COMPONENT MODE SYNTHESIS AND LARGE DEFLECTION VIBRATION OF COMPLEX STRUCTURES

VOLUME 1: EXAMPLES OF NASTRAN® MODAL SYNTHESIS CAPABILITY

By
Chuh Mei, Principal Investigator
and
Mo-How Shen

Final Report
For the period ended January 31, 1987

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
Research Grant NAG-1-301
Mr. Joseph E. Walz, Technical Monitor
SDD-Structural Dynamics Branch

August 1987
DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
COLLEGE OF ENGINEERING AND TECHNOLOGY
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA 23508

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Old Dominion University Research Foundation
P.O. Box 6369
Norfolk, Virginia 23508

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This report illustrates the use of NASTRAN® modal synthesis capability for some small examples. A classical truss problem is examined and the results for accuracy are compared to existing results from other methods. This problem is examined using both fixed interface modes and free interface modes. The solution is carried out for an applied dynamic load down as far as recovery of forces in individual members as a function of time. Another small beam problem is used to compare different means of "combining" substructures.

INTRODUCTION

During the past twenty years, a body of technology has developed within the general field of structural dynamics that has been identified by the term modal synthesis. Modal synthesis is a Rayleigh-Ritz approach using systematically derived displacement functions. It is used to formulate and solve the large eigen problems which arise in dynamic analysis of complex structural systems. Solutions are approximate in the sense that the motion of the structure is constrained to linear combinations of a limited number of modes or displacement functions characterizing the behavior of independent substructures.
Several researchers have formulated various modal synthesis procedures in an attempt to reduce computation errors and minimize computer costs. Hurty developed the first modal synthesis method capable of analyzing structures with redundant interface connections in references 1 and 2. He treated the structure as an assembly of connected components, or substructures, each of which is analyzed separately to derive a set of modes or displacement shapes from which a set of generalized coordinates applicable to the complete structure is synthesized. Craig and Bampton (ref. 3) simplified Hurty's formulation by combining two groups of coordinate functions which Hurty had defined separately. A number of survey papers have been written by Hou, Goldman, Benfield and Hruda in references 4 to 7. Some methods are found to be more suitable for certain applications than others. Yet, experience has shown that no single approach is generally preferred over the others.

The complexity of aerospace structures increased enormously during the last two decades. A new challenge is presented by the proposed space station (ref. 8) in that it is an evolving structure that cannot be ground tested because final configuration may not be known when the first component is put into space. Therefore, the component mode synthesis method may be applied for the dynamic analysis of such large structure system in space. A widely used tool for structural analysis, the NASTRAN® computer program, contains a modal synthesis capability but, other than the demonstration problem presented in reference 5, little is publicly known about its capabilities.

The purpose of the present report is to examine some of the capabilities of this program. This is done by examining two simple problems, a truss and a beam.

NUMERICAL EXAMPLES

The modal synthesis procedure in NASTRAN® is applied to two simple structures. One is a redundant truss confined to lie in a plane but free to move in this plane. It is composed entirely of ROD elements (no bending stiffness for all). This example is used to examine convergence character-
istics of the modal synthesis procedure and also to illustrate the transient response capability all the way down to obtaining stresses in rod members as a function of time. The second example is a free-free beam. It is used to examine different ways to "combine" substructures to yield frequency for the total structure.

**Truss Example**

The redundant truss example is the one used in reference 5 to compare eight different modal synthesis procedures. The full truss model is shown in figure 1(a) and its two components shown in figure 1(b). Component A consists of five equal bays and has a total of 18 joints. Component B consists of four equal bays and has a total of 15 joints. All members in the components have identical properties. At the interface of the components in the full truss model, the vertical member has twice the area of other members. Basic geometric and material properties are presented in table I along with the prescribed load for a transient response analysis. An additional run was made with the full model subdivided into three components with three bays in each component.

The basic run sequence and substructure operation are shown in figure 2. In the figure capitalized letters inside of rectangular blocks indicate names of pseudostructures used in the analysis. Capitalized letters adjacent to, or on, the flow diagram indicate the names of modules that perform a certain function in the computer program. At the top of figure 2, the Phase 1 operations formulate the finite element stiffness and mass matrices using Rigid Format 2. For the convergence study the Phase 2 runs on Rigid Format 3 were repeated using a different number of modes from the individual components. Also Phase 2 runs were using free interface modes as well as the fixed interface modes. A limited amount of data is presented for three components and naturally a Phase 1 run must be made for this component.

A transient response analysis was made on this free-free truss structure for an axial load applied to the right end of the truss. The load was applied for 0.12 seconds and then removed. In order to apply a load at grid point 42 in component B, this grid point must be included on a BOUNDARY
card. Thus, additional degrees of freedom are created corresponding to this point. The structure was represented by eight modes from component A, six modes from component B, and the eight interface modes for a total of twenty-two modes. The modes for the individual component were determined with the interface fixed. The standard procedure will obtain displacements back in the individual component. However, member forces and stresses are not determined automatically, but can be obtained through a simple procedure in a few steps. In the first step a run is made with DIAG 17 turned on to put the DMAP sequence on the punch file with an EXIT scheduled after statement 1. A small substructure deck is included to allow the appropriate commands that interface to the Substructures Operating File (SOF) to be generated. This punch file is subsequently saved and altered to replace the RECOVER module with the SDR2 module which can recover element forces and stresses. The listing of this DMAP sequence and run stream is contained in Appendix A.

Beam Example

This example consists of a beam composed of seven components as shown in figure 3(a). All subbeams have a constant length, area and uniform mass properties. Each component consists of ten equal elements and has a total of 11 joints as shown in figure 3(b). Basic geometric and material properties for each subbeam are presented in table I. A lumped mass formulation is used (no rotary inertia) and, therefore, there are 213 stiffness degrees of freedom in the problem, but only 142 eigenvalues.

Three different ways of "combining" substructures are illustrated in figures 3(c), 3(d), and 3(e). The basic run sequences and substructure operations for each case are shown in figures 4 thru 6. For all cases, the substructuring Phase 1 operations formulate the finite element stiffness and mass matrices for subbeam A using Rigid Format 3. The structural matrices contained in BBASIC, CBASIC, ..., FBASIC are generated as needed by using EQUIV operation. The basic subbeams are reduced to modal coordinates and combined together following the procedures shown in figures 4 thru 6. The eigenvalues of the total beam are obtained by using the MRECOVER command. The driver decks and sample bulk data for cases 1, 2 and 3 are listed in Appendices B, C and D. Only fixed interface modes were used but two sets of runs were made using a different number of modes from the subbeams.
RESULTS

For assessing the accuracy of the modal synthesis procedure, two and three truss components with fixed or free interface connection are run to determine frequencies and compared to results for full model. Percentage errors in frequency for the combined systems of 12, 20, 28 and 36 degrees of freedom are shown in tables III thru VI. Here degrees of freedom include not only the number of flexible modes used but also any interface modes. Thus, for example, for 12 degrees of freedom results, since there are six interface modes, only six flexible modes can be shown. Based on the lowest frequency criterion then four modes were chosen from component A and two modes from component B.

Figures 7 thru 11 are nondimensional plots that indicate the relative accuracy obtained by modal synthesis procedures. Also shown on the figures are results taken directly from reference 5 in which several other procedures are compared. From figures 7 to 10 it can be seen that modes derived with the interfaces fixed yield better results than modes derived with the interface free.

For the transient response run the percentage error in displacement for grid points 41, 42, and 43 of component B are shown in table VII. These results were produced from the 20 degrees of freedom model. The axial force in elements 111-113 and 143 of component B are shown in table VIII.

The full beam shown in figure 2 was run to determine its natural frequencies and used as a comparison of results obtained with the various "combination" procedures. Table IX shows the percentage error in frequency for the various "combination" procedures when 62 degrees of freedom are used. These 62 degrees of freedom correspond to approximately 47% of the total degrees of freedom in the full model. All three "combination" procedures yield good results. However, case 1 uses considerably less CYBER 75 CPU time than the other two cases (53.8 CPU seconds corresponds to 65.3 seconds, 59.1 seconds, respectively). Another run for case 1 was made using 19% of total degrees of freedom, and 55% frequencies were obtained with less than 1% error in frequency.
ACKNOWLEDGEMENT

This work was sponsored by the NASA-Langley Research Center under Grant NAG-1-301. The work was monitored under the supervision of Dr. Jerrold M. Housner and Mr. Joseph E. Walz, Structural Dynamics Branch, Structures and Dynamics Division. Mr. Mo-How Shen, a graduate student in the Department of Mechanical Engineering and Mechanics, Old Dominion University, carried out most of the detailed studies. The author would like to thank Mr. Joe Walz for many valuable suggestions and assistance.
REFERENCES


Table I. Truss Geometric and Material Properties

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<th>Value</th>
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<td>Typical frame height (see fig. 1(b))</td>
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<tr>
<td>Density</td>
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<td>Transient loads</td>
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Table IV. Frequency for Full Truss and Percent Error in Frequency for Two Modal Synthesis Models Using 20 Degrees of Freedom

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Table VII. Transient Response and Percent Error in Displacement

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<th>Full Truss</th>
<th>B. Substr.</th>
<th>F-B % F</th>
<th>Full Truss</th>
<th>B Substr.</th>
<th>F-B % F</th>
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Table VIII. The Axial Force in Elements of B Substructure

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<td>(%)</td>
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<td>---------------</td>
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Table IX. Percent Frequency Error Using 62 Degrees of Freedom (concluded)

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<th>(%)</th>
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Figure 1. Truss Model.
ABASIC

MREDUCE

Eigenvector Data MA

COMBINE

MCOMB

MREDUCE

Eigenvector Data MB

MREDUCE

MRECOVER

RTRUSS

Eigenvector Data MCOMB

SOLVE

Transient Data RTRUSS

RECOVER

Transient Data ABASIC Displacements

RECOVER

Transient Data ABASIC FORCES

Runs 1 and 2 Phase 1, RF2

Run 3 Phase 2, RF3

Run 4 Phase 3, RF9

Run 5 DMAP

Figure 2. Substructure Formulation Tree and Solution Sequence
(a) Total beam model.

(b) Representative finite element model of any component.

(c) Case 1 - All components combined simultaneously.

(d) Case 2 - Components combined sequentially.

(e) Case 3 - Components combined in pairs. Pairs then combined sequentially.

Figure 3. Total Beam Model and Various Subdivided Representations.
Figure 4. Case 1 Subbeam Formulation Tree and Solution Sequence
Figure 5. Case 2 Subbeam Formulation Tree and Solution Sequence.
Figure 6. Case 3 Subbeam Formulation Tree and Solution Sequence
Figure 7. Comparison of Methods with Frequency Error of 0.1%.
Legend:
NR1 = Free-interface NASTRAN
NR2 = Fixed-interface NASTRAN
BH1 = Benfield-Hruda, Free-Free
BH2 = Benfield-Hruda, Constrained
BH3 = Benfield-Hruda, Free-Free, Interface Loading
BH4 = Benfield-Hruda, Constrained, Interface Loading
H = Hurty
BF = Bajan-Feng
CB = Craig-Bampton
HO = Hou
G = Goldman

Figure 8. Comparison of Methods with Frequency Error of 0.5%.
Figure 9. Comparison of Methods with Frequency Error of 1.0%.
Figure 10. Comparison of Methods with Frequency Error of 5.0%.
Figure 11. Comparison of Methods with Frequency Error of 10.0%.
APPENDIX A. Driver decks and sample bulk data for two components truss problem.

NASTRAN FILES = UMF & CDC AND IBM
ID = DEMO31+ NASTRAN
APP DISP = SUBS
SOL 2,0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MOLSYN
SOF (1) = FTLY, 500, NEW & CDC AND IBM
NAME = A BASIC
SOFPROM = T0C
ENDSUBS
TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RF 2
SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-1
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CROD 12 1 1 12 13
CROD 21 1 1 21 22
CROD 22 1 1 22 23
CROD 31 1 1 31 32
CROD 32 1 1 32 33
CROD 41 1 1 41 42
CROD 42 1 1 42 43
CROD 51 1 1 51 52
CROD 52 1 1 52 53
CROD 111 1 1 1 11 12
CROD 112 1 1 2 22
CROD 113 1 1 3 33
CROD 121 1 1 11 22
CROD 122 1 1 22 33
CROD 123 1 1 33 44
CROD 131 1 1 44 55
CROD 132 1 1 55 66
CROD 133 1 1 66 77
CROD 141 1 1 77 88
CROD 142 1 1 88 99
CROD 143 1 1 99 100
CROD 151 1 1 100 111
CROD 152 1 1 111 122
CROD 153 1 1 122 133
CROD 211 1 1 133 144
CROD 212 1 1 144 155
CROD 213 1 1 155 166
CROD 221 1 1 166 177
CROD 222 1 1 177 188
CROD 231 1 1 188 199
CROD 232 1 1 199 200
CROD 241 1 1 200 211
CROD 242 1 1 211 222
CROD 251 1 1 222 233
CROD 252 1 1 233 244
CROD 30 1 1 244 255
CROD 31 1 1 255 333
CROD 32 1 1 333 444
CROD 33 1 1 444 555
CROD 34 1 1 555 666
CROD 35 1 1 666 777
CROD 36 1 1 777 888
CROD 37 1 1 888 999
CROD 38 1 1 999 1000
CROD 39 1 1 1000 456
CROD 40 1 1 3456 3456
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GRID 2 0 0 -30.0 0
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ENDDATA

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APP DISP = SUBS
SOL 2,0
TIME 3
CEND
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PASSWORD = MOLSYN
SOF(1) = FTLY+$ CDC AND IBM
NAME = BBASIC
SOFPRINT T0C
CEND
BEGIN 
BULK
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CROD | 2 | 1 | 2 | 3 |
CROD | 1 | 1 | 12 | 13 |
CROD | 1 | 1 | 21 | 22 |
CROD | 1 | 1 | 22 | 23 |
CROD | 31 | 1 | 31 | 32 |
CROD | 32 | 1 | 32 | 33 |
CROD | 41 | 1 | 41 | 42 |
CROD | 42 | 1 | 42 | 43 |
CROD | 111 | 1 | 11 | 12 |
CROD | 112 | 1 | 12 | 13 |
CROD | 113 | 1 | 13 | 14 |
CROD | 121 | 1 | 11 | 12 |
CROD | 122 | 1 | 12 | 13 |
CROD | 123 | 1 | 13 | 14 |
CROD | 131 | 1 | 11 | 12 |
CROD | 132 | 1 | 12 | 13 |
CROD | 133 | 1 | 13 | 14 |
CROD | 141 | 1 | 11 | 12 |
CROD | 142 | 1 | 12 | 13 |
CROD | 143 | 1 | 13 | 14 |
CROD | 144 | 1 | 14 | 15 |
CROD | 211 | 1 | 21 | 22 |
CROD | 212 | 1 | 22 | 23 |
CROD | 221 | 1 | 21 | 22 |
CROD | 222 | 1 | 22 | 23 |
CROD | 231 | 1 | 23 | 24 |
CROD | 232 | 1 | 24 | 25 |
CROD | 241 | 1 | 25 | 26 |
CROD | 242 | 1 | 26 | 27 |
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GROSET | 3 | -30.0 | 0.0 | 0.0 |
GROSET | 4 | 30.0 | 40.0 | 0.0 |
GROSET | 5 | 0.0 | 40.0 | 0.0 |

3456
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GRID    23  -30.0  80.0   .0
GRID    32    30.0 120.0   .0
GRID    33  -30.0 120.0   .0
GRID    41    30.0 160.0   .0
GRID    42    .0  160.0   .0
GRID    43  -30.0 160.0   .0
MAT1    43 10.0+6   .3   .3 2.5-4

$ NASTRAN FILES = UMF & CDC AND IBM
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SOL 3,0
TIME 5
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OPTIONS K*M*P
SOFPRINT TOC
MREDUCE ABASIC
NAME MA
BOUNDARY 5
FIXED 5
METHOD 19
OUTPUT 1,5,6,9,10
SOFPRINT TOC
MREDUCE BBASIC
NAME MB
BOUNDARY 4
FIXED 4
METHOD 29
OUTPUT 1,5,6,9,10
SOFPRINT TOC
MREDUCE CBASIC
NAME MC
BOUNDARY 7
FIXED 7
METHOD 39
OUTPUT 1,5,6,9,10
SOFPRINT TOC
COMBINE MA*MB*MC
NAME MCOMB
TOLERANCE 0.001
OUTPUT 2,7,12
COMPONENT MB
TRANSFORM 20
COMPONENT MC
TRANSFORM 40
SOFPRINT TOC
MREDUCE MCOMB
NAME MTRUSS
BOUNDARY 42
METHOD 90
NMAX 18
OUTPUT 1,5,6,9,10
SOFPRINT TOC
ENDSUBS
TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-4
LABEL = MODAL REDUCE, COMBINE, MODAL RECOVERY, RUN 4, PHASE 2, RF 3
BEGIN BULK

32
BEGIN DISP 09 - DIRECT TRANSIENT RESPONSE ANALYSIS - APR. 1982 $ 

PRECHK ALL $ 
FILE UDV T=APPEND/TOL=APPEND$ 
PARAM /*MPY*/CARDNO/0/0$ 
GP1 GEOM1,GEOM2,GPL,EQE XIN,GPDT,CS TM,BGP DT,SIL/S,N,LUSE T/S,N,$ 
NOGP D/T/AL/ways=-1 $ 
PLTTRAN BGP DT/SIL/BGP DT/STP/LUSE T/S,N,LUSE P$ 
PURGE USE T/STK,MAA,MAA,K4AA,PST,PK3,OP,EST,EYT/PLTSETX,PLTPAR,$ 
GP SETS,ELSETS/NOGP DT$ 
COND LBL5/NOGP DT$ 
GPZ GEOM2,EQE XIN/EYT$ 
PARAM PCDB/*PRE S*////JUMPPLOT$ 
PURGE PLTSETX,PLTPAR,GP SETS,ELSETS/JUMPPLOT$ 
COND P1,JUMPPLOT$ 
PLTSET PCDB,EQE XIN/EYT/PLTSETX,PLTPAR,GP SETS,ELSETS/S,N,NSIL/S,N,$ 
JUMPPLOT=-1 $ 
PRT MSG PLTSETX/ $ 
PARAM /*MPY*/PLTFLG/1/1$ 
PARAM /*MPY*/PFILE/0/0$ 
COND P1,JUMPPLOT$ 
PLOT PLTPAR,GP SETS,ELSETS,CASECC,BGP DT,EQE XIN,SI L,**/PLOTX1/$ 
NSIL/LUSE T/S,N,JUMPPLOT/S,N,PLTFLG/S,N,PFILE$ 
PRT MSG PLOTX1/ $ 
LABEL P1$ 
GP3 GEOM3,EQE X IN,GEOM2/SLT,GP TT/NOGR A V$ 
TA1 ECT,EPT,BGP DT,SIL,BGTT,CS FM/EST,GEI,GPECT,**/LUSE T/S,N,NOSIMP= 
-1/1/S,N,NOGENL=-1/S,N,GENEL$ 
PURGE K4GG,GP ST,OGPS T,GGG,BGG,K4NN,K4FF,K4AA,MNN,MFF,MAA,BN N,BFF,BAA,$ 
<KGA/NOSIMP/OGPS T/GENEL$ 
COND LBL1,NOSIMP$
ORIGINAL PAGE IS OF POOR QUALITY

```
JUMP FINIS $  
LABEL ERROR1 $  
PRTPARM /-1/DIRTRD $  
LABEL ERROR3 $  
PRTPARM /-3/DIRTRD $  
LABEL FINIS $  
PURGE DUMMY/ALWAYS$  
END $  
TIME 3  
DIAG 14  
END  
SUBSTRUCTURE PHASE3  
PASSWORD = MOLSYN  
SOF(1) = FT19500 $ CDC AND IBM  
ENDSUBS  
TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS  
DLOAD = 101  
TSTEP = 40  
QLOAD = ALL  
DISP = ALL  
FORCE = ALL  
BEGIN BULK  
CROD 1 1 1 2  
CROD 2 1 2 3  
CROD 11 1 11 12  
CROD 12 1 12 13  
CROD 21 1 21 22  
CROD 22 1 22 23  
CROD 31 1 31 32  
CROD 32 1 32 33  
CROD 41 1 41 42  
CROD 42 1 42 43  
CROD 111 1 1 11  
CROD 112 1 2 12  
CROD 113 1 3 13  
CROD 121 1 11 21  
CROD 122 1 12 22  
CROD 123 1 13 23  
CROD 131 1 21 31  
CROD 132 1 22 32  
CROD 133 1 23 33  
CROD 141 1 31 41  
CROD 142 1 32 42  
CROD 143 1 33 43  
CROD 212 1 2 13  
CROD 211 1 2 11  
CROD 221 1 12 21  
CROD 222 1 12 23  
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### APPENDIX A. (concluded)

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EOF

11.30.27, UCLP, DDLT0094B, 0.423 KINS.
APPENDIX B. Driver decks and sample bulk data for beam problem of case 1.

NASTRAN FILES = UMF & CDC AND IBM
ID DEM2031*NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MDLSYN
SOF(1) = FT17.500 & NEW & CDC AND IBM
NAME = ABASIC
SOFPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RC 2
BEGIN BULK
BAR0R 11 1 1 2
CBAR 21 1 2 3
CBAR 31 1 3 4
CBAR 41 1 4 5
CBAR 51 1 5 6
CBAR 61 1 6 7
CBAR 71 1 7 8
CBAR 81 1 8 9
CBAR 91 1 9 10
CBAR 10 1 10 11
GRIDSET GRID 1 0 0 0
GRID 2 10 0 0
GRID 3 20 0 0
GRID 4 30 0 0
GRID 5 40 0 0
GRID 6 50 0 0
GRID 7 60 0 0
GRID 8 70 0 0
GRID 9 80 0 0
GRID 10 90 0 0
GRID 11 100 0 0
PBAR 1 1 56 63
MAT 1 10.0*6 10.0 0.0 1
ENDDATA

NASTRAN FILES = UMF & CDC AND IBM
ID DEM2032*NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 5
CEND
SUBSTRUCTURE PHASE 2
PASSWORD = MDLSYN
SOF(1) = FT17.500 & NEW & CDC AND IBM
EQUIV ABASIC, BBASIC
PREFIX B
EQUIV ABASIC, GBASIC
PREFIX G
MREDUCE ABASIC
NAME MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE BBASIC
NAME MB
BOUNDARY 2
APPENDIX B. (concluded)

FIXED 2
METHOD 2
MREDUCE GBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
EQUIV MB, MC
PREFIX C
EQUIV MB, MD
PREFIX D
EQUIV ME, ME
PREFIX E
EQUIV MB, MF
PREFIX F
COMBINE MA*MB*MC*MD*ME*MF*MG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MB
TRANSFORM 2
COMPONENT MC
TRANSFORM 3
COMPONENT MD
TRANSFORM 4
COMPONENT ME
TRANSFORM 5
COMPONENT MF
TRANSFORM 6
COMPONENT MG
TRANSFORM 7
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 22
OUTPUT 1, 5, 6, 9, 10
SOFPRINT TOC
ENDSUBF
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE, COMBINE, MODAL, RECOVERY, RUN2, PHASE2
BEGIN BULK
BDYC 5  G BASIC 50
BDYC 3  G BASIC 40
BDYC 20  ABASIC 30
BDYS1 30  126  11
BDYS1 40  126  1
BDYS1 50  126  1
TRANS 2  150.  0.  0.  100.  0.  0.  1.
TRANS 3  150.  0.  0.  200.  0.  0.  1.
TRANS 4  150.  0.  0.  300.  0.  0.  1.
TRANS 5  150.  0.  0.  400.  0.  0.  1.
TRANS 6  150.  0.  0.  500.  0.  0.  1.
TRANS 7  150.  0.  0.  600.  0.  0.  1.
EIGR 1  INV  .0  3000.00  10  10
EIGR 2  MAX  .0  3000.00  10  10
EIGR 3  INV  .0  3000.00  10  10
EIGR 22  MAX  .0  2000.00  40  40
ENDDATA
#
APPENDIX C. Driver decks and sample bulk data for beam problem of case 2.

NASTRAN FILES = UMF & CDC AND IBM
ID DEM2031, NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MDLSYN
SOF (1) = FT17,500, NEW & CDC AND IBM
NAME = ABASIC
SOFPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, R6 2
BEGIN BULK
BAROR
1 1
CBAR 2 1 2
CBAR 3 1 3
CBAR 4 1 4
CBAR 5 1 5
CBAR 6 1 6
CBAR 7 1 7
CBAR 8 1 8
CBAR 9 1 9
CBAR 10 1 10
GRIDSET
GRID 1
GRID 2
GRID 3
GRID 4
GRID 5
GRID 6
GRID 7
GRID 8
GRID 9
GRID 10
GRID 11
PBAR 1
MAT 1
ENDDATA
&
NASTRAN FILES = UMF & CDC AND IBM
ID DEM2032, NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 5
CEND
SUBSTRUCTURE PHASE 2
PASSWORD = MDLSYN
SOF (1) = FT17, 500 & CDC AND IBM
EQUIV ABASIC, DBASIC
PREFIX B
EQUIV ABASIC, CBASIC
PREFIX C
EQUIV ABASIC, DBASIC
PREFIX D
EQUIV ABASIC, EBASIC
PREFIX E
EQUIV ABASIC, FBASIC
PREFIX F
EQUIV ABASIC, GBASIC
PREFIX G
MREDUCE ABASIC
NAME MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE BBASIC
NAME MB
BOUNDARY 2
FIXED 2
METHOD 2
MREDUCE CBASIC
NAME MC
BOUNDARY 7
FIXED 7
METHOD 2
MREDUCE DBASIC
NAME MD
BOUNDARY 8
FIXED 8
METHOD 2
MREDUCE EBASIC
NAME ME
BOUNDARY 9
FIXED 9
METHOD 2
MREDUCE GBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
COMBINE MBA,MB
NAME AB
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MD
TRANSFORM 2
MREDUCE AB
NAME MBA
BOUNDARY 10
FIXED 10
METHOD 22
COMBINE MBA,MC
NAME ABC
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MC
TRANSFORM 3
MREDUCE ABC
NAME MABC
BOUNDARY 21
FIXED 21
METHOD 22
COMBINE MABC,MD
NAME ABCD
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MD
TRANSFORM 4
MREDUCE ABCD
NAME MABCD
BOUNDARY 22
FIXED 22
METHOD 22
COMBINE MACD E, ME
NAME ABCDE
TOLERANCE 0.01
OUTPUT 2 7, 12
COMPONENT ME
TRANSFORM S
MREDUCE ABCDE
NAME MACD E
BOUNDARY 23
FIXED 23
METHOD 22
COMBINE MACD E, MF
NAME ABCDEF
TOLERANCE 0.01
OUTPUT 2 7, 12
COMPONENT MF
TRANSFORM 6
MREDUCE ABCDEF
NAME MACD E
BOUNDARY 24
FIXED 24
METHOD 22
COMBINE MACD E, MG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2 7, 12
COMPONENT MG
TRANSFORM 7
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 22
OUTPUT 1 5, 6, 9, 10
SOFLPRINT TOC
ENDSUBS
TITLE BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL MODAL REDUCE, COMBINE, MODAL, RECOVSRY, RUN2, P8 S52
BEGIN Bulk
BDYC 7  CBASIC  50
BDYC 9  CBASIC  50
BDYC 21  CBASIC  30
BDYC 22  DBASIC  30
BDYC 23  DBASIC  30
BDYC 24  FBASIC  30
BDYC 11  FBASIC  30
BDYC 18  DBASIC  30
BDYC 10  BBASIC  30
BDYC 15  CBASIC  40
BDYC 22  BBASIC  50
BDYC 33  GBASIC  40
BDYC 30  ABASIC  30
BDYS1 30  126  11
BDYS1 40  126  1
BDYS1 50  126  1
TRANS 4  350.0  0.0  300.0  0.0  0.0  300.0  0.0  1.0
+T4 350.0  0.0  500.0  0.0  0.0  500.0  0.0  1.0
+T6
+T2 150.0  0.0  100.0  0.0  0.0  100.0  0.0  1.0
+T2
+T3 250.0  0.0  200.0  0.0  0.0  200.0  0.0  1.0
+T3
+T5 450.0  0.0  400.0  0.0  0.0  400.0  0.0  1.0
+T5
END
APPENDIX C. (concluded)

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+E2+  
+E3+  
+E22+  

INDDATA
APPENDIX D. Driver decks and sample bulk data for beam problem of case 3.

NASTRAN FILES = UMF $ CDC AND IBM
ID = DEMZ932,NASTRAN
APP DISP, SUBS
SOL 30
TIME 10
CEND
SUBSTRUCTURE PHASE2
PASSWORD = MDLSYN
SOF(1) = FT1,9500,NEW $ CDC AND IBM
NAME = ABASIC
SOPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RG 2
BEGIN BULK
BARO
CBAR 1 1 1 2
CBAR 2 1 2 3
CBAR 3 1 3 4
CBAR 4 1 4 5
CBAR 5 1 5 6
CBAR 6 1 6 7
CBAR 7 1 7 8
CBAR 8 1 8 9
CBAR 9 1 9 10
CBAR 10 1 10 11
GRIDSET
GRID 1 0 1 0
GRID 2 10 0 0
GRID 3 20 0 0
GRID 4 30 0 0
GRID 5 40 0 0
GRID 6 50 0 0
GRID 7 60 0 0
GRID 8 70 0 0
GRID 9 80 0 0
GRID 10 90 0 0
PBAR 1 100 0 0
PBAR 2 50 0 0
PBAR 3 63 0 0
MAT 1 10 0.6 4
ENDDATA
&

NASTRAN FILES = UMF $ CDC AND IBM
ID = DEMZ932,NASTRAN
APP DISP, SUBS
SOL 30
TIME 10
CEND
SUBSTRUCTURE PHASE2
PASSWORD = MDLSYN
SOF(1) = FT1,9500,NEW $ CDC AND IBM
EQUIV ABASIC, BBASIC
PREFIX B
EQUIV ABASIC, CBASIC
PREFIX C
EQUIV ABASIC, DBASIC
PREFIX D
EQUIV ABASIC, EBASIC
PREFIX E
EQUIV ABASIC, FBASIC
PREFIX F
EQUIV ABASIC, GBASIC
PREFIX G
MREDUCE ABASIC

2.591-4
NAME  MA
BOUNDARY  20
FIXED  20
METHOD  1
MREDUCE BBASIC
NAME  MB
BOUNDARY  2
FIXED  2
METHOD  2
MREDUCE CBASIC
NAME  MC
BOUNDARY  7
FIXED  7
METHOD  2
MREDUCE DBASIC
NAME  MD
BOUNDARY  8
FIXED  8
METHOD  2
MREDUCE EBASIC
NAME  ME
BOUNDARY  9
FIXED  9
METHOD  2
MREDUCE FBASIC
NAME  MF
BOUNDARY  11
FIXED  11
METHOD  2
MREDUCE GBASIC
NAME  MG
BOUNDARY  3
FIXED  3
METHOD  3
COMBINE MA*MB
NAME  AB
TOLERANCE  0.01
OUTPUT  2,7,12
COMPONENT  MB
TRANSFORM  2
MREDUCE AB
NAME  MAB
BOUNDARY  10
FIXED  10
METHOD  22
COMBINE MC*MD
NAME  CD
TOLERANCE  0.01
OUTPUT  2,7,12
COMPONENT  MD
TRANSFORM  2
MREDUCE CD
NAME  MCD
BOUNDARY  15
FIXED  15
METHOD  22
COMBINE ME*MF
NAME  EF
TOLERANCE  0.01
OUTPUT  2,7,12
COMPONENT  MF
TRANSFORM  2
MREDUCE EF
NAME  MEF
BOUNDARY  25
FIXED  25
METHOD  22
COMBINE MAB*MCD
NAME ABCD
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MCD
TRANSFORM 3
MREDUCE ABCD
NAME ABCD
BOUNDARY 30
FIXED 30
METHOD 25
COMBINE MEF*MG
NAME EFG
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MG
TRANSFORM 3
MREDUCE EFG
NAME EFG
BOUNDARY 35
FIXED 35
METHOD 25
COMBINE MABC,MEFG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MEF
TRANSFORM 5
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 25
OUTPUT 1,5,6,9,10
SOPRINT TOC
END SUBS
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE,COMBINE,MODAL,RECOVERY, RUN2, PHASE2
BEGIN BULK
BDYC 30  DBASIC 30
BDYC 35  DBASIC 40
BDYC 30  DBASIC 50
BDYC 11  DBASIC 50
BDYC 25  DBASIC 40
BDYC 8  DBASIC 50
BDYC 10  DBASIC 30
BDYC 5  DBASIC 40
BDYC 22  DBASIC 50
BDYC 35  DBASIC 40
BDYC 20  DBASIC 30
BDYS 1  126  11
BDYS 1  126  11
TRANS 2  0.  0.  0.  0.  100.  0.  1.  +T2
TRANS 3  0.  0.  200.  0.  0.  200.  0.  1.  +T3
TRANS 5  0.  0.  0.  0.  400.  0.  0.  1.  +T5
TRANS 7  0.  0.  600.  0.  0.  600.  0.  1.  +T7
EIGR  1  INV  3000.00  10  10
EIGR  2  INV  3000.00  10  10

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APPENDIX D. (concluded)

| +E2  | MAX | INV | .0 | 3000.00 | 10  | 10 |
|+E3  | MAX | INV | 0.0| 1000.0 | 40  | 40 |
|+E25 | MAX | INV | .0 | 2000.0 | 40  | 40 |

ENDDATA
**Title and Subtitle**

NASTRAN® MODAL SYNTHESIS CAPABILITY

**Author(s)**

Chuh Mei, Principal Investigator and Mo-How Shen

**Performing Organization Name and Address**

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P.O. Box 6369
Norfolk, Virginia 23508

**Sponsoring Agency Name and Address**

National Aeronautics and Space Administration
Hampton, Virginia

**Abstract**

This report compares the accuracy between NASTRAN® modal synthesis, full structure NASTRAN® and several other modal synthesis results (truss only). The results are based on a truss or beam having redundant or point interface connections. Each component substructure is reduced to modal and boundary degrees of freedom prior to the substructure combine operation. The combination structure, formulated in terms of the component modes, is also reduced to modal degrees of freedom for solution by the transient analysis rigid format.

**Distribution Statement**

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