COMPONENT MODE SYNTHESIS AND LARGE DEFLECTION VIBRATION OF COMPLEX STRUCTURES
VOLUME 1: EXAMPLES OF NASTRAN® MODAL SYNTHESIS CAPABILITY

By
Chuh Mei, Principal Investigator
and
Mo-How Shen

Final Report
For the period ended January 31, 1987

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
Research Grant NAG-1-301
Mr. Joseph E. Walz, Technical Monitor
SDD-Structural Dynamics Branch

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Old Dominion University Research Foundation
P.O. Box 6369
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SUMMARY

This report illustrates the use of NASTRAN® modal synthesis capability for some small examples. A classical truss problem is examined and the results for accuracy are compared to existing results from other methods. This problem is examined using both fixed interface modes and free interface modes. The solution is carried out for an applied dynamic load down as far as recovery of forces in individual members as a function of time. Another small beam problem is used to compare different means of "combining" substructures.

INTRODUCTION

During the past twenty years, a body of technology has developed within the general field of structural dynamics that has been identified by the term modal synthesis. Modal synthesis is a Rayleigh-Ritz approach using systematically derived displacement functions. It is used to formulate and solve the large eigen problems which arise in dynamic analysis of complex structural systems. Solutions are approximate in the sense that the motion of the structure is constrained to linear combinations of a limited number of modes or displacement functions characterizing the behavior of independent substructures.
Several researchers have formulated various modal synthesis procedures in an attempt to reduce computation errors and minimize computer costs. Hurty developed the first modal synthesis method capable of analyzing structures with redundant interface connections in references 1 and 2. He treated the structure as an assembly of connected components, or substructures, each of which is analyzed separately to derive a set of modes or displacement shapes from which a set of generalized coordinates applicable to the complete structure is synthesized. Craig and Bampton (ref. 3) simplified Hurty's formulation by combining two groups of coordinate functions which Hurty had defined separately. A number of survey papers have been written by Hou, Goldman, Benfield and Hruda in references 4 to 7. Some methods are found to be more suitable for certain applications than others. Yet, experience has shown that no single approach is generally preferred over the others.

The complexity of aerospace structures increased enormously during the last two decades. A new challenge is presented by the proposed space station (ref. 8) in that it is an evolving structure that cannot be ground tested because final configuration may not be known when the first component is put into space. Therefore, the component mode synthesis method may be applied for the dynamic analysis of such large structure system in space. A widely used tool for structural analysis, the NA STRAN® computer program, contains a modal synthesis capability but, other than the demonstration problem presented in reference 5, little is publicly known about its capabilities.

The purpose of the present report is to examine some of the capabilities of this program. This is done by examining two simple problems, a truss and a beam.

**NUMERICAL EXAMPLES**

The modal synthesis procedure in NA STRAN® is applied to two simple structures. One is a redundant truss confined to lie in a plane but free to move in this plane. It is composed entirely of ROD elements (no bending stiffness for all). This example is used to examine convergence character-
istics of the modal synthesis procedure and also to illustrate the transient response capability all the way down to obtaining stresses in rod members as a function of time. The second example is a free-free beam. It is used to examine different ways to "combine" substructures to yield frequency for the total structure.

**Truss Example**

The redundant truss example is the one used in reference 5 to compare eight different modal synthesis procedures. The full truss model is shown in figure 1(a) and its two components shown in figure 1(b). Component A consists of five equal bays and has a total of 18 joints. Component B consists of four equal bays and has a total of 15 joints. All members in the components have identical properties. At the interface of the components in the full truss model, the vertical member has twice the area of other members. Basic geometric and material properties are presented in table I along with the prescribed load for a transient response analysis. An additional run was made with the full model subdivided into three components with three bays in each component.

The basic run sequence and substructure operation are shown in figure 2. In the figure capitalized letters inside of rectangular blocks indicate names of pseudostructures used in the analysis. Capitalized letters adjacent to, or on, the flow diagram indicate the names of modules that perform a certain function in the computer program. At the top of figure 2, the Phase 1 operations formulate the finite element stiffness and mass matrices using Rigid Format 2. For the convergence study the Phase 2 runs on Rigid Format 3 were repeated using a different number of modes from the individual components. Also Phase 2 runs were using free interface modes as well as the fixed interface modes. A limited amount of data is presented for three components and naturally a Phase 1 run must be made for this component.

A transient response analysis was made on this free-free truss structure for an axial load applied to the right end of the truss. The load was applied for 0.12 seconds and then removed. In order to apply a load at grid point 42 in component B, this grid point must be included on a BOUNDARY
card. Thus, additional degrees of freedom are created corresponding to this point. The structure was represented by eight modes from component A, six modes from component B, and the eight interface modes for a total of twenty-two modes. The modes for the individual component were determined with the interface fixed. The standard procedure will obtain displacements back in the individual component. However, member forces and stresses are not determined automatically, but can be obtained through a simple procedure in a few steps. In the first step a run is made with DIAG 17 turned on to put the DMAP sequence on the punch file with an EXIT scheduled after statement 1. A small substructure deck is included to allow the appropriate commands that interface to the Substructures Operating File (SOF) to be generated. This punch file is subsequently saved and altered to replace the RECOVER module with the SDR2 module which can recover element forces and stresses. The listing of this DMAP sequence and run stream is contained in Appendix A.

Beam Example

This example consists of a beam composed of seven components as shown in figure 3(a). All subbeams have a constant length, area and uniform mass properties. Each component consists of ten equal elements and has a total of 11 joints as shown in figure 3(b). Basic geometric and material properties for each subbeam are presented in table II. A lumped mass formulation is used (no rotary inertia) and, therefore, there are 213 stiffness degrees of freedom in the problem, but only 142 eigenvalues.

Three different ways of "combining" substructures are illustrated in figures 3(c), 3(d), and 3(e). The basic run sequences and substructure operations for each case are shown in figures 4 thru 6. For all cases, the substructuring Phase 1 operations formulate the finite element stiffness and mass matrices for subbeam A using Rigid Format 3. The structural matrices contained in BBASIC, CBASIC, ..., FBASIC are generated as needed by using EQUIV operation. The basic subbeams are reduced to modal coordinates and combined together following the procedures shown in figures 4 thru 6. The eigenvalues of the total beam are obtained by using the MRECOVER command. The driver decks and sample bulk data for cases 1, 2 and 3 are listed in Appendices B, C and D. Only fixed interface modes were used but two sets of runs were made using a different number of modes from the subbeams.
RESULTS

For assessing the accuracy of the modal synthesis procedure, two and three truss components with fixed or free interface connection are run to determine frequencies and compared to results for full model. Percentage errors in frequency for the combined systems of 12, 20, 28 and 36 degrees of freedom are shown in tables III thru VI. Here degrees of freedom include not only the number of flexible modes used but also any interface modes. Thus, for example, for 12 degrees of freedom results, since there are six interface modes, only six flexible modes can be shown. Based on the lowest frequency criterion then four modes were chosen from component A and two modes from component B.

Figures 7 thru 11 are nondimensional plots that indicate the relative accuracy obtained by modal synthesis procedures. Also shown on the figures are results taken directly from reference 5 in which several other procedures are compared. From figures 7 to 10 it can be seen that modes derived with the interfaces fixed yield better results than modes derived with the interface free.

For the transient response run the percentage error in displacement for grid points 41, 42, and 43 of component B are shown in table VII. These results were produced from the 20 degrees of freedom model. The axial force in elements 111-113 and 143 of component B are shown in table VIII.

The full beam shown in figure 2 was run to determine its natural frequencies and used as a comparison of results obtained with the various "combination" procedures. Table IX shows the percentage error in frequency for the various "combination" procedures when 62 degrees of freedom are used. These 62 degrees of freedom correspond to approximately 47% of the total degrees of freedom in the full model. All three "combination" procedures yield good results. However, case 1 uses considerably less CYBER 75 CPU time than the other two cases (53.8 CPU seconds corresponds to 65.3 seconds, 59.1 seconds, respectively). Another run for case 1 was made using 19% of total degrees of freedom, and 55% frequencies were obtained with less than 1% error in frequency.
ACKNOWLEDGEMENT

This work was sponsored by the NASA-Langley Research Center under Grant NAG-1-301. The work was monitored under the supervision of Dr. Jerrold M. Housner and Mr. Joseph E. Walz, Structural Dynamics Branch, Structures and Dynamics Division. Mr. Mo-How Shen, a graduate student in the Department of Mechanical Engineering and Mechanics, Old Dominion University, carried out most of the detailed studies. The author would like to thank Mr. Joe Walz for many valuable suggestions and assistance.
REFERENCES


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<td>Cross-sectional area of members</td>
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<td>Young's modulus</td>
<td>$E = 1.422 \times 10^4 \frac{\text{Kg}}{\text{m}^2} (10^7 \text{ psi})$</td>
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<td>Density</td>
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Table V. Frequency for Full Truss and Percent Error in Frequency for Two Modal Synthesis Models Using 28 Degrees of Freedom

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Table VII. Transient Response and Percent Error in Displacement

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<th>B. Substr.</th>
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Table IX. Percent Frequency Error Using 62 Degrees of Freedom

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Figure 2. Substructure Formulation Tree and Solution Sequence
Figure 3. Total Beam Model and Various Subdivided Representations.
Figure 4. Case 1 Subbeam Formulation Tree and Solution Sequence
Figure 7. Comparison of Methods with Frequency Error of 0.1%.
Legend:
NR1 = Free-interface NASTRAN
NR2 = Fixed-interface NASTRAN
BH1 = Benfield-Hruda, Free-Free
BH2 = Benfield-Hruda, Constrained
BH3 = Benfield-Hruda, Free-Free, Interface Loading
BH4 = Benfield-Hruda, Constrained, Interface Loading
H = Hurty
BF = Bajan-Feng
CB = Craig-Bampton
HO = Hou
G = Goldman

Figure 8. Comparison of Methods with Frequency Error of 0.5%.
Figure 9. Comparison of Methods with Frequency Error of 1.0%.
Legend:
NR1 = Free-interface NASTRAN
NR2 = Fixed-interface NASTRAN
BH1 = Benfield-Hruda, Free-Free
BH2 = Benfield-Hruda, Constrained
BH3 = Benfield-Hruda, Free-Free, Interface Loading
BH4 = Benfield-Hruda, Constrained, Interface Loading
H = Hurty
BF = Bajan-Feng
CB = Craig-Bampton
HO = Hou
G = Goldman

Figure 10. Comparison of Methods with Frequency Error of 5.0%.
Figure 11. Comparison of Methods with Frequency Error of 10.0%.
APPENDICES
**APPENDIX A.** Driver decks and sample bulk data for two components truss problem.

**NASTRAN FILES = UYF & CDC AND IBM**

ID = UEM2031, NASTRAN

APP DISP = SUBS

SOL 20

TIME 3

CEND

SUBSTRUCTURE PHASE 1

PASSWORD = MOLSYN

SOF(1) = FITY9500, NEW & CDC AND IBM

NAME = ABASIC

SOFPRI = TOC

ENDSUBS

TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS

LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RF 2

SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-1

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**NASTRAN FILES = UMF & CDC and IBM**

**APP DISP.SUBS**

**INPUT**

**ENDINPUT**

**TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS**

**SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2**

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PROD  1  &
ENDDATA &

& NASTRAN FILES = UMF $ CDC AND IBM
ID  DEM2033, NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 5
END
SUBSTRUCTURE PHASE2
PASSWORD = MDLSYN
SOF(1) = FT 195 300 $ CDC AND IBM
OPTIONS K, M, P
SOFPRINT TOC
MREDUCE A: BASIC
NAME MA
BOUNDARY 5
FIXED 9
METHOD 19
OUTPUT 1, 5, 6, 9, 10
SOFPRINT TOC
MREDUCE B: BASIC
NAME MB
BOUNDARY 4
FIXED 4
METHOD 29
OUTPUT 1, 5, 6, 9, 10
SOFPRINT TOC
MREDUCE C: BASIC
NAME MC
BOUNDARY 7
FIXED 7
METHOD 39
OUTPUT 1, 5, 6, 9, 10
SOFPRINT TOC
COMBINE MA MB MC
NAME MCOMB
TOLERANCE 0.001
OUTPUT 2, 7, 12
COMPONENT MB
TRANSFORM 20
COMPONENT MC
TRANSFORM 40
SOFPRINT TOC
MREDUCE MCOMB
NAME RTRUSS
BOUNDARY 42
METHOD 90
NMAX 18
OUTPUT 1, 5, 6, 9, 10
SOFPRINT TOC
ENDSUBS
TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-4
LABEL = MODAL REDUCE, COMBINE, MODAL RECOVERY, RUN 4, PHASE 2, RF 3
BEGIN BULK
NASTRAN FILES = UMF $ CDC AND IBM
ID OSM2035*NASTRAN
APP OMAP*SUBS
BEGIN DISP 09 - DIRECT TRANSIENT RESPONSE ANALYSIS - APR. 1982 $
PRECHK ALL $  
FILE UDVT=APPEND/TOL=APPEND $ 
PARAM //*MPY*/CARDNO/0/0 $ 
GP1 GEOM1, GEOM2, /GPL,EQEXIN,GPDT, CSTM, BGPD, SIL/S,N,LUSET/ S,N, NOGPDT/ALWAYS=-1 $ 
PLTTRAN BGPD,SIL/BGPD,SIP/LUSET/S,N,LUSEP $ 
PURGE USET,GM,GQ,KAA,BAA,MAA,K4AA,PST,KFS,GP,EST,ECT,PLTSETX,PLTPAR, GSETS,ELSETS/NOGPDT $ 
COND LBL5,NNOGPDT $ 
GP2 GEOM2,EQEXIN/ECT $ 
PARAM PCD8/*PRES/*///JUMPPLOT $ 
PURGE PLTSETX,PLTPAR,GSETS,ELSETS/JUMPPLOT $ 
COND P1,JUMPPLOT $ 
PLTSET PCD8,EQEXIN,ECT/PLTSETX,PLTPAR,GSETS,ELSETS/S,N,NSIL/S,N, JUMPPLOT=-1 $ 
PRTMSG PLTSETX/ $ 
PARAM //*MPY*/PLTFLG/!1/1 $ 
PARAM //*MPY*/PFILE/0/0 $ 
COND P1,JUMPPLOT $ 
PLOT PLTPAR,GSETS,ELSETS,CASECC,BGPD, EQEXIN,SIL,ECT, /PLOTX1/ NSIL/LUSET/S,N,JUMPPLOT/S,N,PLTFLG/S,N,PFILE $ 
PRTMSG PLOTX1/ $ 
LABEL P1 $ 
GP3 GEOM3,EQEXIN,GEOM2/SLT,GPTT/NOGRAV $ 
TA1 ECT,EPT,BGPD,SIL,GPTT,CSTM/EST,GEI,GPECT, /LUSET/S,N,NOSIMP= -1/1/S,N,NOGENL=1/S,N,GENEL $ 
PURGE K4GG,GPS,T,OGST,BGG,K4NN,K4FF,K4AA,MNN,MFF,MAA,BNN,BFF,BAA, 
KG1/NOSIMP/OGST/GENEL $ 
COND LBL1,NOSIMP $
ORIGINAL PAGE IS
OF POOR QUALITY

SCF1 \set* \knn* \mnn* \bnn* \k4nn* \kff* \kfs* \mff* \bff* \k4ff
LABEL \l8l3 $
EQUIV \kff* \kaa/OMIT $
EQUIV \mff* \maa/OMIT $
EQUIV \bff* \baa/OMIT $
EQUIV \k4ff* \k4aa/OMIT $
COND \l8l5* /OMIT $
SMPL \usft* \kff* /GO* \kaa* \kod* \l00* /OMIT $
COND \l8l6* /NOGG $
SMPL \usft* \go* \mff* \maa $
LABEL \l8l1 $ $
COND \l8l5* /NOGG $
SMPL \usft* \go* \bff* /BAA $
LABEL \l8l5 $ 
COND \l8l5* /NOGG $
SMPL \usft* \go* \bff* /BAA $
LABEL \l8l5 $ 
COND \l8l5* /NOGG $
LABEL \l8l5 $ 
PARAM //ADP* /DRY/1 /O $ 
LABEL \l8l5 $ 
DPR \dynamic* \gpl* \stl* \uset* \gp1* \stl* \uset* \tpf* \q1* \nlft* \trd* 
\Eqdyn* / \uset* / \stl* / \uset* / \trd* \nofel* / \sn* \nodl* \norl* \norl* \norl* 
\nft* / \sn* \norl* \norl* / \sn* \norl* \norl* $ 
COND \error* \norl* $
PURGE \pnld* / \norl* $ 
EQUIV \go* \god* / \norl* / \gm* \gmd* / \norl* $ 
PARAM //NOPP/ALWAYS=0 $ 
SSG1 \slt* \bgpd* \cstm* \stl* \est* \mpd* \gpt* \dtt* \mgg* \casecc* \dtt* / \uset* 
/ \nskip $ 
SSG2 \usft* \gm* \kfs* \go* \bgpd* / \norl* \norl* \norl* $ 
RCQVR3 \pg* \ps* / / \udv* \qas* \ppt* \pst* $ 
SDR1 \uset* / \udv* / \god* \gm* \nst* \kfs* / \up* \q1* / \dynamic* $ 
LABEL \l8l5 $ 
EQUIV \casecc* \casexx* / \always $ 
SDR2 \casexx* \cstm* \est* \mpd* \dtt* \eqdyn* \sld* \bgpd* / \toll* / \up* / \est* \xycd* 
PPT / \op1* / \op2* / \upv* / \oes1* / \off1* / \pugv* / \tranresp $ 
SD3 \op1* / \op2* / \upv* / \oes1* / \of1* / \op2* / \op2* / \upv* / \oes2* / \of2* / 
QPP \op2* / \op2* / \upv* / \oes2* / \of2* / 
COND \p2* / \jumpplot $ 
PLOT \pltpar* \opsets* / \elsets* \casexx* \bgpd* \eqexin* \sip* / \pugv* \gpec* \oes1* 
/ \plotx* / \nsil* / \uset* / \jumpplot* / \plotflg* / \sn* \pfile $ 
PRTMSG \plot* $ $ 
LABEL \p2 $ 
XTRAN \xycd* / \op2* / \op2* / \upv* / \oes2* / \off2* / \xyplot* / \tran* / \opsets* / \sn* \pfile$ 
/ \sn* \cardo* $ 
XYPLT \xyplot* / $ 
LABEL \l8l18 $ 
SOFUT //DRY* / TDC $ $ 
LABEL \l8lend $ 

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APPENDIX A. (concluded)

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<td>2.3-4</td>
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11.30.27, UCLP, DDLP00948, 0.423KINS.
APPENDIX B. Driver decks and sample bulk data for beam problem of case 1.

NASTRAN FILES = UMF $ CDC AND IBM
ID: DEM2031$NASTRAN
APP DISP$SUBS
SOL 3,0
TIME 3
CEND
SUBSTRUCTURE PHASE1
PASSWORD = MDLSYN
SOF(1) = FT17$500$NEW $ CDC AND IBM
NAME = ABASIC
SOFPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RC 2
BEGIN BULK
BAROR
BAR 1 1 1 2
BAR 2 2 2 3
BAR 3 3 3 4
BAR 4 4 4 5
BAR 5 5 5 6
BAR 6 6 6 7
BAR 7 7 7 8
BAR 8 8 8 9
BAR 9 9 9 10
BAR 10 10 10 11
GRIDSET
GRID 1 0 0 0
GRID 2 10 0 0
GRID 3 20 0 0
GRID 4 30 0 0
GRID 5 40 0 0
GRID 6 50 0 0
GRID 7 60 0 0
GRID 8 70 0 0
GRID 9 80 0 0
GRID 10 90 0 0
GRID 11 100 0 0
PBAR 1 1 0.56 63.3 2.591-4
ENDDATA

NASTRAN FILES = UMF $ CDC AND IBM
ID: DEM2032$NASTRAN
APP DISP$SUBS
SOL 3,0
TIME 5
CEND
SUBSTRUCTURE PHASE2
PASSWORD = MDLSYN
SOF(1) = FT17$500$NEW $ CDC AND IBM
EQUIV ABASIC$BBASIC
PREFIX B
EQUIV ABASIC$GBASIC
PREFIX G
MREDUCE ABASIC
NAME MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE BBASIC
NAME MB
BOUNDARY 2

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APPENDIX B. (concluded)

FIXED 2
METHOD 2
MREDUCE GBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
EQUIV MB, MC
PREFIX C
EQUIV MB, MD
PREFIX D
EQUIV ME, ME
PREFIX E
EQUIV MB, MF
PREFIX F
COMBINE MA*MBA*MC*MD*ME*MFM*MG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MB
TRANSFORM 2
COMPONENT MC
TRANSFORM 3
COMPONENT MD
TRANSFORM 4
COMPONENT ME
TRANSFORM 5
COMPONENT MF
TRANSFORM 6
COMPONENT MG
TRANSFORM 7
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 22
OUTPUT 1, 5, 6, 9, 10
SOFPRINT TOC
ENDSUB
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE, COMBINE, MODAL, RECOVERY, RUN2, PHASE2
BEGIN BULK
BDYC 5 BBASIC 50
BDYC 3 GBASIC 40
BDYC 20 ABASIC 30
BDYS1 30 126 11
BDYS1 40 126 1
BDYS1 50 126 1 11
TRANS 2 150 0 0 100 0 0 0 0 0 0 100 0 0 0 1
TRANS 3 250 0 0 200 0 0 0 0 0 0 200 0 0 1
TRANS 4 350 0 0 300 0 0 0 0 0 0 300 0 0 1
TRANS 5 450 0 0 400 0 0 0 0 0 0 400 0 0 1
TRANS 6 550 0 0 500 0 0 0 0 0 0 500 0 0 1
TRANS 7 650 0 0 600 0 0 0 0 0 0 600 0 0 1
EIGR 1 INV 0 3000.0 0 0 0 0 0 0 3000.0 0 0 0 1
EIGR 2 INV 0 3000.0 0 0 0 0 0 0 3000.0 0 0 0 0
EIGR 3 INV 0 3000.0 0 0 0 0 0 0 3000.0 0 0 0 0
EIGR 22 INV 0 2000.0 40 0 0 0 0 0 0 2000.0 40 0 0 0
ENDDATA

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APPENDIX C. Driver decks and sample bulk data for beam problem of case 2.

**NASTRAN FILES = UMF & CDC AND IBM**
ID DEM2031,NASTRAN
APP DISP, SUBS
SOL 3,0
TIME 3
END
SUBSTRUCTURE PHASE 1
PASSWORD = MDLSYN
SOF (1) = FT17,500, NEW & CDC AND IBM
NAME = ABASIC
SOFPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, R6 2
BEGIN BULK
BAROR
CBAR 1 1 2 10.0
CBAR 2 1 2 3 10.0
CBAR 3 1 3 4 10.0
CBAR 4 1 4 5 10.0
CBAR 5 1 5 6 10.0
CBAR 6 1 6 7 10.0
CBAR 7 1 7 8 10.0
CBAR 8 1 8 9 10.0
CBAR 9 1 9 10 10.0
CBAR 10 1 10 11 10.0
GRIDSET
GRID 1 1 0.0 0.0 0.0
GRID 2 2 10.0 0.0 0.0
GRID 3 3 20.0 0.0 0.0
GRID 4 4 30.0 0.0 0.0
GRID 5 5 40.0 0.0 0.0
GRID 6 6 50.0 0.0 0.0
GRID 7 7 60.0 0.0 0.0
GRID 8 8 70.0 0.0 0.0
GRID 9 9 80.0 0.0 0.0
GRID 10 10 90.0 0.0 0.0
GRID 11 11 100.0 0.0 0.0
PBAR 1 1 10.0 0.0 0.0 0.0 2.591-4
ENDDATA
&
NASTRAN FILES = UMF & CDC AND IBM
ID DEM2032,NASTRAN
APP DISP, SUBS
SOL 3,0
TIME 5
END
SUBSTRUCTURE PHASE 2
PASSWORD = MDLSYN
SOF (1) = FT17,500 & CDC AND IBM
EQUIV ABASIC,EBASIC
PREFIX B
EQUIV ABASIC,CBASIC
PREFIX C
EQUIV ABASIC,DBASIC
PREFIX D
EQUIV ABASIC,EBASIC
PREFIX E
EQUIV ABASIC,FBASIC
PREFIX F
EQUIV ABASIC,G BASIS
PREFIX G
MREDUCE ABASIC
NAME MA
BOUNDARY 20
Fixed 20
Method 1
Mreduce BBASIC
Name MB
Boundary 2
Fixed 2
Method 2
Mreduce CBASIC
Name MC
Boundary 7
Fixed 7
Method 2
Mreduce DBASIC
Name MD
Boundary 8
Fixed 8
Method 2
Mreduce EBASIC
Name ME
Boundary 9
Fixed 9
Method 2
Mreduce FBASIC
Name MF
Boundary 11
Fixed 11
Method 2
Mreduce GBASIC
Name MG
Boundary 3
Fixed 3
Method 3
Combine MB,MB
Name AB
Tolerance 0.01
Output 2,7,12
Component MB
Transform 2
Mreduce AB
Name MAB
Boundary 10
Fixed 10
Method 22
Combine MAB,MC
Name ABC
Tolerance 0.01
Output 2,7,12
Component MC
Transform 3
Mreduce ABC
Name MABC
Boundary 21
Fixed 21
Method 22
Combine MABC,MD
Name ABCD
Tolerance 0.01
Output 2,7,12
Component MD
Transform 4
Mreduce ABCD
Name MABCD
Boundary 22
Fixed 22
Method 22
COMBINE MBADE, ME
NAME ABCDE
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT ME
TRANSFORM 5
MREDUCE ABCDE
NAME MBADE
BOUNDARY 23
FIXED 23
METHOD 22
COMBINE MBADE, MF
NAME ABCDEF
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MF
TRANSFORM 6
MREDUCE ABCDEF
NAME MBADE
BOUNDARY 24
FIXED 24
METHOD 22
COMBINE MBADE, MG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MG
TRANSFORM 7
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 22
OUTPUT 1, 5, 6, 9, 10
STOP
PRINT TOC
ENDSUB

TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMOnSTRATION PROBLEM No. 2-3-2
LABEL=MODAL REDUCE, COMBINE, MODAL, RECOVSRY, RUN2, P8 SS2
BEGIN BULK
BDYC 7  CBASIC 50
BDYC 9  CBASIC 50
BDYC 21  CBASIC 30
BDYC 22  DBASIC 30
BDYC 23  EBASIC 30
BDYC 24  FBASIC 30
BDYC 11  FBASIC 50
BDYC 8  DBASIC 50
BDYC 10  BBASIC 30
BDYC 15  CBASIC 40
BDYC 2  BBASIC 50
BDYC 30  GBASIC 40
BDYC 30  ABASIC 30
BDYS1 30  126  11
BDYS1 40  126  1
BDYS1 50  126  1

TRANS 4  350.0  0.0  0.0  0.0  300.0  0.0  1.0  +T4
TRANS 6  350.0  0.0  0.0  0.0  500.0  0.0  1.0  +T6
TRANS 2  150.0  0.0  0.0  100.0  0.0  100.0  0.0  1.0  +T2
TRANS 3  250.0  0.0  0.0  200.0  0.0  200.0  0.0  1.0  +T3
TRANS 5  450.0  0.0  0.0  400.0  0.0  400.0  0.0  1.0  +T5

DBASIC 30
### APPENDIX C. (concluded)

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\[ \text{FOR DATA} \]

\[ \text{FOR DATA} \]

\[ \text{FOR DATA} \]

\[ \text{FOR DATA} \]

\[ \text{FOR DATA} \]
APPENDIX D. Driver decks and sample bulk data for beam problem of case 3.

NASTRAN FILES = UMF $ CDC AND IBM
ID = DEM2031.NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MDLSYN
SOF(1) = FT17$500$NEW $ CDC AND IBM
NAME = A B A S I C
SOPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, R6 2
BEGIN BULK
BAROR
BAR 1 1 1 2
BAR 2 1 2 3
BAR 3 1 3 4
BAR 4 1 4 5
BAR 5 1 5 6
BAR 6 1 6 7
BAR 7 1 7 8
BAR 8 1 8 9
BAR 9 1 9 10
BAR 10 1 10 11
GRDSET
GRID 1 0. 0. 0.
GRID 2 10. 0. 0.
GRID 3 20. 0. 0.
GRID 4 30. 0. 0.
GRID 5 40. 0. 0.
GRID 6 50. 0. 0.
GRID 7 60. 0. 0.
GRID 8 70. 0. 0.
GRID 9 80. 0. 0.
GRID 10 90. 0. 0.
GRID 11 100. 0. 0.
PBAR 1 1 10. 6. 8.
MAT 1 1 10. 6. 3.
ENDATA

NASTRAN FILES = UMF $ CDC AND IBM
ID = DEM2032.NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 10
CEND
SUBSTRUCTURE PHASE 2
PASSWORD = MDLSYN
SOF(1) = FT17$500 $ CDC AND IBM
EQUIV ABASIC, BBASIC
PREFIX B
EQUIV ABASIC, CBASIC
PREFIX C
EQUIV ABASIC, DBASIC
PREFIX D
EQUIV ABASIC, EBASIC
PREFIX E
EQUIV ABASIC, FBASIC
PREFIX F
EQUIV ABASIC, GBASIC
PREFIX G
MRREDUCE ABASIC
NAME MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE BBASIC
NAME MB
BOUNDARY 2
FIXED 2
METHOD 2
MREDUCE CBASIC
NAME MC
BOUNDARY 7
FIXED 7
METHOD 2
MREDUCE CBASIC
NAME MD
BOUNDARY 8
FIXED 8
METHOD 2
MREDUCE CBASIC
NAME ME
BOUNDARY 9
FIXED 9
METHOD 2
MREDUCE CBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
COMBINE MA*MB
NAME AB
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MB
TRANSFORM 2
MREDUCE AB
NAME MAB
BOUNDARY 10
FIXED 10
METHOD 22
COMBINE MB*MD
NAME CD
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MD
TRANSFORM 2
MREDUCE CD
NAME MCD
BOUNDARY 15
FIXED 15
METHOD 22
COMBINE ME*MF
NAME EF
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MF
TRANSFORM 2
MREDUCE EF
NAME MEF
BOUNDARY 25
FIXED 25
METHOD 22
COMBINE MAB*MCD
NAME ABCD
TOLERANCE 0.01
OUTPUT 2.7.12
COMPONENT MCD
TRANSFORM 3
MREDUCE ABCD
NAME MABCD
BOUNDARY 30
FIXED 30
METHOD 25
COMBINE MEF*MG
NAME MEF
TOLERANCE 0.01
OUTPUT 2.7.12
COMPONENT MG
TRANSFORM 3
MREDUCE EFG
NAME MEF
BOUNDARY 35
FIXED 35
METHOD 25
COMBINE MABCD*MEFG
NAME MABCEF
TOLERANCE 0.01
OUTPUT 2.7.12
COMPONENT MEFG
TRANSFORM 5
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 25
OUTPUT 1.5.6.9.10
SOPRINT TOC
ENDSUBS
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE*COMBINE*MODAL*RECOVERY*RUN2*PHASE2
BEGIN BULK

BDY1 30
BDY1 35
BDY1 9
BDY1 11
BDY1 25
BDY1 8
BDY1 10
BDY1 5
BDY1 22
BDY1 5
BDY1 20
BDY1 30
BDYS1 10
BDYS1 30
BDYS1 40
BDYS1 50
TRANS 150. 0. 0. 0. 100. 0. 1.
TRANS 2250. 0. . 0. 0. 200. 0. 1.
TRANS 450. 0. . 0. 0. 400. 0. 1.
TRANS 650. 0. . 0. 0. 600. 0. 1.
EIGR 1 INV .0 3000.00 10 10
EIGR 2 INV .0 3000.00 10 10

FBASIC 30
DBASIC 30

END

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APPENDIX D. (concluded)

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#
This report compares the accuracy between NASTRAN modal synthesis, full structure NASTRAN and several other modal synthesis results (truss only). The results are based on a truss or beam having redundant or point interface connections. Each component substructure is reduced to modal and boundary degrees of freedom prior to the substructure combine operation. The combination structure, formulated in terms of the component modes, is also reduced to modal degrees of freedom for solution by the transient analysis rigid format.