruption. If the function $f_{SF}$ is sufficiently nonlinear ($s>2$), and the cloud mean mass is near the threshold value $m_{SF}$, this extra growth translates into a great deal more (and presumably more efficient) star formation, i.e. build-up to a burst. Roughly one delay time later the system suffers the consequences of these excesses - severe cloud disruption. The energy input is quickly dissipated in (possibly disruptive) collisions, then the system finds itself in a shredded, quiescent state, and begins to regrow through coalescence.

The key step is the overshoot, or excess cloud growth past the equilibrium mean mass (see Fig. 2). This step is similar to the triple-alpha nuclear reaction in that a third-body (or more for the clouds) collision occurs before the outcome of the original collision is resolved.

If we consider a series of closed box models with successively larger values of $T>T_{cr}$, we find that both the amplitude and the period of the limit cycle increase in the system phase space. This implies an increased starburst amplitude, and a longer time between bursts (up to 10 times the burst duration, see Fig. 3). The burst duration is of order $T$ at $T=T_{cr}$, i.e. of order a cloud collision time $= 3 \times 10^7$ yr. It varies quite slowly with $T$. It has been suggested (e.g. Loose et al. 1982) that starbursts are of short duration because the gas is blown out of the region (e.g. a galactic nucleus) by the resulting winds and supernovae. These one-zone models imply instead that the fundamental reason may be that the cloud system is simply broken down to a state where it is no longer capable of forming stars efficiently. The apparent low mass of the molecular clouds in the core of M82 (Knapp et al. 1980, Stark 1982, Olofsson and Rydbeck 1984), together with the possible polar outflow (Ungar et al. 1984), probably indicate that a combination of both processes is at work in that galaxy.

The density dependence of the bifurcation sequence, together with the notion of high gas consumption in bursts (see references in section I), implies another relaxation process, one which tends to drive the cloud system below the burst threshold. For example, consider the case of rapid gas infall into a galactic nucleus induced by a tidal encounter which boosts the gas density and time delay parameter above the value for the onset of chaos. With the system in the burst mode a large fraction of the time, gas consumption will be rapid, which in turn lowers the gas density. Figure 4 shows that a calculation in the limit cycle regime, and including gas consumption, yields rapid damping of burst amplitude. The gas consumption in these calculations is scaled to $f_{SF}$, and can range up to 30%. (Star formation is clearly very efficient.) Thus, galactic cloud systems are probably found in the chaotic regime only rarely (protogalaxies excepted?), and even successive limit cycle bursts are strongly damped. This calculation also indicates that the gas consumption timescale does not determine the duration of the bursts in the limit cycle or chaos regime (It is the breakdown of the clouds that turns off the burst even if there is still an ample gas supply.) This implies that apparent gas consumption timescales in galaxies are relatively meaningless if the galaxies burst.
Figure 3. Time series of the function $f_{\text{SF}}$, an indicator of the relative star formation rate, is shown (solid line) for six closed box calculations with increasing values of the parameter $T$ (from Struck-Marcell and Scalo 1986). Note the increasingly oscillatory response of the model, which leads eventually to the limit cycle bifurcation. Also shown (dashed lines) are the relative collisional coalescence-disruption rates.
Figure 4. Time series of the function $f_{{\text{SF}}}$, as in Fig. 3, but for a calculation with gas depletion included (from Struck-Marcell and Scalo 1986).

C. Other Consequences and Parameter Dependences

The model also yields several other observationally relevant consequences. First of all, even within the limit cycle-starburst regime, the model has a strong tendency to remain nearly isothermal. There is some observational evidence for this result (see Lewis 1984, Gallagher and Hunter 1984). Even under conditions that show large variations in $m$, $n$, and $f_{{\text{SF}}}$, the velocity dispersion varies only by a factor of about a few.

Secondly, we note that, in the same way that the model predicts starbursts for $T>T_{{\text{Cr}}}$, it also predicts bursts of energy dissipation in cloud collisions. These dissipation bursts might produce measurable shock emission, e.g. from H$_2$ and [OI]. Estimates are given in Struck-Marcell and Scalo 1986. An interesting complication is that in the models, the dissipation burst and star formation burst are out of phase.

Thirdly, there is commonly a delay of order several equilibrium cloud collision times between the occurrence of a disturbance in the cloud system and the starburst it triggers. Most disturbances act to break down the clouds initially and the delay is roughly equal to the regrowth timescale. This regrowth time depends on how severe the cloud breakdown is, and thus on the magnitude of the disturbance. Clearly, this effect can help account for the observation that many interacting galaxies do not have enhanced SFRs.
We conclude this discussion of the closed box model with a few comments on the role of parameters other than $T$. Apparently, the chief role of the parameter $m_{SF}/m_0$, is in determining the maximum value of the star formation function: $(f_{SF}/f_{SF}(m_0))_{\text{max}} = (m_{SF}/m_0)^8$. Since $m_{SF}/m_0$ ($m_0$ mean equilibrium cloud mass) is an essentially unknown quantity, the model cannot predict absolute star formation rate amplitudes. On the other hand, the parameter $c_0/c_{cr}$ plays the dominant role in determining variations in $c$. Small values of this parameter (i.e. when the equilibrium value $c$ is much less than the cloud collisional disruption threshold), yield nearly perfect isothermality. Finally, decreasing the dissipation efficiency parameter increases the value of $T_{cr}$ somewhat, and decreases $(f_{SF})_{\text{max}}$ for a given $T>T_{cr}$. This is not surprising since dissipation is an important part of the coalescence overshoot phenomenon.

IV. APPLICATIONS II: THE HYDRODYNAMICS OF RING GALAXIES

A. Why Study Rings?

The chief result of the preceding section was that above a critical value of the time delay parameter, i.e. above a threshold gas density, starbursts are a generic behavior of an Oort-type system. Taken together with the observational evidence that strong disturbances, e.g. strong density waves, induce starbursts, this result cries out for numerical modeling of the gas dynamics in interacting galaxies. In general, this modeling is a very formidable undertaking, although there have been several pioneering efforts (Theys and Spiegel 1977, Icke 1985, and Noguchi and Ishibashi 1986). Thus, in attempting numerical modeling with the Oort cloud fluid we have chosen to focus on what is probably the simplest case - ring galaxies.

Since ring galaxies are relatively rare objects, it is worthwhile to elaborate on the reasons why, among all types of interacting system, they are deserving of detailed study. These reasons include the following.

1. Relatively unambiguous observation comparisons.

Detailed studies of the optical morphology and kinematics of several ring systems have been carried out, including the Cartwheel galaxy (Fosbury and Hawarden 1977), the Lindsey-Shapley ring (Few, Madore and Arp 1982), and the Vela ring (Taylor and Atherton 1984). These studies revealed radially propagating rings of HII regions with companions at a distance of about one ring diameter. Thus, in these cases at least, the circumstantial evidence for a recent collision, and a resulting density wave (as in the Toomre 1978 models) is strong. The strength of the wave depends on the relative masses of the target and intruder galaxies, and, if the encounter is essentially impulsive, to a much lesser degree on orbital passage time.

The possibility of obtaining direct observational estimates of the density wave amplitude is clearly important, and is the first strong argument for studying rings. (Consider the long debate over the amplitude of density waves in spiral galaxies.)
2. Symmetry.

In a direct, head-on collision between a purely stellar intruder galaxy and a target system containing a cold disk, the formation of a cylindrically symmetric ring in the disk can be approximated as a one-dimensional problem. Indeed, even the examination of the evolution of an isolated fluid element, driven by a time-dependent external perturbation can be useful in the ring galaxy problem (see Appleton et al. 1985). In general, some symmetry remains even in the more likely case of an off-center collision. In this case the response of the disk can be treated as approximately two dimensional, i.e. in the plane of the disk. Multidimensional numerical hydrodynamic calculations require a great deal of computer time, so having relevant low-dimensional approximations, within which it is practical to vary parameters and perform many computational runs, is extremely helpful.

3. Range of perturbation amplitude.

Collisions with small companions, as well as with larger intruders are of interest, as is the variation of response with companion mass. From a practical point of view, small disturbances are usually easier to model stably and accurately, so it is a useful check of the stability of the numerical approximation to verify that model behavior changes continuously as the disturbance is increased. Thus, both numerical and astronomical considerations converge here.

4. Extended starbursts.

It appears, on the basis of IRAS infrared luminosity and color temperature $$(S_{100}/S_{60})$$, that many of the rings are undergoing moderately strong, and of course, highly extended starbursts (see Appleton and Struck-Marcell 1986a). The large extent of the star formation in rings is not only intrinsically interesting, but also of practical importance for providing a class of galaxy where any nuclear activity is generally separated from starburst activity. The spatial extent also makes the more nearby rings good candidates for near-IR, radio continuum, 21 cm, and molecular observations of spatial variations in star formation and cloud characteristics. Some work is already underway (e.g. Ghigo et al. 1986).

In summary, the ring galaxies appear to possess a number of unique advantages for facilitating the comparison between observation and theory of star formation in galaxies, as well as being especially amenable to numerical modeling.

B. Approximations for Numerical Modeling

We have begun a two-pronged effort to model ring galaxies using the Oort model cloud fluid. The first part of this program consists of one-dimensional numerical hydrodynamic modeling of cylindrically symmetric rings, using a Lagrangian (moving) grid, and explicitly calculating the time-delay effects. The second part of the program consists of two-dimensional hydrodynamic calculations of the formation and evolution of both symmetric and off-center rings. Both computer programs use the well-known Flux-Corrected Transport algorithm (see Book 1981); details will be published elsewhere.
In the relatively (and only relatively!) inexpensive one-dimensional calculations, it is possible to include the full details of the cloud interaction model (e.g. density-dependent time delay parameters calculated accurately at each grid point), and to do a large enough number of calculations to fairly sample parameters and initial conditions. The two-dimensional calculations are better suited to study kinematic and dynamic questions (e.g. angular momentum transport), but they become impractical without some simplifications in the cloud fluid terms. For example, we typically assume isothermality and use a very coarse calculation of the memory effects in the two-dimensional calculations. Since here we are primarily interested in star formation in the ring(s), most of the discussion below will be limited to results from representative one-dimensional calculations.

We have not yet coupled a realistic treatment of the stellar dynamics to the hydrodynamic calculations (nor included self-gravity in the gas), but instead, assume that the potential of the target galaxy is dominated by a softened point mass, with a softening length of typically 40 equilibrium cloud mean free paths (e.g. 13(\lambda/0.3 \text{ kpc.}) kpc.). This large softening length yields a rather flat rotation curve, over a large range in radius. The companion galaxy is assumed to be a gas-free, softened point mass (softening length of 1/4 - 1/2 that of the target disk), on a free-fall trajectory. In the one-dimensional calculations, angular momentum is assumed to be conserved in each Lagrangian fluid element (i.e. each discrete radial ring).

Finally, in all of the calculations reported below, the gas density was assumed to be constant across the disk initially, with a value well below the threshold for bursts.

C. Numerical Results in One-Dimension

Several of the most interesting numerical results are well-illustrated by considering, either individually or in comparison, two representative one-dimensional calculations. These calculations have initial conditions as described above, with a companion mass equal to 20% of the softened point mass in the target galaxy. The companion orbit is such that it falls through the center very rapidly, yielding a somewhat unrealistically impulsive disturbance, but in this case the details of the orbit are unimportant. The values of the cloud fluid parameters are as follows: \( c_0/c_{cr} = 0.924 \), the equilibrium velocity dispersion is quite near the disruption threshold; \( m_0/m_{SP} = 0.1 \), the equilibrium mean mass is well below the threshold for efficient star formation; the dissipation efficiency is of order unity (i.e. almost completely inelastic collisions); and \( r=s=4 \), implying steep thresholds in \( f_c \) and \( f_{SP} \). The only difference between the two calculations is that the initial, equilibrium value of the time delay parameter is 0.3 of the critical value in the first case, and 0.75 the critical value in the second. (For brevity we will refer to these as the 0.3 and 0.75 models.)

Figures 5 and 6 show the radial profiles of the mass density, star formation rate indicator \( f_{SP}/f_{SP}(m_0) \), and radial velocity at one representative time in the 0.3 and 0.75 models, respectively. Although we will not discuss detailed dynamical questions here (see Appleton and Struck-Marcell 1986b), we note several very basic features in these figures. First, the radial velocity profile shows infall outside of the ring, positive velocity within the ring,
Figure 5. Radial profiles of gas mass density (solid curve), star formation function $f_{\text{SF}}$ (dashed curve), and radial velocity (dotted curve) from a one-dimensional hydrodynamic ring galaxy calculation, with parameter $T=0.3 \, T_c$ initially, are shown at a representative time after the collision. All quantities are dimensionless, with $\rho$ and $f_{\text{SF}}$ normalized to their initial values, radius measured in units of equilibrium cloud mean free path, and radial velocity measured in free paths per mean collision time.

Figure 6. Radial profiles as in Fig. 5, but for a calculation with $T=0.75 \, T_c$ initially, i.e. with a gas density near to the critical value for the limit cycle-starburst bifurcation.
and, at late times, infall behind the ring into a second ring. Secondly, the
density profiles show that most of the density variations are limited to within
a factor of two of the initial value. This is one indication of the fact that
the ring propagates through the gas; it is not a shell.

These comments on the density structure also provide a starting place for
studying the star formation in the first or primary ring. It is clear from
Figure 5 that the enhancement of the SFR in the primary ring is also typically
no more than a factor of a few. A relatively small increase is expected on the
basis of the closed box results: since the density increase is about a factor of
two, the time delay parameter only increases through the density wave from
0.3 to about 0.6 of the critical value for the starburst bifurcation, so we
expect rapid damping to equilibrium following a disturbance.

The situation in Figure 6 is quite different. Shortly after its appear-
ce in this case, the primary ring exceeds the density enhancement factor of
4/3 needed to increase the time delay parameter beyond the critical value.
Interestingly, the ring does not burst immediately. The disturbance is still
modest, and we expect that, in the analogous closed box case, the system would
execute several cycles of a growing oscillation to evolve from near the (now
unstable) equilibrium out to the limit cycle. From the double-peaked structure
of the star formation profile at the intermediate time shown in Figure 6 we see
evidence of such an oscillation. At that time, it appears that a given fluid
element passes out of the overdensity part of the wave, into the rarefaction
zone where $T < T_{cr}$, before it can 'grow' a burst. The multiple peak structure is
interesting in its own right, since several rings seem to show such small-scale
filagree.

Later in the run the density profile steepens, making for a more unstable
cloud system and a stronger disturbance (see Struck-Marcell and Appleton in
preparation). The result is a starburst near the peak of the wave, with echo
bursts behind, each with decreasing amplitude as the density decreases. The
energy input from the bursts generates significant pressure, which begins to
effect the density wave profile.

Up to this point, the simple closed box model has proved to be a useful
tool for helping to understand the numerical hydrodynamic calculations, at least
qualitatively. At the same time, the numerical hydrodynamics reaffirms, in a
more general context, the existence of the starburst bifurcation, which was
discovered in the closed box. However, the hydrodynamic flows are fairly modest
at the times shown in Figures 5 and 6, at least in the primary ring.

Once the primary ring has propagated through a good fraction of the disk, a
second ring forms and begins to move outward. It is apparent in Figures 5 and 6
that the infall velocities generated in the rarefaction behind the primary ring
are larger than those in front of the primary. In this case the hydrodynamic
flow times can become comparable to cloud interaction timescales, and the closed
box analogue without external driving may no longer be very accurate (Appleton
and Struck-Marcell 1986b). Thus, the compression is larger in the second ring,
and the density enhancement soon exceeds that in the primary, leading to
strongly enhanced star formation or bursts.
It is an interesting question whether third, fourth, etc. rings can form. We do not have a definitive answer at this time, but we know that the radial oscillation period of an individual fluid element is roughly equal to its free-fall time, which decreases with radius. Thus, the oscillations of adjacent fluid elements (or stars) grow progressively out of phase. This dispersion, coupled with dissipation damping, and the additional incoherence due to the pressure waves generated by starbursts will cancel out the coherent radial waves fairly rapidly.

The models above provide information on the spatial distribution of relative SFR, which may be usefully compared to optical observations and radio continuum maps. However, if there is substantial obscuration, the optical observations may not reveal all of the star formation. On the other hand, many of the nearby rings were detected at 60 and 100 μm by IRAS, although with its large beam size IRAS can provide no spatial resolution. The IRAS observations show that the integrated FIR luminosity of the rings is typically 2-6 times that of normal galaxies (see Appleton and Struck-Marcell 1986b). Are the models consistent with this result? To provide a partial answer to this question we integrate the SFR over the disk and compare to the initial unperturbed disk. Of course, the result of this integration depends on the choice of the outer radius (and on the value of the parameter $m_{\text{SF}}/m_0$), so it can only provide an estimate.

The first result of this integration exercise is that if the ring bursts, the net SFR can in fact reach a value of a few to 10 times the initial value. Most of this star formation does originate in the ring(s). Moreover, the models imply that to get a strongly enhanced net SFR requires a burst in a ring to offset the suppression in rarefaction regions. Such bursts can only occur in the model if there is a finite time delay, i.e. only if the local gas densities are sufficiently large.

**D. Two-Dimensional Calculations**

In order to treat more realistically the propagation of the density wave within a differentially rotating disk galaxy we have performed somewhat simplified cloud fluid calculations in two-dimensions. We consider both centered and off-center collisions of the companion with the disk (see Appleton and Struck-Marcell 1986b for details). The principal difference between the one- and two-dimensional calculations is the transport of angular momentum within the ring which leads to more compression of the outer edge of the ring and to stronger rarefaction behind the ring. Interesting behavior of the cloud fluid is found when the ring compression timescale becomes comparable with the cloud collision time. Even in the case when the massive cloud lifetime is zero (instantaneous cloud recycling), the models show that significant differences can exist between the spatial distribution of newly formed stars and the amplitude to the density wave. The situation is even more interesting when the amplitude of the density wave varies with position around the ring, as in the off-center collisions. As an example we show in Figure 7 the star formation rate distribution resulting from an off-center collision of a 1/5 mass companion galaxy. Observations of SFRs around off-center ring galaxies will be an important test of the cloud fluid models.

In the future, we plan to include the full set of Oort cycle interactions in the two-dimensional calculations. Eventually, we also intend to incorporate
a better treatment of the stellar gravity, and more realistic modeling of cloud interactions (e.g. of the processes wind-driven fragmentation and magnetic dissipation) in the cloud fluid equations.

Figure 7. Contours of the star formation function $f_{\text{SF}}$ from a two-dimensional hydrodynamic calculation of an off-center collision. As in collisions along the symmetry axis, star formation is clearly concentrated in a ring. However, in this case the ring is noncircular, and the star formation varies strongly with angle around the ring. The base level of the contours is $f_{\text{SF}}/f_{\text{SF}}^0 = 0.5$, with increments of 1.0.

V. CONCLUSION: BEYOND RINGS

In conclusion, we recall that the arguments for studying ring galaxies were not only based on their intrinsic interest, but also in the hope that they might serve as a representative of many types of tidal interaction. This notion is a potentially rich vein that we have hardly begun to mine. There are some direct applications of course. Ring formation is probably the beginning of a small
impact parameter galaxy merger. The evolution from ring to completed merger could be explored by putting the companion in the ring calculations in a damping orbit, allowing multiple, nonimpulsive encounters. The elliptical rings produced in off-center collisions are first cousins to interaction-induced spiral waves. More specifically, we might hope that the results above on star formation in rings can be extended to arbitrary density waves.

The Oort model results suggest that, in general, the nature of the star formation and dynamics of a local cloud system will depend on which of several important timescales are commensurate. The first two of these timescales are the cloud collision time and the massive cloud lifetime. The former is basically the relaxation time of a dissipative cloud system, while the ratio of the latter to the former, $T$, provides a measure of the instability of the system. It was found in both the general applications, that above a critical value, $T_{cr}$, there is no longer a single stable equilibrium state, instead the system tends to 'relax' to an oscillatory attractor in phase space. A third timescale is the local dynamical or flow time, which is typically of the order as the local free-fall time. A fourth timescale, which is closely related to the third in many cases, is the global dynamical timescale, e.g. the time between perigalactic passages of a bound companion. If the local cloud environment is effected by strong disturbances, these timescales can be roughly commensurate with the cloud collision time, and the local cloud system can be forced far out of equilibrium. Even if the time delay parameter is small, so that the system is not unstable to oscillatory behavior, this nonequilibrium behavior can yield enhancements or suppressions of star formation, which do not necessarily correspond to the peaks and valleys in the gas density. If, however, all of the first three or all four of the timescales are comparable, then the system is driven relative to an 'equilibrium' that is inherently oscillatory. In this case, depending on the regularity of the driving forces relative to the natural system oscillation time, the dynamical evolution can appear quite stochastic.

Such behavior seems likely in strong interactions and mergers. In the burst regime of the model in general, and in the case of commensurate large-scale dynamical and cloud system timescales in particular, the dispersion in SFR, as a function of gas density for example, is large. Hopefully, these timescale considerations will be useful in interpreting observations, though the task will be complex.

Finally, these results suggest answers to some of the questions posed in the introduction, at least within the context of the Oort model. The mechanism of starbursts is the limit cycle bifurcation, or coalescence overshoot instability, which is a qualitatively different process than 'normal', equilibrium star formation. Unfortunately, the possibility of driven, non-equilibrium behavior superimposed on the limit cycle may confuse the application of this result to complex systems. However, a number of interesting comparisons between theory and observation should be possible in the simpler cases, like the ring galaxies.

The canonical description of the ring galaxies is that they are like dropping a pebble in a pond. For strongly interacting galaxies in general, the models suggest a better analogy might be to a storm at sea, with starbursts as the froth of a breaking wave.
REFERENCES


Is there a regime where the model breaks down due to energy input by stars formed in a major burst?

Struck-Marcell: It is possible that in a violent burst most of the clouds are broken down completely, and all the material distributed more uniformly in an intercloud phase (e.g. Ikeuchi et al. 1984, and Bodiford and de Loore 1985). In the Oort model discussed here, there is a breakdown of the system into a large number of very low mass clouds following a strong burst. The clouds regrow by coalescence on a long timescale. It is tempting to think that this is at least part of the explanation for the tiny molecular clouds in the core of M82.
Begelman: What is your prescription for injecting kinetic energy into the clouds? Are your results sensitive to the details of this prescription?

Struck-Marcell: At the end of its lifetime a massive cloud is supposed to break up into $N_c$ fragments, which fly off in random directions with a mean velocity dispersion $c_{B3}$. The generic behavior of the model is not sensitive to the precise value of any parameter except the ratio of the cloud lifetime to the collision time, $T$. However, the quantities $N_c$, $c_{B3}$ are not allowed to vary freely, but are tied to the equilibrium values of the model and the dissipation efficiency, since dissipation balances energy input in equilibrium.