COMPONENT MODE SYNTHESIS AND LARGE DEFLECTION VIBRATION OF COMPLEX STRUCTURES
VOLUME 1: EXAMPLES OF NASTRAN® MODAL SYNTHESIS CAPABILITY

By
Chuh Mei, Principal Investigator
and
Mo-How Shen

Final Report
For the period ended January 31, 1987

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
Research Grant NAG-1-301
Mr. Joseph E. Walz, Technical Monitor
SDD-Structural Dynamics Branch

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Submitted by the
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P.O. Box 6369
Norfolk, Virginia 23508

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SUMMARY

This report illustrates the use of NASTRAN® modal synthesis capability for some small examples. A classical truss problem is examined and the results for accuracy are compared to existing results from other methods. This problem is examined using both fixed interface modes and free interface modes. The solution is carried out for an applied dynamic load down as far as recovery of forces in individual members as a function of time. Another small beam problem is used to compare different means of "combining" substructures.

INTRODUCTION

During the past twenty years, a body of technology has developed within the general field of structural dynamics that has been identified by the term modal synthesis. Modal synthesis is a Rayleigh-Ritz approach using systematically derived displacement functions. It is used to formulate and solve the large eigen problems which arise in dynamic analysis of complex structural systems. Solutions are approximate in the sense that the motion of the structure is constrained to linear combinations of a limited number of modes or displacement functions characterizing the behavior of independent substructures.
Several researchers have formulated various modal synthesis procedures in an attempt to reduce computation errors and minimize computer costs. Hurty developed the first modal synthesis method capable of analyzing structures with redundant interface connections in references 1 and 2. He treated the structure as an assembly of connected components, or substructures, each of which is analyzed separately to derive a set of modes or displacement shapes from which a set of generalized coordinates applicable to the complete structure is synthesized. Craig and Bampton (ref. 3) simplified Hurty's formulation by combining two groups of coordinate functions which Hurty had defined separately. A number of survey papers have been written by Hou, Goldman, Benfield and Hruda in references 4 to 7. Some methods are found to be more suitable for certain applications than others. Yet, experience has shown that no single approach is generally preferred over the others.

The complexity of aerospace structures increased enormously during the last two decades. A new challenge is presented by the proposed space station (ref. 8) in that it is an evolving structure that cannot be ground tested because final configuration may not be known when the first component is put into space. Therefore, the component mode synthesis method may be applied for the dynamic analysis of such large structure system in space. A widely used tool for structural analysis, the NASTRAN® computer program, contains a modal synthesis capability but, other than the demonstration problem presented in reference 5, little is publicly known about its capabilities.

The purpose of the present report is to examine some of the capabilities of this program. This is done by examining two simple problems, a truss and a beam.

**Numerical Examples**

The modal synthesis procedure in NASTRAN® is applied to two simple structures. One is a redundant truss confined to lie in a plane but free to move in this plane. It is composed entirely of ROD elements (no bending stiffness for all). This example is used to examine convergence character-
istics of the modal synthesis procedure and also to illustrate the transient response capability all the way down to obtaining stresses in rod members as a function of time. The second example is a free-free beam. It is used to examine different ways to "combine" substructures to yield frequency for the total structure.

Truss Example

The redundant truss example is the one used in reference 5 to compare eight different modal synthesis procedures. The full truss model is shown in figure 1(a) and its two components shown in figure 1(b). Component A consists of five equal bays and has a total of 18 joints. Component B consists of four equal bays and has a total of 15 joints. All members in the components have identical properties. At the interface of the components in the full truss model, the vertical member has twice the area of other members. Basic geometric and material properties are presented in table I along with the prescribed load for a transient response analysis. An additional run was made with the full model subdivided into three components with three bays in each component.

The basic run sequence and substructure operation are shown in figure 2. In the figure capitalized letters inside of rectangular blocks indicate names of pseudostructures used in the analysis. Capitalized letters adjacent to, or on, the flow diagram indicate the names of modules that perform a certain function in the computer program. At the top of figure 2, the Phase 1 operations formulate the finite element stiffness and mass matrices using Rigid Format 2. For the convergence study the Phase 2 runs on Rigid Format 3 were repeated using a different number of modes from the individual components. Also Phase 2 runs were using free interface modes as well as the fixed interface modes. A limited amount of data is presented for three components and naturally a Phase 1 run must be made for this component.

A transient response analysis was made on this free-free truss structure for an axial load applied to the right end of the truss. The load was applied for 0.12 seconds and then removed. In order to apply a load at grid point 42 in component B, this grid point must be included on a BOUNDARY
card. Thus, additional degrees of freedom are created corresponding to this point. The structure was represented by eight modes from component A, six modes from component B, and the eight interface modes for a total of twenty-two modes. The modes for the individual component were determined with the interface fixed. The standard procedure will obtain displacements back in the individual component. However, member forces and stresses are not determined automatically, but can be obtained through a simple procedure in a few steps. In the first step a run is made with DIAG 17 turned on to put the DMAP sequence on the punch file with an EXIT scheduled after statement 1. A small substructure deck is included to allow the appropriate commands that interface to the Substructures Operating File (SOF) to be generated. This punch file is subsequently saved and altered to replace the RECOVER module with the SDR2 module which can recover element forces and stresses. The listing of this DMAP sequence and run stream is contained in Appendix A.

Beam Example

This example consists of a beam composed of seven components as shown in figure 3(a). All subbeams have a constant length, area and uniform mass properties. Each component consists of ten equal elements and has a total of 11 joints as shown in figure 3(b). Basic geometric and material properties for each subbeam are presented in table II. A lumped mass formulation is used (no rotary inertia) and, therefore, there are 213 stiffness degrees of freedom in the problem, but only 142 eigenvalues.

Three different ways of "combining" substructures are illustrated in figures 3(c), 3(d), and 3(e). The basic run sequences and substructure operations for each case are shown in figures 4 thru 6. For all cases, the substructuring Phase 1 operations formulate the finite element stiffness and mass matrices for subbeam A using Rigid Format 3. The structural matrices contained in BBASIC, CBASIC, ..., FBASIC are generated as needed by using EQUIV operation. The basic subbeams are reduced to modal coordinates and combined together following the procedures shown in figures 4 thru 6. The eigenvalues of the total beam are obtained by using the MRECOVER command. The driver decks and sample bulk data for cases 1, 2 and 3 are listed in Appendices B, C and D. Only fixed interface modes were used but two sets of runs were made using a different number of modes from the subbeams.
RESULTS

For assessing the accuracy of the modal synthesis procedure, two and three truss components with fixed or free interface connection are run to determine frequencies and compared to results for full model. Percentage errors in frequency for the combined systems of 12, 20, 28 and 36 degrees of freedom are shown in tables III thru VI. Here degrees of freedom include not only the number of flexible modes used but also any interface modes. Thus, for example, for 12 degrees of freedom results, since there are six interface modes, only six flexible modes can be shown. Based on the lowest frequency criterion then four modes were chosen from component A and two modes from component B.

Figures 7 thru 11 are nondimensional plots that indicate the relative accuracy obtained by modal synthesis procedures. Also shown on the figures are results taken directly from reference 5 in which several other procedures are compared. From figures 7 to 10 it can be seen that modes derived with the interfaces fixed yield better results than modes derived with the interface free.

For the transient response run the percentage error in displacement for grid points 41, 42, and 43 of component B are shown in table VII. These results were produced from the 20 degrees of freedom model. The axial force in elements 111-113 and 143 of component B are shown in table VIII.

The full beam shown in figure 2 was run to determine its natural frequencies and used as a comparison of results obtained with the various 'combination' procedures. Table IX shows the percentage error in frequency for the various 'combination' procedures when 62 degrees of freedom are used. These 62 degrees of freedom correspond to approximately 47% of the total degrees of freedom in the full model. All three 'combination' procedures yield good results. However, case 1 uses considerably less CYBER 75 CPU time than the other two cases (53.8 CPU seconds corresponds to 65.3 seconds, 59.1 seconds, respectively). Another run for case 1 was made using 19% of total degrees of freedom, and 55% frequencies were obtained with less than 1% error in frequency.
ACKNOWLEDGEMENT

This work was sponsored by the NASA-Langley Research Center under Grant NAG-1-301. The work was monitored under the supervision of Dr. Jerrold M. Housner and Mr. Joseph E. Walz, Structural Dynamics Branch, Structures and Dynamics Division. Mr. Mo-How Shen, a graduate student in the Department of Mechanical Engineering and Mechanics, Old Dominion University, carried out most of the detailed studies. The author would like to thank Mr. Joe Walz for many valuable suggestions and assistance.
REFERENCES


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<th>Table I. Truss Geometric and Material Properties</th>
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</tr>
<tr>
<td><strong>Typical frame height</strong>&lt;br&gt;(see fig. 1(b))</td>
</tr>
<tr>
<td><strong>Cross-sectional area of members</strong></td>
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<tr>
<td><strong>Young's modulus</strong></td>
</tr>
<tr>
<td><strong>Density</strong></td>
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</table>
| **Transient loads**                            | $P_{42} = 2.2 \times 10^3 \text{Kg(10}^3\text{lbf}) \ 0 < t < 0.12s$
|                                                | $0 \quad t > 0.12s$ |
Table II. Beam Geometric and Material Properties

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<td>Cross section of beam</td>
<td>( A = 3.613 \text{ cm}^2 (0.56 \text{ in}^2) )</td>
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<td>Young's modulus</td>
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<tr>
<td>Density</td>
<td>( \rho = 282.437 \text{ \frac{Kg\cdot\text{sec}^2}{m^4}} ) (2.591 \times 10^{-4} \text{ \frac{lbf\cdot\text{sec}^2}{in^4}})</td>
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<td>Total beam length</td>
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Table III. Frequency for Full Truss and Percent Error in Frequency for Two Modal Synthesis Models Using 12 Degrees of Freedom

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Table VII. Transient Response and Percent Error in Displacement

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<th>Full Truss</th>
<th>B. Substr.</th>
<th>F-B % F</th>
<th>Full Truss</th>
<th>B Substr.</th>
<th>F-B % F</th>
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<td>4.793882</td>
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Table VIII. The Axial Force in Elements of B Substructure

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Table IX. Percent Frequency Error Using 62 Degrees of Freedom

<table>
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<tr>
<th>Mode No.</th>
<th>Full Beam (Hz)</th>
<th>Case 1 (Hz)</th>
<th>(%)</th>
<th>Case 2 (Hz)</th>
<th>(%)</th>
<th>Case 3 (Hz)</th>
<th>(%)</th>
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<tbody>
<tr>
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Table IX. Percent Frequency Error Using 62 Degrees of Freedom (concluded)

<table>
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<th>Mode No.</th>
<th>Full Beam (Hz)</th>
<th>Case 1 (Hz)</th>
<th>(%)</th>
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<th>Case 2 (%)</th>
<th>Case 3 (Hz)</th>
<th>Case 3 (%)</th>
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</tbody>
</table>
Figure 1. Truss Model.
Figure 2. Substructure Formulation Tree and Solution Sequence
(a) Total beam model.

(b) Representative finite element model of any component.

(c) Case 1 - All components combined simultaneously.

(d) Case 2 - Components combined sequentially.

(e) Case 3 - Components combined in pairs. Pairs then combined sequentially.

Figure 3. Total Beam Model and Various Subdivided Representations.
Figure 4. Case 1 Subbeam Formulation Tree and Solution Sequence
Figure 5. Case 2 Subbeam Formulation Tree and Solution Sequence.
Figure 6. Case 3 Subbeam Formulation Tree and Solution Sequence
Legend:
NR1 = Free-Interface NASTRAN
NR2 = Fixed-Interface NASTRAN
BH1 = Benfield-Hruda, Constrained
BH2 = Benfield-Hruda, Free-Fixed
BH3 = Benfield-Hruda, Free-Free, Interface Loading
BH4 = Harty
BF = Bajan-Feng
CB = Craig-Bampton
H = Ho
G = Goldman

Figure 7. Comparison of Methods with Frequency Error of 0.1%.
Figure 8. Comparison of Methods with Frequency Error of 0.5%.
Figure 9. Comparison of Methods with Frequency Error of 1.0%.
Figure 10. Comparison of Methods with Frequency Error of 5.0%.
Figure 11. Comparison of Methods with Frequency Error of 10.0%.
APPENDIX A. Driver decks and sample bulk data for two components truss problem.

NASTRAN FILES = UMF & CDC AND IBM
ID  = DEME2031*NASTRAN
APP  = SUBS
SOL  = 2, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MOLSYN
SOF(1) = FLY, 500, NEW & CDC AND IBM
NAME = ABASIC
SOFPRT = TOC
ENDSUBS
TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RF 2
SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-1
BEGIN SOL
CRD  1   1   1   2
CRD  2   1   1   2  3
CRD  11   1   11   12
CRD  12   1   12   13
CRD  21   1   21   22
CRD  22   1   22   23
CRD  31   1   31   32
CRD  32   1   32   33
CRD  41   1   41   42
CRD  42   1   42   43
CRD  51   1   51   52
CRD  52   1   52   53
CRD  111   1   111   112
CRD  112   1   112   113
CRD  113   1   113   114
CRD  121   1   121   122
CRD  122   1   122   123
CRD  123   1   123   124
CRD  131   1   131   132
CRD  132   1   132   133
CRD  133   1   133   134
CRD  141   1   141   142
CRD  142   1   142   143
CRD  143   1   143   144
CRD  151   1   151   152
CRD  152   1   152   153
CRD  153   1   153   154
CRD  211   1   211   212
CRD  212   1   212   213
CRD  213   1   213   214
CRD  221   1   221   222
CRD  222   1   222   223
CRD  223   1   223   224
CRD  231   1   231   232
CRD  232   1   232   233
CRD  233   1   233   234
CRD  241   1   241   242
CRD  242   1   242   243
CRD  243   1   243   244
CRD  251   1   251   252
CRD  252   1   252   253
GROSET 1   1   .0   -30.0  0
GRID  3   1   .0   30.0  .0
GRID  11  1   .0   -30.0  0
GRID  12  1   40.0  -30.0  .0
GRID  13  1   40.0   30.0  .0
GRID  21  1   80.0  -30.0  .0
GRID 22
GRID 23
GRID 31
GRID 32
GRID 33
GRID 41
GRID 42
GRID 43
GRID 51
GRID 52
GRID 53
MAT 1
PROD 1

ENDDATA

NASTRAN FILES = UMF $ CDC AND IBM
ID = DEM2032=NASTRAN
APP DISP = SUBS
SOL 2.0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MOLSYN
SOF(1) = FTLY+500 $ CDC AND IBM
NAME = BBASIC
SOFPRINT TOC
ENDSUBS
TITLE = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE = NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL = SUBSTRUCTURE 2, RUN 2, PHASE 1, RF 2
BEGIN BULK
CROD 1 1 1 2
CROD 2 1 1 2 3
CROD 11 1 1 12 13
CROD 12 1 1 12 13
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CROD 32 1 1 32 33
CROD 41 1 1 41 42
CROD 42 1 1 42 43
CROD 11 1 1 11 12
CROD 11 1 1 11 12
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CROD 13 1 1 13 14
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CROD 22 1 1 22 23
CROD 31 1 1 31 32
CROD 32 1 1 32 33
CROD 41 1 1 41 42
CROD 42 1 1 42 43

GROSET
GRID 3
GRID 5
GRID 11
GRID 12

3.456

31
NASTRAN FILES = UMF $ CDC AND IBM
ID DEM2035$NASTRAN
APP DMAP$SBS
BEGIN DSP 09 - DIRECT TRANSIENT RESPONSE ANALYSIS - APR. 1982 $
PRECHK ALL $
FILE UDV=APPEND/TOL=APPEND $
PARAM /*$MPY*/CARDNO/0/0 $
GP1 GEO1,GEOM2,$GP$EQU$GEP$T$CSTM$BGPD$T$SIL$S$N$LUSE$T$S$N$,
NOGP$T$ALW$AYS=-1 $
PLT$TRAN $BGPD$T$SIL$BGPD$T$SIP$LUSE$T$S$N$LUSEP $
PURGE USE$T$G$M$G$KAA$BAA$MAA$K4$A$PST$KFS$OP$EST$FCT$PLT$SET$X$PLTPAR$,
CE$SETS$EL$SETS$NOGP$D$T $
COND LBL5$NOGP$D$T $
GPZ GEO$M2$EQU$IN$FCT $
PARAM PCD$B/*$PRES$///$JUMPPLOT $
PURGE PLT$SET$X$PLTPAR$CE$SETS$EL$SETS$JUMPPLOT $
COND P1$JUMPPLOT $
PLT$SET PCD$B$EQU$IN$FCT$PLT$SET$X$PLTPAR$CE$SETS$EL$SETS$S$N$NSIL$S$N$,
JUMPPLOT=-1 $
PRT$MSG PLT$SET$X/$
PARAM /*$MPY*/PLT$FLG/1/1 $
PARAM /*$MPY*/PF$ILE/0/0 $
COND P1$JUMPPLOT $
PLOT PLTPAR$CE$SETS$EL$SETS$CASECC$BGPD$T$EQU$IN$SIL$E$ECT$/PLT$X$PLT$FLG$S$N$,
PFILE $
PRT$MSG PLOT$X$/ $
LABEL P1 $
GP3 GEO$M3$EQU$IN$GEOM$SLT$GPTT$NOGRAV $
TAL ECT$EPT$BGPD$T$SIL$GPTT$CSTM$EST$GEI$GPECT$/$LUSE$T$S$N$NOSIM$P$=
-1/1/S$N$NOGENL=-1/S$N$GENEL $
PURGE K$4$G$G$PST$OGP$ST$MGG$B$GG$K$NN$K4$FF$K4$A$MNN$MFF$MAA$BNN$BFF$BAA$,
<GGX$NOSIM$P$=0$G$P$ST$GENEL $
COND LBL1$NOSIM$P $
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**TIME 3**

**DIAG 14**

**GEN**

**SUBSTRUCTURE PHASE 3**

**PASSWORD** = MOLSYN

**SOF (1)** = FT19500 $ CDC AND IBM

**ENDSUBS**

**TITLE** = TRUSS DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS

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**QLOAD** = ALL

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APPENDIX B. Driver decks and sample bulk data for beam problem of case 1.

NASTRAN FILES = UMF & CDC AND IBM
ID = DEM2031,NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MDLSYN
SOF (1) = FT17500, NEW & CDC AND IBM
NAME = ABASIC
SOFPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RC 2
BEGIN BULK
BAROR
BAR 1 1 1 2
BAR 2 1 2 3
BAR 3 1 3 4
BAR 4 1 4 5
BAR 5 1 5 6
BAR 6 1 6 7
BAR 7 1 7 8
BAR 8 1 8 9
BAR 9 1 9 10
BAR 10 1 10 11
GRIDSET
GRID 1 0. 0. 0.
GRID 2 10. 0. 0.
GRID 3 20. 0. 0.
GRID 4 30. 0. 0.
GRID 5 40. 0. 0.
GRID 6 50. 0. 0.
GRID 7 60. 0. 0.
GRID 8 70. 0. 0.
GRID 9 80. 0. 0.
GRID 10 90. 0. 0.
GRID 11 100. 0. 0.
PBAR 1 1 1 2.591-4
MAT 1 10.0 63.0 2.591-4
ENDDATA

NASTRAN FILES = UMF & CDC AND IBM
ID = DEM2032,NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 5
CEND
SUBSTRUCTURE PHASE 2
PASSWORD = MDLSYN
SOF (1) = FT17900, NEW & CDC AND IBM
EQUIV, ABASIC, BBASIC
PREFIX B
EQUIV, ABASIC, GBASIC
PREFIX G
MREDUCE, ABASIC
NAME = MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE, BBASIC
NAME = MB
BOUNDARY 2

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APPENDIX B. (concluded)

FIXED 2
METHOD 2
MREDUCE GBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
EQUIV MB, MC
PREFIX C
EQUIV MB, MD
PREFIX D
EQUIV ME, ME
PREFIX E
EQUIV MB, MF
PREFIX F
COMBINE MA*MB*MC*MD*ME*MF*MG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MB
TRANSFORM 2
COMPONENT MC
TRANSFORM 3
COMPONENT MD
TRANSFORM 4
COMPONENT ME
TRANSFORM 5
COMPONENT MF
TRANSFORM 6
COMPONENT MG
TRANSFORM 7
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 22
OUTPUT 1, 5, 6, 9, 10
SOFPRT TOC
ENDSUBF
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE, COMBINE, MODAL, RECOVERY, RUN2, PHASE2
BEGIN BULK
BDYC 5 BBASIC 50
BDYC 30 ABASIC 40
BDYC 20 ABASIC 30
BDYS 30 126 11
BDYS 40 126 1
BDYS 50 126 1
TRANS 2 150. 0. 0. 0. 0. 0. 100. 0. 1. 1. +T2
TRANS 3 250. 0. 0. 0. 0. 0. 200. 0. 1. 1. +T3
TRANS 4 350. 0. 0. 0. 0. 0. 300. 0. 1. 1. +T4
TRANS 5 450. 0. 0. 0. 0. 0. 400. 0. 1. 1. +T5
TRANS 6 550. 0. 0. 0. 0. 0. 500. 0. 1. 1. +T6
TRANS 7 650. 0. 0. 0. 0. 0. 600. 0. 1. 1. +T7
EIGR 1 INV 0. 3000. 10 0. 1. +E1
EIGR 2 MAX 0. 3000. 10 0. 1. +E2
EIGR 3 INV 0. 3000. 10 0. 1. +E3
EIGR 22 MAX 0. 2000. 40 40 1. +E22
ENDDATA

#
APPENDIX C. Driver decks and sample bulk data for beam problem of case 2.

NASTRAN FILES = UMF $ CDC AND IBM
ID DEM2031$NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = M0LSYN
SOF (1) = F17$500, NEW $ CDC AND IBM
NAME = ABASIC
SOFPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, R6 2
BEGIN BULK
BAROR 1 1 1 2
BAR 2 1 2 3
BAR 3 1 3 4
BAR 4 1 4 5
BAR 5 1 5 6
BAR 6 1 6 7
BAR 7 1 7 8
BAR 8 1 8 9
BAR 9 1 9 10
BAR 10 1 10 11
GRDSET 3 4 5
GRID 1 0. 0. 0.
GRID 2 10. 0. 0.
GRID 3 20. 0. 0.
GRID 4 30. 0. 0.
GRID 5 40. 0. 0.
GRID 6 50. 0. 0.
GRID 7 60. 0. 0.
GRID 8 70. 0. 0.
GRID 9 80. 0. 0.
GRID 10 90. 0. 0.
GRID 11 100. 0. 0.
PBAR 1 100. 6. 63.
MAT 1 0. 0. 6. 2.591-4
ENDDATA

&
NASTRAN FILES = UMF $ CDC AND IBM
ID DEM2032$NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 5
CEND
SUBSTRUCTURE PHASE 2
PASSWORD = M0LSYN
SOF (1) = F17$500 $ CDC AND IBM
EQUIV ABASIC, CBASIC
PREFIX B
EQUIV ABASIC, CBASIC
PREFIX C
EQUIV ABASIC, DBASIC
PREFIX D
EQUIV ABASIC, EBASIC
PREFIX E
EQUIV ABASIC, FBASIC
PREFIX F
EQUIV ABASIC, GBASIC
PREFIX G
MREDUCE ABASIC
NAME MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE BBASIC
NAME MB
BOUNDARY 2
FIXED 2
METHOD 2
MREDUCE CBASIC
NAME MC
BOUNDARY 7
FIXED 7
METHOD 2
MREDUCE DBASIC
NAME MD
BOUNDARY 8
FIXED 8
METHOD 2
MREDUCE EBASIC
NAME ME
BOUNDARY 9
FIXED 9
METHOD 2
MREDUCE FBASIC
NAME MF
BOUNDARY 11
FIXED 11
METHOD 2
MREDUCE GBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
COMBINE MBA, MB
NAME AB
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MB
TRANSFORM 2
MREDUCE AB
NAME MAB
BOUNDARY 10
FIXED 10
METHOD 22
COMBINE MAB, MC
NAME ABC
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MC
TRANSFORM 3
MREDUCE ABC
NAME MABC
BOUNDARY 21
FIXED 21
METHOD 22
COMBINE MABC, MD
NAME ABCD
TOLERANCE 0.01
OUTPUT 2, 7, 12
COMPONENT MD
TRANSFORM 4
MREDUCE ABCD
NAME MABCD
BOUNDARY 22
FIXED 22
METHOD 22
COMBINE MABCDEF,ME
NAME ABCDE
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT ME
TRANSFORM 5
MREDUCE ABCDE
NAME MABCDE
BOUNDARY 23
FIXED 23
METHOD 22
COMBINE MABCDEF,MF
NAME ABCDEF
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MF
TRANSFORM 6
MREDUCE ABCDEF
NAME MABCDEF
BOUNDARY 24
FIXED 24
METHOD 22
COMBINE MABCDEF,ME
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 1,5,6,9,10
SOFTPRT TOC
ENDSUBS
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE,COMBINE,MODAL,RECOVRSY,RUN2,P8 S52
BEGIN BULK
BDYC  7  CBASIC  50
BDYC  9  EBASIC  50
BDYC 21  EBASIC  30
BDYC 22  DBASIC  30
BDYC 23  EBASIC  30
BDYC 24  FBASIC  30
BDYC 11  FBASIC  50
BDYC  8  DBASIC  50
BDYC 10  BBASIC  30
BDYC 15  CBASIC  40
BDYC 22  BBASIC  50
BDYC 30  ABASIC  30
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BDYS1 40  126  1
BDYS1 50  126  1
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TRANS  6  350.0  0.0  0.0  0.0  500.0  0.0  1.0  +T6
TRANS  4  550.0  0.0  0.0  0.0  500.0  0.0  1.0  +T2
TRANS  4  150.0  0.0  0.0  0.0  100.0  0.0  1.0  +T2
TRANS  3  250.0  0.0  0.0  0.0  200.0  0.0  1.0  +T3
TRANS  5  450.0  0.0  0.0  0.0  400.0  0.0  1.0  +T5

END

DBASIC 30
APPENDIX C. (concluded)

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INDATA
APPENDIX D. Driver decks and sample bulk data for beam problem of case 3.

NASTRAN FILES = UMF $ CDC AND IBM
ID DEMZ031.NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 3
CEND
SUBSTRUCTURE PHASE 1
PASSWORD = MDLSYN
SOF(1) = FT17.500, NEW $ CDC AND IBM
NAME = ABASIC
SOPPRINT TOC
ENDSUBS
TITLE = BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1, RS 2
BEGIN BULK
BAR
BAR 1 1 1 1 2
BAR 2 1 1 2 3
BAR 3 1 1 3 4
BAR 4 1 1 4 5
BAR 5 1 1 5 6
BAR 6 1 1 6 7
BAR 7 1 1 7 8
BAR 8 1 1 8 9
BAR 9 1 1 9 10
BAR 10 1 1 10 11
GRID
GRID 1 0 0 0
GRID 2 10 0 0
GRID 3 20 0 0
GRID 4 30 0 0
GRID 5 40 0 0
GRID 6 50 0 0
GRID 7 60 0 0
GRID 8 70 0 0
GRID 9 80 0 0
GRID 10 90 0 0
GRID 11 100 0 0
PBAR
PBAR 1 0.56 63
MAT1
MAT1 1 10.0 10.0 0.0 1
ENDDATA

&

NASTRAN FILES = UMF $ CDC AND IBM
ID DEMZ032.NASTRAN
APP DISP, SUBS
SOL 3, 0
TIME 10
CEND
SUBSTRUCTURE PHASE 2
PASSWORD = MDLSYN
SOF(1) = FT17.500, NEW $ CDC AND IBM
EQUIV ABASIC, BBASIC
PREFIX B
EQUIV ABASIC, CBASIC
PREFIX C
EQUIV ABASIC, DBASIC
PREFIX D
EQUIV ABASIC, EBASIC
PREFIX E
EQUIV ABASIC, FBASIC
PREFIX F
EQUIV ABASIC, GBASIC
PREFIX G
MREDUCE ABASIC
NAME MA
BOUNDARY 20
FIXED 20
METHOD 1
MREDUCE EBASIC
NAME MB
BOUNDARY 2
FIXED 2
METHOD 2
MREDUCE EBASIC
NAME MC
BOUNDARY 7
FIXED 7
METHOD 2
MREDUCE EBASIC
NAME MD
BOUNDARY 8
FIXED 8
METHOD 2
MREDUCE EBASIC
NAME ME
BOUNDARY 9
FIXED 9
METHOD 2
MREDUCE EBASIC
NAME MF
BOUNDARY 11
FIXED 11
METHOD 2
MREDUCE EBASIC
NAME MG
BOUNDARY 3
FIXED 3
METHOD 3
COMBINE MA*MB
NAME AB
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MB
TRANSFORM 2
MREDUCE AB
NAME MA8
BOUNDARY 10
FIXED 10
METHOD 22
COMBINE MC*MD
NAME CD
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MD
TRANSFORM 2
MREDUCE CD
NAME MCD
BOUNDARY 15
FIXED 15
METHOD 22
COMBINE ME*MF
NAME EF
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MF
TRANSFORM 2
MREDUCE EF
NAME MEF
BOUNDARY 25
FIXED 25
METHOD 22
COMBINE MA8*MCD
NAME ABCD
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MCD
TRANSFORM 3
MREDUCE ABCD
NAME MABCD
BOUNDARY 30
FIXED 30
METHOD 25
COMBINE MEF*MG
NAME EFG
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MG
TRANSFORM 3
MREDUCE EFG
NAME MEF
BOUNDARY 35
FIXED 35
METHOD 25
COMBINE MABCD*MEFG
NAME ABCDEFG
TOLERANCE 0.01
OUTPUT 2,7,12
COMPONENT MEFG
TRANSFORM 5
MREDUCE ABCDEFG
NAME BEAM
BOUNDARY 20
METHOD 25
OUTPUT 1,5,6,9,10
SOPRINT TOC
ENDSUBS
TITLE=BEAM DYNAMIC ANALYSIS USING AUTOMATED MODAL SYNTHESIS
SUBTITLE=NASTRAN DEMONSTRATION PROBLEM NO. 2-3-2
LABEL=MODAL REDUCE, COMBINE, MODAL RECOVERY, RUN2, PHASE 2
BEGIN BULK
BDYC 30 DBASIC 30
BDYC 35 EBASIC 40
BDYC 9 EBASIC 50
BDYC 9 FBASIC 50
BDYC 25 EBASIC 40
BDYC 5 EBASIC 50
BDYC 10 BBASIC 30
BDYC 15 CBASIC 40
BDYC 22 BBASIC 50
BDYC 30 GBASIC 40
BDYC 20 ABASIC 30
BDYS1 30 126 11
BDYS1 40 126 1
BDYS1 50 126 1
TRANS 250 100 0 0 100 0 1
TRANS 3 200 0 0 200 0 1
TRANS 250 0 0 400 0 1
TRANS 5 450 0 0 400 0 1
TRANS 7 650 0 0 600 0 1
+E1 1 INV .0 3000.00 10 10
+E2 2 INV .0 3000.00 10 10
+F

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APPENDIX D. (concluded)

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ENDDATA
### 4. Title and Subtitle

**NASTRAN® MODAL SYNTHESIS CAPABILITY**

### 7. Author(s)

Chuh Mei, Principal Investigator and Mo-How Shen

### 9. Performing Organization Name and Address

Old Dominion University Research Foundation  
P.O. Box 6369  
Norfolk, Virginia 23508

### 12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration  
Hampton, Virginia

### 15. Supplementary Notes

Langley Technical Monitor: Joseph E. Walz

### 16. Abstract

This report compares the accuracy between NASTRAN® modal synthesis, full structure NASTRAN® and several other modal synthesis results (truss only). The results are based on a truss or beam having redundant or point interface connections. Each component substructure is reduced to modal and boundary degrees of freedom prior to the substructure combine operation. The combination structure, formulated in terms of the component modes, is also reduced to modal degrees of freedom for solution by the transient analysis rigid format.