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A THEORETICAL STUDY OF
PHOTOVOLTAIC CONVERTERS

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Laser Photovoltaic Energy Converters

Mathematical models for the photovoltaic conversion of laser power have been developed. These models simulate the operation of planar and vertical junction photovoltaic converters and are described in detail within the references 1 and 2.

Another parameter which can effect the operation of a laser power converter is a graded junction. To study the effect of graded junctions we will consider a one dimensional model. With reference to the Fig. 1, let \( f = N_D - N_A = \alpha \xi \) denote the graded junction. For charge neutrality we must have

\[
\int_{-\xi_p}^{0} f(\xi) d\xi = \int_{0}^{\xi_n} f(\xi) d\xi .
\]  

(1)

This gives

\[
\frac{\alpha \xi_p^2}{2} = \frac{\alpha \xi_n^2}{2} \quad \text{or} \quad \alpha (\xi_n + \xi_p) (\xi_n - \xi_p) = 0
\]  

(2)

This requires that \( \xi_n = \xi_p = W/2 \), where \( W = \xi_n + \xi_p \) is the width of the depletion region. For Gauss's law we use the linear approximation

\[
\frac{d\xi}{dx} = \frac{q}{\varepsilon} (p - n + N_d - N_A) = \frac{q}{\varepsilon} \alpha x.
\]  

(3)
This produces

\[
\varepsilon(\xi) = \begin{cases} 
  \int_{-\xi}^{\xi_n} q \varepsilon(\xi) \, d\xi, & -\xi_p < \xi < 0 \\
  -\int_{-\xi}^{\xi_n} q \varepsilon(\xi) \, d\xi, & 0 < \xi < \xi_n 
\end{cases}
\]  

(4)

with \( \varepsilon(-\xi_p) = \varepsilon(\xi_n) = 0 \).

The maximum electric field is

\[
|\varepsilon_m| = \frac{q}{\epsilon} \int_0^{\xi_n} \varepsilon(\xi) \, d\xi = \frac{q}{\epsilon} \int_{-\xi_p}^{0} \varepsilon(\xi) \, d\xi
\]  

(5)

and the potential distribution is given by

\[
\frac{d\psi}{d\xi} = -\varepsilon(\xi) \quad \text{or} \quad \psi = -\int_{-\xi_p}^{\xi} \varepsilon(\xi) \, d\xi, \quad -\xi_p < \xi < \xi_n
\]  

(6)

by the mean value theorem for integrals we can write

\[
\\w \cdot \varepsilon_m = \int_{-\xi_p}^{\xi_n} \varepsilon(\xi) \, d\xi = V_{b_i} = \text{built in voltage},
\]  

(7)

using \( f = a\xi \) we calculate
\[ \epsilon(\xi) = \frac{aq}{2e} \left( \xi^2 - \xi_p^2 \right), \quad -\xi_p < \xi < 0 \]

\[ \quad - \frac{aq}{2e} \left( \xi_n^2 - \xi^2 \right), \quad 0 < \xi < \xi_n \]

with maximum field

\[ |\epsilon_m| = \frac{aq}{2e} \xi_p^2 \]

and the voltage is given by

\[ \psi(\xi) = \begin{cases} 
-\frac{aq}{2e} \left( \frac{\xi}{3} - \frac{\xi_p^2}{\xi} - 2 \frac{\xi}{3} \right), & -\xi_p < \xi < 0 \\
\frac{aq}{2e} \left( \frac{\xi_n^2}{\xi} - \frac{\xi}{3} + \frac{\alpha q w^3}{24e} \right), & 0 < \xi < \xi_n 
\end{cases} \]

we find that

\[ \psi(\xi_n) = \frac{\alpha q w^3}{12e} = \frac{q}{v_b} = \frac{kt}{q} \ln \left[ \frac{\frac{aw}{n_i^2}}{\left( \frac{aw}{n_i} \right)^2} \right] \]

or

\[ w^3 = \frac{12e}{aq} \left( \frac{2kT}{q} \right) \ln \left( \frac{aw}{2n_i^2} \right) \]

From this equation we can obtain the relationship between the junction depth and the slope of the graded junction which can then be related to the built in voltage. These relationships are depicted in the Fig. 2. On a semi-log plot the built in voltage increases linearly with slope of the graded junction. Also as the slope of the graded junction increases the
Figure 2: Junction Depth and Built-in Voltage vs Slope of Graded Junction
width of the depletion region decreases.

A theoretical analysis of silicon p-n junction solar cells predicts a limiting upper value for the open circuit voltage $V_{oc}$. This limiting value is in the neighborhood of 700 mV. The efficiency $\eta$ of a silicon p-n junction solar cell is limited by the value obtained for $V_{oc}$. A graded junction is one factor which influences the value of the open circuit voltage, another factor is the magnitude of the bandgap narrowing $\Delta E_g$.

From Fig. 2, we can see that higher levels of doping concentration can significantly influence the device performance of a laser photoconverter.

**Iodine Lasers**

The References 3, 4 contain models for the chemical kinetics associated with Iodine Lasers. The model includes the basic kinetic processes for the photodissociative iodine laser including all known chemical reactions. A scaled version of these chemical kinetic equations is obtained by letting

\[
\begin{align*}
B_1 y_1 &= [RI] \\
B_2 y_2 &= [R] \\
B_3 y_3 &= [R_2] \\
B_4 y_4 &= [I_2] \\
B_5 y_5 &= [I^*] \\
B_6 y_6 &= [I] \\
B_7 y_7 &= [p]
\end{align*}
\]

where $B_i$, $i=1, \ldots, 7$ are scale factors and $[ ]$ represents density of states and $R$ represents one of the perflouralkyl iodides such as CF$_3$I, C$_2$F$_5$I, i-C$_3$F$_7$I, n-C$_3$F$_7$I. The scaled chemical kinetic equations are:

\[
\frac{dy_1}{dt} = k_1 \frac{B_2 B_5}{B_1} y_2 y_5 + k_2 \frac{B_2 B_6}{B_1} y_2 y_6 - \xi_1(t) y_1 - K_4 \frac{B_2 B_1}{B_1} y_1 y_2
\]
\[
\frac{dy_2}{dt} = -k_1 B_5 y_1 y_5 - k_2 B_6 y_2 y_6 - 2k_3 B_2 y_2^2 + \xi_1(t) \frac{B_1}{B_2} y_1 - k_4 B_1 y_1 y_2
\]
\[
\frac{dy_3}{dt} = k_3 \frac{B_2^2}{B_3} y_2^2 + k_4 \frac{B_1 B_2}{B_3} y_1 y_2
\]
\[
\frac{dy_4}{dt} = \frac{C_1 B_1 B_5 B_6}{B_4} y_1 y_5 y_6 + \frac{C_2 B_1 B_5^2}{B_4} y_1 y_6^2 + C_3 B_6 B_5 y_4 y_5 y_6
\]
\[-\xi_2(t) y_4 + C_4 B_6^2 y_4 y_6^2\]
\[
\frac{dy_5}{dt} = -k_1 B_2 y_2 y_5 - C_1 B_1 B_6 y_1 y_5 y_6 - C_3 B_4 B_6 y_4 y_5 y_6 - Q_1 B_1 y_1 y_5
\]
\[-Q_2 B_4 y_4 y_5 - A_0 y_5 - A_0 y_5 + \xi_1(t) \frac{B_1 y_1}{B_5} + \xi_2(t) \frac{B_4 y_4}{B_5} - \frac{r_{max}}{B_5}\]
\[
\frac{dy_6}{dt} = \xi_2(t) \frac{B_4}{B_6} y_4 + Q_1 \frac{B_1 B_5}{B_6} y_1 y_5 + Q_2 \frac{B_4 B_5}{B_6} y_4 y_5 + A_0 \frac{B_5 y_5}{B_6}
\]
\[-C_1 B_1 B_5 y_1 y_5 y_6 + \frac{r_{max}}{B_6} + k_4 \frac{B_1 B_2}{B_6} y_1 y_2 - 2C_2 B_1 B_6 y_1 y_6
\]
\[-C_3 B_4 B_5 y_4 y_5 y_6 - k_2 B_2 y_2 y_6 - A_0 y_6 - 2C_4 B_4 B_6 y_4 y_6^2\]
\[
\frac{dy_7}{dt} = \frac{r_{max}}{2.77} \frac{\pi}{L} - \frac{1}{L/c} B_7 y_7 + g A_0 B_5 y_5
\]
with \( r_{\text{max}} = \frac{C B_7 y_7 (B_5 y_5 - 0.5 B_6 y_6)}{(A_{00} + B_0 B_1 y_1)} \)

In the above equations \( \xi_1(t), \xi_2(t) \) are the photodissociation rates of the laser gases for visible and ultraviolet light. We use

\[
\xi_1(t) = \begin{cases} 
E_{01} C_0 (e^{x_{11}} - e^{x_{12}}) & 0 < t < r_0 \\
0 & t > r_0
\end{cases}
\]

and

\[
\xi_2(t) = \begin{cases} 
E_{02} C_0 (e^{x_{21}} - e^{x_{22}}) & 0 < t < r_0 \\
0 & t > r_0
\end{cases}
\]

with \( \xi_i(t+\Delta t) = \xi_i(t) \) for \( i = 1,2 \) where \( \Delta t \) is the period of the light pulses. Here

\[
X_{i1} = -E_{i1}(t-0.5 r_0)^2 \quad i = 1,2
\]

\[
X_{i2} = -E_{i2} \frac{r_0^2}{4} \quad i = 1,2
\]

\[
E_{01} = 2(3.04 \times 10^{-3}), \quad \Delta t = 10 \text{ seconds}
\]

\[
E_{02} = 2(3.38 \times 10^{-2}) \quad r_0 = 7 \times 10^{-4}
\]

\[
C_0 = 2 E_4
\]

\[
E_{12} = E_{11} = 4 \ln(10)/r_0^2
\]

\[
E_{21} = E_{22} = E_{11}
\]

the remaining parameters are: \( B = (3.5E16)P \) where \( P \) is the pressure in torr and
The system of scaled chemical kinetic equations are solved numerically using the Runge-Kutta-Fehlberg numerical procedure applied to the nonlinear system of 7 equations. Nominal results are illustrated in the Fig. 3 over the time interval 0<\(t<1.2(10^{-3})\) seconds. An analysis of parameter effects on the solution behavior is to be studied using a computer.
FIGURE 3 NOMINAL RESULTS FROM IODINE EQUATIONS
REFERENCES


