CENTER FOR COMPUTER AIDED DESIGN

College of Engineering
The University of Iowa
Iowa City, Iowa  52242
Revised
Technical Report 86-2

DESIGN SENSITIVITY ANALYSIS USING EAL:
Part I: Conventional Design Parameters

by

B. Dopker
K.K. Choi
J. Lee

Center for Computer Aided Design
College of Engineering
The University of Iowa
Iowa City, IA 52242

Research Supported by NASA Grant NAG-1-215

January, 1986
Revised August 1986
ABSTRACT

A numerical implementation of design sensitivity analysis of built-up structures is presented, using the versatility and convenience of an existing finite element structural analysis code and its database management system. The finite element code used in the implementation presented is the Engineering Analysis Language (EAL), which is based on a hybrid method of analysis. It has been shown that design sensitivity computations can be carried out using the database management system of EAL, without writing a separate program and a separate database.

Conventional (sizing) design parameters such as cross-sectional area of beams or thickness of plates and plane elastic solid components are considered in this report. Compliance, displacement, and stress functionals are considered as performance criteria. The method presented in this paper is being extended to implement shape design sensitivity analysis using a domain method and a design component method. Results of shape design sensitivity analysis will be reported in Part II.
## TABLE OF CONTENTS

**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>iv</td>
<td></td>
</tr>
</tbody>
</table>

**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td></td>
</tr>
</tbody>
</table>

**CHAPTER**

I. **INTRODUCTION**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1.1. Purpose</td>
<td>1</td>
</tr>
<tr>
<td>1.2. Continuum Approach of Design Sensitivity Analysis</td>
<td>2</td>
</tr>
</tbody>
</table>

II. **DESIGN SENSITIVITY ANALYSIS METHOD**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2.1. Membranes</td>
<td>7</td>
</tr>
<tr>
<td>2.2. Bending of Plates</td>
<td>14</td>
</tr>
<tr>
<td>2.3. Beams</td>
<td>20</td>
</tr>
<tr>
<td>2.4. Built-Up Structures</td>
<td>26</td>
</tr>
<tr>
<td>2.5. Coupling of Bending and Membrane Effect</td>
<td>27</td>
</tr>
<tr>
<td>2.5.1. Formulation of Membrane Plus Bending</td>
<td>27</td>
</tr>
<tr>
<td>2.5.2. Numerical Examples</td>
<td>29</td>
</tr>
<tr>
<td>2.6. Design Sensitivity Analysis of Pointwise Stress Functional</td>
<td>32</td>
</tr>
<tr>
<td>2.6.1. Membranes</td>
<td>32</td>
</tr>
<tr>
<td>2.6.2. Plates</td>
<td>34</td>
</tr>
<tr>
<td>2.6.3. Numerical Examples</td>
<td>35</td>
</tr>
</tbody>
</table>

III. **PROGRAM ASPECTS**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>38</td>
</tr>
<tr>
<td>3.1. The EAL Database Management System</td>
<td>38</td>
</tr>
<tr>
<td>3.2. Program Organization</td>
<td>41</td>
</tr>
</tbody>
</table>

IV. **NUMERICAL EXAMPLES**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>47</td>
</tr>
<tr>
<td>4.1. Membranes</td>
<td>47</td>
</tr>
<tr>
<td>4.2. Bending of Plates</td>
<td>52</td>
</tr>
<tr>
<td>4.3. Beams</td>
<td>56</td>
</tr>
<tr>
<td>4.4. Built-Up Structures</td>
<td>59</td>
</tr>
</tbody>
</table>

V. **CONCLUSIONS**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>68</td>
</tr>
</tbody>
</table>

**REFERENCES**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70</td>
</tr>
</tbody>
</table>

**APPENDIX**

<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1. Design Sensitivity Vector</td>
<td>72</td>
</tr>
<tr>
<td>A2. Source Program</td>
<td>80</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.</td>
<td>Design Sensitivity Check for von Mises' Stress of Membrane plus Bending Plate with δt = 0.01t</td>
</tr>
<tr>
<td>2.</td>
<td>Design Sensitivity Check for Displacement of Membrane plus Bending Plate, δt = 0.05t</td>
</tr>
<tr>
<td>3.</td>
<td>Membrane Design Sensitivity Check for Pointwise Stress at the First Gauss Point, δt = 0.01t</td>
</tr>
<tr>
<td>4.</td>
<td>Plate Design Sensitivity Check for Pointwise Stress at the First Gauss Point, δt = 0.001t</td>
</tr>
<tr>
<td>5.</td>
<td>Membrane Design Sensitivity Check for Compliance</td>
</tr>
<tr>
<td>6.</td>
<td>Membrane Design Sensitivity Check for Displacements</td>
</tr>
<tr>
<td>7.</td>
<td>Membrane Design Sensitivity Check for Stress</td>
</tr>
<tr>
<td>8.</td>
<td>Plate Design Sensitivity Check for Compliance</td>
</tr>
<tr>
<td>9.</td>
<td>Plate Design Sensitivity Check for Displacement</td>
</tr>
<tr>
<td>10.</td>
<td>Plate Design Sensitivity Check for Stress</td>
</tr>
<tr>
<td>11.</td>
<td>Beam Design Sensitivity Check for Compliance</td>
</tr>
<tr>
<td>12.</td>
<td>Beam Design Sensitivity Check for Displacement</td>
</tr>
<tr>
<td>13.</td>
<td>Beam Design Sensitivity Check for Stress</td>
</tr>
<tr>
<td>15.</td>
<td>Built-Up Structure Design Sensitivity Check for Displacement</td>
</tr>
<tr>
<td>16.</td>
<td>Built-Up Structure Design Sensitivity Check for Plate Stress</td>
</tr>
<tr>
<td>17.</td>
<td>Built-Up Structure Design Sensitivity Check for Beam Stress</td>
</tr>
<tr>
<td>18.</td>
<td>Sensitivity Vectors for the Compliance Constraint</td>
</tr>
<tr>
<td>19.</td>
<td>Sensitivity Vectors for the Displacement Constraint</td>
</tr>
<tr>
<td>20.</td>
<td>Sensitivity Vectors for the Beam Stress Constraint</td>
</tr>
<tr>
<td>21.</td>
<td>Sensitivity Vectors for the Plate Stress Constraint</td>
</tr>
</tbody>
</table>

iv
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Flow Chart of Design Sensitivity Computation</td>
<td>6</td>
</tr>
<tr>
<td>2.2</td>
<td>Clamped Plane Elastic Solid of Variable Thickness</td>
<td>8</td>
</tr>
<tr>
<td>2.3</td>
<td>Clamped Plate of Variable Thickness</td>
<td>14</td>
</tr>
<tr>
<td>2.4</td>
<td>Beam of Variable Cross-Sectional Area</td>
<td>21</td>
</tr>
<tr>
<td>2.5</td>
<td>Clamped Plate of Variable Thickness</td>
<td>27</td>
</tr>
<tr>
<td>2.6</td>
<td>Membrane plus Bending Plate Finite Element Model</td>
<td>30</td>
</tr>
<tr>
<td>2.7</td>
<td>2-by-2 Gauss Points</td>
<td>35</td>
</tr>
<tr>
<td>3.1</td>
<td>Data Flow in EAL</td>
<td>39</td>
</tr>
<tr>
<td>3.2</td>
<td>Program Organization</td>
<td>43</td>
</tr>
<tr>
<td>4.1</td>
<td>Plane Elastic Solid Finite Element Model</td>
<td>48</td>
</tr>
<tr>
<td>4.2</td>
<td>Bending Plate Finite Element Model</td>
<td>53</td>
</tr>
<tr>
<td>4.3</td>
<td>Beam Finite Element Model</td>
<td>57</td>
</tr>
<tr>
<td>4.4</td>
<td>Built-Up Structure Finite Element Model</td>
<td>60</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

1.1. Purpose

To date there exist a wide variety of finite element structural analysis programs that are used as reliable tools for structural analysis. They give the designer pertinent information such as stresses, strains, and displacements of the structure being modeled. However, if this information reveals that the structure does not meet specified constraint requirements, the designer must make intuitive estimates on how to improve the design. If the structure is complex, it becomes very difficult to decide what step must be taken to improve the design. There is however, substantial literature [1] on the theory of design sensitivity analysis, which predicts the effect that structural design changes have on the performance of a structure.

The purpose of this work is to develop and implement structural design sensitivity analysis, using the adjoint variable method that takes advantage of the versatility and convenience of an existing finite element structural analysis code and its database management system and the theoretical foundation of structural design sensitivity analysis that is found in Ref. 1. The finite element code that will be used is the Engineering Analysis Language EAL [2].
Using the full capabilities of the EAL system, design sensitivities can be calculated within the program, without knowing the source code of the program. This has the advantage that the user deals with only one program, with only one database and no interfaces between different programs [3,4,5].

1.2. Continuum Approach of Design Sensitivity Analysis

A number of methods could be used to implement structural design sensitivity analysis, but the most powerful is the continuum approach [1]. This method can be implemented outside an existing finite element code [3,4,5], using only postprocessing data. This approach is convenient, because design sensitivity analysis software does not have to be embedded in an existing finite element code. The continuum approach to design sensitivity analysis calculation can also be implemented using a powerful database management system such as the Engineering Analysis Language (EAL). Using the database management system of EAL, only one database is necessary for computation of design sensitivity information. That is, it is not necessary to create interfaces and other datafiles to compute sensitivity information. Information on element shape functions used in the finite element model is, however, necessary for design sensitivity calculation.

The continuum approach to design sensitivity analysis can easily be extended to complex structural systems that have more than one structural component [6]. The design sensitivity vector is the derivative of a constraint functional with respect to design parameters. The magnitude of each component reflects how sensitive the
constraint functional is to a change in the associated design parameter. If a component of the vector is negative, the corresponding design parameter should be decreased to increase the value of the constraint functional. If a component of the vector is positive, the corresponding design parameter should be increased to increase the value of the constraint functional. In addition, if the magnitude of a component of the vector is large, then the corresponding design parameter will have a more substantial effect on design improvement.

When a designer uses a finite element structural analysis code in design of a structure, it is most likely that a number of program runs are necessary before a substantially improved design is obtained. With the aid of a design sensitivity vector, the designer will know what direction to take to improve the design most efficiently.
CHAPTER II
DESIGN SENSITIVITY ANALYSIS METHOD

A detailed treatment of methods and procedures for calculating design sensitivities are given in Ref. 1, for constraint functionals such as compliance, displacement, stress, and eigenvalues. For compliance and eigenvalue functionals, the adjoint equation is the same as the state equation, thus no adjoint equation needs to be solved. Each displacement and stress functional requires an adjoint load computation and an adjoint equation must be solved. Note that the state equation and the adjoint equation differ only in their load terms. Using the reload or multiload option of an existing finite element code, the adjoint equation can be solved efficiently [3]. For the displacement functional, the adjoint load is a unit point load acting at the point where the displacement constraint is imposed. For the stress functional the shape function of the finite element code is necessary to calculate equivalent nodal forces of the adjoint load.

The equivalent nodal force computation of the adjoint load for the stress functional can be based on different principles. To be consistent with EAL, a hybrid formulation should be used, which requires computation of the applied loads in terms of stress coefficients. This is impractical, because EAL is formulated in terms of nodal displacement coefficients. Another method, which is consistent with the hybrid formulation of EAL, is based on the modified Hellinger-Reissner principle [10,11]. Here the equivalent nodal forces are expressed in
terms of nodal displacement coefficients. To alleviate this difficulty, an equivalent nodal force computation is based on the formulation used in the displacement method of finite element analysis. That is, once the degrees of freedom (nodal displacements) of the element in EAL are known, a compatible displacement shape function that is defined on the same finite element and has the same degrees of freedom can be used for computation of equivalent nodal forces.

The flow chart of Fig. 2.1 shows the overall procedure. First, the model is defined by identifying the design variables, constraint functionals, finite element model, and loadings. In the next step, EAL is used to obtain structural response. With the structural response obtained, an adjoint load is calculated, external to EAL, using assumed displacement shape functions. The adjoint load is input to EAL, to obtain an adjoint response for each constraint using reload option. Using the original structural response and the adjoint response, design sensitivity information is computed for each constraint, by numerically integrating the design sensitivity expressions. The process is convenient, since it uses data that are available or easily computable in the database.

To give the basic idea of implementation of design sensitivity analysis and computation procedures, simple prototype structural components are investigated. Once design sensitivity analysis of structural components is completed, the design component method of Ref. 6 can be used for design sensitivity analysis of built-up structures. The procedures and equations necessary for analysis of
Figure 2.1. Flow Chart of Design Sensitivity Computation
structural components as membranes, plates, and beams are shown in the following sections.

2.1. Membranes

Consider a thin elastic clamped solid shown in Fig. 2.2, with thickness \( u = h(x) \) of the membrane as a design parameter where \( x = (x_1, x_2) \). The energy bilinear form and load linear form are [1],

\[
a_u(z, \overline{z}) = \iint_{\Omega} h(x) \sum_{i,j=1}^{2} \sigma_{ij}(z) \varepsilon_{ij}(\overline{z}) d\Omega \\
\text{(2.1.1)}
\]

and

\[
\xi_u(\overline{z}) = \iint_{\Omega} \sum_{i=1}^{2} F_i z_i^{1} d\Omega + \int_{\Gamma} \sum_{i=1}^{2} T_i z_i^{1} d\Gamma \\
\text{(2.1.2)}
\]

where \( z = [z_1, z_2]^T \) is the displacement field, \( F = [F_1, F_2]^T \) is the applied body force, \( T = [T_1, T_2]^T \) is the boundary traction, and \( \sigma_{ij}(z) \) and \( \varepsilon_{ij}(\overline{z}) \) are the stress and strain fields associated with the displacement \( z \) and the virtual displacement \( \overline{z} \), respectively. The variational state equation is [1]

\[
a_u(z, \overline{z}) = \xi_u(\overline{z}) \quad \text{(2.1.3)}
\]

for all kinematically admissible virtual displacements \( \overline{z} \).

First consider the functional that represents compliance of the structure,

\[
\Psi_1 = \iint_{\Omega} \sum_{i=1}^{2} F_i z_i^{1} d\Omega + \int_{\Gamma} \sum_{i=1}^{2} T_i z_i^{1} d\Gamma \\
\text{(2.1.4)}
\]

The first variation of Eq. 2.1.4 is

\[
\Psi'_1 = \iint_{\Omega} \sum_{i=1}^{2} F_i z_i^{1'} d\Omega + \int_{\Gamma} \sum_{i=1}^{2} T_i z_i^{1'} d\Gamma \\
\text{(2.1.5)}
\]
In order to eliminate the dependence on variation of the state variable in Eq. 2.1.5, it is necessary to define the adjoint equation as [1]

$$a_u(\lambda, \lambda) = \iint_\Omega \sum_{i=1}^{2} T^{i\lambda_i} d\Omega + \int_\Gamma \sum_{i=1}^{2} T^{i\lambda_i} d\Gamma$$

(2.1.6)

for all kinematically admissible virtual displacements $\lambda$. Since the right side of Eqs. 2.1.3 and 2.1.6 are identical if $\lambda = \bar{z}$, the adjoint equation does not need to be solved. Using the adjoint variable method, the design sensitivity is [1]

$$\psi_1 = -\int_\Omega \sum_{i,j=1}^{2} \sigma^{ij}(z) \varepsilon^{ij}(z) \delta h d\Omega$$

(2.1.7)

since $z = \lambda$ for the compliance functional.
To numerically integrate Eqs. 2.1.4 and 2.1.7, a two-by-two Gauss-point integration procedure is used. The equations then become

\[
\psi_k = \frac{N}{L} \sum_{\ell=1}^{4} \sum_{h=1}^{2} \left[ \frac{2}{L} (F^\ell(z^\ell)) W^\ell J^\ell \right]^k + \sum_{\ell=1}^{4} \sum_{h=1}^{2} \left[ \frac{2}{L} (T^\ell(z^\ell)) W^\ell J^\ell \right]^k
\]

and

\[
\psi'_1 = \frac{N}{L} \sum_{\ell=1}^{4} \sum_{1,j=1}^{2} \left[ -\frac{2}{L} \sigma^{ij}(z)^\ell (\varepsilon^{ij}(z)^\ell) W^\ell J^\ell \right]^k \delta_{h}^k
\]

respectively, where \( J \) is the Jacobian, \( N \) is the total number of elements, superscript \( \ell \) is the counter for the number of Gauss points, superscript \( k \) is the counter for the element number, \( W \) is the weighting constant for the \( \ell \)th Gauss point, and superscript \( h \) is the direction of the force and the displacement.

Since EAL gives only the boundary displacement shape function for the 4-noded membrane element E41, a bilinear shape function for displacement is adopted for integration in Eqs. 2.1.4 and 2.1.7. Using stress computation of membrane element E41 in EAL, stresses and strains can be expressed in matrix form as

\[
\begin{bmatrix} \sigma^{11}(z), \sigma^{22}(z), \sigma^{12}(z) \end{bmatrix}^T = [p][\beta]
\]

\[
\begin{bmatrix} \varepsilon^{11}(z), \varepsilon^{22}(z), \varepsilon^{12}(z) \end{bmatrix}^T = [E]^{-1}[\sigma]
\]

where

\[
[p] = \begin{bmatrix} 0 & 0 & 1 & x_2 & 0 \\ 0 & 1 & 0 & 0 & x_1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
\[ [\beta] = [b_1, b_2, b_3, b_4, b_5]^T \]  

(2.1.13)

\[ [E] = \frac{E}{(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1 - \nu}{2} \end{bmatrix} \]  

(2.1.14)

and \([\pi]\) is the position coordinate matrix, which determines points where the stresses are obtained, \([\beta]\) is the stress coefficient vector, and \([E]\) is the elasticity matrix for a plane stress problem [8,9].

Next consider the functional that represents the displacement \(z\) at a discrete point \(x\),

\[ \psi_2 = z(\hat{x}) = \iint_{\Omega} \delta(x - \hat{x})z(x) \, d\Omega \]  

(2.1.15)

where \(\delta(x)\) is the Dirac delta. The first variation of Eq. 2.1.15 is

\[ \psi'_2 = \iint_{\Omega} \delta'(x - \hat{x})z'(x) \, d\Omega \]  

(2.1.16)

The adjoint equation in this case is [1,3]

\[ a_u(\lambda, \bar{\lambda}) = \iint_{\Omega} \delta(x - \hat{x})\bar{\lambda}(x) \, d\Omega \]  

(2.1.17)

for all kinematically admissible displacements \(\bar{\lambda}\). This equation has a unique solution \(\lambda\), which is the displacement field due to a unit point load acting at a point \(\hat{x}\). Using the adjoint variable method, design sensitivity of the displacement functional is

\[ \psi'_2 = -\iint_{\Omega} \frac{2}{2} \sigma^{ij}(z) \varepsilon^{ij}(\lambda) \delta_{hd\Omega} \]  

(2.1.18)

where \(\lambda\) is the solution of Eq. 2.1.17.
Using the same numerical techniques applied in the compliance constraint case, Eq. 2.1.18 can be evaluated as

\[ \psi_2 = \sum_{k=1}^{N} \left[ \sum_{l=1}^{2} \sum_{i,j=1}^{4} (\sigma_{ij}(z))^l \{ \delta_{ij}(\lambda) \}^l \delta_{ij}^l \right] \delta_h^k \]  

(2.1.19)

where

\[ \{ \varepsilon(\lambda) \} = [E]^{-1} \{ \sigma(\lambda) \} \]

\[ = [E]^{-1} [p] \{ \theta \} \]  

(2.1.20)

Finally consider the general functional that represents a locally averaged stress on an element as

\[ \psi_3 = \iint_{\Omega} g(\sigma(z)) m_p d\Omega \]  

(2.1.21)

where \( m_p \) is a characteristic function, defined on a finite element \( \Omega_p \) as

\[ m_p = \begin{cases} \frac{1}{\iint_{\Omega_p} d\Omega} & x \in \Omega_p \\ 0 & x \notin \Omega_p \end{cases} \]  

(2.1.22)

and \( g \) is the stress function. The first variation of Eq. 2.1.21 is

\[ \psi'_3 = \iint_{\Omega} \sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma_{ij}}(z) \sigma_{ij}(z') [m_p d\Omega] \]  

(2.1.23)

Replacing the variation in state \( z' \) by a virtual displacement \( \lambda \), the adjoint equation is obtained as [1,3]

\[ a_u(\lambda, \lambda) = \iint_{\Omega} \sum_{i,j=1}^{2} \frac{\partial g}{\partial \sigma_{ij}}(z) \sigma_{ij}(\lambda) [m_p d\Omega] \]  

(2.1.24)

for all kinematically admissible displacements \( \lambda \). Equation 2.1.24 has a
unique solution for a displacement field $\lambda$. Using the adjoint variable method, design sensitivity of the stress functional is

$$
\psi^3 = \int_\Omega \left[ \sum_{i,j=1}^{2} \sigma^{ij}(z) \varepsilon^{ij}(\lambda) \right] \delta h d\Omega
$$

(2.1.25)

where $\lambda$ is the solution of Eq. 2.1.24.

Using the same numerical techniques applied in the compliance constraint case, Eqs. 2.1.21 and 2.1.25 become

$$
\psi^3 = \frac{4}{k} \sum_{k=1}^{N} \left[ g^k [\sigma^k(z)] m_p \lambda \right] W J ^k
$$

(2.1.26)

and

$$
\psi^3 = \sum_{k=1}^{N} \left[ \sum_{i,j=1}^{2} \sigma^{ij}(z) \varepsilon^{ij}(\lambda) W J ^k \right] \delta h ^k
$$

(2.1.27)

The right side of Eq. 2.1.24 can be evaluated as

$$
\frac{4}{k} \sum_{l=1}^{2} \sum_{i,j=1}^{2} \frac{\partial g^k}{\partial \sigma^{ij}} \sigma^{ij}(z) m_p \lambda W J ^k
$$

(2.1.28)

After the adjoint load of Eq. 2.1.17 is calculated, the adjoint displacement field and resulting adjoint stress and strain fields are evaluated. The adjoint strains are then used in evaluating Eq. 2.1.27 for sensitivity of the constraint functional.

If von Mises' stress criteria is selected in the constraint functional, then

$$
g = \left[ (\sigma^{11})^2 + (\sigma^{22})^2 + 3(\sigma^{12})^2 - \sigma^{11} \sigma^{22} \right]^{1/2}
$$

(2.1.29)

and
\[
\frac{\partial g}{\partial \sigma^{11}} = \frac{(2\sigma^{11} - \sigma^{22})}{2g} \\
\frac{\partial g}{\partial \sigma^{22}} = \frac{(2\sigma^{22} - \sigma^{11})}{2g} \\
\frac{\partial g}{\partial \sigma^{12}} = \frac{3\sigma^{12}}{g}
\]

which can be written in vector form as

\[
\left[ \frac{\partial g}{\partial \sigma^i} \right] = \left[ \frac{\partial g}{\partial \sigma^{11}}, \frac{\partial g}{\partial \sigma^{22}}, \frac{\partial g}{\partial \sigma^{12}} \right]^T
\]

(2.1.31)

The equivalent nodal force is computed, based on the modified Hellinger-Reissner principle. The stress can be written as

\[
\{\sigma\} = [p]\{\beta\}
\]

(2.1.32)

(see also Eq. 2.1.10). The stress coefficients \{\beta\} can be expressed in terms of the nodal displacement coefficients \{q\} as [7]

\[
\{\beta\} = [H^{-1}][T]\{q\}
\]

(2.1.33)

if \{\nabla \sigma\} = \{0\}, where

\[
\{\nabla \sigma\} = \left[ \frac{\partial \sigma^{11}}{\partial x_1}, \frac{\partial \sigma^{22}}{\partial x_1}, \frac{\partial \sigma^{12}}{\partial x_1} \right]^T \quad i=1,2
\]

(2.1.34)

\{\nabla \sigma\} can be written as

\[
\{\nabla \sigma\} = [\nabla p]\{\beta\}
\]

(2.1.35)

where

\[
[\nabla p] = \begin{bmatrix}
\frac{\partial}{\partial x_1}(0 & 0 & 1 & x_2 & 0) \\
\frac{\partial}{\partial y_2}(0 & 1 & 0 & 0 & x_1) \\
\frac{\partial}{\partial x_1}(1 & 0 & 0 & 0 & 0)
\end{bmatrix}
\]

\(i=1,2\)
Using Eq. 2.1.34, \([\nabla p]\) in Eq. 2.1.35 becomes zero. Note that, if \([\nabla p]\) is not zero, Eq. 2.1.33 is not valid. Using Eq. 2.1.33, Eq. 2.1.32 can be written as

\[
\{\sigma\} = [P][H^{-1}][T]\{q\} \quad (2.1.36)
\]

Using Eqs. 2.1.27, 2.1.31, and 2.1.36, the equivalent nodal forces \(\{F\}\) can be written as

\[
\{F\} = \sum_{l=1}^{4} \left[ \frac{\partial^2}{\partial x^l} \right] \{P\}[H^{-1}][T]\{W\} [J]^l \quad (2.1.37)
\]

which is valid only if \([\nabla p]\) is zero.

2.2. Bending of Plates

Consider the clamped plate in Fig. 2.3 of variable thickness 
\(u = t(x)\), with a distributed load \(f(x)\). For this design independent

Figure 2.3. Clamped Plate of Variable Thickness
loading, the energy bilinear form and the load linear form for the plate are given as [1]

\[ a_u(z, \overline{z}) = \iint_\Omega \overline{\delta(u)} \left[ z_{11} \overline{z}_{11} + z_{22} \overline{z}_{22} + \nu (z_{22} \overline{z}_{11} + z_{11} \overline{z}_{22}) \right] + 2(1 - \nu) z_{12} \overline{z}_{12} \, d\Omega \]  

(2.2.1)

and

\[ \ell(\overline{z}) = \iint_\Omega \overline{f} d\Omega \]  

(2.2.2)

where \( \overline{\delta(u)} = Et/[12(1 - \nu^2)] \) is flexural rigidity, \( E \) is Young's modulus, \( \nu \) is the Poisson's ratio, and \( f \) is externally applied pressure. The governing state equation is [1]

\[ a_u(z, \overline{z}) = \ell(\overline{z}) \]  

(2.2.3)

for all kinematically admissible displacements \( \overline{z} \).

First consider the functional that represents compliance of the structure,

\[ \psi_4 = \iint_\Omega \overline{f} z d\Omega \]  

(2.2.4)

The first variation of Eq. 2.2.4 is

\[ \psi'_4 = \iint_\Omega \overline{f} z' d\Omega \]  

(2.2.5)

The adjoint equation is defined as [1]

\[ a_u(\lambda, \overline{\lambda}) = \iint_\Omega \overline{f} \overline{\lambda} d\Omega \]  

(2.2.6)

for all kinematically admissible displacements \( \overline{\lambda} \). As in the membrane
case, the right sides of Eqs. 2.2.3 and 2.2.6 is identical, so \( \bar{z} = \bar{z} \).

Using the adjoint variable method, design sensitivity of the compliance functional is \([1,3,4]\)

\[
\psi' = \iint_{\Omega} \left\{ -Et^2 \left[ \frac{z_{11}^2}{2} + z_{22}^2 + 2\nu z_{11}z_{22} + 2(1 - \nu)z_{12}^2 \right] / 4(1 - \nu^2) \right\} \delta t d\Omega
\]

\[
= - \iint_{\Omega} \left\{ \sum_{i,j=1}^{2} \sigma_{ij}(z) \epsilon_{ij}(z) \right\} \delta t d\Omega \tag{2.2.7}
\]

where \( \sigma_{ij}(z) \) and \( \epsilon_{ij}(z) \) are stress and strain of the extreme fiber, given as

\[
\epsilon_{ij} = -\frac{t z_{1j}}{2}, \quad i,j = 1, 2 \tag{2.2.8}
\]

and

\[
\sigma^{11} = -\frac{Et}{2(1 - \nu^2)} (z_{11} + \nu z_{22})
\]

\[
\sigma^{22} = -\frac{Et}{2(1 - \nu^2)} (z_{22} + \nu z_{11}) \tag{2.2.9}
\]

\[
\sigma^{12} = -\frac{Et}{2(1 - \nu^2)} z_{12}
\]

To evaluate Eqs. 2.2.4 and 2.2.7 numerically, a two-by-two Gauss-point integration procedure is used. Equations 2.2.4 and 2.2.7 become

\[
\psi' = \sum_{k=1}^{N} \left\{ \sum_{l=1}^{4} \left[ f(z) \right]_{k} \right\} \delta t \frac{K}{J} \tag{2.2.10}
\]

and

\[
\psi' = -\sum_{k=1}^{N} \left[ \sum_{l=1}^{2} \left( \sigma_{ij}(z) \right)_{k} \epsilon_{ij}(z) \right] \frac{l}{K} \delta t \frac{1}{J} \tag{2.2.11}
\]

For the Gauss integration, \( \sigma_{ij}(z) \) and \( \epsilon_{ij}(z) \) are obtained at each Gauss point, using the same moment formulation as in EAL. The moment vector
[M] is defined as

\[ [M^{11}(z), M^{22}(z), M^{12}(z)]^T = [P][\beta] \]  \hspace{1cm} (2.2.12)

where

\[
[P] = \begin{bmatrix}
1 & 0 & 0 & x_2 & 0 & x_1 & 0 & 0 & 0 & x_1x_2 & 0 \\
0 & 1 & 0 & 0 & x_1 & 0 & x_2 & 0 & 0 & 0 & x_1x_2 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & x_2 & x_1 & 0 & 0
\end{bmatrix} \hspace{1cm} (2.2.13)
\]

\[ [\beta] = [b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}]^T \hspace{1cm} (2.2.14) \]

Thus,

\[ [\sigma^{11}(z), \sigma^{22}(z), \sigma^{12}(z)]^T = \frac{6}{t^2} [M^{11}(z), M^{22}(z), -M^{12}(z)]^T \hspace{1cm} (2.2.15) \]

\[ [\varepsilon^{11}(z), \varepsilon^{22}(z), \varepsilon^{12}(z)]^T = [E]^{-1} [\sigma^{11}(z), \sigma^{22}(z), \sigma^{12}(z)]^T \hspace{1cm} (2.2.16) \]

where \([E]^{-1}\) is given in Eq. 2.1.14.

Next consider the functional that represents displacement \(z\) at a discrete point \(x\),

\[ \psi_5 \equiv z(x) = \iint_\Omega \delta(x - \hat{x})z(x)d\Omega \hspace{1cm} (2.2.17) \]

The first variation of Eq. 2.2.17 is

\[ \psi'_5 = \iint_\Omega \delta(x - \hat{x})z'(x)d\Omega \hspace{1cm} (2.2.18) \]

The adjoint equation is defined as [1]

\[ a_u(\lambda, \overline{\lambda}) = \iint_\Omega \delta(x - \hat{x})\overline{\lambda}(x)d\Omega \hspace{1cm} (2.2.19) \]

for all kinematically admissible displacements \(\overline{\lambda}\). This equation has a
unique solution \( \lambda \), which is the plate displacement field due to a unit vertical load at point \( x \). Using the adjoint variable method [1], design sensitivity of the displacement functional is

\[
\psi'_s = \iiint_{\Omega} \left\{ -E \left[ z_{11} \lambda_{11} + z_{22} \lambda_{22} + \nu(z_{11} \lambda_{11} + z_{22} \lambda_{22}) + 2(1 - \nu)z_{12} \lambda_{12} \right]/4(1 - \nu^2) \right\} \delta t d\Omega
\]  

(2.2.20)

Using Eqs. 2.2.8 and 2.2.9, Eq. 2.2.20 can be rewritten as

\[
\psi'_s = -\iiint_{\Omega} \frac{2}{\varepsilon} \sigma^{ij}(z) \epsilon^{ij}(\lambda) \delta \varepsilon d\Omega
\]  

(2.2.21)

Using the same numerical integration as in the compliance case, Eq. 2.2.21 becomes

\[
\psi'_s = \sum_{k=1}^{N} \left\{ -\sum_{i,j=1}^{2} \left[ \sum_{k=1}^{2} (\sigma^{ij}(z)) \epsilon^{ij}(\lambda) \right] \right\} \delta \varepsilon d\Omega
\]  

(2.2.22)

where \( \epsilon^{ij}(\lambda) \) are strains obtained by applying the adjoint load. The procedure is the same as in Eqs. 2.2.12 thru 2.2.16.

Finally, consider the functional that represents a locally averaged stress at the extreme fiber of the plate,

\[
\psi'_6 = \iint_{\Omega} g(\sigma(z)) m_p d\Omega
\]  

(2.2.23)

where \( g(\sigma(z)) \) is chosen as the von Mises' stress criteria and \( m_p \) is a characteristic function on finite element \( \bar{\Omega}_p \), defined as

\[
m_p = \begin{cases} 
\frac{1}{\iiint_{\bar{\Omega}_p} d\Omega} & x \in \bar{\Omega}_p \\
0 & x \not\in \bar{\Omega}_p
\end{cases}
\]  

(2.2.24)
The first variation of Eq. 2.2.23 is
\[
\psi' = \int \int_{\Omega} \left[ - \frac{2}{\sigma^{ij}(z)} \sigma^{ij}(z') \right] m_p d\Omega
\]
(2.2.25)

The adjoint equation is defined as [1]
\[
a_u(\lambda, \overline{\lambda}) = \int \int_{\Omega} \left[ - \int \frac{2}{\sigma^{ij}(z)} \sigma^{ij}(\lambda) \right] d\Omega + \int \int \frac{\partial g}{\partial \lambda} m_p d\Omega
\]
(2.2.26)

for all kinematically admissible displacement \( \overline{\lambda} \). Using the adjoint variable method, design sensitivity of the stress functional is
\[
\psi' = \int \int_{\Omega} \left[ - \int \frac{2}{\sigma^{ij}(z)} \sigma^{ij}(\lambda) \right] d\Omega + \int \int \frac{\partial g}{\partial \lambda} m_p d\Omega
\]
(2.2.27)

where \( \lambda \) is the solution of Eq. 2.2.26. Using two-by-two Gauss-point integration, Eqs 2.2.23 and 2.2.27 can be expressed as
\[
\psi = \sum_{l=1}^{4} \left[ g(\sigma^l(z)) m_p \right] W^l J^l
\]
(2.2.28)

and
\[
\psi = \sum_{k=1}^{N} \sum_{l=1}^{4} \left[ - \int \frac{2}{\sigma^{ij}(z)} \sigma^{ij}(\lambda) \right] W^l J^l + \sum_{k=1}^{4} \left[ \frac{\partial g}{\partial \lambda} \right] W^l J^l \delta^l_k
\]
(2.2.29)

The right side of Eq. 2.2.26 can be written as
\[
\sum_{l=1}^{4} \left[ \int \int_{\Omega} \left[ - \frac{2}{\sigma^{ij}(z)} \sigma^{ij}(\lambda) \right] m_p W^l J^l
\]
(2.2.30)

For the von Mises' stress criteria, the last term on side right side of Eq. 2.2.29 becomes
\[
\sum_{l=1}^{4} \left[ 0.5 \left[ (\sigma^{11})^2 + (\sigma^{22})^2 + 3(\sigma^{12})^2 - \sigma^{11} \sigma^{22} \right] \right] W^l J^l m_p
\]
(2.2.31)
For the displacement method, the equivalent nodal force for the adjoint load can be computed in a consistent way, by using the displacement shape functions of the code and then using the same procedure as in Refs. 3, 4 and 5. For the hybrid method, Reissner’s principle cannot be used, because \([\nabla P]\) is not necessarily zero. To alleviate this difficulty, it is proposed to select an acceptable displacement shape function for the adjoint load calculation. That is, once degrees of freedom of the element in EAL are known, a compatible shape function that has the same degrees of freedom can be selected. The term \(\frac{\partial g}{\partial \tilde{\eta}(j)}\) is the same as for membrane elements (Eq. 2.1.31).

Stresses in a displacement formulation are given as [8, 9]

\[
\{\sigma\} = [E][B]q
\]  \hspace{1cm} (2.2.32)

Using Eqs. 2.1.31, 2.2.29, and 2.2.32, the equivalent nodal forces \([F]\) can be evaluated as

\[
[F] = \sum_{k=1}^{N} \left[ \frac{\partial g}{\partial \tilde{\eta}(j)} \right]^T [E][B]m_p \hat{w}_k^J \hat{J}^k
\]  \hspace{1cm} (2.2.33)

2.3. Beams

Consider a beam with variable width and height, as shown in Fig. 2.4. Width and height are the design parameters; i.e., \(u = [b(x_3), h(x_3)]^T\).

The energy bilinear form and the load linear form of the beam are

\[
\begin{align*}
\mathcal{A}_m(z, \overline{z}) &= \int_0^L \left[ E \left( bh \overline{w_3 \overline{v_3}} + \frac{hb^3}{12} \overline{(v_{33} \overline{v_{33}})} \right) + \frac{bh^3}{12} \overline{(u_{33} \overline{u_{33}})} \right] dx_3 \hspace{1cm} (2.3.1) \\
\mathcal{L}(z) &= \int_0^L F^T z dx_3 \hspace{1cm} (2.3.2)
\end{align*}
\]
where $F = [f_1, f_2, f_3]$ is the distributed load, $E$ is Young's modulus, $hb^3/12$ is the moment of inertia with respect to $x_1$-axis [2], $bh^3/12$ is the moment of inertia with respect to $x_2$-axis, $z = [\bar{u}, \bar{v}, \bar{w}]^T$ is the

![Figure 2.4. Beam of Variable Thickness and Width](image)

...displacement field, in $x_1$-, $x_2$-, and $x_3$-directions, respectively, $\bar{u}_{33}$ and $\bar{v}_{33}$ are curvatures of the displacement field, and $\bar{v}_{33}$ and $\bar{u}_{33}$ are curvatures of the virtual displacement field $\bar{z} = [\bar{u}, \bar{v}, \bar{w}]^T$.

In EAL [2], the beam element E21 is based on Timoshenko beam theory and includes torsion. The derivation here is based on technical beam theory without torsion. Thus, if the energy bilinear form of Eq. 2.3.1 is used, results of sensitivity analysis are applicable only if shear and torsion are negligible. However, it can be easily extended to include torsional effects.

Note that the load is applied in the positive direction of the coordinate axis. The state equation is [1]
\[ a_u(z, \overline{z}) = \mathcal{L}(\overline{z}) \quad (2.3.3) \]

for all kinematically admissible displacements \( \overline{z} \).

First consider the functional that represents compliance of the structure,

\[ \psi_7 = \int_0^L F^T z dx_3 \quad (2.3.4) \]

The first variation of Eq. 2.3.4 is

\[ \psi'_7 = \int_0^L F^T z' dx_3 \quad (2.3.5) \]

By replacing the variation of \( z' \) in Eq. 2.3.5 by a virtual displacement \( \lambda' \), an adjoint equation is defined as [1]

\[ a_u(\lambda, \overline{\lambda}) = \int_0^L F^T \overline{\lambda} dx \quad (2.3.6) \]

for all kinematically admissible displacements \( \overline{\lambda} \). Since Eq. 2.3.2 is identical to Eq. 2.3.6, \( \lambda = z \). Using the adjoint variable method of design sensitivity analysis,

\[ \psi_7 = - \int_0^L \left\{ Eh w^2 + \frac{3Eb^2}{12} v_3^2 + \frac{Eh^3}{12} u_3^2 \right\} \delta b dx_3 \]

\[ - \int_0^L \left\{ Eb w^2 + \frac{Eh^3}{12} v_3^2 + \frac{3Ebh^2}{12} u_3^2 \right\} \delta h dx_3 \quad (2.3.7) \]

A cubic displacement shape function is assumed for beam bending and a linear displacement shape function is assumed for axial displacement.

To numerically integrate Eqs. 2.3.4 and 2.3.7, a two-point Gaussian integration procedure is used. Equations 2.3.4 and 2.3.7 can be evaluated as
and

\[
\psi_\gamma = \sum_{k=1}^{N} \sum_{l=1}^{2} \left[ \frac{1}{\varepsilon} \int \psi_{l}^T z J W \right] \delta_{b^k}
\]  \hspace{1cm} (2.3.8)

Next, consider the functional that represents displacement \(z\) at a discrete point \(x_3\),

\[
\psi_8 \equiv z(x_3) = \int_{0}^{L} \delta(x_3 - x_3)z(x_3)dx_3
\]  \hspace{1cm} (2.3.10)

where \(\delta\) is the Dirac delta. The first variation of Eq. 2.3.10 is

\[
\psi_8' = \int_{0}^{L} \delta(x_3 - x_3)z'(x_3)dx_3
\]  \hspace{1cm} (2.3.11)

The adjoint equation is defined as [1]

\[
a(\lambda, \overline{\lambda}) = \int_{0}^{L} \delta(x_3 - x_3)\overline{\lambda}(x_3)dx_3
\]  \hspace{1cm} (2.3.12)

for all kinematically admissible displacements \(\overline{\lambda}\). Equation 2.3.12 has a unique solution \(\lambda\), where \(\lambda = [\lambda^u, \lambda^v, \lambda^w]^T\) is beam displacement due to a unit load acting at a point \(x_3\). Using the adjoint variable method the design sensitivity analysis is
Using the two-point Gaussian integration procedure, Eq. 2.3.13 becomes

$$
\psi_8 = - \int_0^l \left\{ E w_3 \lambda_3^w + \frac{E h b^2}{4} v_{33} \lambda_{33}^v + \frac{h^3}{12} u_{33} \lambda_{33}^u \right\} \delta b dx_3 \\
- \int_0^l \left\{ E b w_3 \lambda_3^w + \frac{E b^3}{12} v_{33} \lambda_{33}^v + \frac{E h^2 b}{4} u_{33} \lambda_{33}^u \right\} \delta h dx_3 \quad (2.3.13)
$$

Finally, consider the functional that represents extreme fiber stresses in the beam,

$$
\psi_9 = \int_0^l \left\{ w_3 - \text{sign}(b) \frac{b}{2} v_{33} - \text{sign}(h) \frac{h}{2} u_{33} \right\} m_p dx_3 \quad (2.3.15)
$$

where \( h/2 \) is the half-depth of the beam, \( b/2 \) is the half width of the beam, \( \text{sign}(b) \) and \( \text{sign}(h) \) are +1 or -1 and indicate at what extreme fiber the sensitivity is computed, and \( m_p \) is a characteristic function that is defined on a finite element \( dx_p \) as

$$
m_p = \begin{cases} 
\frac{1}{dx_p} & x \in dx_p \\
0 & x \notin dx_p 
\end{cases} \quad (2.3.16)
$$

The first variation of Eq 2.3.15 is
Replacing the first variation $z'$ in Eq. 2.3.17 by a virtual displacement $\tilde{\lambda} = [\tilde{\lambda}^u, \tilde{\lambda}^v, \tilde{\lambda}^w]^T$, the adjoint equation is defined as

$$a(\lambda, \tilde{\lambda}) = \int_{\Omega} \left[ E_{hw} \lambda^w_3 + \frac{E_b}{2} \lambda^v_{33} + \frac{E_h}{4} \lambda^3 \lambda^3_{u_33} \right] \delta b dx_3 - \int_{\Omega} \left[ \frac{E_b}{12} \lambda^v_{33} \right] \delta h dx_3$$

for all kinematically admissible displacements $\tilde{\lambda}$. Equation 2.3.18 has a unique solution for a displacement field $\lambda$. Using the adjoint variable method of design sensitivity analysis,

$$\psi_9 = - \int_{\Omega} \left[ E_{hw} \lambda^w_3 + \frac{E_b}{2} \lambda^v_{33} + \frac{E_h}{4} \lambda^3 \lambda^3_{u_33} \right] \delta b dx_3 - \int_{\Omega} \left[ \frac{E_b}{12} \lambda^v_{33} \right] \delta h dx_3$$

where $\lambda$ is the solution of Eq. 2.3.18. With the two-point Gauss integration procedure, integrals in Eqs. 2.3.15 and 2.3.19 are evaluated as

$$\psi_9 = \sum_{k=1}^{N} \sum_{l=1}^{N} \left[ \frac{2}{L} \left( E_{hw} \lambda^w_3 + \frac{E_b}{2} \lambda^v_{33} + \frac{E_h}{4} \lambda^3 \lambda^3_{u_33} \right) \frac{b}{L} v^l_{33} - \frac{b}{L} v^l_{33} \right] \delta b_k$$

$$\psi_9' = - \sum_{k=1}^{N} \sum_{l=1}^{N} \left[ \frac{2}{L} \left( E_{bw} \lambda^w_3 + \frac{E_b}{2} \lambda^v_{33} + \frac{E_h}{4} \lambda^3 \lambda^3_{u_33} \right) \frac{b}{L} v^l_{33} - \frac{b}{L} v^l_{33} \right] \delta h_k$$

$$- \sum_{k=1}^{N} \sum_{l=1}^{N} \left[ \frac{2}{L} \left( E_{hw} \lambda^w_3 + \frac{E_b}{2} \lambda^v_{33} + \frac{E_h}{4} \lambda^3 \lambda^3_{u_33} \right) \frac{b}{L} v^l_{33} - \frac{b}{L} v^l_{33} \right] \delta b_k$$
The right hand side of Eq. 2.3.18 can be written as

\[ \sum_{k=1}^{N} \sum_{l=1}^{m_p} \left\{ \sum_{n=1}^{2} \left[ \frac{E}{2} \lambda_{33}^{l} - \text{sign}(b) \frac{h}{2} \lambda_{33}^{l} \right] \frac{w}{2} \lambda_{33}^{l} - \text{sign}(b) \frac{h}{2} \lambda_{33}^{l} \right\} \lambda_{2}^{l} \lambda_{1}^{l} \right\} = (2.3.22)

### 2.4. Built-Up Structures

A general structure is a collection of structural components that are interconnected by kinematic constraints at their boundaries. Results stated in this section are from Refs. 1 and 6. The energy bilinear form of a general system, consisting of beams, membranes, and plates can be written as

\[ a_u(z, \overline{z}) = \sum [a_u(z, \overline{z})]^M + \sum [a_u(z, \overline{z})]^P + \sum [a_u(z, \overline{z})]^B \]  (2.4.1)

where \([a_u(z, \overline{z})]^M, [a_u(z, \overline{z})]^P,\) and \([a_u(z, \overline{z})]^B\) are given in Eqs. 2.1.1, 2.2.1, and 2.3.1, respectively. The load linear form of a general system can be written as

\[ l_u(\overline{z}) = \sum [l_u(\overline{z})]^M + \sum [l_u(\overline{z})]^P + \sum [l_u(\overline{z})]^B \]  (2.4.2)

where \([l_u(\overline{z})]^M, [l_u(\overline{z})]^P,\) and \([l_u(\overline{z})]^B\) are given in Eqs. 2.1.2, 2.2.2, and 2.3.2, respectively. The state equation is [1]

\[ a_u(z, \overline{z}) = l_u(\overline{z}) \]  (2.4.3)

for all kinematically admissible virtual displacements \(\overline{z}\). Since the energy bilinear and load linear forms of the state equation are just the sum of energy bilinear and load linear forms of each structural component, the design sensitivity equation of the system is a simple additive process [1,6]. The generalized design sensitivity of a built-up structure is
\[ \psi' = \psi'_b \delta b + \psi'_h \delta h + \psi'_t \delta t \]  

(2.4.4)

which is the sum of the sensitivities of each structural component, given in Sections 2.1, 2.2, and 2.3, respectively.

2.5. Coupling of Bending and Membrane Effect

2.5.1. Formulation of Membrane Plus Bending

A clamped plate combining bending and stretching is shown in Fig. 2.5 with the laterally distributed load \( f(x) \) and in-plane traction load \( T = [T^1, T^2]^T \).

Assuming that bending and stretching are decoupled, one can obtain the energy bilinear form by adding the plate and membrane energy bilinear forms as in the case of the built-up structure.

\[ a_u(z, \bar{z}) = [a_u(z, \bar{z})]^M + [a_u(z, \bar{z})]^P \]  

(2.5.1)

where \([a_u(z, \bar{z})]^M\) and \([a_u(z, \bar{z})]^P\) are given in Eqs. 2.1.1 and 2.2.1, respectively. The membrane thickness \( h(x) \) has to be replaced by \( t(x) \) in this case. Likewise, the load linear form can be expressed as

\[ \ell_u(\bar{z}) = [\ell_u(\bar{z})]^M + [\ell_u(\bar{z})]^P \]  

(2.5.2)
where \([\mathcal{L}_u(z)]^M\) and \([\mathcal{L}_u(z)]^P\) are given in Eqs. 2.1.2 and 2.2.2, respectively. The state equation is

\[ a_u(z,\bar{z}) = \mathcal{L}_u(z) \quad (2.5.3) \]

for all kinematically admissible virtual displacements \(\bar{z}\).

Consider the functional representing the allowable stresses in the middle plane due to stretching and on the surface due to bending as

\[ \psi_{10} = \psi_3 + \psi_6 \quad (2.5.4) \]

where \(\psi_3\) and \(\psi_6\) are given in Eqs. 2.1.21 and 2.2.23, respectively.

Forces and moments of the component are [8]

\[ \{N, M\} = \int_{-t/2}^{t/2} \begin{bmatrix} \sigma_M \\ \sigma_B \end{bmatrix} \, dx_3 \quad (2.5.5) \]

where \(\{\sigma_M\} = [E]\{\varepsilon_M\}\) are the membrane stresses and \(\{\sigma_B\} = x_3[E]\{\kappa\}\) are the bending stresses. The curvatures \(\{\kappa\}\) are

\[ \{\kappa\} = \begin{bmatrix} z_{11}^3 \\ z_{22}^3 \\ 2z_{12}^3 \end{bmatrix} \quad (2.5.6) \]

Membrane and bending stress resultants can be decoupled if the plate is symmetric with respect to the \(x_1-x_2\) plane. In EAL, moments and forces are related as

\[ \begin{bmatrix} N \\ M \end{bmatrix} = [C] \begin{bmatrix} \varepsilon_M \\ \kappa \end{bmatrix} = \begin{bmatrix} C_{mm} & C_{mb} \\ C_{bm} & C_{bb} \end{bmatrix} \begin{bmatrix} \varepsilon_M \\ \kappa \end{bmatrix} \quad (2.5.7) \]

where \([C]\) is the coupling coefficient matrix [2].
Design sensitivity analysis of the stress functional is chosen to illustrate the procedure. The sensitivity is the summation of membrane and plate sensitivity as

\[ \psi'_10 = \psi'_3 + \psi'_6 \]

\[ = \iint \left[ - \frac{2}{E} \sigma^{ij}(z)\varepsilon^{ij}(\lambda) \right] d\Omega + \iint \left[ - \frac{2}{G} \tau^{ij}(z)\tau^{ij}(\lambda) \right] d\Omega \]

\[ + \iint \frac{\partial}{\partial t} \left[ a(z) \right] M_p \delta t d\Omega \quad (2.5.8) \]

which are given in Eqs. 2.1.25 and 2.2.27. Membrane stresses and strains are expressed in Eqs. 2.1.10 and 2.1.11, respectively and bending stresses and strains on the surface are expressed in Eqs. 2.2.15 and 2.2.16, respectively.

For the design sensitivity of the displacement functional, Eq. 2.1.18 can be used for the in-plane displacements due to stretching and Eq. 2.2.22 for the displacement \( z_3 \) due to bending.

2.5.2. Numerical Examples

To demonstrate the numerical accuracy of the approach, a numerical example is tested. The finite element plate model is given in Fig. 2.6 which is restrained at one side and loaded with a distributed tensile load in the positive \( x_1 \) direction and nodal loads of 501b at nodes 21, 42, 63, 84 and 105 in the negative \( x_3 \) direction as shown in Fig. 2.6. It contains 80 E43 elements (EAL bending plus membrane element type), 105 nodal points, and 500 degrees of freedom with the design variable
thickness $u = t(x_1, x_2)$. The material properties are the same as the one in Section 4.1.

Design sensitivity results of the von Mises' stress functional are given in Table 1 for Membrane, Bending and Total sensitivities with the perturbation of 0.01t. Since the stress resultants can be decoupled, the membrane sensitivity is expected to be the same as that of the original membrane model given in Table 7. However slightly different numerical values are obtained because numerical difference can occur in the decomposition process of membrane plus bending stiffness matrix when the process is compared to the membrane stiffness matrix alone.

![Figure 2.6. Membrane plus Bending Plate Finite Element Model](image)

The same nodal points as in Table 6 are selected to check the accuracy of the design sensitivity of the displacement functional in the $x_1$, $x_2$ and negative $x_3$ directions. Design sensitivity predictions and finite differences, with $\delta t = 0.05t$ are given in Table 2.
Table 1. Design Sensitivity Check for von Mises' Stress of Membrane plus Bending Plate with $\delta t = 0.01t$

<table>
<thead>
<tr>
<th>El. No.</th>
<th>$\psi(t)$</th>
<th>$\psi(t+\Delta t)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 1</td>
<td>10053.27</td>
<td>9953.71</td>
<td>-99.56</td>
<td>-100.53</td>
<td>101.0</td>
</tr>
<tr>
<td>B</td>
<td>12459.37</td>
<td>12194.25</td>
<td>-265.12</td>
<td>-249.86</td>
<td>94.3</td>
</tr>
<tr>
<td>T</td>
<td>22512.63</td>
<td>22147.96</td>
<td>-364.67</td>
<td>-350.93</td>
<td>96.2</td>
</tr>
<tr>
<td>M 10</td>
<td>9995.58</td>
<td>9896.59</td>
<td>-98.99</td>
<td>-99.96</td>
<td>101.0</td>
</tr>
<tr>
<td>B</td>
<td>7897.13</td>
<td>7732.11</td>
<td>-165.01</td>
<td>-158.86</td>
<td>96.3</td>
</tr>
<tr>
<td>T</td>
<td>17892.71</td>
<td>17628.71</td>
<td>-264.00</td>
<td>-258.81</td>
<td>98.0</td>
</tr>
<tr>
<td>M 20</td>
<td>9999.86</td>
<td>9900.97</td>
<td>-98.89</td>
<td>-100.00</td>
<td>101.1</td>
</tr>
<tr>
<td>B</td>
<td>748.51</td>
<td>731.58</td>
<td>-16.93</td>
<td>-16.72</td>
<td>98.9</td>
</tr>
<tr>
<td>T</td>
<td>10748.38</td>
<td>10632.55</td>
<td>-115.82</td>
<td>-116.72</td>
<td>100.8</td>
</tr>
<tr>
<td>M 21</td>
<td>8570.37</td>
<td>8485.51</td>
<td>-84.86</td>
<td>-85.7</td>
<td>101.0</td>
</tr>
<tr>
<td>B</td>
<td>14344.51</td>
<td>14040.40</td>
<td>-304.05</td>
<td>-289.13</td>
<td>95.1</td>
</tr>
<tr>
<td>T</td>
<td>22914.89</td>
<td>22525.91</td>
<td>-388.89</td>
<td>-374.84</td>
<td>96.4</td>
</tr>
<tr>
<td>M 30</td>
<td>10019.71</td>
<td>9920.49</td>
<td>-99.21</td>
<td>-100.19</td>
<td>101.0</td>
</tr>
<tr>
<td>B</td>
<td>7727.56</td>
<td>7565.95</td>
<td>-161.61</td>
<td>-155.44</td>
<td>96.2</td>
</tr>
<tr>
<td>T</td>
<td>17747.27</td>
<td>17486.45</td>
<td>-260.82</td>
<td>-255.64</td>
<td>98.0</td>
</tr>
<tr>
<td>M 40</td>
<td>9999.20</td>
<td>9900.91</td>
<td>-98.30</td>
<td>-99.99</td>
<td>101.7</td>
</tr>
<tr>
<td>B</td>
<td>675.18</td>
<td>642.77</td>
<td>-14.41</td>
<td>-13.94</td>
<td>96.7</td>
</tr>
<tr>
<td>T</td>
<td>10657.08</td>
<td>10543.67</td>
<td>-113.41</td>
<td>-113.93</td>
<td>100.5</td>
</tr>
</tbody>
</table>

M: membrane, B: bending, and T: total
Table 2. Design Sensitivity Check for Displacement of Membrane plus Bending Plate, $\delta t = 0.05t$

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Dir.</th>
<th>$\psi(t)$</th>
<th>$\psi(t+\Delta t)$</th>
<th>$\Delta\psi$</th>
<th>$\psi'$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>$x_1$</td>
<td>6.631E-3</td>
<td>6.315E-3</td>
<td>-3.157E-4</td>
<td>-3.316E-4</td>
<td>105.0</td>
</tr>
<tr>
<td>63</td>
<td>$x_3$</td>
<td>-2.585E-1</td>
<td>-2.214E-1</td>
<td>3.710E-2</td>
<td>3.895E-2</td>
<td>105.0</td>
</tr>
<tr>
<td>74</td>
<td>$x_1$</td>
<td>3.297E-3</td>
<td>3.141E-3</td>
<td>-1.570E-4</td>
<td>-1.649E-4</td>
<td>105.0</td>
</tr>
<tr>
<td>74</td>
<td>$x_2$</td>
<td>-2.011E-4</td>
<td>-1.915E-4</td>
<td>0.961E-5</td>
<td>1.004E-5</td>
<td>105.0</td>
</tr>
<tr>
<td>74</td>
<td>$x_3$</td>
<td>-7.995E-2</td>
<td>-6.842E-2</td>
<td>1.153E-2</td>
<td>1.205E-2</td>
<td>104.5</td>
</tr>
<tr>
<td>105</td>
<td>$x_1$</td>
<td>6.631E-3</td>
<td>6.315E-3</td>
<td>-3.157E-4</td>
<td>-3.316E-4</td>
<td>105.0</td>
</tr>
<tr>
<td>105</td>
<td>$x_2$</td>
<td>-3.996E-4</td>
<td>-3.804E-4</td>
<td>1.914E-5</td>
<td>1.995E-5</td>
<td>104.2</td>
</tr>
<tr>
<td>105</td>
<td>$x_3$</td>
<td>-2.582E-1</td>
<td>-2.211E-1</td>
<td>3.707E-2</td>
<td>3.895E-2</td>
<td>105.1</td>
</tr>
</tbody>
</table>

Since the displacements can be decoupled in membrane plus bending elements, the sensitivities for $x_1$ and $x_2$ directions are the same as that of Table 6. The sensitivity for $x_2$ direction displacement at node 63 is not considered since the displacement is zero due to the symmetry of loads and structure.

2.6. Design Sensitivity Analysis of Pointwise Stress Functional

2.6.1. Membranes

Consider the general functional that represents a locally averaged stress on an element as in Eq. 2.1.21.

$$\psi = \iint_{\Omega} g(\sigma(z)) m_p \, d\Omega$$  \hspace{1cm} (2.6.1)
where \( m_p \) is defined on a finite element \( \Omega_p \) in Eq. 2.1.22. If we have a smooth problem so the stress is continuous, we can consider pointwise constraint. In this case, by letting the test area shrink to a point \( \hat{x} \), \( m_p \) becomes a Dirac delta measure. Thus

\[
\psi_{11} = g(\sigma(z(\hat{x})))
\]

\[
= \iint g(\sigma(z(\hat{x}))) \delta(x - \hat{x}) d\Omega
\]

The first variation of Eq. 2.6.2 is

\[
\psi'_{11} = \sum_{i,j=1}^{2} \frac{\partial g(z(\hat{x}))}{\partial \sigma_{ij}} \sigma_{ij}(z'(\hat{x}))
\]

\[
= \iint \sum_{i,j=1}^{2} \frac{\partial g(z(\hat{x}))}{\partial \sigma_{ij}} \sigma_{ij}(z'(\hat{x})) \delta(x - \hat{x}) d\Omega
\]

Replacing the variation in state \( z' \) by a virtual displacement \( \lambda \), the adjoint equation is obtained as

\[
a_u(\lambda, \lambda) = \iint \sum_{i,j=1}^{2} \frac{\partial g(z(\hat{x}))}{\partial \sigma_{ij}} \sigma_{ij}(\lambda) \delta(x - \hat{x}) d\Omega
\]

for all kinematically admissible virtual displacements \( \lambda \). Using the adjoint variable method, the design sensitivity of the pointwise stress functional is

\[
\psi_{11}' = -\iint \left[ \sum_{i,j=1}^{2} \frac{\partial \sigma_{ij}(z(\lambda))}{\partial \varepsilon_{ij}(\lambda))} \right] \delta \theta d\Omega
\]

which can be computed using information at Gauss points as

\[
\psi_{11}' = \sum_{k=1}^{N} \left[ \frac{4}{\ell} \left[ -\sum_{i,j=1}^{2} \left( \sigma_{ij}(z(\lambda)) \varepsilon_{ij}(\lambda)) \right] \delta \theta^k \right] \right] k \delta \theta^k
\]
where $N$ is the total number of elements, $i$ is the counter for the number of Gauss points, $k$ is the counter for the element number, $J$ is the Jacobian, and $W$ is the weighting constant for the $\ell$th Gauss point.

If von Mises' stress functional is selected as the constraint functional, the formulations for stresses are given in Section 2.1.

2.6.2 Plates

Consider the functional that represents a locally averaged stress on the surface of the plate as in Eq. 2.2.23. If we consider the pointwise stress, the stress functional and its first variation will become

$$\psi_{12} = g(\sigma(z(x)))$$

$$= \iint_{\Omega} g(\sigma(z)) \delta(x - \hat{x}) d\Omega$$

and

$$\psi'_{12} = \sum_{i,j=1}^{2} \frac{\partial g(z(x))}{\partial \sigma_{ij}} \sigma_{ij}(z'(\hat{x}))$$

$$= \iint_{\Omega} \sum_{i,j=1}^{2} \frac{\partial g(z(x))}{\partial \sigma_{ij}} \sigma_{ij}(z'(\hat{x})) \delta(x - \hat{x}) d\Omega$$

The adjoint equation can be defined as

$$a_u(\lambda, \overline{\lambda}) = \sum_{i,j=1}^{2} \frac{\partial g(z(x))}{\partial \sigma_{ij}} \sigma_{ij}(\overline{\lambda}(x))$$

$$= \iint_{\Omega} \sum_{i,j=1}^{2} \frac{\partial g(z(x))}{\partial \sigma_{ij}} \sigma_{ij}(\overline{\lambda}(x)) \delta(x - \hat{x}) d\Omega$$

Then the design sensitivity of the pointwise stress functional can be obtained using the adjoint variable method as
\[
\psi_{12} = \frac{1}{2} \left[ \sum_{i,j=1}^{2} \sigma^{ij}(z) \varepsilon^{ij}(\lambda) \right] \delta t \, d\Omega + \int_{\Omega} \frac{\partial \sigma}{\partial t} (\sigma(z)) \delta(x - \hat{x}) d\Omega ...
\]

which can be computed as

\[
\psi_{12} = \sum_{k=1}^{N} \left[ \sum_{l=1}^{4} \left( \sigma^{ij}(z) \right)^{l} \left( \varepsilon^{ij}(\lambda) \right)^{l} \right] w^{k} j^{l} + \sum_{i,j=1}^{2} \frac{\partial \sigma^{ij}(z)}{\partial t} \delta(x - \hat{x}) \right] k \delta t^{k}
\]

where \( N, \ell, \) and \( k \) are explained in Section 2.6.1. In case of von Mises' stress constraint, the formulations of stresses for plate are given in Section 2.2.

### 2.6.3. Numerical Examples

The examples given in Sections 4.1 and 4.2 are used to test the accuracy of design sensitivity of the pointwise stress constraint. For the pointwise stress sensitivity, the first Gauss point shown in Fig. 2.7 is selected as \( \hat{x} \).

![2-by-2 Gauss Points](image)

Figure 2.7. 2-by-2 Gauss Points

To check sensitivity of the stress constraint of Eq. 2.6.2, the equivalent nodal forces of the adjoint load are computed using right side of Eq. 2.6.4. Design sensitivity analysis results for the von Mises' pointwise stress are given in Table 3 for several finite elements with the perturbation of \( \delta t = 0.01t \).
Accuracy of the sensitivity analysis results of the pointwise stresses at the first Gauss point of elements are almost equal to that of averaged stresses except at the first element. The design sensitivity results compared to the finite difference approximations are excellent. Especially the sensitivity of pointwise stress of the first element is better than that of the averaged stress in Table 7.

The design sensitivities of the pointwise stress of plate problem are computed using Eq. 2.6.11. The adjoint load of Eq. 2.6.9 is used to get the equivalent nodal forces. Design sensitivity results of the von Mises' functional at the first Gauss point are given in Table 4 for several finite elements. The design perturbation for the finite difference computation is \( \delta t = 0.001t \). The sensitivity results of pointwise stress are not quite excellent when compared to Table 10. It is found that the bending stresses are not evenly distributed on the plate surface for a given finite element element. That is, values of stress at four Gauss points are rather different from the average value.

<table>
<thead>
<tr>
<th>El. No.</th>
<th>( \psi(t) )</th>
<th>( \psi(t+\Delta t) )</th>
<th>( \Delta \psi )</th>
<th>( \psi' )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10496.95</td>
<td>10392.99</td>
<td>-103.95</td>
<td>-104.97</td>
<td>101.0</td>
</tr>
<tr>
<td>10</td>
<td>9990.33</td>
<td>9891.39</td>
<td>-98.94</td>
<td>-99.90</td>
<td>101.0</td>
</tr>
<tr>
<td>20</td>
<td>9999.99</td>
<td>9900.96</td>
<td>-99.03</td>
<td>-100.0</td>
<td>101.0</td>
</tr>
<tr>
<td>21</td>
<td>8453.86</td>
<td>8370.15</td>
<td>-83.71</td>
<td>-84.54</td>
<td>101.0</td>
</tr>
<tr>
<td>30</td>
<td>10015.01</td>
<td>9915.84</td>
<td>-99.17</td>
<td>-100.15</td>
<td>101.0</td>
</tr>
<tr>
<td>40</td>
<td>9999.93</td>
<td>9900.94</td>
<td>-98.99</td>
<td>-99.99</td>
<td>101.0</td>
</tr>
</tbody>
</table>
Table 4. Plate Design Sensitivity Check for Pointwise Stress at the first Gauss point, $\delta t = 0.001t$

<table>
<thead>
<tr>
<th>El. No.</th>
<th>$\psi(t)$</th>
<th>$\psi(t+\Delta t)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.64</td>
<td>35.57</td>
<td>-0.071</td>
<td>-0.074</td>
<td>104.7</td>
</tr>
<tr>
<td>2</td>
<td>830.29</td>
<td>828.62</td>
<td>-1.661</td>
<td>-1.665</td>
<td>100.2</td>
</tr>
<tr>
<td>3</td>
<td>1841.86</td>
<td>1838.17</td>
<td>-3.691</td>
<td>-3.609</td>
<td>97.8</td>
</tr>
<tr>
<td>4</td>
<td>2716.80</td>
<td>2711.37</td>
<td>-5.432</td>
<td>-5.331</td>
<td>98.1</td>
</tr>
<tr>
<td>5</td>
<td>3252.14</td>
<td>3245.63</td>
<td>-6.510</td>
<td>-6.374</td>
<td>97.9</td>
</tr>
<tr>
<td>7</td>
<td>1157.15</td>
<td>1154.83</td>
<td>-2.314</td>
<td>-2.376</td>
<td>102.6</td>
</tr>
<tr>
<td>8</td>
<td>1273.81</td>
<td>1271.26</td>
<td>-2.553</td>
<td>-2.473</td>
<td>95.7</td>
</tr>
<tr>
<td>9</td>
<td>1169.60</td>
<td>1167.25</td>
<td>-2.341</td>
<td>-2.112</td>
<td>90.2</td>
</tr>
<tr>
<td>10</td>
<td>1038.10</td>
<td>1036.02</td>
<td>-2.078</td>
<td>-1.735</td>
<td>83.5</td>
</tr>
<tr>
<td>13</td>
<td>1310.30</td>
<td>1307.68</td>
<td>-2.620</td>
<td>-2.670</td>
<td>101.9</td>
</tr>
<tr>
<td>14</td>
<td>1151.13</td>
<td>1148.83</td>
<td>-2.307</td>
<td>-2.432</td>
<td>105.4</td>
</tr>
<tr>
<td>15</td>
<td>901.55</td>
<td>899.75</td>
<td>-1.805</td>
<td>-1.985</td>
<td>109.9</td>
</tr>
<tr>
<td>19</td>
<td>1402.82</td>
<td>1400.01</td>
<td>-2.813</td>
<td>-3.057</td>
<td>108.7</td>
</tr>
<tr>
<td>20</td>
<td>1533.27</td>
<td>1530.20</td>
<td>-3.069</td>
<td>-3.291</td>
<td>107.0</td>
</tr>
<tr>
<td>25</td>
<td>1883.25</td>
<td>1879.47</td>
<td>-3.77</td>
<td>-3.901</td>
<td>103.5</td>
</tr>
</tbody>
</table>
CHAPTER III
PROGRAMMING ASPECTS

So far analytical results and numerical algorithms for design sensitivity analysis have been stated. This Chapter outlines the basic organization of the EAL database management system and a structural design sensitivity analysis program that has been implemented using EAL.

3.1. The EAL Database Management System

Design sensitivity analysis of structural components and built-up structures can be implemented using a database management system. A survey of database management systems can be found in Ref. 12. EAL can be considered as a database management system [12], as well as a higher order programming language with advanced programming concepts [13].

EAL is a set of independent processors that communicate with each other through a random access database (Fig. 3.1). The database is manipulated according to user commands. The commands can be combined in a runstream, which can be stored in the database as a runstream dataset. Runstream datasets are driven by the Execution Control System (ECS), which also allows branching and looping within a runstream dataset. The ECS allows the user to call a runstream dataset from the database within a called runstream dataset. The ECS also initiates the execution of a new processor.
Figure 3.1  Data Flow in EAL
Besides the regular database, EAL has a set of registers, which are stored in core. The user can assign a name, a type code (real, integer, or alphanumeric), and a value to each register. The contents of a register can be manipulated by register action commands. The registers, together with the ECS, enable controlled branching to be performed in runstream datasets. Register action commands have a higher priority than other commands, so whenever a register action command is given, the regular command procedure will be interrupted and the register command will be executed first.

To manage the database, a set of database utility functions are provided. The database consists of one or more libraries, where each library contains a set of named datasets. The contents of each library are stored in a separate table, which for each dataset stores the name, data type, block size, number of columns, total number of words in a dataset, and location in the library. If many datasets are stored in one library, database overhead disk I/O can be very large. Database utility functions allow the user to change information in the table of contents, copy datasets from one library to another, and erase information in libraries.

To manipulate information within each library, EAL provides the Arithmetic Utility System (AUS). AUS allows a wide range of vector and matrix manipulations as well as storing new datasets that are specified by the user. Matrices can be stored as full matrices or in a sparse storage forum.
Besides the database management system, EAL contains a variety of processors common to finite element analysis programs. Although EAL's element library is limited, it contains one, two, and three dimensional elements. The program is able to solve static as well as buckling problems. The dynamic analysis part can handle eigenvalue and eigenvector solution and forced dynamic response analysis. Substructuring and graphical pre- and post-processing are also available. Because the global system matrices (stiffness, mass, and geometrical stiffness matrices) are usually very large and sparse, a sparse storage technique for hypermatrices is used for global matrices. Thus, the user does not have to worry about bandwidth (as in SAP IV [14]) or wave front length (as in ANSYS [5]). For the solution of a large system of equations, the global stiffness matrix is factored according to Ref. 15.

EAL also allows the user to write his own program and combine it with the EAL database system. Because the user can use Fortran callable data handling routines, it is relatively easy to create new processors for the EAL database management system [16]. Writing a separate processor for a specific task is time consuming, but the result will be a more efficient system than just using the registers and the arithmetic utility processor. No separate processor is written in this work, because the goal is to show the feasibility of combining the design sensitivity analysis method with the EAL database management system.

3.2. Program Organization

Using the Engineering Analysis Language EAL, a general purpose design sensitivity analysis program has been written. The program can
handle three types of constraint functionals; compliance, displacement, and stress. Three types of elements (element types E21, E41, and E42) can be used to model a structure and evaluate design sensitivities. A flow chart of the program is given in Figure 3.2. To use the program, the user sets the system control parameter, gives information about the design variables, specifies the constraints, sets up his finite element model. The finite element model is described in the runstream dataset INIT MODL 0 0, where all other information is given in the runstream dataset PARA SET 0 0. After that, the program automatically computes sensitivities for the given constraints and design variables. System control parameters are as follows:

- **NLST** - Number of load cases
- **LCAS** - Actual load case
- **NBTD** - Number of independent beam/truss design variables
- **NMDV** - Number of independent membrane design variables
- **NPDV** - Number of independent plate design variables
- **NDV** - Total number of design variables
- **DE21** - Number of beam/truss elements
- **DE41** - Number of membrane elements
- **DE42** - Number of plate elements
- **DETO** - Total number of elements

For simplification of the checking process only, the program allows only one control parameter per finite element. For beam elements, the
Figure 3.2 Program Organization
user can chose the height, the width, or a combination of both as independent design parameter. Weighting values for beam design parameter are introduced for selecting the height or width or any combination as design variables. The weighting values for beam design parameters are given in the table DESV VALU 0 0.

The relationship between elements and design parameters are defined in the lists given below. For each element, there is one entry in the appropriate table that gives the design variable group.

ED21 REL 0 0 Relationship for beam/truss elements
ED41 REL 0 0 Relationship for membrane elements
ED42 REL 0 0 Relationship for plate elements

The constraint control parameters are

CCOM - Compliance constraint
CDIS - Number of displacement constraints
CS21 - Number of stress constraints for element typ E21
CS41 - Number of stress constraints for element typ E41
CS42 - Number of stress constraints for element typ E42
CTOT - Total number of constraints

For every constraint group, with the exception of the compliance constraint, a table is required to describe the location of the constraint. For displacement constraints, two entries are needed for each constraint. The first entry is the node number on which the displacement sensitivity is evaluated and the second entry is the
direction of the displacement constraint. For each stress constraint group (E21, E41, and E42) a separate table is needed that gives the element numbers on which the stress constraint functionals are evaluated. The tables are

ST21 LIST 0 0 - Gives the constraints for beam/truss elements
ST41 LIST 0 0 - Gives the constraints for membrane elements
ST43 LIST 0 0 - Gives the constraints for plate elements

For the stress constraint in a beam element (E21), the maximal stress is at one of the four corners of the beam cross section. The program computes stresses at four corners, finds the corner with the largest absolute value of stress, and computes sensitivity of the maximum stress.

Additional parameters are;

IDGP=1 - Pointwise stress at Gauss point case
IDGP=0 - Averaged stress case

(1) For E43 elements,
DE43  - Number of E43 plate elements
ED43 REL 0 0 Relationship for E43 plate elements
CS43  - Number of stress constraints for element type E43

For results,
CIND has an additional number between 60,000 and 70,000 indicating the stress constraint of E43 element.

(2) For pointwise stress constraint,
GPNO LIST 0 0 Indicates the Gauss point at which pointwise stress will be evaluated
For results,

DVAL E42 1 1 The value of pointwise stress constraint functional at Gauss point listed in input data set of E42

DVAL E41 1 1 The value of pointwise stress constraint functional at Gauss point listed in input data set of E41
CHAPTER IV
NUMERICAL EXAMPLES

In order to check whether the design sensitivity information obtained is accurate, a comparison is made with the finite difference $\Delta \psi$. An appropriate design perturbation $\Delta u$ must be selected, in order to obtain a meaningful finite difference of the constraint functional. That is, if $\Delta u$ is too small, $\Delta \psi = \psi(u + \Delta u) - \psi(u)$ may be inaccurate, due to loss of significant digits in the difference. On the other hand, if $\Delta u$ is too large, $\Delta \psi$ will contain nonlinear terms and the comparison with $\psi'$ will be meaningless. The design sensitivity $\psi'$ of a constraint functional is the scalar product of the design sensitivity vector $\frac{\partial \psi}{\partial u}$ and the design variable perturbation vector $\delta u$. That is, the perturbation of an element design parameter is multiplied by the corresponding sensitivity component and the sum of all products is the design sensitivity $\psi'$.

4.1. Membrane

The finite element membrane model in Fig. 4.1 is a simple plane elastic solid that is restrained at one end and loaded with a distributed tensile load at the other end. It contains 80 isoparametric elements (EAL plane stress element, type E41), 105 nodal points, and 200 degrees of freedom, with the design variable thickness $u = h(x)$. Young's modulus and Poisson's ratio are given as $E = 3 \times 10^7$ psi and $\nu = 0.3$, respectively. Each finite element has uniform thickness,
Figure 4.1 Plane Elastic Solid Finite Element Mode
so that a maximum possible number of design variables is 80. For simplicity, a uniform thickness of \( h = 0.5 \text{ in.} \) is used for the sensitivity check.

Compliance sensitivity results are shown in Table 5, where

\[
\Delta \psi = \psi(h + \Delta h) - \psi(h) \quad \text{and} \quad \psi' \text{ is the predicted value computed from Eq. 2.1.9, with design perturbations of} \ \delta h = 0.01h \ \text{and} \ \delta h = 0.05h. \quad \text{The percent accuracy of the sensitivity prediction is computed using the ratio} \ \psi' \times 100/\Delta \psi.
\]

Table 5. Membrane Design Sensitivity Check for Compliance

<table>
<thead>
<tr>
<th>( \delta h )</th>
<th>( \psi(h) )</th>
<th>( \psi(h+\Delta h) )</th>
<th>( \Delta \psi )</th>
<th>( \psi' )</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01h</td>
<td>265.24</td>
<td>262.62</td>
<td>-2.63</td>
<td>-2.65</td>
<td>101.0%</td>
</tr>
<tr>
<td>0.05h</td>
<td>265.24</td>
<td>252.62</td>
<td>-12.63</td>
<td>-13.26</td>
<td>105.0%</td>
</tr>
</tbody>
</table>

Several discrete nodal points shown in Fig. 4.1 are selected to check accuracy of design sensitivity of the displacement functional of Eq. 2.1.19. In order to compute this equation, the adjoint strain \( \varepsilon_{ij}^a(\lambda) \) due to the adjoint load is needed. For each direction of displacement on a node, there is a separate sensitivity calculation that produces an adjoint strain \( \varepsilon_{ij}^a(\lambda) \). Design sensitivity predictions and differences, with \( \delta h = 0.05h \), are given in Table 6.
Table 6. Membrane Design Sensitivity Check for Displacement

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Dir.</th>
<th>$\psi(h)$</th>
<th>$\psi(h+\Delta h)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>74</td>
<td>X</td>
<td>3.297E-3</td>
<td>3.141E-3</td>
<td>-1.570E-4</td>
<td>-1.649E-4</td>
<td>105.0</td>
</tr>
<tr>
<td>74</td>
<td>Y</td>
<td>-2.011E-4</td>
<td>-1.915E-4</td>
<td>0.961E-5</td>
<td>1.004E-5</td>
<td>105.0</td>
</tr>
<tr>
<td>105</td>
<td>X</td>
<td>6.631E-3</td>
<td>6.315E-3</td>
<td>-3.157E-4</td>
<td>-3.315E-4</td>
<td>105.0</td>
</tr>
<tr>
<td>105</td>
<td>Y</td>
<td>-3.996E-4</td>
<td>-3.804E-4</td>
<td>1.914E-5</td>
<td>1.995E-5</td>
<td>104.2</td>
</tr>
<tr>
<td>63</td>
<td>X</td>
<td>6.631E-3</td>
<td>6.315E-3</td>
<td>-3.157E-4</td>
<td>-3.316E-4</td>
<td>105.0</td>
</tr>
</tbody>
</table>

To check the stress constraint sensitivity of Eq. 2.1.27, the equivalent nodal force of the adjoint load on the right of Eq. 2.1.37 is computed for the finite element adjoint analysis and $e^{ij}(\lambda)$ is obtained for each constrained element. Design sensitivity results for von Mises' stress functionals are given in Table 7, for several finite elements. Perturbations are $\delta h = 0.01h$ and $\delta h = 0.05h$, for the von Mises' stress criteria.

With all three constraint functionals, design sensitivity results compared to the finite difference approximation are excellent. It is interesting to note that in Tables 5, 6, and 7, the finite difference approximation is nearly 1% of the constraint functional when $\delta h = 0.01h$ and nearly 5% of the constraint functional when $\delta h = 0.05h$. These results also show that as $\delta h$ approaches zero, the ratio $\psi'/\Delta \psi$ approaches one.
Table 7. Membrane Design Sensitivity Check for Stress
(a) von Mises' Stress with $\delta h = 0.01h$

<table>
<thead>
<tr>
<th>El. No.</th>
<th>$\psi(h)$</th>
<th>$\psi(h+\Delta h)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10053.402</td>
<td>9953.863</td>
<td>-99.539</td>
<td>-106.804</td>
<td>107.3</td>
</tr>
<tr>
<td>10</td>
<td>9995.646</td>
<td>9896.680</td>
<td>-98.966</td>
<td>-99.957</td>
<td>101.0</td>
</tr>
<tr>
<td>20</td>
<td>10000.141</td>
<td>9901.130</td>
<td>-99.011</td>
<td>-99.999</td>
<td>101.0</td>
</tr>
<tr>
<td>21</td>
<td>8570.358</td>
<td>8485.503</td>
<td>-84.855</td>
<td>-86.209</td>
<td>101.0</td>
</tr>
<tr>
<td>30</td>
<td>10019.743</td>
<td>9920.537</td>
<td>-99.206</td>
<td>-100.197</td>
<td>101.0</td>
</tr>
<tr>
<td>40</td>
<td>10000.065</td>
<td>9901.054</td>
<td>-99.011</td>
<td>-99.996</td>
<td>100.9</td>
</tr>
</tbody>
</table>

(b) von Mises' Stress with $\delta h = 0.05h$

<table>
<thead>
<tr>
<th>El. No.</th>
<th>$\psi(h)$</th>
<th>$\psi(h+\Delta h)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10053.402</td>
<td>9574.668</td>
<td>-478.734</td>
<td>-534.021</td>
<td>111.5</td>
</tr>
<tr>
<td>10</td>
<td>9995.646</td>
<td>9519.663</td>
<td>-475.983</td>
<td>-499.958</td>
<td>105.0</td>
</tr>
<tr>
<td>20</td>
<td>10000.141</td>
<td>9523.944</td>
<td>-476.147</td>
<td>-499.996</td>
<td>105.0</td>
</tr>
<tr>
<td>21</td>
<td>8570.358</td>
<td>8162.246</td>
<td>-408.112</td>
<td>-431.046</td>
<td>105.6</td>
</tr>
<tr>
<td>30</td>
<td>10019.743</td>
<td>9542.612</td>
<td>-477.131</td>
<td>-500.983</td>
<td>105.0</td>
</tr>
<tr>
<td>40</td>
<td>10000.065</td>
<td>9523.871</td>
<td>-476.194</td>
<td>-499.982</td>
<td>105.0</td>
</tr>
</tbody>
</table>
4.2 Bending of Plates

The clamped plate element model shown in Fig. 4.2 is uniformly loaded with a pressure $f(x) = -1.5 \text{ lb/in}$ in the $z$ direction. Since the model is symmetric with respect to the center and symmetric boundary conditions are applied, only one quarter of the plate is analyzed. The quarter model contains 25 4-node quadrilateral thin plate elements of type E42. It has 36 nodal points and 85 degrees of freedom.

The design variable is the plate thickness $u = t(x)$. Young's Modulus and Poisson's ratio are $E = 30.5 \times 10^7 \text{ psi}$ and $\nu = 0.3$, respectively. The constant plate thickness is $t = 0.4 \text{ in}$. Self-weight of the plate is neglected.

Compliance sensitivity results are shown in Table 8, where

$\Delta \psi \psi(t + \Delta t) - \psi(t)$ and $\psi'$ is the predicted value that is computed from Eq. 2.2.11, with design perturbations $\delta_t = 0.01t$ and $\delta_t = 0.05t$. The 1% design perturbation for the compliance constraint functional gives good correlation between design sensitivity and the finite difference approximation. However the 5% perturbation shows nonlinearity in the compliance of the plate element.

Table 8. Plate Design Sensitivity Check for Compliance

<table>
<thead>
<tr>
<th>$\Delta t$</th>
<th>$\psi(t)$</th>
<th>$\psi(t+\Delta t)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01t</td>
<td>5.0016</td>
<td>4.8545</td>
<td>-0.1471</td>
<td>-0.1501</td>
<td>102.0</td>
</tr>
<tr>
<td>0.05t</td>
<td>5.0016</td>
<td>4.3205</td>
<td>-0.6811</td>
<td>-0.7502</td>
<td>110.2</td>
</tr>
</tbody>
</table>
Figure 4.2 Bending Plate Finite Element Model
Several discrete points in Fig. 4.2 are selected to check design sensitivity accuracy for the displacement functional in Eq. 2.2.22. In order to compute this equation, as in the membrane case, the adjoint strain $\varepsilon_{ij}^a(\lambda)$ due to the adjoint load is needed. Some displacement results are shown in Table 9 for a design perturbation of $\Delta t = 0.01t$. The design sensitivity results of Table 9 agree very well with the finite difference approximation.

Table 9. Plate Design Sensitivity Check for Displacement

<table>
<thead>
<tr>
<th>Node No.</th>
<th>$\psi(t)$</th>
<th>$\psi(t+\Delta t)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.9287E-3</td>
<td>1.8720E-3</td>
<td>-5.6720E-5</td>
<td>-5.7861E-5</td>
<td>102.0</td>
</tr>
<tr>
<td>10</td>
<td>3.0860E-3</td>
<td>2.9952E-3</td>
<td>-9.0760E-5</td>
<td>-9.2579E-5</td>
<td>102.0</td>
</tr>
<tr>
<td>14</td>
<td>1.9287E-3</td>
<td>1.8720E-3</td>
<td>-5.6720E-5</td>
<td>-5.7861E-5</td>
<td>102.0</td>
</tr>
<tr>
<td>17</td>
<td>1.1349E-2</td>
<td>1.1015E-2</td>
<td>-3.3380E-4</td>
<td>-3.4046E-4</td>
<td>102.0</td>
</tr>
<tr>
<td>20</td>
<td>3.0860E-3</td>
<td>2.9952E-3</td>
<td>-9.0760E-5</td>
<td>-9.2579E-5</td>
<td>102.0</td>
</tr>
<tr>
<td>23</td>
<td>1.8585E-2</td>
<td>1.8038E-2</td>
<td>-5.4660E-4</td>
<td>-5.5753E-4</td>
<td>102.0</td>
</tr>
<tr>
<td>27</td>
<td>1.1349E-2</td>
<td>1.1015E-2</td>
<td>-3.3380E-4</td>
<td>-3.4046E-4</td>
<td>102.0</td>
</tr>
<tr>
<td>28</td>
<td>1.8585E-2</td>
<td>1.8038E-2</td>
<td>-5.4660E-4</td>
<td>-5.5753E-4</td>
<td>102.0</td>
</tr>
<tr>
<td>32</td>
<td>4.1105E-3</td>
<td>3.9896E-3</td>
<td>-1.2088E-4</td>
<td>-1.2331E-4</td>
<td>102.0</td>
</tr>
<tr>
<td>35</td>
<td>2.5256E-2</td>
<td>2.4514E-2</td>
<td>-7.4280E-4</td>
<td>-7.5767E-4</td>
<td>102.0</td>
</tr>
<tr>
<td>36</td>
<td>2.7124E-2</td>
<td>2.6327E-2</td>
<td>-7.9780E-4</td>
<td>-8.1372E-4</td>
<td>102.0</td>
</tr>
</tbody>
</table>
To check sensitivity of the constraint functional of Eq. 2.2.29, the equivalent nodal forces of the adjoint load are computed with Eq. 2.2.33. Design sensitivity results for the von Mises' stress functional are given in Table 10, for several different elements. The perturbation for the finite difference calculation is $\delta t = 0.001t$. Note that even though the equivalent nodal force calculation for the adjoint load is not consistent with the hybrid method, since a displacement shape function is used, design sensitivity accuracy is excellent.

Table 10. Design Sensitivity Check for Stress

<table>
<thead>
<tr>
<th>El. No.</th>
<th>$\psi(t)$</th>
<th>$\psi(t+\Delta t)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>451.13</td>
<td>450.23</td>
<td>-0.90</td>
<td>-0.889</td>
<td>98.7</td>
</tr>
<tr>
<td>2</td>
<td>1128.39</td>
<td>1126.14</td>
<td>-2.25</td>
<td>-2.185</td>
<td>97.1</td>
</tr>
<tr>
<td>3</td>
<td>1762.13</td>
<td>1758.61</td>
<td>-3.52</td>
<td>-3.476</td>
<td>98.7</td>
</tr>
<tr>
<td>4</td>
<td>2268.37</td>
<td>2263.84</td>
<td>-4.53</td>
<td>-4.518</td>
<td>99.7</td>
</tr>
<tr>
<td>5</td>
<td>2549.64</td>
<td>2544.54</td>
<td>-5.10</td>
<td>-5.097</td>
<td>99.9</td>
</tr>
<tr>
<td>7</td>
<td>1253.26</td>
<td>1250.76</td>
<td>-2.50</td>
<td>-2.543</td>
<td>101.7</td>
</tr>
<tr>
<td>8</td>
<td>1248.91</td>
<td>1246.42</td>
<td>-2.49</td>
<td>-2.471</td>
<td>99.2</td>
</tr>
<tr>
<td>9</td>
<td>1034.00</td>
<td>1031.93</td>
<td>-2.07</td>
<td>-1.967</td>
<td>95.0</td>
</tr>
<tr>
<td>10</td>
<td>809.66</td>
<td>808.04</td>
<td>-1.62</td>
<td>-1.460</td>
<td>90.5</td>
</tr>
<tr>
<td>13</td>
<td>1288.46</td>
<td>1285.89</td>
<td>-2.57</td>
<td>-2.548</td>
<td>99.1</td>
</tr>
<tr>
<td>14</td>
<td>1210.50</td>
<td>1208.09</td>
<td>-2.41</td>
<td>-2.380</td>
<td>98.8</td>
</tr>
<tr>
<td>15</td>
<td>1084.12</td>
<td>1081.96</td>
<td>-2.16</td>
<td>-2.134</td>
<td>98.8</td>
</tr>
</tbody>
</table>
Table 10--continued

<table>
<thead>
<tr>
<th></th>
<th>1534.12</th>
<th>1531.06</th>
<th>-3.06</th>
<th>-3.057</th>
<th>99.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1680.28</td>
<td>1676.92</td>
<td>-3.36</td>
<td>-3.358</td>
<td>99.9</td>
</tr>
<tr>
<td>25</td>
<td>1963.52</td>
<td>1959.61</td>
<td>-3.91</td>
<td>-3.927</td>
<td>100.4</td>
</tr>
</tbody>
</table>

4.3 Beams

A cantilever beam finite element model shown in Fig. 4.3 is loaded with a force $F = \begin{bmatrix} 0.0 & 10.0 & -10.0 \end{bmatrix}$ lb. at the tip. It contains 20 2-node beam elements of type E21 and 21 nodal points with six degrees of freedom each. The beam has a rectangular cross-section with constant width and height, $b = 0.5$ in. and $h = 0.25$ in., respectively. Young's modulus and Poisson's ratio are $E = 30.5 \times 10^7$ psi and $\nu = 0.3$, respectively. Self weight is excluded in the analysis.

Compliance sensitivity results are shown in Table 11, where $\Delta \psi = \psi(u + \Delta u) - \psi(u)$ and $\psi'$ is the predicted value calculated from Eq. 2.3.9, with design perturbations $\delta b = 0.01b$ and $\delta h = 0.01h$.

Table 11. Beam Design Sensitivity Check for Compliance

<table>
<thead>
<tr>
<th>$\delta h$</th>
<th>$\delta b$</th>
<th>$\psi(u)$</th>
<th>$\psi(u + \Delta u)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01h</td>
<td>0.01b</td>
<td>22.524</td>
<td>21.646</td>
<td>-0.8789</td>
<td>-0.9010</td>
<td>102.5</td>
</tr>
</tbody>
</table>
Several discrete points along the beam are selected to check accuracy of design sensitivity of the displacement functional of Eq. 2.3.14. In order to compute Eq. 2.3.14, the beam curvature of the adjoint displacement field is needed. Displacement results are shown in Table 12 for design perturbations of $\delta b = 0.01b$ and $\delta h = 0.01h$. Results show that design sensitivity predictions are close to the finite difference approximation.
Table 12. Beam Design Sensitivity Check for Displacement

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Direction</th>
<th>$\psi(u)$</th>
<th>$\psi(u+\Delta u)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>-0.0272</td>
<td>-0.0261</td>
<td>0.00106</td>
<td>0.00109</td>
<td>102.6</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>-0.1613</td>
<td>-0.1550</td>
<td>0.00629</td>
<td>0.00645</td>
<td>102.6</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>-0.3904</td>
<td>-0.3752</td>
<td>0.01523</td>
<td>0.01562</td>
<td>102.5</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>-0.6955</td>
<td>-0.6684</td>
<td>0.02713</td>
<td>0.02783</td>
<td>102.5</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>-1.0577</td>
<td>-1.0164</td>
<td>0.04130</td>
<td>0.04231</td>
<td>102.4</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>-1.4578</td>
<td>-1.4009</td>
<td>0.05690</td>
<td>0.05831</td>
<td>102.5</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>-1.8769</td>
<td>-1.8037</td>
<td>0.07320</td>
<td>0.07508</td>
<td>102.6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.00546</td>
<td>0.00525</td>
<td>-0.2126E-3</td>
<td>-0.2192E-3</td>
<td>103.1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0.03229</td>
<td>0.03104</td>
<td>-0.001259</td>
<td>-0.001294</td>
<td>102.8</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>0.07814</td>
<td>0.07509</td>
<td>-0.003046</td>
<td>-0.003129</td>
<td>102.7</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>0.13918</td>
<td>0.13375</td>
<td>-0.005430</td>
<td>-0.005572</td>
<td>102.6</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>0.21162</td>
<td>0.20337</td>
<td>-0.008250</td>
<td>-0.008470</td>
<td>102.6</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>0.29167</td>
<td>0.28030</td>
<td>-0.011370</td>
<td>-0.011670</td>
<td>102.6</td>
</tr>
<tr>
<td>21</td>
<td>2</td>
<td>0.37551</td>
<td>0.36087</td>
<td>-0.014640</td>
<td>-0.015028</td>
<td>102.6</td>
</tr>
</tbody>
</table>

To check stress constraint sensitivity of Eq. 2.3.21, the equivalent nodal force for the adjoint load and the right side of Eq. 2.3.18 must be computed for finite element adjoint analysis and the adjoint displacement field $\lambda$ must be obtained for each constraint element. Stress results for several finite elements are shown in Table 13 for design perturbations $\delta h = 0.01h$ and $\delta b = 0.01b$. 
The reduced accuracy in correlation between the finite difference calculation and the first variation for the element number 20 results from an inaccurate finite difference calculation.

Table 13. Beam Design Sensitivity Check for Stress

<table>
<thead>
<tr>
<th>El. Fiber No.</th>
<th>$\psi(u)$</th>
<th>$\psi(u+\Delta u)$</th>
<th>$\Delta \psi$</th>
<th>$\psi''$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60613.5</td>
<td>58831.9</td>
<td>-1781.6</td>
<td>-1818.0</td>
<td>102.0</td>
</tr>
<tr>
<td>5</td>
<td>48187.0</td>
<td>46770.9</td>
<td>-1416.1</td>
<td>-1445.2</td>
<td>102.0</td>
</tr>
<tr>
<td>10</td>
<td>32654.9</td>
<td>31695.6</td>
<td>-959.3</td>
<td>-979.1</td>
<td>102.1</td>
</tr>
<tr>
<td>15</td>
<td>17121.3</td>
<td>16621.7</td>
<td>-499.6</td>
<td>-512.8</td>
<td>102.6</td>
</tr>
<tr>
<td>20</td>
<td>1592.9</td>
<td>1541.7</td>
<td>-51.2</td>
<td>-62.3</td>
<td>121.8</td>
</tr>
</tbody>
</table>

4.4 Built-Up Structures

A built-up structure that uses beams and plates is shown in Fig. 4.4. The structure is clamped on two edges, with symmetric boundary conditions applied along the other two edges. A uniform pressure $f(x) = 1.5$ lb/in is applied on the top surface of the plate. The model contains 20 beam elements (E21) and 25 plate elements (E42). Beam width and beam height are two dependent design variables of the first independent design parameter, and the plate thickness is the second independent design parameter. The initial values are $b = 0.05$ in., $h = 0.40$ in., and $t = 0.1$ in. Young's modulus and Poisson's ratio for the beams and plates are $E = 30.5 \times 10^7$ psi and $\gamma = 0.3$, respectively.
Figure 4.4 Built-Up Structure Finite Element Model
Self weight is neglected. An example of the design sensitivity vector \( \frac{\partial \psi}{\partial u} \) is given in Appendix A1.

Compliance sensitivity results are shown in Table 14, where

\[ \Delta \psi = \psi(u+\Delta u) - \psi(u) \] and \( \psi' \) is the predicted value computed from Eq. 2.4.4, with the design perturbations \( \delta b = 0.01b \) and \( \delta h = 0.01h \) for the first control parameter and perturbation of \( \delta t = 0.01t \) for the second control parameter.

Table 14. Built-Up Structure Design Sensitivity Check for Compliance

<table>
<thead>
<tr>
<th>Control Parameter</th>
<th>( \psi(u) )</th>
<th>( \psi(u+\Delta u) )</th>
<th>( \Delta \psi )</th>
<th>( \psi' )</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110.084</td>
<td>109.086</td>
<td>-0.998</td>
<td>-0.9893</td>
<td>99.1</td>
</tr>
<tr>
<td>2</td>
<td>110.084</td>
<td>107.560</td>
<td>-2.524</td>
<td>-2.5577</td>
<td>101.3</td>
</tr>
</tbody>
</table>

Several discrete points in Fig. 4.4 are selected to check design sensitivity accuracy for the displacement functional in Eq. 2.4.4. In order to compute this equation, just as in the case of single components, the adjoint strain \( c^{ij}(\lambda) \) is needed. Some displacement results are shown in Table 15 for the design perturbations \( \delta b = 0.01b \) and \( \delta h = 0.01h \) for the first control parameter and a perturbation of \( \delta t = 0.01t \) for the second control parameter.
Table 15. Built-Up Structure Design Sensitivity for Displacement

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Control Parameter</th>
<th>$\psi(u)$</th>
<th>$\psi(u+\Delta u)$</th>
<th>$\Delta \psi$</th>
<th>$\psi'$</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>$-0.02115$</td>
<td>$-0.02131$</td>
<td>$0.000197$</td>
<td>$0.000196$</td>
<td>99.6</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>$-0.02115$</td>
<td>$-0.02102$</td>
<td>$0.000492$</td>
<td>$0.000498$</td>
<td>101.2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>$-0.06217$</td>
<td>$-0.06157$</td>
<td>$0.000605$</td>
<td>$0.000606$</td>
<td>100.1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>$-0.06217$</td>
<td>$-0.06078$</td>
<td>$0.001394$</td>
<td>$0.001420$</td>
<td>101.3</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>$-0.10086$</td>
<td>$-0.09992$</td>
<td>$0.000941$</td>
<td>$0.000937$</td>
<td>99.6</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>$-0.10086$</td>
<td>$-0.09857$</td>
<td>$0.002295$</td>
<td>$0.002324$</td>
<td>101.3</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>$-0.12491$</td>
<td>$-0.12372$</td>
<td>$0.001190$</td>
<td>$0.001192$</td>
<td>100.2</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>$-0.12491$</td>
<td>$-0.12209$</td>
<td>$0.002820$</td>
<td>$0.002855$</td>
<td>101.2</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>$-0.13473$</td>
<td>$-0.13348$</td>
<td>$0.001250$</td>
<td>$0.001242$</td>
<td>99.3</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>$-0.13473$</td>
<td>$-0.13166$</td>
<td>$0.003070$</td>
<td>$0.003111$</td>
<td>101.3</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>$-0.18204$</td>
<td>$-0.18032$</td>
<td>$0.001720$</td>
<td>$0.001715$</td>
<td>99.7</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>$-0.18204$</td>
<td>$-0.17792$</td>
<td>$0.004120$</td>
<td>$0.004174$</td>
<td>101.3</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>$-0.29650$</td>
<td>$-0.29375$</td>
<td>$0.002750$</td>
<td>$0.002727$</td>
<td>99.1</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>$-0.29650$</td>
<td>$-0.28974$</td>
<td>$0.006760$</td>
<td>$0.006845$</td>
<td>101.3</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>$-0.37298$</td>
<td>$-0.36955$</td>
<td>$0.003430$</td>
<td>$0.003407$</td>
<td>99.3</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>$-0.37298$</td>
<td>$-0.36447$</td>
<td>$0.008510$</td>
<td>$0.008626$</td>
<td>101.4</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>$-0.40049$</td>
<td>$-0.39683$</td>
<td>$0.003660$</td>
<td>$0.003628$</td>
<td>99.1</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>$-0.40049$</td>
<td>$-0.39133$</td>
<td>$0.009160$</td>
<td>$0.009285$</td>
<td>101.4</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>$-0.48626$</td>
<td>$-0.48186$</td>
<td>$0.004400$</td>
<td>$0.004365$</td>
<td>99.2</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
<td>$-0.48626$</td>
<td>$-0.47511$</td>
<td>$0.011150$</td>
<td>$0.011300$</td>
<td>101.3</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>$-0.61524$</td>
<td>$-0.60970$</td>
<td>$0.005540$</td>
<td>$0.005478$</td>
<td>98.9</td>
</tr>
</tbody>
</table>
To check design sensitivity of the stress constraint functional for plate elements in the built-up structure, the equivalent nodal force for the adjoint load of each constraint must be computed. Design sensitivity results for the von Mises' stress functional are given in Table 16, for several different elements. The perturbations $\delta h = 0.01h$ and $\delta b = 0.01b$ are for finite difference calculation of the first control parameter and $\delta t = 0.01t$ is the perturbation of the second control parameter. Note that the equivalent nodal force calculation for the adjoint load is done with shape functions that are inconsistent with the hybrid method, but this has no effect on accuracy of design sensitivity calculations.

![Table 15--continued]

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>2</td>
<td>-0.61524</td>
<td>-0.60110</td>
<td>0.014140</td>
<td>0.014329</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>-0.66123</td>
<td>-0.65533</td>
<td>0.005900</td>
<td>0.005840</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>-0.66123</td>
<td>-0.64600</td>
<td>0.015230</td>
<td>0.015434</td>
</tr>
<tr>
<td>29</td>
<td>1</td>
<td>-0.78180</td>
<td>-0.77486</td>
<td>0.006940</td>
<td>0.006861</td>
</tr>
<tr>
<td>29</td>
<td>2</td>
<td>-0.78180</td>
<td>-0.76377</td>
<td>0.018030</td>
<td>0.018279</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>-0.84074</td>
<td>-0.83332</td>
<td>0.007420</td>
<td>0.007334</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>-0.84074</td>
<td>-0.82131</td>
<td>0.019430</td>
<td>0.019688</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
<td>-0.90440</td>
<td>-0.89647</td>
<td>0.007930</td>
<td>0.007836</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
<td>-0.90440</td>
<td>-0.88347</td>
<td>0.020930</td>
<td>0.021217</td>
</tr>
</tbody>
</table>
Table 16. Design Sensitivity Check for Plate Stress
Von Mises' Stress with $\delta t = 0.01t$

<table>
<thead>
<tr>
<th>El. Control No.</th>
<th>Parameter</th>
<th>$\psi(u)$</th>
<th>$\psi(u+\Delta u)$</th>
<th>$\Delta\psi$</th>
<th>$\psi'$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3669.32</td>
<td>3635.34</td>
<td>-33.98</td>
<td>-34.11</td>
<td>100.4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3669.32</td>
<td>3621.67</td>
<td>-47.65</td>
<td>-47.95</td>
<td>100.6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>9094.17</td>
<td>9005.53</td>
<td>-89.17</td>
<td>-86.17</td>
<td>96.6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>9094.17</td>
<td>8979.13</td>
<td>-115.04</td>
<td>-111.05</td>
<td>96.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>14410.77</td>
<td>14276.46</td>
<td>-134.31</td>
<td>-132.65</td>
<td>98.8</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>14410.77</td>
<td>14223.77</td>
<td>-187.00</td>
<td>-184.91</td>
<td>98.9</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>18484.13</td>
<td>18309.74</td>
<td>-174.39</td>
<td>-172.25</td>
<td>98.8</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>18484.13</td>
<td>18245.95</td>
<td>-238.18</td>
<td>-238.23</td>
<td>100.0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>20882.59</td>
<td>20688.57</td>
<td>-194.02</td>
<td>-192.30</td>
<td>99.1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>20882.59</td>
<td>20611.15</td>
<td>-271.44</td>
<td>-272.81</td>
<td>100.5</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>10370.80</td>
<td>10277.77</td>
<td>-93.03</td>
<td>-92.73</td>
<td>99.4</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>10370.80</td>
<td>10233.58</td>
<td>-137.22</td>
<td>-140.06</td>
<td>102.1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>10381.71</td>
<td>10288.66</td>
<td>-93.05</td>
<td>-91.21</td>
<td>98.0</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>10381.71</td>
<td>10244.29</td>
<td>-137.42</td>
<td>-137.10</td>
<td>99.8</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>8802.67</td>
<td>8730.74</td>
<td>-71.93</td>
<td>-68.44</td>
<td>95.1</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>8802.67</td>
<td>8681.11</td>
<td>-121.56</td>
<td>-115.74</td>
<td>95.2</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>6956.20</td>
<td>6900.33</td>
<td>-55.87</td>
<td>-51.49</td>
<td>92.2</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>6956.20</td>
<td>6859.40</td>
<td>-96.80</td>
<td>-87.96</td>
<td>90.9</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>10757.59</td>
<td>10664.65</td>
<td>-92.94</td>
<td>-90.63</td>
<td>97.5</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>10757.59</td>
<td>10612.64</td>
<td>-144.95</td>
<td>-143.94</td>
<td>99.3</td>
</tr>
</tbody>
</table>
To check stress constraint sensitivity for beam elements in the built-up structure, the equivalent nodal force for the adjoint load on beam elements must be computed, so that the adjoint displacement field $\lambda$ can be calculated. Stress results for several beam elements are shown in Table 17.

The forward finite difference method has been used so far to check the accuracy of the design sensitivity prediction. The prediction of the gradient for the first design parameter is very small for the beam element, which implies that the function has a nearly zero slope. For better finite difference approximation, the central finite difference method is used, to compare the accuracy of the prediction. The central finite difference method is defined as
\[ \Delta \psi = \frac{\psi(u + \Delta u) - \psi(u - \Delta u)}{2} \]

The perturbations are \( \delta b = 0.01b \) and \( \delta h = 0.01h \) for the first control parameter \( \Delta u \), and \( \delta t = 0.01t \) is the perturbation for second control parameter.

**Table 17. Design Sensitivity Check for Beam Stress**

<table>
<thead>
<tr>
<th>El. Fiber Control No.</th>
<th>Parameter</th>
<th>( \psi(u) )</th>
<th>( \psi(u+\Delta u) )</th>
<th>( \psi(u-\Delta u) )</th>
<th>( \Delta \psi )</th>
<th>( \psi' )</th>
<th>Ratio %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 4 1</td>
<td></td>
<td>40182.0</td>
<td>40191.2</td>
<td>40161.1</td>
<td>15.1</td>
<td>17.8</td>
<td>118.7</td>
</tr>
<tr>
<td>1 4 2</td>
<td></td>
<td>40182.0</td>
<td>39277.9</td>
<td>41109.3</td>
<td>-915.7</td>
<td>-915.7</td>
<td>100.0</td>
</tr>
<tr>
<td>2 4 1</td>
<td></td>
<td>7430.8</td>
<td>7457.8</td>
<td>7401.8</td>
<td>28.0</td>
<td>27.9</td>
<td>99.9</td>
</tr>
<tr>
<td>2 4 2</td>
<td></td>
<td>7430.8</td>
<td>7245.7</td>
<td>7621.7</td>
<td>-188.0</td>
<td>-188.0</td>
<td>100.0</td>
</tr>
<tr>
<td>3 4 1</td>
<td></td>
<td>-9945.1</td>
<td>-9947.6</td>
<td>-9939.7</td>
<td>-3.9</td>
<td>-4.6</td>
<td>117.4</td>
</tr>
<tr>
<td>3 4 2</td>
<td></td>
<td>-9945.1</td>
<td>-9721.0</td>
<td>-10174.6</td>
<td>226.8</td>
<td>226.8</td>
<td>100.0</td>
</tr>
<tr>
<td>4 4 1</td>
<td></td>
<td>-17328.3</td>
<td>-17337.9</td>
<td>-17313.5</td>
<td>-12.2</td>
<td>-13.1</td>
<td>107.4</td>
</tr>
<tr>
<td>4 4 2</td>
<td></td>
<td>-17328.3</td>
<td>-16934.5</td>
<td>-17732.3</td>
<td>398.9</td>
<td>399.0</td>
<td>100.0</td>
</tr>
<tr>
<td>5 4 1</td>
<td></td>
<td>-20339.1</td>
<td>-20363.2</td>
<td>-20309.4</td>
<td>-26.9</td>
<td>-28.1</td>
<td>104.5</td>
</tr>
<tr>
<td>5 4 2</td>
<td></td>
<td>-20339.1</td>
<td>-19867.9</td>
<td>-20823.6</td>
<td>477.9</td>
<td>477.8</td>
<td>100.0</td>
</tr>
<tr>
<td>6 4 1</td>
<td></td>
<td>82011.8</td>
<td>82049.4</td>
<td>81950.6</td>
<td>49.4</td>
<td>56.0</td>
<td>113.4</td>
</tr>
<tr>
<td>6 4 2</td>
<td></td>
<td>82011.8</td>
<td>80152.9</td>
<td>83919.1</td>
<td>-1883.1</td>
<td>-1883.1</td>
<td>100.0</td>
</tr>
<tr>
<td>7 4 1</td>
<td></td>
<td>17977.1</td>
<td>18035.4</td>
<td>17914.1</td>
<td>60.6</td>
<td>62.9</td>
<td>103.8</td>
</tr>
<tr>
<td>7 4 2</td>
<td></td>
<td>17977.1</td>
<td>17534.3</td>
<td>18433.3</td>
<td>-449.5</td>
<td>-449.6</td>
<td>100.0</td>
</tr>
<tr>
<td>8 4 1</td>
<td></td>
<td>-18646.6</td>
<td>-18639.5</td>
<td>-18648.1</td>
<td>4.3</td>
<td>1.7</td>
<td>40.0</td>
</tr>
<tr>
<td>8 4 2</td>
<td></td>
<td>-18646.6</td>
<td>-18234.8</td>
<td>-19067.7</td>
<td>416.5</td>
<td>416.5</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>1</td>
<td>-36874.1</td>
<td>-36910.3</td>
<td>-36648.1</td>
<td>-41.5</td>
<td>-43.5</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>2</td>
<td>-36874.1</td>
<td>-36024.9</td>
<td>-37746.3</td>
<td>860.7</td>
<td>860.8</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>1</td>
<td>-44467.5</td>
<td>-44534.1</td>
<td>-44388.5</td>
<td>-72.8</td>
<td>-77.2</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>2</td>
<td>-44467.5</td>
<td>-43426.8</td>
<td>-45537.6</td>
<td>1055.4</td>
<td>1055.3</td>
</tr>
</tbody>
</table>

Table 17--continued
CHAPTER V
CONCLUSIONS

Results of this study show that it is possible to combine the design sensitivity algorithms of Ref. 1 with the database management system of EAL. For stress constraints as performance criteria, it is necessary to compute equivalent nodal forces for the adjoint load. For plate elements, the EAL finite element analysis is based on a hybrid formulation, but a displacement finite element formulation is used for evaluating the equivalent adjoint nodal forces. Nevertheless, results of the design sensitivity analysis are very accurate, which indicates that it is not necessary to compute equivalent nodal forces for the adjoint load using exactly the same shape functions that are employed in finite element analysis.

A database management system with a finite element capability and the adjoint variable method of design sensitivity analysis, permit implementation of a design sensitivity analysis method that does not require differentiation of element stiffness and mass matrices. It is shown that a database management system can be used to implement design sensitivity analysis, so only one program with one database is necessary.

Work is progressing to extend the methods presented in this report to include shape (geometric) design parameters. A domain method [17] for shape design sensitivity analysis and a design component method [6] for sensitivity analysis of built-up structures are used for software
implementation. Numerical implementation and results of shape design sensitivity analysis will be reported in Part II: Shape Design Parameters.
REFERENCES


APPENDIX A1 - DESIGN SENSITIVITY VECTORS

This appendix lists the design sensitivity vectors $\frac{\partial \psi}{\partial u}$ for the built-up structure for the compliance constraint, for the displacement constraint at node 36, for the stress constraint in beam element 1, and for the stress constraint in plate element 25.

Table 18. Sensitivity Vectors for the Compliance Constraint

<table>
<thead>
<tr>
<th>Beam element</th>
<th>$\frac{\partial \psi}{\partial b_1}$</th>
<th>$\frac{\partial \psi}{\partial b_2}$</th>
<th>Plate element</th>
<th>$\frac{\partial \psi}{\partial t_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-11.62</td>
<td>-30.97</td>
<td>1</td>
<td>-9.83</td>
</tr>
<tr>
<td>2</td>
<td>-0.62</td>
<td>-1.65</td>
<td>2</td>
<td>-47.94</td>
</tr>
<tr>
<td>3</td>
<td>-0.73</td>
<td>-1.94</td>
<td>3</td>
<td>-127.21</td>
</tr>
<tr>
<td>4</td>
<td>-1.97</td>
<td>-5.26</td>
<td>4</td>
<td>-219.05</td>
</tr>
<tr>
<td>5</td>
<td>-2.72</td>
<td>-7.26</td>
<td>5</td>
<td>-282.67</td>
</tr>
<tr>
<td>6</td>
<td>-47.55</td>
<td>-126.80</td>
<td>6</td>
<td>-47.94</td>
</tr>
<tr>
<td>7</td>
<td>-3.13</td>
<td>-8.34</td>
<td>7</td>
<td>-50.04</td>
</tr>
<tr>
<td>8</td>
<td>-2.51</td>
<td>-6.68</td>
<td>8</td>
<td>-50.25</td>
</tr>
<tr>
<td>9</td>
<td>-8.94</td>
<td>-23.85</td>
<td>9</td>
<td>-38.58</td>
</tr>
<tr>
<td>10</td>
<td>-12.97</td>
<td>-34.58</td>
<td>10</td>
<td>-27.41</td>
</tr>
<tr>
<td>11</td>
<td>-11.62</td>
<td>-30.97</td>
<td>11</td>
<td>-127.20</td>
</tr>
<tr>
<td>12</td>
<td>-0.62</td>
<td>-1.65</td>
<td>12</td>
<td>-50.25</td>
</tr>
<tr>
<td>13</td>
<td>-0.73</td>
<td>-1.94</td>
<td>13</td>
<td>-59.09</td>
</tr>
<tr>
<td>14</td>
<td>-1.97</td>
<td>-5.26</td>
<td>14</td>
<td>-62.54</td>
</tr>
</tbody>
</table>
In Table 18, the sum of the sensitivity components $\frac{\partial \psi}{\partial u_i}$ for $i=1, \ldots, n$ is given in the third row from the bottom. When multiplying the sum of the sensitivity components with the perturbation $\delta u$ of the design variable $u$, one gets the first variation, which is given in the last row of Table 18. Note that this result coincides with results given in Table 14. Results in Tables 19, 20, and 21 coincide with results given in Tables 15, 16, and 17, respectively.

<table>
<thead>
<tr>
<th></th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.72</td>
<td>-7.26</td>
<td>15</td>
<td>-59.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>-47.55</td>
<td>-126.80</td>
<td>16</td>
<td>-219.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>-3.13</td>
<td>-8.34</td>
<td>17</td>
<td>-38.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>-2.51</td>
<td>-6.68</td>
<td>18</td>
<td>-62.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>-8.94</td>
<td>-23.85</td>
<td>19</td>
<td>-114.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-12.97</td>
<td>-34.58</td>
<td>20</td>
<td>-145.47</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\sum \psi' _i = -185.52 -494.66 \quad \sum \psi' = -2557.65$$

$$\delta u = 0.004 \quad 0.0005 \quad \delta u = 0.001$$

$$\psi' = -0.7421 -0.2473 \quad \psi' = -2.557$$
Table 19. Sensitivity Vectors for the Displacement Constraint

<table>
<thead>
<tr>
<th>Beam element</th>
<th>( \frac{\partial \psi}{\partial b} )</th>
<th>( \frac{\partial \psi}{\partial h} )</th>
<th>Plate element</th>
<th>( \frac{\partial \psi}{\partial t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0604</td>
<td>0.1610</td>
<td>1</td>
<td>0.040</td>
</tr>
<tr>
<td>2</td>
<td>0.0074</td>
<td>0.0197</td>
<td>2</td>
<td>0.251</td>
</tr>
<tr>
<td>2</td>
<td>0.0010</td>
<td>0.0028</td>
<td>3</td>
<td>0.755</td>
</tr>
<tr>
<td>4</td>
<td>0.0144</td>
<td>0.0386</td>
<td>4</td>
<td>1.426</td>
</tr>
<tr>
<td>5</td>
<td>0.0286</td>
<td>0.0763</td>
<td>5</td>
<td>1.934</td>
</tr>
<tr>
<td>6</td>
<td>0.3205</td>
<td>0.8547</td>
<td>6</td>
<td>0.251</td>
</tr>
<tr>
<td>7</td>
<td>0.0405</td>
<td>0.1079</td>
<td>7</td>
<td>0.355</td>
</tr>
<tr>
<td>8</td>
<td>-0.0068</td>
<td>-0.0181</td>
<td>8</td>
<td>0.429</td>
</tr>
<tr>
<td>9</td>
<td>0.0587</td>
<td>0.1564</td>
<td>9</td>
<td>0.394</td>
</tr>
<tr>
<td>10</td>
<td>0.2098</td>
<td>0.5595</td>
<td>10</td>
<td>0.311</td>
</tr>
<tr>
<td>11</td>
<td>0.0604</td>
<td>0.1610</td>
<td>11</td>
<td>0.755</td>
</tr>
<tr>
<td>12</td>
<td>0.0074</td>
<td>0.0197</td>
<td>12</td>
<td>0.429</td>
</tr>
<tr>
<td>13</td>
<td>0.0010</td>
<td>0.0028</td>
<td>13</td>
<td>0.448</td>
</tr>
<tr>
<td>14</td>
<td>0.0144</td>
<td>0.0386</td>
<td>14</td>
<td>0.457</td>
</tr>
<tr>
<td>15</td>
<td>0.0286</td>
<td>0.0763</td>
<td>15</td>
<td>0.389</td>
</tr>
<tr>
<td>16</td>
<td>0.3205</td>
<td>0.8547</td>
<td>16</td>
<td>1.426</td>
</tr>
<tr>
<td>17</td>
<td>0.0405</td>
<td>0.1079</td>
<td>17</td>
<td>0.394</td>
</tr>
<tr>
<td>18</td>
<td>-0.0068</td>
<td>-0.0181</td>
<td>18</td>
<td>0.457</td>
</tr>
<tr>
<td>19</td>
<td>0.0589</td>
<td>0.1564</td>
<td>19</td>
<td>0.958</td>
</tr>
<tr>
<td>20</td>
<td>0.2098</td>
<td>0.5594</td>
<td>20</td>
<td>1.490</td>
</tr>
</tbody>
</table>
Table 19--continued

<table>
<thead>
<tr>
<th></th>
<th>21</th>
<th>1.934</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>0.389</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>1.490</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>3.744</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Σ</th>
<th>1.469</th>
<th>3.9176</th>
<th>Σ</th>
<th>21.217</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>δu</td>
<td>0.004</td>
<td>0.0005</td>
<td>δu</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>ψr</td>
<td>0.005876</td>
<td>0.0001958</td>
<td>ψr</td>
<td>0.021217</td>
</tr>
</tbody>
</table>
Table 20. Sensitivity Vectors for the Beam Stress Constraint

<table>
<thead>
<tr>
<th>Beam element</th>
<th>$\frac{\partial \psi}{\partial b}$</th>
<th>$\frac{\partial \psi}{\partial h}$</th>
<th>Plate element</th>
<th>$\frac{\partial \psi}{\partial c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>77168</td>
<td>-62100</td>
<td>1</td>
<td>-15002</td>
</tr>
<tr>
<td>2</td>
<td>9230</td>
<td>24614</td>
<td>2</td>
<td>-52185</td>
</tr>
<tr>
<td>2</td>
<td>-652</td>
<td>-1739</td>
<td>3</td>
<td>-80766</td>
</tr>
<tr>
<td>4</td>
<td>603</td>
<td>1610</td>
<td>4</td>
<td>-87753</td>
</tr>
<tr>
<td>5</td>
<td>1108</td>
<td>2955</td>
<td>5</td>
<td>-83260</td>
</tr>
<tr>
<td>6</td>
<td>-18728</td>
<td>-49942</td>
<td>6</td>
<td>-88775</td>
</tr>
<tr>
<td>7</td>
<td>-97</td>
<td>-258</td>
<td>7</td>
<td>-13790</td>
</tr>
<tr>
<td>8</td>
<td>-1728</td>
<td>-4608</td>
<td>8</td>
<td>4890</td>
</tr>
<tr>
<td>9</td>
<td>-2162</td>
<td>-5765</td>
<td>9</td>
<td>3402</td>
</tr>
<tr>
<td>10</td>
<td>-1163</td>
<td>-3102</td>
<td>10</td>
<td>-1870</td>
</tr>
<tr>
<td>11</td>
<td>-13565</td>
<td>-36174</td>
<td>11</td>
<td>-111076</td>
</tr>
<tr>
<td>12</td>
<td>618</td>
<td>1648</td>
<td>12</td>
<td>23378</td>
</tr>
<tr>
<td>13</td>
<td>-2072</td>
<td>-5524</td>
<td>13</td>
<td>-45076</td>
</tr>
<tr>
<td>14</td>
<td>587</td>
<td>1565</td>
<td>14</td>
<td>-21198</td>
</tr>
<tr>
<td>15</td>
<td>1405</td>
<td>3746</td>
<td>15</td>
<td>-4547</td>
</tr>
<tr>
<td>16</td>
<td>-15697</td>
<td>-41858</td>
<td>16</td>
<td>-104176</td>
</tr>
<tr>
<td>17</td>
<td>-709</td>
<td>-1889</td>
<td>17</td>
<td>1248</td>
</tr>
<tr>
<td>18</td>
<td>-1428</td>
<td>-3807</td>
<td>18</td>
<td>-29999</td>
</tr>
<tr>
<td>19</td>
<td>-2440</td>
<td>-6506</td>
<td>19</td>
<td>-30033</td>
</tr>
<tr>
<td>20</td>
<td>-1815</td>
<td>-4841</td>
<td>20</td>
<td>-20950</td>
</tr>
</tbody>
</table>
Table 20—continued

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-97175</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>137</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>-7950</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>-23568</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>-29563</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Σ</td>
<td>28463</td>
<td>-191975</td>
</tr>
<tr>
<td>Δυ</td>
<td>0.004</td>
<td>0.0005</td>
</tr>
<tr>
<td>ψ'</td>
<td>113.85</td>
<td>-95.99</td>
</tr>
</tbody>
</table>

\[
\sum \quad 28463 \quad -191975 \quad \sum \quad -915657 \\
\delta_u \quad 0.004 \quad 0.0005 \quad \delta_u \quad 0.001 \\
\psi' \quad 113.85 \quad -95.99 \quad \psi' \quad -915.657
\]
Table 21. Sensitivity Vectors for the Plate Stress Constraint

<table>
<thead>
<tr>
<th>Beam element</th>
<th>$\frac{\partial \psi}{\partial b}$</th>
<th>$\frac{\partial \psi}{\partial h}$</th>
<th>Plate element</th>
<th>$\frac{\partial \psi}{\partial e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-641</td>
<td>-1711</td>
<td>1</td>
<td>-361</td>
</tr>
<tr>
<td>2</td>
<td>-117</td>
<td>-312</td>
<td>2</td>
<td>-2660</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>51</td>
<td>3</td>
<td>-8478</td>
</tr>
<tr>
<td>4</td>
<td>-201</td>
<td>-538</td>
<td>4</td>
<td>-16560</td>
</tr>
<tr>
<td>5</td>
<td>-444</td>
<td>-1185</td>
<td>5</td>
<td>-22898</td>
</tr>
<tr>
<td>6</td>
<td>-3784</td>
<td>-10090</td>
<td>6</td>
<td>-2660</td>
</tr>
<tr>
<td>7</td>
<td>-865</td>
<td>-2307</td>
<td>7</td>
<td>-4650</td>
</tr>
<tr>
<td>8</td>
<td>1316</td>
<td>3509</td>
<td>8</td>
<td>-6543</td>
</tr>
<tr>
<td>9</td>
<td>2632</td>
<td>7018</td>
<td>9</td>
<td>-6805</td>
</tr>
<tr>
<td>10</td>
<td>-10611</td>
<td>-28298</td>
<td>10</td>
<td>-5715</td>
</tr>
<tr>
<td>11</td>
<td>-641</td>
<td>-1711</td>
<td>11</td>
<td>-8477</td>
</tr>
<tr>
<td>12</td>
<td>-117</td>
<td>-312</td>
<td>12</td>
<td>-6543</td>
</tr>
<tr>
<td>13</td>
<td>19</td>
<td>51</td>
<td>13</td>
<td>-7439</td>
</tr>
<tr>
<td>14</td>
<td>-201</td>
<td>-538</td>
<td>14</td>
<td>-6114</td>
</tr>
<tr>
<td>15</td>
<td>-444</td>
<td>-1185</td>
<td>15</td>
<td>4866</td>
</tr>
<tr>
<td>16</td>
<td>-3784</td>
<td>-10090</td>
<td>16</td>
<td>-16560</td>
</tr>
<tr>
<td>17</td>
<td>-865</td>
<td>-2307</td>
<td>17</td>
<td>-6805</td>
</tr>
<tr>
<td>18</td>
<td>1316</td>
<td>3509</td>
<td>18</td>
<td>-6115</td>
</tr>
<tr>
<td>19</td>
<td>2632</td>
<td>7018</td>
<td>19</td>
<td>-803</td>
</tr>
<tr>
<td>20</td>
<td>-10612</td>
<td>-28299</td>
<td>20</td>
<td>2472</td>
</tr>
</tbody>
</table>
Table 21—continued

<table>
<thead>
<tr>
<th></th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-22897</td>
<td>-5714</td>
<td>4864</td>
<td>2472</td>
<td>-79195</td>
</tr>
</tbody>
</table>

| $\Sigma$   | -25392 | -67726 | $\Sigma$   | -229328 |
| $\delta_u$ | 0.004  | 0.0005 | $\delta_u$ | 0.001  |
| $\psi'$    | -101.57 | -33.86 | $\psi'$    | -229.328 |
APPENDIX A2 - SOURCE PROGRAM

*INIT U1

$-----------------------------------------------

$ Program for computing the first variation of stress, displacement
$ and compliance constraints

$ Written by B. DOPKER

$-----------------------------------------------

*(29 DRV GLOB)       ENDDRIV

* DCALL, (29 PARA SET 0 0)
* DCALL, (29 INIT MODL 0 0)
* DCALL, (29 DSA COM 0 0)
*
*RETURN
*
*ENDDRIV

$-----------------------------------------------

*(29 PARA SET 0 0)       ENDPARA

!NLST=1  $ NUMBER OF LOAD CASES

!LCAS=1  $ NORMAL LOAD CASE


!NBD=1  $ NUMBER OF INDEPENDENT BEAM/TRUSS DESIGN VARIABLES
!NMDV=0  $ NUMBER OF INDEPENDENT MEMBRANE DESIGN VARIABLES
!NPDV=1  $ NUMBER OF INDEPENDENT PLATE DESIGN VARIABLES

!NDV=NBD+NMV+NPV

!DE21=20  $ NUMBER OF BEAM/TRUSS ELEMENTS
!DE41=0  $ NUMBER OF MEMBRANE ELEMENTS
!DE42=25  $ NUMBER OF PLATE ELEMENTS

!DEID=DE21+DE41+DE42

$ DEFINE INITIAL VALUE FOR ALL DESIGN VARIABLES


*AUT AUS
TABLE(NI=2, NJ="NBD"): DESV VALU 0 0
J=1: 1 0 1.0

$ DEFINE RELATION BETWEEN ELEMENTS AND DESIGN VARIABLES (SIZING)

$ BEAM/TRUSS
*J2(DE21, 10)
TABLE(NJ="DE21", TYPE=0): ED21 REL 0 0
```
# LABEL 10
* MEMBRANE

# Z (DE41:30)
TABLE (N=":DE41", TYPE=0) : ED41 REL 0 0
  J=1  : 1
  J=2  : 1
  J=3  : 1
  J=4  : 1
  J=5  : 1
  J=6  : 1
  J=7  : 1
  J=8  : 1
  J=9  : 1
  J=10 : 1
  J=11 : 1
  J=12 : 1
  J=13 : 1
  J=14 : 1
  J=15 : 1
  J=16 : 1
  J=17 : 1
  J=18 : 1
  J=19 : 1
  J=20 : 1

# LABEL 30
* PLATE

# Z (DE42:40)
TABLE (N=":DE42", TYPE=0) : ED42 REL 0 0
  J=1  : 2
  J=2  : 2
  J=3  : 2
  J=4  : 2
  J=5  : 2
  J=6  : 2
  J=7  : 2
```
J=8  :  2
J=9  :  2
J=10 :  2
J=11 :  2
J=12 :  2
J=13 :  2
J=14 :  2
J=15 :  2
J=16 :  2
J=17 :  2
J=18 :  2
J=19 :  2
J=20 :  2
J=21 :  2
J=22 :  2
J=23 :  2
J=24 :  2
J=25 :  2

*LABEL 40

*CONSTRAINT DEFINITION

'CUM=0  $  COMPLIANCE CONSTRAINTS
'CDIS=0  +  NUMBER OF DISPLACEMENT CONSTRAINTS
'CS21=0  +  NUMBER OF STRESS CONSTRAINTS IN BEAM/TRUSS ELEMENTS
'CS41=0  +  NUMBER OF STRESS CONSTRAINTS IN MEMBRANE ELEMENTS
'CS42=1  +  NUMBER OF STRESS CONSTRAINTS IN PLATE ELEMENTS
'CSTR=CS21+CS41+CS42

'CUT=CUM+CDIS+CSTR

* TABLES FOR CONSTRAINT NODES/ELEMENTS
*JZ(CDIS, 100)
  TABLE(NI=2, NJ="CDIS", TYPE=0): DICO LIST 1 1
    J=1 : 36  3  * FIRST NUMBER IS THE NODE, SECOND IS DIRECTION
    J=2 :  9  3
    J=3 : 10  3
    J=4 : 11  3
    J=5 : 12  3

*LABEL 100

*JZ(CS21, 110)
  TABLE(NJ="CS21", TYPE=0): ST21 LIST
    J=1 : 1

*LABEL 110

*JZ(CS41, 130)
  TABLE(NJ="CS41", TYPE=0): ST41 LIST
    J=1 : 1
    J=2 : 25
    J=3 : 10
    J=4 : 15
    J=5 : 21
    J=6 : 31
    J=7 : 61
    J=8 : 64

*LABEL 130

*JZ(CS42, 140)
  TABLE(NJ="CS42", TYPE=0): ST42 LIST
ORIGINAL PAGE IS OF POOR QUALITY
CLCC=CLC-CIOT
+JVZ(CLCC,500)

RETURN

ENDDSAC

...(29 ADDO COMP 0 0)

TEMP=CLC-1
JENT=0
JGZ(TEMP,B70)
JZ(CLCC,B70)

$ COMPUTE COMPLIANCE

*XOT DCU
TOCC 1 STAT DISP "LCAS" 1: N3="ADLC"
JUMP 1000
LABEL B70

TEMP=CLC-CLC-COM CDIS
JGZ(TEMP,B80)

$ COMPUTE ADJOINT FOR DISPLACEMENTS

TEMP=CLC-CLC-COM
$GET TEMP'S ENTRY OF DICO LIST
JENT=DS 1,"TEMP",1 (1 DICO LIST 1 1)
IDIR=DS 2,"TEMP",1 (1 DICO LIST 1 1)

*XOT AUS
SYSVEC: APPL FORC "ADLC" 1
I="IDIR"
J="JENT"
0

IDIR=FREE()
JUMP 1000
LABEL B80

$ COMPUTE STRESS ADJOINTS

$ FOR $21 STRESS CONSTRAINTS
TEMP=CLC-CLC-COM-CD15-CS21
TEMP
JGZ(TEMP,B90)

TEMP=CLC-CLC-COM-CDIS
TEMP
DEC21
$GET TEMP'S ENTRY OF ST21 LIST
JENT=DS "TEMP",1,1 (1 ST21 LIST 1 1)

ORIGINAL PAGE IS
OF POOR QUALITY.
* COMPUTE STRESS CONSTR FOR ELEMENT: JENT E21
*DCALL, (29 ADJO E21 0 0)
*JUMP 1000
*LABEL DYO

FOR E41 STRESS CONSTRAINTS

* TEMP=CLC-CCOM-CDIS-CS21-CS41
* TEMP
* JGZ (TEMP, 910)
* TEMP
* TEMP=CLC-CCOM-CDIS-CS21
* TEMP
*GET TEMP'S ENTRY OF ST41 LIST
*DE41
*JENT=DS "TEMP", 1 ,1 (1 ST41 LIST 1 1)
*
* DCALL, (29 ADJO E41 0 0)
*JUMP 1000
*LABEL 910
*TEMP=FREE()

* FOR E42 STRESS CONSTRAINTS
* TEMP=CLC-CCOM-CDIS-CS21-CS41-CS42
* TEMP
* JGZ (TEMP, 1000)
* TEMP
* TEMP=CLC-CCOM-CDIS-CS21-CS41
* TEMP
*DE42
*GET TEMP'S ENTRY OF ST42 LIST
*JENT=DS "TEMP", 1 ,1 (1 ST42 LIST 1 1)
*
* DCALL, (29 ADJO E42 0 0)
*LABEL 1000
*TEMP=FREE()
*JENT=FREE()
*RETURN
*
*ENDADCO

* -------------------------------
*(29 ADJO SOLU 0 0) ENDA9SO
* TEMP=CLC-1
* JGZ (TEMP, 895)
/*JZ(CCMM.BBS)
 *
 * THIS JUMP IS ONLY FOR COMPLIANCE CONSTRAINTS
 *
 *JUMP RBL
 *
 */

LABEL BS0
*GT SSOL
RESET SET="ADLC"
*
*GT VPRT
*
PRINT 1 STAT DISP "ADLC"
*
LABEL BS6
*
!TEMP=DETO-DEP1
*JZ(TEMP,136)
*JZ(DE41,136)
*GT ES
PROC=2
OUTLIB=1
OUTPUT TYPE=ESB
U= STAT DISP "ADLC" 1
E41 1
*LABEL 136
*JZ(DE42,137)
*GT ES
PROC=2
OUTLIB=1
OUTPUT TYPE=ESB
U= STAT DISP "ADLC" 1
E42 1
*LABEL 137
*LABEL 158
!TEMP=FREE()
*
*RETURN
*
ENDADSO
*

*(24 SENS CALC O O) ENDSECA
*
!CDV=1
!TEMP=O
*LABEL 300
!PSI=O 0
*
*JZ(DE21,140)
*DCALL, (29 SECA E21 O O)
*LABEL 140
*
*JZ(DE41,150)
*DCALL, (29 SECA E41 O O)
*LABEL 150
*
*JZ(DE42,160)
*DCALL, (29 SECA E42 O O)
*LABEL 160
"ORIGINAL PAGE IS OF POOR QUALITY"

```plaintext
!ICDV
!NDV
!ICDV=ICDV+1
!TEMP=NDV-ICDV+1
!ICDV
!TEMP
!
jg2(temp,300)
!
!temp=free()
!icdv=free()
!
return
!
endsec

! (29 prep e21 0 0) endpe21
!
! this subroutine is for element typ e21
!
! the principal library is 13
! temporary library is 17. will be deleted at the end of this routine
!
xgt e11
!
extract: source=e21: content spec: geom 1 $ length of elements
create 13 length e21 0 0
!
extract: source=e21: content spec: geom 5,13 $ direction cosines
create 13 direct e21 0 0
!
extract: source=e21: content spec: geom 14,31 $ element-nodal relation
create 13 elno e21 0 0
!
extract: source=e21: content spec: mate 1,2 $ material properties
create 13 matp e21 0 0
!
extract: source=e21: content spec: inte 1,18 $ connectivity
create 13 conn e21 0 0
!
extract: source=e21: content spec: section 16,23 $ stress point location
create 13 splo e21 0 0
!
*xgt dco

toc 13 conn e21 0 0: type=0
!
if not all nodal information is in global coordinates use ltoq
!
*xgt aus
!
icou=0
!
label 100
!
icou=icou+1
!
p1=ds.13,"icou",1 (13 conn e21 0 0)
!
p2=ds.14,"icou",1 (13 conn e21 0 0)
!
u1=ds.1,"p1",1 (1 stat disp "lcas" 1)
!
v1=ds.2,"p1",1 (1 stat disp "lcas" 1)
!
w1=ds.3,"p1",1 (1 stat disp "lcas" 1)
!
x1=ds.4,"p1",1 (1 stat disp "lcas" 1)
!
y1=ds.5,"p1",1 (1 stat disp "lcas" 1)
```

91

*LABEL 100

ICOU=ICOU+1
TEMP=DS 1,"ICOU",1 (1 ED21 REL 0 0)
!ICOU
TEMP
TEMP=TEMP-ICOU
*JNZ(TEMP, 200)
!P1=DS.13,"ICOU",1 (13 CONN E21 0 0)
!P2=DS.14,"ICOU",1 (13 CONN E21 0 0)
!UX1=DS.1,"P1",1 (1 STAT DISP "ADLC" 1)
!UY1=DS.2,"P1",1 (1 STAT DISP "ADLC" 1)
!UZ1=DS.3,"P1",1 (1 STAT DISP "ADLC" 1)
!RX1=DS.4,"P1",1 (1 STAT DISP "ADLC" 1)
!RY1=DS.5,"P1",1 (1 STAT DISP "ADLC" 1)
!RZ1=DS.6,"P1",1 (1 STAT DISP "ADLC" 1)
!UX2=DS.1,"P2",1 (1 STAT DISP "ADLC" 1)
!UY2=DS.2,"P2",1 (1 STAT DISP "ADLC" 1)
!UZ2=DS.3,"P2",1 (1 STAT DISP "ADLC" 1)
!RX2=DS.4,"P2",1 (1 STAT DISP "ADLC" 1)
!RY2=DS.5,"P2",1 (1 STAT DISP "ADLC" 1)
!RZ2=DS.6,"P2",1 (1 STAT DISP "ADLC" 1)
!P1=FREE()
!P2=FREE()

TABLE(N1=3,NJ=4): 17 EL21 DISP "ICOU" "ADLC"
J=1: "UX1" "UY1" "UZ1"
J=2: "RX1" "RY1" "RZ1"
J=3: "UX2" "UY2" "UZ2"
J=4: "RX2" "RY2" "RZ2"
DEFINE B11=13 EL21 ROTA "ICOU" 0
DEFINE B12=17 EL21 DISP "ICOU" "ADLC"
17 DI21 ELRF "ICOU" "ADLC"=RPROD(B11,B12)
!UX1=DS. 1,1,1 (17 DI21 ELRF "ICOU" "ADLC")
!UY1=DS. 2,1,1 (17 DI21 ELRF "ICOU" "ADLC")
!UZ1=DS. 3,1,1 (17 DI21 ELRF "ICOU" "ADLC")
!RX1=DS. 1,2,1 (17 DI21 ELRF "ICOU" "ADLC")
!RY1=DS. 2,2,1 (17 DI21 ELRF "ICOU" "ADLC")
!RZ1=DS. 3,2,1 (17 DI21 ELRF "ICOU" "ADLC")
!UX2=DS. 1,3,1 (17 DI21 ELRF "ICOU" "ADLC")
!UY2=DS. 2,3,1 (17 DI21 ELRF "ICOU" "ADLC")
!UZ2=DS. 3,3,1 (17 DI21 ELRF "ICOU" "ADLC")
!RX2=DS. 1,4,1 (17 DI21 ELRF "ICOU" "ADLC")
!RY2=DS. 2,4,1 (17 DI21 ELRF "ICOU" "ADLC")
!RZ2=DS. 3,4,1 (17 DI21 ELRF "ICOU" "ADLC")

"ICOU"

|L=DS. "ICOU",1,1 (13 LENG E21 0 0)
|XGP1=1.0-0. 57735#L=#0. 5
|XGP2=1.0+0. 57735#L=#0. 5
|A1=-1.0/L#UZ1
|A2=1.0/L#UZ2
|LUZ1=A1*A2
|LUZ2=A1*A2
|A1=1.0/L
|A2=A1*A1
|A3=A2*A1
|E1=-6.0*A2
|E2=12.0*XGP1*A3
|E3=4.0*A1
|E4=-6.0*XGP1*A2
*LABEL 155
!TEMP=FREE()
!DELB=FREE()
!DELI=FREE()

$!
!ICOU
!PSI
!DDEL
!HDEL
!A1=FREE()
!A2=FREE()
!E1=FREE()
!E2=FREE()
!L=FREE()
!SIG1=FREE()
!SIG2=FREE()
$

*LABEL 200
!TEMP=ICOU-DE21
*JNZ (TEMP, 100)
!ICOU=FREE()
!TEMP=FREE()
!UZ1=FREE()
!UZ2=FREE()
!WX1=FREE()
!WX2=FREE()
!WY1=FREE()
!WY2=FREE()
!E=FREE()
!B=FREE()
!H=FREE()
!UX1=FREE()
!UY1=FREE()
!UZ1=FREE()
!RX1=FREE()
!RY1=FREE()
!RZ1=FREE()
!UX2=FREE()
!UY2=FREE()
!UZ2=FREE()
!RX2=FREE()
!RY2=FREE()
!RZ2=FREE()
*RETURN

$-----------------------------------------------

$* (29 ADJC E=1 O O) ENDA021
$*XQT AUS
$
* COMPUTE AVERAGE ELEMENT STRESS

COMP=VMCS-VMC2
*JGZ(COMP, 444)
VMST=VMS2
P01N=2
SIB=-1.0
SIR=1.0
*LABEL 444
COMP=VMCS-VMC3
*JGZ(COMP, 445)
VMST=VMS3
P01N=3
SIB=-1.0
SIR=1.0
*LABEL 445
COMP=VMCS-VMC4
*JGZ(COMP, 446)
VMST=VMS4
P01N=4
SIB=1.0
SIR=-1.0
*LABEL 446
VMST
P01N
COMP=FREE()
VMS1=FREE()
VMS2=FREE()
VMS3=FREE()
VMS4=FREE()
VMC1=FREE()
VMC2=FREE()
VMC3=FREE()
VMC4=FREE()
VMCS=FREE()
SIG1=FREE()
SIG2=FREE()
XGP1=1.0-0.57735*L*0.5
XGP2=1.0+0.57735*L*0.5
AD3=1.0*F/L
AD9=1.0*F/L
A1=1.0/L
A2=A1*A1
A3=A2*A1
E1=-6.0*A2
E2=12.0*XGP1*A3
E3=-4.0*A1
E4=6.0*XGP1*A2
E5=E3+0.5
F6=12.0*XGP2*A3
F7=6.0*XGP2*A2
AD=2.0*E1+E2+E6/2.0
AD2=-AD+E*SIB+B/2.0
AD1=-AD+E*SIS*H/2.0
AD=-2.0*E1-E2-E6/2.0
AD8=-AD+E*SIB+B/2.0
AD7=-AD+E*SIS*H/2.0
AD2=2.0*E3+E4+E7/2.0
AD4=AD+E*SIB+B/2.0
AD5=-AD+E*SIS*H/2.0
AD2=2.0*E5+E4+E7/2.0
TABLE(NI=3,NJ=4): 17 ROTA AD21 "ADLC" 0
J=1: "AD1" "AD2" "AD3"
J=2: "AD4" "AD5" 0.0
J=3: "AD7" "AD8" "AD9"
J=4: "AD10" "AD11" 0.0
DEFINE B11=13 EL21 ROTA "JENT" 0
17 EL21 ROTT "JENT" 0=TRAN(B11)
DEFINE B12=17 EL21 ROTT "JENT" 0
DEFINE B11=17 ROTA AD21 "ADLC" 0
17 GLAD E21 "ADLC" 0=RPROD(B12,B11)
!AD1=DS. 1,1,1 (17 GLAD E21 "ADLC" 0)
!AD2=DS. 2,1,1 (17 GLAD E21 "ADLC" 0)
!AD3=DS. 3,1,1 (17 GLAD E21 "ADLC" 0)
!AD4=DS. 1,2,1 (17 GLAD E21 "ADLC" 0)
!AD5=DS. 2,2,1 (17 GLAD E21 "ADLC" 0)
!AD6=DS. 3,2,1 (17 GLAD E21 "ADLC" 0)
!AD7=DS. 1,3,1 (17 GLAD E21 "ADLC" 0)
!AD8=DS. 2,3,1 (17 GLAD E21 "ADLC" 0)
!AD9=DS. 3,3,1 (17 GLAD E21 "ADLC" 0)
!AD10=DS. 1,4,1 (17 GLAD E21 "ADLC" 0)
!AD11=DS. 2,4,1 (17 GLAD E21 "ADLC" 0)
!AD12=DS. 3,4,1 (17 GLAD E21 "ADLC" 0)
!P1=DS.13."JENT".1 (13 CONN E21 0 0)
!P2=DS.14."JENT".1 (13 CONN E21 0 0)
SYSEVE: APPL FORC "ADLC" 0
I=1
J="P1" : "AD1"
J="P2" : "AD7"
1=2
J="P1" : "AD2"
J="P2" : "AD8"
1=3
J="P1" : "AD3"
J="P2" : "AD9"
1=4
J="P1" : "AD4"
J="P2" : "AD10"
1=5
J="P1" : "AD5"
ORIGINAL PAGE IS
OF POOR QUALITY

J="P2" : "AD11"
I=0
J="P1" : "AD6"
J="P2" : "AD12"

!P1=FREE()
!P2=FREE()
!AD1=FREE()
!AD2=FREE()
!AD3=FREE()
!AD4=FREE()
!AD5=FREE()
!AD6=FREE()
!AD7=FREE()
!AD8=FREE()
!AD9=FREE()
!AD10=FREE()
!AD11=FREE()
!AD12=FREE()

*XQT U1
*SHOW
*XQT DCU
ERASE 17
*RETURN
*

ENDAD21

*(29 PREP E41 0 0) ENDPE41

% THIS SUBROUTINE IS FOR ELEMENT TYP E41
%
% THE PRINCIPAL LIBRARY IS 14
% TEMPORARY LIBRARY IS 15 CAN BE DELETED AT THE END OF THIS ROUTINE
%
*XQT EI1
%
EXTRACT: SOURCE=E41: CONTENT SPEC: GEOM 1 * AREA OF ELEMENTS
CREATE 14 AREA E41 0 0
%
EXTRACT: SOURCE=E41: CONTENT SPEC: GEOM 5,12 * LOCAL ELEM. REF. FRAME
CREATE 14 ELRF E41 0 0
%
EXTRACT: SOURCE=E41: CONTENT SPEC: GEOM 22,57 * ELEMENT-NODAL RELATION
CREATE 14 ENLO E41 0 0
%
EXTRACT: SOURCE=E41: CONTENT SPEC: MATE 1,2 * MATERIAL PROPERTIES
CREATE 14 MATP E41 0 0
%
EXTRACT: SOURCE=E41: CONTENT SPEC: INTE 1,16 * CONNECTIVITY
CREATE 14 CONN E41 0 0
%
EXTRACT: SOURCE=E41: CONTENT SPEC: S 1,25 * HINV T
CREATE 14 HIT E41 0 0
%
*XQT DCU
TDCC 14 CONN E41 0 0 : TYPE=0
*XQT U1
'ENT1=0.6220084
'ENT2=1.0/6.0
'ENT3=0.0446582
'TI(14 SHAP FUNC 1 1)
"ENT1" "ENT2" "ENT3" "ENT2"
"ENT2" "ENT1" "ENT2" "ENT3"
"ENT3" "ENT2" "ENT1" "ENT2"
"ENT2" "ENT3" "ENT2" "ENT1"
!ENT1=FREE()
!ENT2=FREE()
!ENT3=FREE()
$
$
*XGT AUS
!ICOU=1
!TEMP=0
*LABEL 300
!TEMP=FREE()
!B1=ICOU-1*B
DEFINE A1=14 ELRF E41 0 0
TABLE(NI=2, NJ=4):15 ELEM REFT "ICOU" 99
TRANSFER (SOURCE=A1, ILIM=8, SBASE="B1")
!B1=FREE()
DEFINE BQ1=15 ELEM REFT "ICOU" 99
14 ELEM REFE "ICOU" 0=RTRAN(BQ1)
$
$
* COMPUTE DETERMINANTS FOR INTEGRATION
!IL=1
'OP1=0.57735
*LABEL 55
'PXI=-1.0*OP1
'PET=-1.0*OP1
!TEMP=IL-2
*JNZ(TEMP, J5)
'PXI=-1.0*OP1
'PET=-1.0*OP1
*LABEL 55
!TEMP=IL-3
*JNZ(TEMP, J7)
'PXI=+1.0*OP1
'PET=+1.0*OP1
*LABEL 37
!TEMP=IL-4
*JNZ(TEMP, J9)
'PXI=-1.0*OP1
'PET=+1.0*OP1
*LABEL 39
'DUM1=-1.0+PET*0.25
'DUM2=-1.0+PXI*0.25
'DUM3=+1.0-PET*0.25
'DUM4=-1.0-PXI*0.25
'DUM5=+1.0+PET*0.25
'DUM6=-1.0+PXI*0.25
'DUM7=-1.0-PET*0.25
'DUM8=+1.0-PXI*0.25
TABLE(NI=2, NJ=4):15 HELP E41 "IL" 99
J=1: "DUM1" "DUM2"
J=2: "DUM3" "DUM4"
J=3: "DUM5" "DUM6"
J=4: "DUM7" "DUM8"
DEFINE E1=15 HELP E41 "IL" 99
DEFINE BB2=14 ELEM REFE "ICOU" 0
15 JAC E41 "ICOU" "IL"=RPRD (E1, BB2)
'DUM1=DS 1, 1, 1 (15 JAC E41 "ICOU" "IL")
'DUM2=DS 1, 2, 1 (15 JAC E41 "ICOU" "IL")
SECTION 41 "ICOU" 0

J=1: 1.0 0.0 0.0 1.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0 0.0 1.0
J=2: 0.0 1.0 0.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 0.0 1.0
J=3: 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 0.0
J=4: "YGP1" 0.0 0.0 "YGP2" 0.0 0.0 "YGP3" 0.0 0.0 "YGP4" 0.0 0.0
J=5: 0.0 "XGP1" 0.0 0.0 "XGP2" 0.0 0.0 "XGP3" 0.0 0.0 "XGP4" 0.0

! YGP1=FREE()
! YGP2=FREE()
! YGP3=FREE()
! YGP4=FREE()

! Table (NI=12, NJ=5): 14 PMAT E41 "ICOU" 0
J=1: 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 0.0 1.0
J=2: 0.0 1.0 0.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 0.0 1.0
J=3: 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 0.0
J=4: "YGP1" 0.0 0.0 "YGP2" 0.0 0.0 "YGP3" 0.0 0.0 "YGP4" 0.0 0.0
J=5: 0.0 "XGP1" 0.0 0.0 "XGP2" 0.0 0.0 "XGP3" 0.0 0.0 "XGP4" 0.0

! YGP1=FREE()
! YGP2=FREE()
! YGP3=FREE()
! YGP4=FREE()

! Table (NI=12, NJ=12): 14 SIEP E41 "ICOU" 0
J=1: 0.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=2: "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=3: 0.0 0.0 "NU1" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=4: 0.0 0.0 0.0 1.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=5: 0.0 0.0 0.0 0.0 "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=6: 0.0 0.0 0.0 0.0 0.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=7: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 "NU" 0.0 0.0 0.0 0.0
J=8: 0.0 0.0 0.0 0.0 0.0 "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0
J=9: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 "NU" 0.0 0.0 0.0
J=10: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 "NU" 0.0 0.0
J=11: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=12: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

! NU=DS 2, "ICOU", 1 (14 MATP E41 0 0)
! NUP1=NU+1.0
! NUP2=NU+2.0

! Table (NI=12, NJ=12): 14 SIEP E41 "ICOU" 0
J=1: 1.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=2: "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=3: 0.0 0.0 "NU1" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=4: 0.0 0.0 0.0 1.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=5: 0.0 0.0 0.0 0.0 "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=6: 0.0 0.0 0.0 0.0 0.0 "NU1" 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=7: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 "NU" 0.0 0.0 0.0 0.0
J=8: 0.0 0.0 0.0 0.0 0.0 "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0
J=9: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 "NU" 0.0 0.0 0.0
J=10: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 "NU" 0.0 0.0
J=11: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 0.0 0.0
J=12: 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

! NU=DS 2, "ICOU", 1 (14 MATP E41 0 0)
! NUP1=NU+1.0
! NUP2=NU+2.0

! Table (NI=3, NJ=1): 15 BBB E41 "ICOU" 0

! TEMP=FREE()
! B1=ICOU-1

DEFINE A2=ESB E41 1 1

TABLE (NI=3, NJ=1): 15 BBB E41 "ICOU" 0

DEFINE BB1=14 PMAT E41 "ICOU" 0
DEFINE BB2=15 BBB E41 "ICOU" 0

! SAGR=DS 8, "ICOU", 1 (14 CONN E41 0 0)
! OOTH=DS 26, "SAGR", 1 (1 SA BTAB 2 13)

14 QST E41 "ICOU" 0= RPROD ("OOTH" BB1, BB2)

! OOTH=FREE()
! SAGR=FREE()

! ICOU=ICOU+1
! TEMP=DE41-ICOU+1
! JCZ (TEMP, 200)
! B1=FREE()
! ICOU=FREE()

! TEMP=FREE()
!

! XGT DCU
ERASE 15

! RETURN
* (29 SEC A E41 0 0) ENDSEC41

* XQT AUS
* ICDU=1
* TEMP=0
* LABCL: 200
* ELGR=DS 1, "ICOU", 1 (1 ED41 REL 0 0)
* TEMP=ELGR-ICDV
* JNZ ((TEMP, 175)
* B1=ICDU-1#9
DEFINE A2=1 ESB E41 "ADLC" 1
TABLE(MI=5, NJ=1):15 BBB E41 "ICOU" "CLC"
TRANSFERN(SOURCE=A2, ILIM=5, SBASE="B1")
B1=FREE()

DEFINE BB1=14 PMAT E41 "ICOU" 0
DEFINE BB2=15 BBB E41 "ICOU" "CLC"
SACR=DS 8, "ICOU", 1 (14 CONN E41 0 0)
DOOTH=DS 26, "SAQR", 1 (1 SA BTAB 2 13)
15 G PST E41 "ICOU" "CLC" = RPROD ("DOOTH" BB1, BB2)
DOOTH=FREE()
SACR=FREE()

DEFINE B11=14 SIEP E41 "ICOU" 0
DEFINE B12=15 OPST E41 "ICOU" "CLC"
14 G PEP E41 "ICOU" "CLC" = RPROD (B11, B12)

'SI1 =DS 1,1,1 (14 OPST E41 "ICOU" 0)
'SI2 =DS 2,1,1 (14 OPST E41 "ICOU" 0)
'SI3 =DS 3,1,1 (14 OPST E41 "ICOU" 0)
'SI4 =DS 4,1,1 (14 OPST E41 "ICOU" 0)
'SI5 =DS 5,1,1 (14 OPST E41 "ICOU" 0)
'SI6 =DS 6,1,1 (14 OPST E41 "ICOU" 0)
'SI7 =DS 7,1,1 (14 OPST E41 "ICOU" 0)
'SI8 =DS 8,1,1 (14 OPST E41 "ICOU" 0)
'SI9 =DS 9,1,1 (14 OPST E41 "ICOU" 0)
'SI10=DS 10,1,1 (14 OPST E41 "ICOU" 0)
'SI11=DS 11,1,1 (14 OPST E41 "ICOU" 0)
'SI12=DS 12,1,1 (14 OPST E41 "ICOU" 0)
'EP1 =DS 1,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP2 =DS 2,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP3 =DS 3,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP4 =DS 4,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP5 =DS 5,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP6 =DS 6,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP7 =DS 7,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP8 =DS 8,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP9 =DS 9,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP10=DS 10,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP11=DS 11,1,1 (14 GPEP E41 "ICOU" "CLC")
'EP12=DS 12,1,1 (14 GPEP E41 "ICOU" "CLC")

'EP1=SI1*EP1
'EP2=SI2*EP2
'SI 122 = SI 122 = 0.5 * DE TE
'VMST = SI 13 + SI 6 + SI 9 + SI 12
'AREA = DS 1, "ICOU", 1 (15 AREA E41 0 0)
'VMST = VMST / AREA
'AREA = FREE()
'VMST

*  
'SI 21 = FREE()
'SI 54 = FREE()
'SI 187 = FREE()
'SI 01 = FREE()
'SI 1 = FREE()
'SI 2 = FREE()
'SI 3 = FREE()
'SI 4 = FREE()
'SI 5 = FREE()
'SI 6 = FREE()
'SI 7 = FREE()
'SI 8 = FREE()
'SI 9 = FREE()
'SI 10 = FREE()
'SI 11 = FREE()
'SI 12 = FREE()
'L11 = FREE()
'L12 = FREE()
'L13 = FREE()
'L14 = FREE()
'L15 = FREE()
'L16 = FREE()
'L17 = FREE()
'L18 = FREE()
'L19 = FREE()
'L20 = FREE()
'L21 = FREE()
'L22 = FREE()
'L23 = FREE()
'L24 = FREE()
'L25 = FREE()
'L26 = FREE()
'L27 = FREE()
'L28 = FREE()
'L29 = FREE()
'L30 = FREE()
'L31 = FREE()
'L32 = FREE()
'L33 = FREE()
'L34 = FREE()
'L35 = FREE()
'L36 = FREE()
'L37 = FREE()
'L38 = FREE()
'L39 = FREE()
'L40 = FREE()
'L41 = FREE()
*L42 = FREE()

* 

* LABEL 195
'ICOU = ICOU + 1
'TEMP = DE41 - ICOU + 1
*JGZ (TEMP, 200)
'B1 = FREE()
'ICOU = FREE()
'TEMP = FREE()

* 

*XQT U1
*SHOW
*RETURN
*

ENDSC41

* 

* (29 ADJO E41 0 0)

ENDAD41

* 

*XQT AUS
'B1 = JENT - 1 * 25
DEFINE A1 = 14 HIT E41 0 0
TABLE (NI = 5, NJ = 5): 15 HITE ELEM "ADLC" 1
TRANSFERR SOURCE = A1, ILIM = 25, SBASE = "B1"
'B1 = FREE()
DEFINE MATRIX A
A21=DS 3, "JENT", 1 (14 ELRF E41 0 0)
A31=DS 5, "JENT", 1 (14 ELRF E41 0 0)
A32=DS 6, "JENT", 1 (14 ELRF E41 0 0)
A41=DS 7, "JENT", 1 (14 ELRF E41 0 0)
A42=DS 8, "JENT", 1 (14 ELRF E41 0 0)
E22=-1.0 *X32/X21
E32=X32/X21
E33=X31/X21
E42=-1.0 *X42/X21
E44=X42/X21
E52=X41/X21-1.0
E54=-X41/X21
\*XGT U1
*TI (15 AMAT E41 "ADLC" 0)
-1.0 -1.0 0.0 0.0
0.0 "EN22" "EN32" "EN42" "EN52"
1.0 0.0 0.0 0.0
0.0 "EN24" "EN34" "EN44" "EN54"
0.0 1.0 0.0 0.0
0.0 0.0 1.0 0.0
0.0 0.0 0.0 1.0
0.0 0.0 0.0 0.0
X21=FREE()
X31=FREE()
X32=FREE()
X41=FREE()
X42=FREE()
EN22=FREE()
EN24=FREE()
EN33=FREE()
EN34=FREE()
EN44=FREE()
EN42=FREE()
EN44=FREE()
EN54=FREE()
\$\n\$\nXGT AUS
DEFINE BAG=15 HITE ELEM "ADLC" 1
DEFINE BAW=15 AMAT E41 "ADLC" 0
15 HITF E41 "ADLC" 0=RPROD(BAG, BAW)
DEFINE BAG=14 PMAT E41 "JENT" 0)
DEFINE BAW=15 HITF E41 "ADLC" 0)
!SAGR=DS 8, "JENT", 1 (14 CONN E41 0 0)
!OOTH=DS 26, "SAGR", 1 (1 SA BTAB 2 13)
15 PHTF E41 "ADLC" 0=RPROD ("OOTH" BAG, BAW)
!OOTH=FREE()
!SAGR=FREE()
!IDOF =1
!LABEL 400
!IDOF
S11 =DS 1, 1, 1 (14 GPST E41 "JENT" 0)
S12 =DS 2, 1, 1 (14 GPST E41 "JENT" 0)
S13 =DS 3, 1, 1 (14 GPST E41 "JENT" 0)
S14 =DS 4, 1, 1 (14 GPST E41 "JENT" 0)
S15 =DS 5, 1, 1 (14 GPST E41 "JENT" 0)
S16 =DS 6, 1, 1 (14 GPST E41 "JENT" 0)
S17 =DS 7, 1, 1 (14 GPST E41 "JENT" 0)
107

\[ \begin{align*}
\text{SI6} &= \text{SI6} \ast 0.5 \\
\text{SI9} &= \text{SI9} \ast 3.0 + \text{SI7} + \text{SI8} - \text{SI8} \\
\text{SI9} &= \text{SI9} \ast 0.5 \\
\text{SI12} &= \text{SI12} \ast 3.0 + \text{SI10} + \text{SI11} - \text{SI1} \\
\text{SI12} &= \text{SI12} \ast 0.5
\end{align*} \]

\$\$
\text{VM1} = \text{VM1} / \text{SI3} \ast \text{EP1}
\text{VM2} = \text{VM2} / \text{SI3} \ast \text{EP2}
\text{VM3} = \text{VM3} / \text{SI3} \ast \text{EP3}
\text{VM4} = \text{VM4} / \text{SI4} \ast \text{EP4}
\text{VM5} = \text{VM5} / \text{SI5} \ast \text{EP5}
\text{VM6} = \text{VM6} / \text{SI6} \ast \text{EP6}
\text{VM7} = \text{VM7} / \text{SI7} \ast \text{EP7}
\text{VM8} = \text{VM8} / \text{SI8} \ast \text{EP8}
\text{VM9} = \text{VM9} / \text{SI9} \ast \text{EP9}
\text{VM10} = \text{VM10} / \text{SI12} \ast \text{EP10}
\text{VM11} = \text{VM11} / \text{SI12} \ast \text{EP11}
\text{VM12} = \text{VM12} / \text{SI12} \ast \text{EP12}
\$

\$\$
\text{DET} = \text{DS 1, 1, 1} \quad (14 \text{ DETE E41 "JENT" 0})
\text{VM3} = \text{VM3} + \text{VM2} + \text{VM1} \ast \text{DET}
\text{DET} = \text{DS 2, 1, 1} \quad (14 \text{ DETE E41 "JENT" 0})
\text{VM6} = \text{VM6} + \text{VM5} + \text{VM4} \ast \text{DET}
\text{DET} = \text{DS 3, 1, 1} \quad (14 \text{ DETE E41 "JENT" 0})
\text{VM9} = \text{VM9} + \text{VM8} + \text{VM7} \ast \text{DET}
\text{DET} = \text{DS 4, 1, 1} \quad (14 \text{ DETE E41 "JENT" 0})
\text{VM12} = \text{VM12} + \text{VM11} + \text{VM10} \ast \text{DET}
\text{ARE} = \text{DS 1, "JENT", 1} \quad (14 \text{ AREA E41 0 0})
\text{VM12} = \text{VM12} + \text{VM9} + \text{VM6} + \text{VM3}
\text{VM12} = \text{VM12} / \text{AREA}
\text{AREA} = \text{FREE()}
\$

\$\$
\text{TEM} = \text{IDDF} - 1
\text{JNZ([TEM2, 100])}
\text{AD} = 1.0 \ast \text{VM12}
\text{JUMP 300}
\text{LABEL 100}
\text{TEM} = \text{IDDF} - 2
\text{JNZ([TEM2, 110])}
\text{AD} = 1.0 \ast \text{VM12}
\text{JUMP 300}
\text{LABEL 110}
\text{TEM} = \text{IDDF} - 3
\text{JNZ([TEM2, 120])}
\text{AD} = 1.0 \ast \text{VM12}
\text{JUMP 300}
\text{LABEL 120}
\text{TEM} = \text{IDDF} - 4
\text{JNZ([TEM2, 130])}
\text{AD} = 1.0 \ast \text{VM12}
\text{JUMP 300}
\text{LABEL 130}
\text{TEM} = \text{IDDF} - 5
\text{JNZ([TEM2, 140])}
\text{AD} = 1.0 \ast \text{VM12}
\text{JUMP 300}
\text{LABEL 140}
\text{TEM} = \text{IDDF} - 6
\text{JNZ([TEM2, 150])}
\text{AD} = 1.0 \ast \text{VM12}
**NOW ASSIGN LOAD TO PROPER DEGREES OF FREEDOM**

!VMST

DEFINE BB2=14 ELNO E41 0 0

TABLE(NI=12, NJ=12): 15 ROTA ELEM "ADLC" 99

: A1=JENT-1*36
'B1=0

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+3
'B1=12

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+6
'B1=24

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+9
'B1=36+3

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+12
'B1=48+3

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+15
'B1=60+3

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+18
'B1=72+6

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+21
'B1=84+6

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+24
'B1=96+6

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+27
'B1=108+9

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+30
'B1=120+9

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=JENT-1*36+33
'B1=132+9

TRANSFR (SOURCE=BB2, ILIM=3, SBASE="A1", DBASE="B1")

'A1=FREE()

'B1=FREE()

TABLE(NI=12, NJ=1): 15 ELL0 E41 "ADLC" 1

J-1: "AD1" "AD2" 0. 0 "AD3" "AD4" 0. 0 "AD5" "AD6" 0. 0 "AD7" "AD8" 0. 0

DEFINE A1 = 15 ROTA ELEM "ADLC" 99

DEFINE BB2 = 15 ELL0 E41 "ADLC" 1

DEFINE A1 = 15 ROTA ELEM "ADLC" 1

14 ADLO VE41 "ADLC" 0 = RPROD ( A1, BB2)

'AD1=DS 1. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD2=DS 2. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD3=DS 3. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD4=DS 4. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD5=DS 5. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD6=DS 6. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD7=DS 7. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD8=DS 8. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD9=DS 9. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD10=DS 10. 1. 1 (14 ADLO VE41 "ADLC" 0)

'AD11=DS 11. 1. 1 (14 ADLO VE41 "ADLC" 0)
110

(AD12=DS 12, 1, 1) (14 ADLO VE41 "ADLC" 0)
J1=DS 13, "JENT", 1 (14 CONN E41 0 0)
J2=DS 14, "JENT", 1 (14 CONN E41 0 0)
J3=DS 15, "JENT", 1 (14 CONN E41 0 0)
J4=DS 16, "JENT", 1 (14 CONN E41 0 0)

SYSVEC: APPL FORC "ADLC"]
I=1
J arsen "J1": "AD1"
J arsen "J2": "AD4"
J arsen "J3": "AD7"
J arsen "J4": "AD10"
I=2
J arsen "J1": "AD2"
J arsen "J2": "AD5"
J arsen "J3": "AD8"
J arsen "J4": "AD11"
I=3
J arsen "J1": "AD3"
J arsen "J2": "AD6"
J arsen "J3": "AD9"
J arsen "J4": "AD12"
AD1=FREE()
AD2=FREE()
AD3=FREE()
AD4=FREE()
AD5=FREE()
AD6=FREE()
AD7=FREE()
AD8=FREE()
AD9=FREE()
AD10=FREE()
AD11=FREE()
AD12=FREE()
J1=FREE()
J2=FREE()
J3=FREE()
J4=FREE()

*XGF DCU
ERASE 13
*RETURN

*--------------------------------------------------------

(29 PREP E42 0 0) ENDPE42

THIS SUBROUTINE IS FOR ELEMENT TYP E42
THE PRINCIPAL LIBRARY IS 16
TEMPORARY LIBRARY IS 17 CAN BE DELETED AT THE END OF THIS ROUTINE
*XGF E11

EXTRACT: SOURCE=E42: CONTENT SPEC: GEOM 1 & AREA OF ELEMENTS
CREATE 16 AREA E42 0 0

EXTRACT: SOURCE=E42: CONTENT SPEC: GEOM 5, 12 & LOCAL ELEM. REF. FRAME
CREATE 16 ELNF E42 0 0
CREATE 16 ELND E42 0 0
*EXTRACT: SOURCE=E42: CONTENT SPEC: GEOM 22, 57 $ ELEMENT-NODAL RELATION
*EXTRACT: SOURCE=E42: CONTENT SPEC: MATE 1, 2 $ MATERIAL PROPERTIES
*EXTRACT: SOURCE=E42: CONTENT SPEC: INTE 1, 16 $ CONNECTIVITY
*XGT DCU
TUCC 16 CONN E42 0 0 : TYPE=0
*XGT U1
'ENT1=0.6220084
'ENT2=0.076
'ENT3=0.0446592
*TI(16 SHAP FUNC 1 1)
"ENT1" "ENT2" "ENT3" "ENT2"
"ENT2" "ENT1" "ENT2" "ENT3"
"ENT3" "ENT2" "ENT1" "ENT2"
"ENT2" "ENT3" "ENT2" "ENT1"
'ENT1=FREE()
'ENT2=FREE()
'ENT3=FREE()
*
*XGT AUS
!ICOU=1
!TEMP=0
#LABEL 200
!TEMP=FREE()
'B1=ICOU-1*B1
DEFINE A1=16 ELRF E42 0 0
TABLE(NI=2, NJ=4): 17 ELEM REFT "ICOU" 99
TRANSFER (SOURCE=A1, ILIM=8, SBASE="B1")
'B1=FREE()
DEFINE BG1=17 ELEM REFT "ICOU" 99
16 ELEM RF E "ICOU" 0=RTRAN(BG1)
*
$ COMPUTE DETERMINANTS FOR INTEGRATION

"IL=1
'GP1=0.57735
#LABEL 35
'PX1=-1.0*GP1
'PET=-1.0*GP1
!TEMP=IL-2
#JNZ(TEMP, 35)
'PX1=-1.0*GP1
'PET=-1.0*GP1
#LABEL 35
'TEMP=IL-3
#JNZ(TEMP, 37)
'PX1=-1.0*GP1
'PET=-1.0*GP1
#LABEL 37
!TEMP=IL-4
#JNZ(TEMP, 39)
'PX1=-1.0*GP1
'PET=-1.0*GP1
#LABEL 39
'DUM1=-1.0*PET*0.25
'DUM2=-1.0*PX1*0.25
Table(NI=2,NJ=4): 17 HELP E42 "IL" 99
J=1   "DUM1" "DUM3"
J=2   "DUM3" "DUM4"
J=3   "DUM5" "DUM6"
J=4   "DUM7" "DUMB"
DEFINE E1=17 HCLP E42 "IL" 99
DEFINE BB2=16 ELEM REFE "ICOU" 0
17 JAC E42 "ICOU" "IL"=RPRD (E1, BB2)
!DUM1=DS 1.1.1 (17 JAC E42 "ICOU" "IL")
!DUM2=DS 1.2.1 (17 JAC E42 "ICOU" "IL")
!DUM3=DS 2.1.1 (17 JAC E42 "ICOU" "IL")
!DUM4=DS 2.2.1 (17 JAC E42 "ICOU" "IL")
!DUM5=DUM1*DUM4
!DUM6=DUM2*DUM3
!DUMB=DUM5-DUM6
!DUMB
!TEMP=IL-1
*JNZ(TEMP, 45)
!DET1=DUMB
*JUMP 53
*LABEL 45
!TEMP=IL-2
*JNZ(TEMP, 47)
!DET2=DUMB
*JUMP 53
*LABEL 47
!TEMP=IL-3
*JNZ(TEMP, 49)
!DET3=DUMB
*JUMP 53
*LABEL 49
!TEMP=IL-4
*JNZ(TEMP, 53)
!DET4=DUMB
*LABEL 53
!IL=IL+1
!TEMP=IL-5
*JNZ(TEMP, 55)
Table(NI=4,NJ=1): 16 DETE E42 "ICOU" 0
J=1   "DET1" "DET2" "DET3" "DET4"
!DET1=FREE()
!DET2=FREE()
!DET3=FREE()
!DET4=FREE()
!DUM1=FREE()
!DUMB=FREE()
!DUM3=FREE()
!DUM4=FREE()
!DUM5=FREE()
!DUM6=FREE()
!DUM7=FREE()
!DUM8=FREE()
!IL=FREE()
!PX1=FREE()
I) SHIFT 16 SHAPE FUNC 1 1
DEFINE BB1=16 ELEM REFE "ICOU" 0
17 GAUS POIN "ICOU" 0= RPROD (BB1, BB2)

\$ XGP1=DS 1.1.1 (17 GAUS POIN "ICOU" 0)
\$ YGP1=DS 1.2.1 (17 GAUS POIN "ICOU" 0)
\$ XGP2=DS 2.1.1 (17 GAUS POIN "ICOU" 0)
\$ YGP2=DS 2.2.1 (17 GAUS POIN "ICOU" 0)
\$ XGP3=DS 3.1.1 (17 GAUS POIN "ICOU" 0)
\$ YGP3=DS 3.2.1 (17 GAUS POIN "ICOU" 0)
\$ XGP4=DS 4.1.1 (17 GAUS POIN "ICOU" 0)
\$/ YGP4=DS 4.2.1 (17 GAUS POIN "ICOU" 0)
\$ XYG1=XCPI1*YGPI
\$ XYG2=XCPI2*YGPI
\$ XYG3=XCPI3*YGPI
\$ XYG4=XCPI4*YGPI

\$ TABLE (NI=12, NJ=11): 16 PMAT E42 "ICOU" 0
J=1 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0
J=2 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0
J=3 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0 0.0 0.0 1.0
J=4 "XGP1" 0.0 0.0 "XGP2" 0.0 0.0 "XGP3" 0.0 0.0 "XGP4" 0.0 0.0 0.0 0.0
J=5 "YGP1" 0.0 0.0 "YGP2" 0.0 0.0 "YGP3" 0.0 0.0 "YGP4" 0.0 0.0 0.0 0.0
J=6 "XYG1" 0.0 0.0 "XYG2" 0.0 0.0 "XYG3" 0.0 0.0 "XYG4" 0.0 0.0 0.0 0.0
J=7 "XGP1" 0.0 0.0 "XGP2" 0.0 0.0 "XGP3" 0.0 0.0 "XGP4" 0.0 0.0 0.0 0.0
J=8 "YGP1" 0.0 0.0 "YGP2" 0.0 0.0 "YGP3" 0.0 0.0 "YGP4" 0.0 0.0 0.0 0.0
J=9 "XYG1" 0.0 0.0 "XYG2" 0.0 0.0 "XYG3" 0.0 0.0 "XYG4" 0.0 0.0 0.0 0.0
J=10 "XGP1" 0.0 0.0 "XGP2" 0.0 0.0 "XGP3" 0.0 0.0 "XGP4" 0.0 0.0 0.0 0.0
J=11 "YGP1" 0.0 0.0 "YGP2" 0.0 0.0 "YGP3" 0.0 0.0 "YGP4" 0.0 0.0 0.0 0.0

\$ NU=DS 2, "ICOU", 1 (16 MATP E42 0 0)
\$ NU=1.0 NU1=1.0
\$ NU=-1.0*NU

\$ TABLE (NI=12, NJ=12): 16 SIGEP E42 "ICOU" 0
J=1 1.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=2 "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=3 0.0 0.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=4 0.0 0.0 0.0 1.0 "NU" 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=5 0.0 0.0 0.0 "NU" 1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=6 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=7 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 1.0 "NU" 0.0 0.0 0.0 0.0
J=8 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=9 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
J=10 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

ORIGINAL PAGE IS
OF POOR QUALITY
J=11: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 "NU" 1 0 0 0
J=12: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 "NUP1"
'NU'=FREE()
'NUP1'=FREE()
$edefine A2=1 ESB E42 11
DEFINE A2=1 ESB E42 11
TABLE(NI=11, NJ=11):17 BBB E42 "ICOU" 0
TRANSFERR(SOURCE=A2, ILIM=11, SBASE="B1")
'B1'=FREE()
$edefine BB1=16 PMAT E42 "ICOU" 0
DEFINE BB1=16 PMAT E42 "ICOU" 0
'SAGR=DS 8,"ICOU", 1 (16 CONN E42 0 0)
'SOTH=DS 26,"SAGR", 1 (1 SA BTAB 2 13)
'SOTH=0OTH*OOTH*6.0
16 GIST E42 "ICOU" O= RPROD ("OOTH" BB1, BB2)
'OOTH'=FREE()
'SAGR'=FREE()
$edefine ICOU=ICOU+1
'TEMP=DE42-ICOU+1
*JGZ (TEMP, 200)
'B1'=FREE()
'ICOU'=FREE()
'TEMP'=FREE()
$
*ERASE DCU
*RETURN
*

ENDPE42

$(29 SECA E42 0 0) ENDS42
$
*edefine AUS
$edefine ICOU=1
'TEMP=0
*LABEL 200
$edefine ELGR=DS 1,"ICOU", 1 (1 ED42 REL 0 0)
'TEMP=ELGR-ICDV
'ELGR
'ICDV
'ELGR'=FREE()
*JNZ (TEMP, 175)
$
'B1=ICOU-1=11
DEFINE A2=1 ESB E42 "ADLC" 1
TABLE(NI=11, NJ=11):17 BBB E42 "ICOU" "CLC"
TRANSFERR(SOURCE=A2, ILIM=11, SBASE="B1")
'B1'=FREE()
$
DEFINE BB2=17 BBB E42 "ICOU" "CLC"
'SAGR=DS 8,"ICOU", 1 (16 CONN E42 0 0)
'SOTH=DS 26,"SAGR", 1 (1 SA BTAB 2 13)
'OOTH=OOTH*OOTH*6.0


17 GPST E42 "ICOU" "CLC" = RPROD ("OOTH" BB1, BB2)
"OOTH=FREE()
"SAGR=FREE()

$ DEFINE B11=16 SIEP E42 "ICOU" 0
DEFINE B12=17 GPST E42 "ICOU" "CLC"
16 GPEP E42 "ICOU" "CLC"= RPROD (B11,B12) $

! $I1 =DS 1,1,1 (16 GPST E42 "ICOU" 0)
! $I2 =DS 2,1,1 (16 GPST E42 "ICOU" 0)
! $I3 =DS 3,1,1 (16 GPST E42 "ICOU" 0)
! $I4 =DS 4,1,1 (16 GPST E42 "ICOU" 0)
! $I5 =DS 5,1,1 (16 GPST E42 "ICOU" 0)
! $I6 =DS 6,1,1 (16 GPST E42 "ICOU" 0)
! $I7 =DS 7,1,1 (16 GPST E42 "ICOU" 0)
! $I8 =DS 8,1,1 (16 GPST E42 "ICOU" 0)
! $I9 =DS 9,1,1 (16 GPST E42 "ICOU" 0)
! $I10=DS 10,1,1 (16 GPST E42 "ICOU" 0)
! $I11=DS 11,1,1 (16 GPST E42 "ICOU" 0)
! $I12=DS 12,1,1 (16 GPST E42 "ICOU" 0)

! $EP1 =DS 1,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP2 =DS 2,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP3 =DS 3,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP4 =DS 4,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP5 =DS 5,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP6 =DS 6,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP7 =DS 7,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP8 =DS 8,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP9 =DS 9,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP10=DS 10,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP11=DS 11,1,1 (16 GPEP E42 "ICOU" "CLC")
! $EP12=DS 12,1,1 (16 GPEP E42 "ICOU" "CLC")

! $I13=-1.0*$I13
! $I16=-1.0*$I16
! $I19=-1.0*$I19
! $EP1=-1.0*$EP1
! $I12=-1.0*$I12
! $EP12=-1.0*$EP12

! $EP1=$I11*EP1
! $EP2=$I12*EP2
! $EP3=2.0*$I13*EP3
! $DETE=DS 1,1,1 (16 DETE E42 "ICOU" 0)
! $EP5=$I15*EP5
! $EP6=2.0*$I16*EP6
! $DETE=DS 2,1,1 (16 DETE E42 "ICOU" 0)
! $EP7=$I17*EP7
! $EP8=$I18*EP8
! $EP9=2.0*$I19*EP9
! $DETE=DS 3,1,1 (16 DETE E42 "ICOU" 0)
! $EP10=$I10*EP10
! $EP11=$I11*EP11
! $DETE=DS 4,1,1 (16 DETE E42 "ICOU" 0)
!PSIS=1.0*0.0
!PSI
!NU=DS 1, "ICOU", 1 (16 MATP E42 0 0)
!PSIS=.:PSIS/11
!PSI=PSI+PSIS
!ICOU
!NU=FREE()
!SAQR=FREE()
!DOOH=FREE()

$!
S121=S12*S11
S154=S15*S14
S187=S18*S17
S101=S110*S11
!
S11=S11*S11
S12=S12*S12
S13=S13*S13
S14=S14*S14
S15=S15*S15
S16=S16*S16
S17=S17*S17
S18=S18*S18
S19=S19*S19
S110=S110*S110
S111=S111*S111
S112=S112*S112
!
S13=S13*3.0+S11+S12-S121
!DETE=DS 1.1.1 (16 DETE E42 "ICOU" 0)
S13=S13**0.5*DETE
S16=S16*3.0+S14+S15-S154
!DETE=DS 2.1.1 (16 DETE E42 "ICOU" 0)
S16=S16**0.5*DETE
S19=S19*3.0+S17+S18-S187
!DETE=DS 3.1.1 (16 DETE E42 "ICOU" 0)
S19=S19**0.5*DETE
S112=S112*3.0+S110+S111-S101
!DETE=DS 4.1.1 (16 DETE E42 "ICOU" 0)
S112=S112**0.5*DETE
VHST=S13+S16+S19+S112
!AREA=DS 1, "ICOU", 1 (16 AREA E42 0 0)
!VMST=VMST/AREA
!VHST
!TEMP=CLC-CCOM-CDIS-CS21-CS41-1
!JLZ(TEMP, 155)
!TEMP=TEMP+1-CS42
!JOZ(TEMP, 155)
!TEMP=CLC-CCOM-CDIS-CS21-CS41
!JENT=DS "TEMP", 1, 1 (1 ST42 LIST 1 1)
!JENT
!TEMP=ICOU-JENT
!JENT=FREE()
!JNZ(TEMP, 155)

$!
SAQR=DS 8, "ICOU", 1 (16 CONN E42 0 0)
DOOH=DS 26, "SAQR", 1 (1 SA BTAB 2 13)
!PSI
PSIS = VMST * OOTH + PSIS
PSI = VMST * OOTH + PSI
PSIS
PSIS
SA = FREE()
OOTH = FREE()
DELT = FREE()

# LABEL 159
TEMP = FREE()

# PSIS
PSIS = FREE()
SIG2 = FREE()
SIG4 = FREE()
SIG7 = FREE()
SIG1 = FREE()
TEM1 = FREE()
SIG11 = FREE()
SIG12 = FREE()
SIG13 = FREE()
SIG14 = FREE()
SIG15 = FREE()
SIG17 = FREE()
SIG18 = FREE()
SIG19 = FREE()
SIG10 = FREE()
SIG11 = FREE()
SIG12 = FREE()
EP1 = FREE()
EP2 = FREE()
EP3 = FREE()
EP4 = FREE()
EP5 = FREE()
EP6 = FREE()
EP7 = FREE()
EP8 = FREE()
EP9 = FREE()
EP10 = FREE()
EP11 = FREE()
DETE = FREE()
AREA = FREE()

# LABEL 199
ICOU = ICOU + 1
TEMP = DE42 - ICOU + 1
JG7 (TEMP, 200)
B1 = FREE()
ICOU = FREE()
TEMP = FREE()

# GT U1
SHOW
RETURN

ENDSC42
IQRIGINAL PAGE IS OF POOR QUALITY
DEFINE A1=17 AROW E42 1 0
DEFINE A2=17 AROW E42 2 0
DEFINE A3=17 AROW E42 3 0
DEFINE A4=17 AROW E42 4 0
DEFINE A5=17 AROW E42 5 0
DEFINE A6=17 AROW E42 6 0
DEFINE A7=17 AROW E42 7 0
DEFINE A8=17 AROW E42 8 0
DEFINE A9=17 AROW E42 9 0
DEFINE A10=17 AROW E42 10 0
DEFINE A11=17 AROW E42 11 0
DEFINE A12=17 AROW E42 12 0
TABLE(N1=12, N1=12) 16 BB E42 "ADLC" 99
TRANSFER(SOURCE=A1, ILIM=12, DBASE=0)
TRANSFER(SOURCE=A2, ILIM=12, DBASE=12)
TRANSFER(SOURCE=A3, ILIM=12, DBASE=24)
TRANSFER(SOURCE=A4, ILIM=12, DBASE=36)
TRANSFER(SOURCE=A5, ILIM=12, DBASE=48)
TRANSFER(SOURCE=A6, ILIM=12, DBASE=60)
TRANSFER(SOURCE=A7, ILIM=12, DBASE=72)
TRANSFER(SOURCE=A8, ILIM=12, DBASE=84)
TRANSFER(SOURCE=A9, ILIM=12, DBASE=96)
TRANSFER(SOURCE=A10, ILIM=12, DBASE=108)
TRANSFER(SOURCE=A11, ILIM=12, DBASE=120)
TRANSFER(SOURCE=A12, ILIM=12, DBASE=132)
DEFINE A2=16 BB E42 "ADLC" 99
16 BB E42 "ADLC" 0=RTRAN(A2)

DEFINE A1=17 DMAT E42 "ADLC" 0
DEFINE A2=16 BB E42 "ADLC" 0
17 DB E42 "ADLC" 0=RPROD(A1,A2)

IDOF=1

LABEL 400
'S11 =DS 1,1,1 (16 GPST E42 "JENT" 0)
'S12 =DS 2,1,1 (16 GPST E42 "JENT" 0)
'S13 =DS 3,1,1 (16 GPST E42 "JENT" 0)
'S14 =DS 4,1,1 (16 GPST E42 "JENT" 0)
'S15 =DS 5,1,1 (16 GPST E42 "JENT" 0)
'S16 =DS 6,1,1 (16 GPST E42 "JENT" 0)
'S17 =DS 7,1,1 (16 GPST E42 "JENT" 0)
'S18 =DS 8,1,1 (16 GPST E42 "JENT" 0)
'S19 =DS 9,1,1 (16 GPST E42 "JENT" 0)
'S110=DS 10,1,1 (16 GPST E42 "JENT" 0)
'S111=DS 11,1,1 (16 GPST E42 "JENT" 0)
'S112=DS 12,1,1 (16 GPST E42 "JENT" 0)
'EP1 =DS 1, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP2 =DS 2, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP3 =DS 3, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP4 =DS 4, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP5 =DS 5, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP6 =DS 6, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP7 =DS 7, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP8 =DS 8, "IDOF", 1 (17 DB E42 "ADLC" 0)
'EP9 =DS 9, "IDOF", 1 (17 DB E42 "ADLC" 0)
'LPI1=DS 10, "IDOF", 1 (17 DB E42 "ADLC" 0)
'LPI1=DS 11, "IDOF", 1 (17 DB E42 "ADLC" 0)
'LPI1=DS 12, "IDOF", 1 (17 DB E42 "ADLC" 0)
!VM6 = VM6/SI6
!VM7 = VM7/SI7
!VM8 = VM8/SI8
!VM9 = VM9/SI9
!VM10 = VM10/SI10
!VM11 = VM11/SI12
!VM12 = VM12/SI12
!VM1 = VM1*EP1
!VM2 = VM2*EP2
!VM3 = VM3*EP3
!VM4 = VM4*EP4
!VM5 = VM5*EP5
!VM6 = VM6*EP6
!VM7 = VM7*EP7
!VM8 = VM8*EP8
!VM9 = VM9*EP9
!VM10 = VM10*EP10
!VM11 = VM11*EP11
!VM12 = VM12*EP12

!DET1 = DS 1.1.1 (16 DETE E42 "JENT" 0)
!DET2 = VM2*VM3+VM1*DET2
!DET3 = DS 2.1.1 (16 DETE E42 "JENT" 0)
!DET4 = VM6*VM2+VM4*DET4
!DET5 = DS 3.1.1 (16 DETE E42 "JENT" 0)
!DET6 = VM9*VM5+VM7*DET6
!DET7 = DS 4.1.1 (16 DETE E42 "JENT" 0)
!DET8 = VM2+VM1+VM10*DET8

!AREA = DS 1. "JENT".1 (16 AREA E42 0 0)
! indust = DS 3. "JENT".1 (16 DCON E42 0 0)

!DO1 = DS 26. "SAOR", 1 (1 SA (TAB 2 13)
!VM12 = VM12*VM4+VM6+VM3
!VM12 = VM12/AREA
!VM12 = 0.5*VM12
!VM12 = VM12/DO1
!ARA = FREE()
!SAGR = FREE()
!OUT1 = FREE()

!TEM2 = IDOF-1
!JNZ (TEM2, 100)
!AD1 = 1.0*VM12
!JUMP 300
!LABEL 100
!TEM2 = IDOF-2
!JNZ (TEM2, 110)
!AD2 = 1.0*VM12
!JUMP 300
!LABEL 110
!TEM2 = IDOF-3
!JNZ (TEM2, 120)
!AD3 = 1.0*VM12
!JUMP 300
!LABEL 120
!TEM2 = IDOF-4
!JNZ (TEM2, 130)
!AD4 = 1.0*VM12
!JUMP 300
!LABEL 130
!TEM2 = IDOF-5
!JNZ (TEM2, 140)
ORIGINAL PAGE IS OF POOR QUALITY
HOW ASSIGN LOAD TO PROPER DEGREES OF FREEDOM

VMST

DEFINE BB2=16 CLNO E42 O 0

TABLE (NI=24, NJ=24): 17 ROTA ELEM "ADLC" 99

\[ A1 = \text{JENT} - 1 + 36 \]

\[ B1 = 0 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 3 \]

\[ B1 = 24 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 6 \]

\[ B1 = 48 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 9 \]

\[ B1 = 72 + 3 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 12 \]

\[ B1 = 96 + 3 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 15 \]

\[ B1 = 120 + 3 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 18 \]

\[ B1 = 144 + 6 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 21 \]

\[ B1 = 168 + 6 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 24 \]

\[ B1 = 192 + 6 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")

\[ A1 = \text{JENT} - 1 + 36 + 27 \]

\[ B1 = 216 + 9 \]

TRANSFER (SOURCE=DB2, ILIM=3, SBASE="A1", DBASE="B1")
'A1=JENT-1*36+30
'B1=240+9
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+3
'B1=264+9
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36
'B1=288+12
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+3
'B1=312+12
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+6
'B1=336+12
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+9
'B1=360+15
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+12
'B1=384+15
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+15
'B1=408+15
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+18
'B1=432+18
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+21
'B1=456+18
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+24
'B1=480+18
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+27
'B1=504+21
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+30
'B1=528+21
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=JENT-1*36+33
'B1=552+21
TRANSFER (SOURCE='BB2', ILIM=3, SBASE='A1', DBASE='B1')
'A1=FREE()
'B1=FREE()
TABLE (NI=1, NJ=24): 17 ELD E42 "ADLC" 99
J=1 : 0.0
J=2 : 0.0
J=3 : "A03"
J=4 : "A03"
J=5 : "A03"
J=6 : 0.0
J=7 : 0.0
J=8 : 0.0
J=9 : "A04"
J=10 : "A05"
J=11 : "A06"
J=12 : 0.0
J=13 : 0.0
J=14 : 0.0
J=15 : "A07"
127

```
J=16 : "AIRB"
J=17 : "AIR9"
J=18 : 0.0
J=19 : 0.0
J=20 : 0.0
J=21 : "AIR0"
J=22 : "AIR1"
J=23 : "AIR2"
J=24 : 0.0

DEFINE A1 = 17 ELLO E42 "ADLC" 99
17 ELLO E42 "ADLC" 1 =RTRAN ( A1 )

DEFINE A1 = 17 ROTA ELEM "ADLC" 99
17 ROTA ELEM "ADLC" 1 =RTHAN ( A1 )

DEFINE BB2 = 17 ELLO E42 "ADLC" 1
DEFINE A1 = 17 ROTA ELEM "ADLC" 1

16 ADLO VE42 "ADLC" 0 = RPROD ( A1, BB2 )
'AD1=DS 1,1,1 (16 ADLO VE42 "ADLC" 0)
'AD2=DS 2,1,1 (16 ADLO VE42 "ADLC" 0)
'AD3=DS 3,1,1 (16 ADLO VE42 "ADLC" 0)
'AD4=DS 7,1,1 (16 ADLO VE42 "ADLC" 0)
'AD5=DS 8,1,1 (16 ADLO VE42 "ADLC" 0)
'AD6=DS 9,1,1 (16 ADLO VE42 "ADLC" 0)
'AD7=DS 13,1,1 (16 ADLO VE42 "ADLC" 0)
'AD8=DS 14,1,1 (16 ADLO VE42 "ADLC" 0)
'AD9=DS 15,1,1 (16 ADLO VE42 "ADLC" 0)
'AD10=DS 19,1,1 (16 ADLO VE42 "ADLC" 0)
'AD11=DS 20,1,1 (16 ADLO VE42 "ADLC" 0)
'AD12=DS 21,1,1 (16 ADLO VE42 "ADLC" 0)

$  
'J1=DS 13,"JENT",1 (16 CONN E42 0 0)
'J2=DS 14,"JENT",1 (16 CONN E42 0 0)
'J3=DS 15,"JENT",1 (16 CONN E42 0 0)
'J4=DS 16,"JENT",1 (16 CONN E42 0 0)

SYSVEC: APPL FORC "ADLC" 1
I=1
J="J1" : "AIR1"
J="J2" : "AIR4"
J="J3" : "AIR7"
J="J4" : "AIR10"
I=2
J="J1" : "AIR2"
J="J2" : "AIR5"
J="J3" : "AIR8"
J="J4" : "AIR11"
I=3
J="J1" : "AIR3"
J="J2" : "AIR6"
J="J3" : "AIR9"
J="J4" : "AIR12"

!AD1=DS 4,1,1 (16 ADLO VE42 "ADLC" 0)
!AD2=DS 5,1,1 (16 ADLO VE42 "ADLC" 0)
!AD3=DS 6,1,1 (16 ADLO VE42 "ADLC" 0)
!AD4=DS 10,1,1 (16 ADLO VE42 "ADLC" 0)
!AD5=DS 11,1,1 (16 ADLO VE42 "ADLC" 0)
!AD6=DS 12,1,1 (16 ADLO VE42 "ADLC" 0)
!AD7=DS 16,1,1 (16 ADLO VE42 "ADLC" 0)
!AD8=DS 17,1,1 (16 ADLO VE42 "ADLC" 0)
!AD9=DS 18,1,1 (16 ADLO VE42 "ADLC" 0)
!AD10=DS 22,1,1 (16 ADLO VE42 "ADLC" 0)
!AD11=DS 23,1,1 (16 ADLO VE42 "ADLC" 0)
```
RESULTS OF THE DESIGN SENSITIVITY ANALYSIS

The results of the design sensitivity analysis program in EAL are all stored in EAL-library file L012.

If a stress constraint design sensitivity of the appropriate element type is specified in the input control parameter, the following data sets are created to store the appropriate stress constraint values:

DVAL E21 1 0 The values of the stress constraint functionals are given for constraints listed in the input data set ST 21 LIST

DVPO E21 1 0 A set of pointers that indicate the location of points where maximum stress occurs in elements are given for constraints listed in input data set ST21 LIST.

DVAL E41 1 0 The values of the stress constraint functionals are given for constraint listed in input data set ST41 LIST

DVAL E42 1 0 The values of the stress constraint functionals are given for constraints listed in input data set ST42 LIST
The names of the data sets for design sensitivity vectors are given in the EAL-library file L012 that have the following basic form:

```
DSVE "ETYP" "CONT" "CIND"
```

where

- **DSVE** Design sensitivity vector

- **"ETYP"** - E21, E41, or E42: "ETYP" indicates the element type for which the design sensitivity vector is stored. For element types E41 and E42, the data set contains one sensitivity vector, indicating design sensitivity with respect to thickness design parameter, \( t \). For element type E21, the data set contains two sensitivity vectors, indicating design sensitivity with respect to width, \( b \), and height, \( h \), of the beam.

- **"CONT"**: In the input file for each element, a control parameter has to be specified as the following arrays:
  
  \[
  \text{ED21 REL 0 0, ED41 REL 0 0 and/or ED42 REL 0 0.}
  \]

  The parameter "CONT" indicates the control parameter for which a design sensitivity vector is computed.
"CIND":  

<table>
<thead>
<tr>
<th>Constraint Indicator</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>Compliance Constraint</td>
</tr>
<tr>
<td>between 21,000 and 22,000</td>
<td>Displacement constraint in the x-direction</td>
</tr>
<tr>
<td>between 22,000 and 23,000</td>
<td>Displacement constraint in the y-direction</td>
</tr>
<tr>
<td>between 23,000 and 24,000</td>
<td>Displacement constraint in the z-direction</td>
</tr>
<tr>
<td>between 30,000 and 40,000</td>
<td>Stress constraint in beam elements</td>
</tr>
<tr>
<td>between 40,000 and 50,000</td>
<td>Stress constraint in membrane elements</td>
</tr>
<tr>
<td>between 50,000 and 60,000</td>
<td>Stress constraint in plate elements</td>
</tr>
</tbody>
</table>

For displacement and stress constraints, the last three digits of the constraint indicator "CIND" give the node number of the displacement constraint and the element number of the stress constraint, respectively.