N87-27244

CALCULATION OF FORCES ON MAGNETIZED BODIES USING COSMIC NASTRAN

JOHN SHEERER

TEXAS INSTRUMENTS DATA SYSTEMS GROUP

INTRODUCTION:

The analogy between the equations of Heat Transfer and Magnetostatics has been used by several researchers to calculate magnetic fields in a variety of applications and shown to give correct results in theoretically verifiable cases. The resultant fields and fluxes have been used to find forces upon moving charged particles in the design of particle accelerators (1) and upon electrical conductors in the design of electrical machinery (2). Magnetostatic forces, due to the action of magnetic fields on magnetically permeable bodies, are a less tractable problem. Analytical methods (3,4,5) generally applied to obtaining an net force on a body of iron in air have produced apparently conflicting expressions for forces based upon the virtual work method, calculation of Maxwell stresses, and use of a free magnetic pole model. Carpenter has reconciled the apparent discrepancies (5) but the expressions obtained are not entirely suitable for finite element analysis, as they are shown by Carpenter to be mathematical devices to produce an equivalent force on a body rather than a true representation of a force distribution. While in many cases an equivalent force is all that is required, such an approach is not appropriate to problems involving structural deformation.

MAGNETOSTATIC EQUATIONS

The equations of Magnetostatics are based upon forces between field sources. In the S.I. system of units, the Ampere is the current required to produce unit force between two long parallel conductors separated by unit distance. In the older c.g.s. e.m.u. system, unit force was produced between unit magnetic poles in vacuo at unit distance. The imaginary but convenient concept of a magnetic pole is difficult to express in the modern system of units . It is convenient to describe the properties of a magnetic material in terms of the applied magnetic field, H, and the volume magnetization, M, where the flux density, B, is given by:

$$\underline{B} = \mu_0 \underline{H} + \underline{M} \qquad (1)$$

where μ_0 is the permeability of free space. This is the Kennelly SI system. In the Sommerfield S.I. system the magnetization is dimensionally the same as the field strength and is also operated on by the permeability constant to obtain B:

$$\underline{B} = \mu_0(\underline{H} + \underline{M}) \tag{2}$$

In order to use the NASTRAN heat transfer solver the equation must be recast in the form :

$$\underline{B} = \mu \underline{H} \tag{3}$$

where μ is the permeability of the material. This formulation is analogous to the heat transfer equation but in no way represents the physical relationship between B and H. At the boundary between materials of different permeability the total magnetic flux, ϕ, is conserved. The normal component of magnetic flux, Bn, and the tangential component of the magnetic field, Hn, are also continuous across the interface. These conditions are essential to all the subsequent force derivations.

MAGNETOMECHANICAL DEVICES

A magnetic field has associated with it an energy given by the equation

$$U = \int \underline{H} \cdot d\underline{B} \tag{4}$$

Given that in equation (3) the value of permeability in a ferromagnetic material is several thousands, it is apparent that the energy required to support a given flux density in an air gap greatly exceeds that required if the same volume contains a ferromagnetic material. Two categories of device utilize this mechanically: devices employing electromagnets generate a mechanical force to reduce the volume of an airgap, thereby minimizing magnetic field energy, when a magnetizing force is generated by a coil, while in stored energy devices a permanent field is generated by a permanent magnet, producing a force that is eliminated by the action of an opposing electromagnet when it is required to remove the mechanical force. The magnetic field energy may be converted to kinetic energy or strain energy.

THE VIRTUAL WORK METHOD

The virtual work method is the most widely known means of force calculation. It is generally applied by calculation of the working gap energy, which is subsequently differentiated with respect to a direction of motion to obtain a force in the direction under consideration. In air, the usual working gap,

$$\underline{B} = \mu_0 \underline{H} \tag{5}$$

Thus, by substitution in equation (4), an expression for air gap energy from which forces may be derived is obtainable in certain geometries. Between two large parallel plates the force per unit area is:

$$F = \tfrac{1}{2}\mu_0 H^2 \tag{6}$$

This formula neglects the energy changes within the magnetic material, and applies to a highly idealised orientation. Further, the result is a global force rather than a distributed force. Physically, the assumptions are that the magnetic material is of infinite permeability and that the source of magnetomotive force (MMF), is infinitely stiff. If the field energy of the magnetic material filling the gap is considered equation (6) becomes

$$F = \tfrac{1}{2}\mu_0 \left(1 - \frac{\mu_0^2}{\mu^2}\right) H^2 \tag{7}$$

ADAPTION FOR FINITE ELEMENT METHOD

The finite element method can be applied to the virtual work model to obtain a higher degree of accuracy. Since the NASTRAN heat transfer solver calculates both B and H it is possible to obtain the energy associated with each element for an initial configuration and geometrical perturbations thereof. If the permeability across the element is constant, as it must be, equation (4) becomes simply,

$$U = \frac{1}{2} B.H \tag{8}$$

If a rigid body is being analysed, perturbation in the six degrees of freedom, with the system energy summed in each case, should in theory provide complete data about the forces acting upon the body. The advantages of the finite element approach over analytical and boundary element methods are:

(I): Account is taken of field energy inside the magnetic materials
(II): The technique comprehends non-uniform fields, leakage flux etc. in the circuit

Since only an equivalent force is derived the approach is inapplicable to problems where deformation occurs and a distributed force is required. It is particularly suited, however,to applications where the displacement or deformation is constrained to a single degree of freedom. Applications such as linear and rotary solenoidal actuators or rotating armature printheads may be modelled using this technique.

THE MAXWELL STRESS METHOD

Equation (8) may be rewritten:

$$U = \frac{1}{2}\mu H^2 \tag{9}$$

For a spatially varying field, the equation has the more general form

$$U = \frac{1}{2}\iiint \mu H^2 \, dx\,dy\,dz \tag{10}$$

giving x,y and z force components of the form

$$F_x = \frac{1}{2}\iiint \frac{\partial \mu}{\partial x} \cdot H^2 \, dx\,dy\,dz \tag{11}$$

If the permeability is constant then equation (11) has a value of zero. Consequently, traction forces are only exerted at the boundaries between regions of different permeability, or where the permeability varies continuously. A spatially varying field will not of itself produce traction forces. If a closed surface is constructed around a magnetized part then, from equation (11) a stress through the surface and a pressure tangential to it are obtained,

$$F_s = \frac{1}{2}\mu H^2 \cdot \underline{\hat{n}} \tag{12}$$

$$F_p = \frac{1}{2}\mu H^2 \times \underline{\hat{n}} \tag{13}$$

The expressions reduce to zero if no discontinuities in H are enclosed. They may be resolved into components normal and tangential to the surface:

$$F_n = \frac{1}{2}\mu \left(H_n^2 - H_t^2\right) \tag{14}$$

$$F_t = \mu H_n H_t \tag{15}$$

If the surface is constructed outside of a magnetic part (in free space) then $\mu = \mu_0$ .Alternatively a surface may be constructed within the part. In this latter case no field sources are enclosed, since all field sources lie at the interface between the part and free space, and a zero resultant force is obtained.

ADAPTION TO THE FINITE ELEMENT METHOD

Equations (14) and (15) may be applied by construction of a surface enclosing a part, located arbitrarily close to the boundary between the part and free space. This force acts thru a closed surface on enclosed field sources, and are not necessarily a physical representation of the true surface force distribution. The case will now be considered where a pill-box is constructed normal to the boundary, of vanishingly small thickness, and extending to both sides of the boundary. By this construction the sources of field are localized within the box and forces may be obtained in direction both out of and into the magnetic body:

$$\mu_0 H_{o_n} = \mu_0 H_{i_n} + M \tag{16}$$

$$H_{o_t} = H_{i_t}$$

where the suffixes o and i denote inside and outside values of the field and permeability. If μ_r is defined as the ratio of internal to external permeability then

$$F_n = \frac{1}{2}\mu_0 \left(1 - \frac{1}{\mu_r}\right) H_{n_o}^2 \tag{17}$$

$$F_t = \mu_0 \left(1 + \frac{1}{\mu_r^2}\right) H_{n_o} H_{t_o} \tag{18}$$

This appears to be a local force, and reduces to equation (7) for the case where the field is normal to the interface. The model, however, does not impart physical insight into the phenomena, and there are implied assumptions in the neglect of the sides of the pill box surface.

THE FREE POLE METHOD

The magnetization process consists of realignment of magnetic domains in a ferromagnetic material so as to orient then with the external magnetic field. Within the material, macroscopically, each North pole produced will be cancelled by a South pole, and there will be no discontinuities in H within the material. At the boundaries of the material, however, the alignment (polarization) of the domains produces free magnetic poles. The magnetization, M, is equal to the free pole density. The free poles are acted upon by the field H with a force in proportion to both H and M,

$$\underline{F} = \underline{H} . \underline{M} \tag{19}$$

The normal component of the force acting on M free poles is obtained by considering the force which must be applied to an increment dM, in order to move it to the interface from an arbitrary noncoincident point. Since the free poles themselves affect the relationship between internal and external values of H, an integration is required to obtain the normal force:

$$F_{i_n} = \int_0^M H_{i_n} \cdot dM = \frac{1}{2}\mu_0 \left(H_{o_n}^2 - H_{i_n}^2\right) \tag{20}$$

In considering the field from the poles alone, it is physically apparent that it must be normal to the surface on which the poles lie. Therefore, the tangential force is simply:

$$F_t = \mu_0 H_t M \tag{21}$$

These equations may be reformulated to be comparable with the results of the other methods discussed:

$$F_n = \frac{1}{2}\mu_o \left(1 - \frac{1}{\mu_r^2}\right) H_{on} \tag{22}$$

$$F_t = \mu_o \left(1 - \frac{1}{\mu_r}\right) H_{on} H_t \tag{23}$$

NASTRAN CALCULATIONS

For the theoretically verifiable case of an air gap between two large parallel plates, forces on the plates have been calculated from NASTRAN heat transfer output by the methods described above. Since the methods reduce to the same theoretical base equation in this geometry, it is not surprising that the results are in agreement with theory and with each other. The problem of finding a non-trivial yet theoretically verifiable test problem remains.

COMPARISON OF DERIVATIONS

Equations for a net magnetic traction force have been obtained by virtual work and Maxwell stress methods. Distributed forces have been obtained by the free pole method and a variation of the Maxwell Stress method. All methods are equivalent for cases where the magnetic field at an interface between the magnetic medium and air is normal to the surface. The equations governing the angle of field on either side of the medium are:

$$H_i \cos\alpha_i = H_o \cos\alpha_o - (M/\mu_o)\cos\alpha_i$$

$$H_i \sin\alpha_i = H_o \sin\alpha_o$$

where α_i is the incident angle of the internal field H_i and α_o is the incident angle of the external field H_o. Thus,

$$\tan\alpha_i = H_o \sin\alpha_o / \left(H_o \cos\alpha_o - (M/\mu_o)\cos\alpha_i\right) \tag{24}$$

For reasonable values of M, $\alpha_i \sim 0$. Thus in all but extermely saturated nmagnetic materials the normal force will be orders of magnitude larger than the tangential forces, and the latter can be ignored for all practical purposes.

Although not admired by theoreticians, the polar model of magnetism is mathematically valid and there is no reason to doubt the results obtained above. The polarization model is based on an analogy between electrostatic and magnetostatic phenomena which is not entirely valid, but which provides the same results in terms of force and energy as for the more correct domain model of ferromagnetism. The use of analogy has been compared unfavourably with the mathematical purity of Maxwell's equations, since it is little known that these were derived using a tortuous mechanical analog of electrostatics and magnetostatics. The Maxwell stress method described above is one of three stress systems presented by Maxwell, one of which is in total contradiction and has been described by Carpenter (5) as completely useless. The discrepancy between the Maxwell and free poles methods is likely due to the neglect of the side surfaces of the pill-box structure described above, and in any case is of negligible magnitude.

CONCLUSIONS

The methods described may be used with a high degree of confidence for calculations of magnetic traction forces normal to a surface. In this circumstance all models agree, and test cases have resulted in the theoretically correct result. It is shown above that the tangential forces are in practice negligible. The surface pole method is preferable to the virtual work method because of the necessity for more than one NASTRAN run in the latter case, and because distributed forces are obtained. The derivation of local forces from the Maxwell stress method involves an undesireable degree of manipulation of the problem and produces a result in contradiction of the surface pole method.

REFERENCES

[1]: J. Simkin and C.W. Trowbridge, IEE Proc., Vol. 127, 368-374 (1980)

[2]: J. L. Brauer, Research and Development, 71-73, (Dec. 1984)

[3]: Maxwell, Electricity and Magnetism, Clarendon Press (1891)

[4]: Lehman, Trans A.I.E.E., 45, 383, (1926)

[5]: Carpenter, Proc. IEE, 107C, 19, (1959)