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# Dynamics of Spacecraft Control Laboratory Experiment (SCOLE) Slew Maneuvers

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# DYNAMICS OF SPACECRAFT CONTROL LABORATORY EXPERIMENT (SCOLE) SLEW MANEUVERS

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## SUMMARY

This is the first report of a set of two reports on the dynamics and control of slewing maneuvers of NASA Spacecraft Control Laboratory Experiment (SCOLE) article. In this report, the dynamics of slewing maneuvers of SCOLE are developed in terms of an arbitrary maneuver about any given axis. The set of dynamical equations incorporate rigid-body slew maneuver and three-dimensional vibrations of the complete assembly comprising the rigid shuttle, the flexible beam, and the reflector with an offset mass. The analysis also includes kinematic nonlinearities of the entire assembly during the maneuver and the dynamics of the interaction between the rigid shuttle and the flexible appendage. The final set of dynamical equations obtained for slewing maneuvers are highly nonlinear and coupled in terms of the flexible modes and the rigid-body modes.

The equations are further simplified and evaluated numerically to include the first ten flexible modes and the SCOLE data to yield a model for designing control systems to perform slew maneuvers.

## 1. INTRODUCTION

The primary control objective of the Spacecraft Control Laboratory Experiment (SCOLE) is to direct the RF Line-Of-Sight (LOS) of the antenna-like configuration towards a fixed target under the conditions of minimum time and limited control authority [1]. This problem of directing the LOS of antenna- like configuration involves both the slewing maneuver of the entire assembly and the vibration suppression of the flexible antenna-like beam. The study of ordinary rigid-body slew maneuvers has received considerable attention in the literature [2,3] due to the fact that any arbitrary large-angle slew maneuver involves kinematic nonlinearities. This is further complicated in the case of SCOLE by virtue of a flexible appendage deployed from the rigid space shuttle. The dynamics of arbitrary large-angle slew maneuvers of SCOLE model are derived in this report as a set of coupled equations with the rigid-body motions including the nonlinear kinematics and the vibratory equations of the flexible appendage.

The dynamical equations of slewing maneuvers of this large flexible spacecraft are developed by writing the total kinetic and potential energy expressions for the entire system. The energy expressions are further utilized in formulating Lagrange's equations which are expressed in terms of non-generalized co-ordinates using an inertial co-ordinate system and a body-fixed co-ordinate system at the point of attachment of the flexible beam to the shuttle. The generic model used for this analysis consists of a distributed parameter beam with two end masses. The three dimensional linear vibration analysis of this free-free beam model with end masses [4] is incorporated together with rigid-slewing maneuver dynamics which are written in terms of four Euler parameters [5] and angular rotation about an arbitrary axis of rotation to yield the final set of highly nonlinear and coupled equations. In the derivation of the equations, it is assumed that the vibratory analysis is for small motions.

## 2. LIST OF SYMBOLS

| | |
|---|---|
| $\underline{a}(z)$ | Position vector of mass element on the beam from the point of attachment |
| $B$ | Damping matrix |
| $C$ | Inertial frame to body-fixed frame transformation |
| $\underline{c}$ | Position vector from the point of attachment to the mass center of the beam |
| $D$ | Mass density of the beam |
| $\underline{d}(z,t)$ | Displacement vector of mass element in the body-fixed frame |
| $E$ | Modulus of Elasticity |
| $\underline{F}_o(t)$ | Force applied at the orbiter mass center |
| $\underline{F}(t)$ | Force applied at the reflector mass center |
| $\underline{G}_o(t)$ | Moment applied about the orbiter mass center |
| $G_\psi$ | Modulus of rigidity for the beam |
| $I$ | Beam cross section moment of inertia |
| $I_x$ | Beam cross section moment of inertia, roll bending |
| $I_y$ | Beam cross section moment of inertia, pitch bending |
| $I_1$ | Mass moment of inertia matrix of the shuttle |
| $I_2$ | Mass moment of inertia matrix of the reflector |
| $J$ | Mass moment of inertia matrix of the beam |
| $L$ | The Length of the beam |
| $M$ | Angular velocity vector transformation |
| $m$ | Total mass of the flexible beam |
| $m_1$ | Mass of the orbiter |
| $m_2$ | Mass of the reflector |
| $n$ | The maximum number of modes considered |
| $q_i$ | Generalized coordinates |

| | |
|---|---|
| $\underline{R}$ | Position vector of the mass center of the orbiter in the inertial frame |
| $\underline{r}$ | Position vector from the orbiter mass center to the point of attachment |
| $r_x$ | x co-ordinate of the reflector mass center in the body-fixed frame |
| $r_y$ | y co-ordinate of the reflector mass center in the body-fixed frame |
| $T$ | Total Kinetic Energy |
| $U$ | Total Potential Energy |
| $u_x(z,t)$ | The beam deflection in x direction referred to the body-fixed frame |
| $u_y(z,t)$ | The beam deflection in y direction referred to the body-fixed frame |
| $u_\psi(z,t)$ | The torsional deflection about z axis in the body-fixed frame |
| $\underline{V}$ | Velocity vector of the mass center of the orbiter in the body-fixed frame |
| $\underline{V_o}$ | Velocity vector of the point of attachment in the body-fixed frame |
| $\rho$ | Mass per unit length of the flexible beam |
| $\underline{\lambda}$ | Vector representing the axis rotation during the slew maneuver |
| $\phi_{xi}$ | i th Eigenfunction corresponding to $u_x$ |
| $\phi_{yi}$ | i th Eigenfunction corresponding to $u_y$ |
| $\phi_{\psi i}$ | i th Eigenfunction corrsponding to $u_\psi$ |
| $\underline{\theta}$ | The attitude of the orbiter in the inertial frame |
| $\xi$ | Slew Angle |
| $\underline{\omega}$ | The angular velocity of the orbiter in the inertial frame |
| $\underline{\Omega}$ | The angular velocity of the reflector in the inertial frame |
| $\zeta$ | Damping ratio |

## 3. ANALYTICS

### Co-ordinate Systems

The motion of SCOLE assembly when considered as a rigid body in space has six dynamic degrees of freedom: three of these define the location of the mass center, and three define the orientation (attitude) of the body. The motion of this rigid body is goverened by newtonion laws of motion expressed in terms of changes in linear momentum and angular momentum. These relationships are valid only when the axes along which the motion is resolved are an inertial frame of reference [9,10]. To define the orientation of the orbiter in space, a set of orthogonal axes fixed in the body is utilized. Then the attitude of the orbiter is defined in terms of the angles $(\theta_1,\theta_2,\theta_3)$ between the body- fixed axes and the inertial co-ordinate axes. The body-fixed frame origin is located at the point of attachment of the flexible appendage with the rigid shuttle for this analysis (Fig. 1).

The transformation from the inertial frame to the body-fixed frame is given by the matrix, $C$ as developed in figure 2 where if $\vec{i}$, $\vec{j}$, $\vec{k}$ represent the dexteral set of orthogonal unit vectors fixed in the body- fixed frame and $\theta_1$ is the rotation about $\vec{i}$, $\theta_2$ is the rotation about $\vec{j}$ and $\theta_3$ is the rotation about $\vec{k}$. These rotations are carried out successively as shown in figure 1 and the matrix $C$ is given as

$$C = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \tag{1}$$

Thus $C^T$ is obtained as

$$C^T = \begin{bmatrix} \cos\theta_2\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2 \\ \sin\theta_1\sin\theta_2\cos\theta_3+\sin\theta_3\cos\theta_1 & -\sin\theta_1\sin\theta_2\sin\theta_3+\cos\theta_3\cos\theta_1 & -\sin\theta_1\cos\theta_2 \\ -\cos\theta_1\sin\theta_2\cos\theta_3+\sin\theta_3\sin\theta_1 & \cos\theta_1\sin\theta_2\sin\theta_3+\cos\theta_3\sin\theta_1 & \cos\theta_1\cos\theta_2 \end{bmatrix} \tag{2}$$

In order to completely define the attitude (orientation), it is needed to relate the rotation angles $\theta_1$, $\theta_2$, and $\theta_3$ to the angular velocity components ($\omega_1$, $\omega_2$, $\omega_3$) of the orbiter. One way of obtaining the required relations is via body-three angles method [5] which was utilized in developing $C$ matrix in equation (1) and these relations are

$$\begin{aligned} \dot{\theta}_1 &= (\omega_1 \cos\theta_3 - \omega_2 \sin\theta_3)/\cos\theta_2 \\ \dot{\theta}_2 &= (\omega_1 \sin\theta_3 + \omega_2 \cos\theta_3) \\ \dot{\theta}_3 &= (-\omega_1 \cos\theta_3 + \omega_2 \sin\theta_3)\tan\theta_2 + \omega_3 \end{aligned} \tag{3}$$

Thus, the angular velocity of the orbiter can be obtained in the inertial frame by means of the following transformation

$$\underline{\omega} = M^T \underline{\dot{\theta}} \tag{4}$$

where the transformation $M^T$ is given as

$$M^T = \begin{bmatrix} \cos\theta_2 \cos\theta_3 & \sin\theta_3 & 0 \\ -\cos\theta_2 \sin\theta_3 & \cos\theta_3 & 0 \\ \sin\theta_2 & 0 & 1 \end{bmatrix} \tag{5}$$

Although the body-three angles method is used here for obtaining the transformations $C$ and $M$, there are three other methods which can be used to obtain the same transformations. A detailed discussion of all the methods is given in reference [5] and a summary of the transformations using the remaining three methods is given in the Appendix.

<u>Kinetic Energy</u>

If the position vector of the mass center of the orbiter in the inertial frame (Fig. 3), $\underline{R}$, is given as

$$\underline{R} = \begin{bmatrix} R_X \\ R_Y \\ R_Z \end{bmatrix} \tag{6}$$

then the velocity of the mass center in the inertial frame is

$$\underline{V}^I(t) = \begin{bmatrix} \dot{R}_X \\ \dot{R}_Y \\ \dot{R}_Z \end{bmatrix} \tag{7}$$

This velocity can be transformed in the body-fixed frame as

$$\underline{V}(t) = C \begin{bmatrix} \dot{R}_X \\ \dot{R}_Y \\ \dot{R}_Z \end{bmatrix} \tag{8}$$

The velocity of the point of attachment in the body-fixed frame is

$$\underline{V_o} = \underline{V} + \underline{\omega} \times \underline{r} \tag{9}$$

where $\underline{r}$ is the vector from orbiter mass center to the point of attachment.

Defining the position vector (Fig. 4), $\underline{a}$, of a mass element on the beam from the point of attachment (origin of the body-fixed frame) before deformation as

$$\underline{a} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} \tag{10}$$

and the displacement vector of this mass element as

$$\underline{d}(z,t) = \begin{bmatrix} u_x(z,t) \\ u_y(z,t) \\ 0 \end{bmatrix} \tag{11}$$

the position vector after deflection is given as $\underline{a}+\underline{d}$. The kinetic energy in the beam [6] is

$$T_1 = (1/2)\, m\underline{V}_o^T\underline{V}_o + (1/2)\,\underline{\omega}^T[J]\underline{\omega} - m\underline{V}_o^T[\underline{\tilde{c}}]\underline{\omega} + (1/2)\int \underline{\dot{d}}^T\underline{\dot{d}}dm$$
$$+ \underline{V}_o^T \int \underline{\dot{d}}dm + \underline{\omega}^T \int \underline{\tilde{a}}\underline{\dot{d}}dm + (1/2)\int \left[\dot{u}_x{}'\ \dot{u}_y{}'\ \dot{u}_\psi\right] dI \begin{bmatrix} \dot{u}_x{}' \\ \dot{u}_y{}' \\ \dot{u}_\psi \end{bmatrix} \qquad (12)$$

where the vector $\underline{c}$ is from the point of attachment to the mass center of the beam and if it is assumed that the beam is a thin rod, then it is given as

$$\underline{c} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$$
$$= (1/m)\int \underline{a}dm = \begin{bmatrix} 0 \\ 0 \\ -L/2 \end{bmatrix} \qquad (13)$$

and using the skew symmetric form for the vector cross product for any two vectors $\underline{c}$ and $\underline{\omega}$ (in the same reference frame) as

$$\underline{c}\times\underline{\omega} = [\underline{\tilde{c}}]\underline{\omega}$$

$$\underline{\tilde{c}} = \begin{bmatrix} 0 & -c_z & c_y \\ c_z & 0 & -c_x \\ -c_y & c_x & 0 \end{bmatrix} \qquad (14)$$

also, the moment of inertia matrix is given as

$$J = (1/3)\,\rho L^3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (15)$$

where $\rho$ is the mass per unit length of the beam. The last term in the equation (12) corresponding to torsional motion is given as

$$(1/2)\int\begin{bmatrix}\dot{u}_x{}' & \dot{u}_y{}' & \dot{u}_\psi\end{bmatrix} dI \begin{bmatrix}\dot{u}_x{}' \\ \dot{u}_y{}' \\ \dot{u}_\psi\end{bmatrix}$$

$$= (1/2)\int\begin{bmatrix}\dot{u}_x{}' & \dot{u}_y{}' & \dot{u}_\psi\end{bmatrix}\begin{bmatrix}1/2(\rho ds)s^2 & 0 & 0 \\ 0 & 1/2(\rho ds)s^2 & 0 \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{u}_x{}' \\ \dot{u}_y{}' \\ \dot{u}_\psi\end{bmatrix} . \tag{16}$$

The kinetic energy equation (12) can be simplified as

$$T_1 = (1/2)\,\rho L \underline{V}_o^T \underline{V}_o + (1/6)\,\rho L^3\left(\omega_1^2+\omega_2^2\right) - \rho L \underline{V}_o^T \underline{\tilde{c}}\,\underline{\omega} + \rho L \sum_{i=1}^{n} \dot{q}_i^{\,2}$$

$$+\underline{V}_o^T \underline{\dot{\alpha}} + \underline{\omega}^T \underline{\dot{\beta}} + (1/4)\,\rho\left[\sum_{i=1}^{n} p_{5i}\,\dot{q}_i^{\,2} + \sum_{i=1}^{n} p_{6i}\,\dot{q}_i^{\,2}\right] \tag{17}$$

where

$$\begin{aligned}
u_x &= \sum_{i=1}^{n} \phi_{xi}(s)q_i(t) \\
u_y &= \sum_{i=1}^{n} \phi_{yi}(s)q_i(t) \\
u_x{}' &= \sum_{i=1}^{n} \phi_{xi}{}'(s)q_i(t) \\
u_y{}' &= \sum_{i=1}^{n} \phi_{yi}{}'(s)q_i(t) \\
u_\psi &= \sum_{i=1}^{n} \phi_{\psi i}(s)q_i(t) \\
p_{1i} &= \int_0^L \phi_{xi}(s)ds \\
p_{2i} &= \int_0^L \phi_{yi}(s)ds \\
p_{3i} &= \int_0^L s\,\phi_{xi}(s)ds \\
p_{4i} &= \int_0^L s\,\phi_{yi}(s)ds \\
p_{5i} &= \int_0^L \left(s\,\phi_{xi}{}'\right)^2 ds \\
p_{6i} &= \int_0^L \left(s\,\phi_{yi}{}'\right)^2 ds
\end{aligned} \tag{18}$$

and

$$\underline{\dot{\alpha}}(t) = \rho \begin{bmatrix} \sum_{i=1}^{n} p_{1i}\dot{q}_i \\ \sum_{i=1}^{n} p_{2i}\dot{q}_i \\ 0 \end{bmatrix} \tag{19}$$

$$\underline{\dot{\beta}}(t) = \rho \begin{bmatrix} \sum_{i=1}^{n} p_{4i}\dot{q}_i \\ \sum_{i=1}^{n} p_{3i}\dot{q}_i \\ 0 \end{bmatrix} . \tag{20}$$

The expressions for $p_{1i}$, $p_{2i}$, $p_{3i}$, $p_{4i}$, $p_{5i}$, and $p_{6i}$ are developed as follows. Note that

$$\phi_{xi}(s) = A_{xi}\sin\beta_i s + B_{xi}\cos\beta_i s + C_{xi}\sinh\beta_i s + D_{xi}\cosh\beta_i s$$

where $\beta_i = \left(\frac{\omega_i^2 \rho}{EI}\right)^{1/4}$ .

Since for SCOLE configuration $EI_x = EI_y$ and $\beta_{ix} = \beta_{iy}$ , $EI$ and $\beta_i$ are used for both $\phi_{xi}(s)$ and $\phi_{yi}(s)$. However, this may not be true for other configurations.

$$p_{1i} = \int_0^L \phi_{xi}(s)ds$$

$$p_{2i} = \int_0^L \phi_{yi}(s)ds$$

$$p_{1i} = \frac{1}{\beta_i L^2}\Big[-A_{xi}\cos\beta_i L + B_{xi}\sin\alpha_i + C_{xi}\cosh\beta_i L + D_{xi}\sinh\beta_i L + A_{xi} - C_{xi}\Big] \tag{21A}$$

Defining $\alpha_i = \beta_i L$

$$p_{1i} = \frac{1}{\alpha_i L}\left[-A_{xi}\cos\alpha_i + B_{xi}\sin\alpha_i + C_{xi}\cosh\alpha_i + D_{xi}\sinh\alpha_i + A_{xi} - C_{xi}\right] \tag{21B}$$

similarly,

$$p_{2i} = \frac{1}{\beta_i L^2}\Big[-A_{yi}\cos\beta_i L + B_{yi}\sin\beta_i L + C_{yi}\cosh\beta_i L + D_{yi}\sinh\beta_i L + A_{yi} - C_{yi}\Big] \tag{22A}$$

$$p_{2i} = \frac{1}{\alpha_i L}\left[-A_{yi}\cos\alpha_i + B_{yi}\sin\alpha_i + C_{yi}\cosh\alpha_i + D_{yi}\sinh\alpha_i + A_{yi} - C_{yi}\right] \tag{22B}$$

$$p_{3i} = \int_0^L s\,\phi_{xi}(s)ds$$

$$p_{4i} = \int_0^L s\,\phi_{yi}(s)ds$$

and these can be given as

$$p_{3i} = A_{xi}\left[\frac{\sin\beta_i L}{\beta_i^2} - \frac{L\cos\beta_i L}{\beta_i}\right] + B_{xi}\left[\frac{\cos\beta_i L}{\beta_i^2} + \frac{L\sin\beta_i L}{\beta_i} - \frac{1}{\beta_i^2}\right] + C_{xi}\left[\frac{L\cosh\beta_i L}{\beta_i} - \frac{\sinh\beta_i L}{\beta_i^2}\right] + D_{xi}\left[\frac{L\sinh\beta_i L}{\beta_i} - \frac{\cosh\beta_i L}{\beta_i^2} + \frac{1}{\beta_i^2}\right] \tag{23A}$$

$$p_{3i} = A_{xi}\left[\frac{L^2\sin\alpha_i}{\alpha_i^2} - \frac{L^2\cos\alpha_i}{\alpha_i}\right] + B_{xi}\left[\frac{L^2\cos\alpha_i}{\alpha_i^2} + \frac{L^2\sin\alpha_i}{\alpha_i} - \frac{L^2}{\alpha_i^2}\right] + C_{xi}\left[\frac{L^2\cosh\alpha_i}{\alpha_i} - \frac{L^2\sinh\alpha_i}{\alpha_i^2}\right] + D_{xi}\left[\frac{L^2\sinh\alpha_i}{\alpha_i} - \frac{L^2\cosh\alpha_i}{\alpha_i^2} + \frac{L^2}{\alpha_i^2}\right] \tag{23B}$$

Similarly,

$$p_{4i} = A_{yi}\left[\frac{\sin\beta_i L}{\beta_i^2} - \frac{L\cos\beta_i L}{\beta_i}\right] + B_{yi}\left[\frac{\cos\beta_i L}{\beta_i^2} + \frac{L\sin\beta_i L}{\beta_i} - \frac{1}{\beta_i^2}\right] + C_{yi}\left[\frac{L\cosh\beta_i L}{\beta_i} - \frac{\sin\beta_i L}{\beta_i^2}\right] + D_{yi}\left[\frac{L\sinh\beta_i L}{\beta_i} - \frac{\cosh\beta_i L}{\beta_i^2} + \frac{1}{\beta_i^2}\right] \tag{24A}$$

$$p_{4i} = A_{yi}\left[\frac{L^2\sin\alpha_i}{\alpha_i^2} - \frac{L^2\cos\alpha_i}{\alpha_i}\right] + B_{yi}\left[\frac{L^2\cos\alpha_i}{\alpha_i^2} + \frac{L^2\sin\alpha_i}{\alpha_i} - \frac{L^2}{\alpha_i^2}\right] + C_{yi}\left[\frac{L^2\cosh\alpha_i}{\alpha_i} - \frac{L^2\sinh\alpha_i}{\alpha_i^2}\right] + D_{yi}\left[\frac{L^2\sinh\alpha_i}{\alpha_i} - \frac{L^2\cosh\alpha_i}{\alpha_i^2} + \frac{L^2}{\alpha_i^2}\right] \tag{24B}$$

$$p_{5i} = \int_0^L \left[ s\,\phi_{xi}'(s) \right]^2 ds$$

$$p_{6i} = \int_0^L \left[ s\,\phi_{yi}'(s) \right]^2 ds$$

and these can be shown to be

$$\begin{aligned}
p_{5i} = {} & A_{xi}^2 \left[ \frac{\beta_i^2 L^3}{6} + \frac{1}{2} \left\{ \frac{L}{2}\cos 2\beta_i L + \left(\frac{L^2\beta_i}{2} - \frac{1}{4\beta_i}\right)\sin 2\beta_i L \right\} \right] \\
& - A_{xi} B_{xi} \left[ \frac{L}{2}\sin 2\beta_i L + \left[ \frac{1}{4\beta_i} - \frac{L^2\beta_i}{2} \right] \cos 2\beta_i L - \frac{1}{4\beta_i} \right] \\
& + A_{xi} C_{xi} \left[ \beta_i L^2 \left\{ (\cos\beta_i L \sinh\beta_i L) + (\sin\beta_i L \cosh\beta_i L) \right\} \right. \\
& \left. - 2L \left\{ \sin\beta_i L \sinh\beta_i L \right\} - \frac{1}{\beta_i} \left\{ (\cos\beta_i L \sinh\beta_i L) - (\sin\beta_i L \cosh\beta_i L) \right\} \right] \\
& + A_{xi} D_{xi} \left[ \beta_i L^2 \left\{ (\cos\beta_i L \cosh\beta_i L) + (\sin\beta_i L \sinh\beta_i L) \right\} \right. \\
& \left. - 2L \left\{ \sin\beta_i L \cosh\beta_i L \right\} - \frac{1}{\beta_i} \left\{ (\cos\beta_i L \cosh\beta_i L) - (\sin\beta_i L \sinh\beta_i L) \right\} + \frac{1}{\beta_i} \right] \\
& + B_{xi}^2 \left[ \frac{\beta_i^2 L^3}{6} - \frac{1}{2} \left\{ \frac{L}{2}\cos 2\beta_i L + \left(\frac{L^2\beta_i}{2} - \frac{1}{4\beta_i}\right)\sin 2\beta_i L \right\} \right] \\
& - B_{xi} C_{xi} \left[ \beta_i L^2 \left\{ (\sin\beta_i L \sinh\beta_i L) - (\cos\beta_i L \cosh\beta_i L) \right\} \right. \\
& + 2L \left\{ \cos\beta_i L \sinh\beta_i L \right\} - \frac{1}{\beta_i} \left\{ (\cos\beta_i L \cosh\beta_i L) \right. \\
& \left. \left. + (\sin\beta_i L \sinh\beta_i L) \right\} + \frac{1}{\beta_i} \right] \\
& - B_{xi} D_{xi} \left[ \beta_i L^2 \left\{ (\sin\beta_i L \cosh\beta_i L) - (\cos\beta_i L \sinh\beta_i L) \right\} \right. \\
& + 2L \left\{ \cos\beta_i L \cosh\beta_i L \right\} - \frac{1}{\beta_i} \left\{ (\cos\beta_i L \sinh\beta_i L) \right. \\
& \left. \left. - (\sin\beta_i L \cosh\beta_i L) \right\} \right] \\
& + C_{xi}^2 \left[ \frac{1}{2} \left\{ \beta_i L \cosh 2\beta_i L + \left(\frac{\beta_i L^2}{2} + \frac{1}{4\beta_i}\right)\sinh^2\beta_i L - \frac{\beta_i^2 L^3}{3} \right\} \right] \\
& + C_{xi} D_{xi} \left[ \frac{\beta_i L^2}{2}\cos 2\beta_i L - \frac{L}{2}\sinh 2\beta_i L + \frac{1}{4\beta_i}\cosh 2\beta_i L - \frac{1}{4\beta_i} \right]
\end{aligned}$$

$$+D_{xi}^2\left[\frac{1}{2}\left\{\beta_i L\cos 2\beta_i L+\left(\frac{\beta_i L^2}{2}+\frac{1}{4\beta_i}\right)\sinh^2\beta_i L+\frac{\beta_i^2 L^3}{3}\right\}\right] \tag{25}$$

$$p_{6i} = A_{yi}^2\left[\frac{\beta_i^2 L^3}{6}+\frac{1}{2}\left\{\frac{L}{2}\cos 2\beta_i L+\left(\frac{L^2\beta_i}{2}-\frac{1}{4\beta_i}\right)\sin 2\beta_i L\right\}\right]$$

$$-A_{yi}B_{yi}\left[\frac{L}{2}\sin 2\beta_i L+\left(\frac{1}{4\beta_i}-\frac{L^2\beta_i}{2}\right)\cos 2\beta_i L-\frac{1}{4\beta_i}\right]$$

$$+A_{yi}C_{yi}\left[\beta_i L^2\left\{(\cos\beta_i L\sinh\beta_i L)+(\sin\beta_i L\cosh\beta_i L)\right\}-2L\left\{\sin\beta_i L\sinh\beta_i L\right\}\right.$$

$$\left.-\frac{1}{\beta_i}\left\{(\cos\beta_i L\sinh\beta_i L)-(\sin\beta_i L\cosh\beta_i L)\right\}\right]$$

$$+A_{yi}D_{yi}\left[\beta_i L^2\left\{(\cos\beta_i L\cosh\beta_i L)+(\sin\beta_i L\sinh\beta_i L)\right\}-2L\left\{\sin\beta_i L\cosh\beta_i L\right\}\right.$$

$$\left.-\frac{1}{\beta_i}\left\{(\cos\beta_i L\cosh\beta_i L)-(\sin\beta_i L\sinh\beta_i L)\right\}+\frac{1}{\beta_i}\right]$$

$$+B_{yi}{}^2\left[\frac{\beta_i^2 L^3}{6}-\frac{1}{2}\left\{\frac{L}{2}\cos 2\beta_i L+\left(\frac{L^2\beta_i}{2}-\frac{1}{4\beta_i}\right)\sin 2\beta_i L\right\}\right]$$

$$-B_{yi}C_{yi}\left[\beta_i L^2\left\{(\sin\beta_i L\sinh\beta_i L)-(\cos\beta_i L\cosh\beta_i L)\right\}+2L\left\{\cos\beta_i L\sinh\beta_i L\right\}\right.$$

$$\left.-\frac{1}{\beta_i}\left\{(\cos\beta_i L\cosh\beta_i L)+(\sin\beta_i L\sinh\beta_i L)\right\}+\frac{1}{\beta_i}\right]$$

$$-B_{yi}D_{yi}\left[\beta_i L^2\left\{(\sin\beta_i L\cosh\beta_i L)-(\cos\beta_i L\sinh\beta_i L)\right\}+2L\left\{\cos\beta_i L\cosh\beta_i L\right\}\right.$$

$$\left.-\frac{1}{\beta_i}\left\{(\cos\beta_i L\sinh\beta_i L)-(\sin\beta_i L\cosh\beta_i L)\right\}\right]$$

$$C_{yi}^2\left[\frac{1}{2}\left\{\beta_i L\cosh 2\beta_i L+\left(\frac{\beta_i L^2}{2}+\frac{1}{4\beta_i}\right)\sinh^2\beta_i L-\frac{\beta_i^2 L^3}{3}\right\}\right]$$

$$C_{yi}D_{yi}\left[\frac{\beta_i L^2}{2}\cos 2\beta_i L-\frac{L}{2}\sinh 2\beta_i L+\frac{1}{4\beta_i}\cos 2\beta_i L-\frac{1}{4\beta_i}\right]$$

$$D_{yi}^2\left[\frac{1}{2}\left\{\beta_i L\cos 2\beta_i L+\left(\frac{\beta_i L^2}{2}+\frac{1}{4\beta_i}\right)\sinh^2\beta_i L+\frac{\beta_i^2 L^3}{3}\right\}\right]\ . \tag{26}$$

The equations (25) and (26) can alternatively be derived by replacing $\beta_i = \frac{\alpha_i}{L}$.

The kinetic energy of the reflector is

$$T_2 = (1/2)\, m_2 \underline{V}_o^T \underline{V}_o - m_2 \underline{V}_o^T \underline{\tilde{a}}(L)\underline{\omega} + m_2 \underline{V}_o^T \underline{\dot{d}}(L) - (1/2)\, m_2 \underline{\omega}^T \underline{\tilde{a}}^T(L) \underline{\tilde{a}}(L)\underline{\omega}$$
$$+ m_2 \underline{\omega}^T \underline{\tilde{a}}(L) \underline{\dot{d}}(L) + (1/2)\, m_2 \underline{\dot{d}}^T(L) \underline{\dot{d}}(L) + (1/2)\, \underline{\Omega}^T I_2 \underline{\Omega} \tag{27}$$

where $m_2$ is the mass of the reflector and $I_2$ is the mass moment of inertia matrix of the reflector. The deflection vector $\underline{d}(L)$ at the mass center of the reflector is given as

$$\underline{d}(L) = \begin{bmatrix} u_x(L) - r_y u_\psi(L) \\ u_y(L) + r_x u_\psi \\ u_x'(L) r_x + u_y'(L) r_y \end{bmatrix} \tag{28}$$

and the position vector from the point of attachment to the reflector mass center is given by

$$\underline{a}(L) = \begin{bmatrix} r_x \\ r_y \\ -L \end{bmatrix} . \tag{29}$$

Thus,

$$\underline{\dot{d}}(L) = \begin{bmatrix} \dot{u}_x(L) - r_y \dot{u}_\psi(L) \\ \dot{u}_y(L) + r_x \dot{u}_\psi(L) \\ \dot{u}_x'(L) r_x + \dot{u}_y'(L) r_y \end{bmatrix} . \tag{30}$$

The angular velocity of the reflector in the inertial co-ordinate system $\underline{\Omega}$ can be shown to be

$$\underline{\Omega} = \underline{\omega} + \begin{bmatrix} \dot{u}_x' \\ \dot{u}_y' \\ \dot{u}_\psi \end{bmatrix}_L . \tag{31}$$

The equation (27) can be simplified as

$$T_2 = (1/2)\, m_2 \underline{V}_o^T \underline{V}_o - m_2 \underline{V}_o^T \underline{\tilde{a}}(L)\underline{\omega} + m_2 \underline{V}_o^T \underline{\dot{d}}(L) + (1/2)\, m_2 L^2 \left[\omega_1^2 + \omega_2^2\right]$$
$$+ m_2 \underline{\omega}^T \underline{\tilde{a}}(L) \underline{\dot{d}}(L) + (1/2)\, m_2 \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \phi_{xi}(L) \phi_{xj}(L) \dot{q}_i \dot{q}_j + \right.$$

$$\left.\sum_{i=1}^{n}\sum_{j=1}^{n}\phi_{yi}(L)\phi_{yj}(L)\dot{q}_i\dot{q}_j\right\}+(1/2)\,\underline{\dot{P}}^T I_2\underline{\dot{P}}+(1/2)\,\underline{\omega}^T I_2\underline{\omega} \tag{32}$$

where

$$\underline{\dot{P}}^T = \left[\dot{u}_x{}'\ \dot{u}_y{}'\ \dot{u}_\psi\right]_L$$
$$= \left[\sum_{i=1}^{n}\phi_{xi}{}'(L)\dot{q}_i(t)\ \sum_{i=1}^{n}\phi_{yi}{}'(L)\dot{q}_i\ \sum_{i=1}^{n}\phi_{\psi i}(L)\dot{q}_i(t)\right] \quad . \tag{33}$$

The kinetic energy of the shuttle, $T_o$ , is given as

$$T_o = (1/2)\,m_1\underline{V}^T\underline{V}+(1/2)\,\underline{\omega}^T\left[I_1\right]\underline{\omega} \tag{34}$$

where $m_1$ is the mass of the shuttle and $I_1$ is the mass moment of inertia matrix of the shuttle.

The total kinetic energy is given as

$$T = T_o+T_1+T_2 \tag{35}$$

This can be simplified as

$$\begin{aligned} T = {} & (1/2)\,m_o\underline{V}^T\underline{V}+\underline{\omega}^T\left[H\right]\underline{V}+(1/2)\,\underline{\omega}^T\left[I_o\right]\underline{\omega}+\rho L\sum_{i=1}^{n}\dot{q}_i{}^2+\underline{V}^T\underline{\dot{\alpha}} \\ & +\underline{\omega}^T\underline{\tilde{r}}\,\underline{\dot{\alpha}}+\underline{\omega}^T\underline{\dot{\beta}}+m_2\underline{V}^T\underline{\dot{d}}(L)+m_2\underline{\omega}^T\underline{\tilde{r}\dot{d}}(L)+ \\ & m_2\underline{\omega}^T\underline{\tilde{a}}(L)\underline{\dot{d}}(L)+(1/2)\,m_2\left[\sum_{i=1}^{n}\left\{\phi_{xi}^2(L)+\phi_{yi}^2(L)\right\}\dot{q}_i{}^2\right] \\ & +(1/2)\,\underline{\dot{P}}^T I_2\underline{\dot{P}}+(1/4)\,\rho\left[\sum_{i=1}^{n}p_{5i}\dot{q}_i{}^2+\sum_{i=1}^{n}p_{6i}\dot{q}_i{}^2\right] \end{aligned} \tag{36}$$

where

$$m_o = m_1+\rho L+m_2$$
$$H = \left(\rho L+m_2\right)\underline{\tilde{r}}+m_2\underline{\tilde{a}}(L)+\rho L\underline{\tilde{c}}$$
$$I_o = I_1+(1/3)\,\rho L^3\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}+I_2+J_2-\rho L\underline{\tilde{r}\tilde{r}}-\rho L\underline{\tilde{r}\tilde{c}}-m_2\underline{\tilde{r}\tilde{r}}-m_2\underline{\tilde{r}\tilde{a}}(L)$$

The term $J_2$ in this equation can be shown to be:

$$J_2 = m_2 \begin{bmatrix} (r_y^2+L^2) & -r_x r_y & r_x L \\ -r_x r_y & (r_x^2+L^2) & r_y L \\ r_x L & r_y L & (r_x^2+r_y^2) \end{bmatrix} .$$

The total kinetic energy expression can be further simplified as

$$T = (1/2)\, m_o \underline{V}^T \underline{V} + \underline{\omega}^T \left[H\right] \underline{V} + (1/2)\, \underline{\omega}^T \left[I_o\right] \underline{\omega} + \underline{V}^T \left[A_1\right] \underline{\dot{q}} + \underline{\omega}^T \left[A_2\right] \underline{\dot{q}} + (1/2)\, \underline{\dot{q}}^T \left[A_3\right] \underline{\dot{q}} \tag{37}$$

where

$$\left[A_1\right] \underline{\dot{q}} = \underline{\dot{\alpha}} + m_2 \underline{\dot{d}}(L)$$

$$\left[A_2\right] \underline{\dot{q}} = \underline{\tilde{r}\,\dot{\alpha}} + \underline{\dot{\beta}} + m_2 \underline{\tilde{r}\dot{d}}(L) + m_2 \underline{\tilde{a}}(L) \underline{\dot{d}}(L)$$

$$\left[A_3\right] = \begin{bmatrix} 0 \\ \rho L + m_2 + p_{5i} + p_{6i} \\ 0 \end{bmatrix} + \left[ \left[\phi'(L)\right]^T I_2 \left[\phi'(L)\right] \right] .$$

In this equation

$$\left[\phi'(L)\right]^T = \begin{bmatrix} \phi_{1x}'(L) & 0 & 0 \\ 0 & \phi_{1y}'(L) & 0 \\ 0 & 0 & \phi_{1\psi}(L) \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \phi_{ix}'(L) & 0 & 0 \\ 0 & \phi_{iy}'(L) & 0 \\ 0 & 0 & \phi_{i\psi}(L) \end{bmatrix}$$

Here i=2,3,.....,n. The number n indicates the total number of flexible modes considered.

<u>Equations of motion</u>

Lagrange's equations of motion for the case of independent generalized co-ordinates $q_k$ are

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} = Q_k - \frac{\partial U}{\partial q_k} \qquad (k=1,2,......,n) \tag{38}$$

where, $T = T(\underline{q},\dot{\underline{q}})$ is the kinetic energy

$U = U(\underline{q})$ is the potential energy, and

$Q_k$ are the generalized forces arising from nonconservative sources.

The generalized co-ordinates are:

$R_X, R_Y, R_Z$ — position of orbiter mass center relative to inertial frame origin.

$\theta_1,\theta_2,\theta_3$ — roll, pitch and yaw angles of orbiter.

$q_1, q_2, ......, q_n$ — modal deformation co-ordinates for the beam.

The previous kinetic energy expression developed in equation (37) is given in terms of nonholonomic velocities $\underline{V}$ and $\underline{\omega}$, and generalized velocities $\dot{\underline{q}}$. Using the notation $\bar{T}(\underline{V},\underline{\omega},\dot{\underline{q}})$ for this kinetic energy expression and $T$ for kinetic energy expression in terms of generalized velocities, the equations of motion are developed. Thus, equation (37) is rewritten as

$$\begin{aligned}\bar{T} = (1/2)\, m_o \underline{V}^T \underline{V} + \underline{\omega}^T \left[H\right] \underline{V} + (1/2)\underline{\omega}^T \left[I_o\right] \underline{\omega} + \underline{V}^T \left[A_1\right] \dot{\underline{q}} \\ + \underline{\omega}^T \left[A_2\right] \dot{\underline{q}} + (1/2)\dot{\underline{q}}^T \left[A_3\right] \dot{\underline{q}}\end{aligned} \tag{37}$$

<u>(a) Translational Equations</u>

From the chain rule applied to equation (37) using equation (8), one gets

$$\begin{bmatrix} \dfrac{\partial T}{\partial \dot{R}_X} \\ \dfrac{\partial T}{\partial \dot{R}_Y} \\ \dfrac{\partial T}{\partial \dot{R}_Z} \end{bmatrix} = C^T \begin{bmatrix} \dfrac{\partial \bar{T}}{\partial V_1} \\ \dfrac{\partial \bar{T}}{\partial V_2} \\ \dfrac{\partial \bar{T}}{\partial V_3} \end{bmatrix} \qquad . \tag{39}$$

Also, the generalized forces are $C\underline{F}(t)$ where

$$\underline{F}(t) = \underline{F}_o(t) + \underline{F}_2(t) \tag{40}$$

$\underline{F}_o(t)$ represents the force applied at the orbiter mass center and $\underline{F}_2(t)$ represents the force applied at the reflector mass center. From Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial \bar{T}}{\partial \underline{V}}\right) + C\dot{C}^T \left(\frac{\partial \bar{T}}{\partial \underline{V}}\right) = \underline{F}(t) \tag{41}$$

and from equation (37)

$$\left(\frac{\partial \bar{T}}{\partial \underline{V}}\right) = m_o \underline{V} - H\underline{\omega} + A_1\dot{\underline{q}} \tag{42}$$

Substituting equation (42) in (41),

$$m_o\dot{\underline{V}} - H\dot{\underline{\omega}} + A_1\ddot{\underline{q}} = -C\dot{C}^T(m_o\underline{V} - H\underline{\omega} + A_1\dot{\underline{q}}) + \underline{F}(t) \tag{43}$$

This can be rewritten as

$$m_o\dot{\underline{V}} - H\dot{\underline{\omega}} + A_1\ddot{\underline{q}} = \underline{N}_1 + \underline{F}(t) \tag{44}$$

where the nonlinear term $\underline{N}_1$ is given as

$$\begin{aligned} \underline{N}_1 &= -C\dot{C}^T(m_o\underline{V} - H\underline{\omega} + A_1\dot{\underline{q}}) \\ &= -\tilde{\underline{\omega}}(m_o\underline{V} - H\underline{\omega} + A_1\dot{\underline{q}}) \end{aligned} \tag{45}$$

Here, $\tilde{\underline{\omega}} = C\dot{C}^T$.

<u>(b) Rotational Equations :</u>

From equation (4)

$$\underline{\omega} = M^T\dot{\underline{\theta}}$$

Again using the chain rule

$$\left(\frac{\partial T}{\partial \dot{\underline{\theta}}}\right) = M\left(\frac{\partial \bar{T}}{\partial \underline{\omega}}\right) \tag{46}$$

Also

$$\begin{bmatrix} \frac{\partial T}{\partial \theta_1} \\ \frac{\partial T}{\partial \theta_2} \\ \frac{\partial T}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{V}^T}{\partial \theta_1} \\ \frac{\partial \underline{V}^T}{\partial \theta_2} \\ \frac{\partial \underline{V}^T}{\partial \theta_3} \end{bmatrix} \left( \frac{\partial \bar{T}}{\partial \underline{V}} \right) + \begin{bmatrix} \frac{\partial \underline{\omega}^T}{\partial \theta_1} \\ \frac{\partial \underline{\omega}^T}{\partial \theta_2} \\ \frac{\partial \underline{\omega}^T}{\partial \theta_3} \end{bmatrix} \left( \frac{\partial \bar{T}}{\partial \underline{\omega}} \right) \quad . \tag{47}$$

It can be shown that

$$\frac{\partial \underline{V}^T}{\partial \theta_i} = \underline{V}^T C \, \frac{\partial C^T}{\partial \theta_i} \qquad i = 1,2,3,\ldots\ldots. \tag{48A}$$

and

$$\frac{\partial \underline{\omega}^T}{\partial \theta_i} = \underline{\omega}^T M^{-1} \, \frac{\partial M}{\partial \theta_i} \qquad i = 1,2,3,\ldots\ldots. \tag{48B}$$

and

$$\left( \frac{\partial T}{\partial \underline{\theta}} \right) = \begin{bmatrix} \underline{V}^T C \frac{\partial C^T}{\partial \theta_1} \\ \underline{V}^T C \frac{\partial C^T}{\partial \theta_2} \\ \underline{V}^T C \frac{\partial C^T}{\partial \theta_3} \end{bmatrix} \left( \frac{\partial \bar{T}}{\partial \underline{V}} \right) + \begin{bmatrix} \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_1} \\ \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_2} \\ \underline{\omega}^T M^{-1} \frac{\partial M}{\partial \theta_3} \end{bmatrix} \left( \frac{\partial \bar{T}}{\partial \underline{\omega}} \right) \quad . \tag{49}$$

From equation (37),

$$\left( \frac{\partial \bar{T}}{\partial \underline{\omega}} \right) = H\underline{V} + I_o \underline{\omega} + A_2 \underline{\dot{q}} \tag{50}$$

and as before

$$\left( \frac{\partial \bar{T}}{\partial \underline{V}} \right) = m_o \underline{V} - H \underline{\omega} + A_1 \underline{\dot{q}} \quad . \tag{42}$$

Using the Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\underline{\theta}}}\right) - \frac{\partial T}{\partial \underline{\theta}} = M\underline{G} \tag{51}$$

where $\underline{G}$ is the net moment about the mass center of the orbiter with respect to the body-fixed frame. It is given as

$$\underline{G} = \underline{G}_o + (\underline{r}+\underline{a})\times\underline{F}_2 \tag{52}$$

$\underline{G}_o$ is the external moment applied about the mass center. Eqation (51) can be simpified by substituting equations (42),(49), and (50) together with the relationship developed in (46) as

$$H\dot{\underline{V}} + I_o\dot{\underline{\omega}} + A_2\ddot{\underline{q}} = \underline{G} + \underline{N}_2 \tag{53}$$

where the nonlinear term $\underline{N}_2$ is given as

$$\underline{N}_2 = M^{-1}\begin{bmatrix} \underline{V}^T C\dfrac{\partial C^T}{\partial\theta_1} \\ \underline{V}^T C\dfrac{\partial C^T}{\partial\theta_2} \\ \underline{V}^T C\dfrac{\partial C^T}{\partial\theta_3}\end{bmatrix}\left(\frac{\partial\bar{T}}{\partial\underline{V}}\right) + M^{-1}\left(\begin{bmatrix} \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial\theta_1} \\ \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial\theta_2} \\ \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial\theta_3}\end{bmatrix} - \dot{M}\right)\left(\frac{\partial\bar{T}}{\partial\underline{\omega}}\right) \quad . \tag{54}$$

<u>(c) Vibration Equations of the Beam</u>

Since $\bar{T}$ in equation (37) is given in terms of $\dot{\underline{q}}$ which is a vector of generalized velocities,

$$\frac{\partial\bar{T}}{\partial\dot{\underline{q}}} = \frac{\partial T}{\partial\dot{\underline{q}}}$$

and

$$\left(\frac{\partial T}{\partial\dot{\underline{q}}}\right) = A_1^T\underline{V} + A_2^T\underline{\omega} + A_3\dot{\underline{q}} \quad . \tag{55}$$

The potential energy in the beam is given by

$$U = (1/2)\underline{q}^T K\underline{q} \tag{56}$$

where the stiffness matrix K is given as

$$K = \begin{bmatrix} \ddots & & 0 \\ & k_{ii} & \\ 0 & & \ddots \end{bmatrix} \tag{57}$$

and

$$k_{ii} = EI\,\beta_i^4 \left[ \int_0^L \phi_{xi}^2(s)ds + \int_0^L \phi_{yi}^2(s)ds \right] + G_\psi \beta_{\psi i}^2 \int_0^L \phi_{\psi i}^2(s)ds$$

$G_\psi$ represents the modulus of rigidity of the beam and $\beta_{\psi i} = \left[\frac{D\,\omega_i^2}{G_\psi}\right]^{\frac{1}{2}}$ where $D$ is the mass per unit volume (mass density) of the beam. Thus,

$$\left(\frac{\partial U}{\partial \underline{q}}\right) = K\underline{q} \quad . \tag{58}$$

Using the Lagrangian Equations (38) and assuming that $\underline{F}_2 = \underline{0}$,

$$A_1^T\dot{\underline{V}} + A_2^T\dot{\underline{\omega}} + A_3\ddot{\underline{q}} = -K\underline{q} \quad . \tag{59}$$

(d) Slewing Equations

If it is considered to perform a slew maneuver about an arbitrary axis $\underline{\lambda}$ and the slew angle to be $\xi$, then the slew maneuver can be expressed in terms of four Euler parameters. These four Euler parameters are defined as

$$\underline{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{bmatrix} = \underline{\lambda} \sin\frac{\xi}{2} \tag{60}$$

$$\epsilon_4 = \cos\frac{\xi}{2} \tag{61}$$

and their derivatives with respect to time are given as

$$\frac{d\,\underline{\epsilon}}{dt} = \frac{1}{2}\left(\epsilon_4\underline{\omega} + \underline{\epsilon}\times\underline{\omega}\right) \tag{62}$$

$$\frac{d\,\epsilon_4}{dt} = -\frac{1}{2}\underline{\omega}\cdot\underline{\epsilon} \quad . \tag{63}$$

If a slew maneuver is considered to be purely rotational, then the translational velocity and acceleration can be shown to be negligible during the slew maneuver and only the rotational and vibration equations are reqired for the analysis and they are simplified by setting $\dot{\underline{V}} \equiv \underline{0}$ in both (53) and (59) and are written as follows

$$I_o\,\dot{\underline{\omega}} + A_2\ddot{\underline{q}} = \underline{G}(t) + \underline{N}_2(\underline{\omega}) \tag{64}$$

$$A_2^T\dot{\underline{\omega}} + A_3\ddot{\underline{q}} = -K\underline{q} \quad . \tag{65}$$

Thus equations (62) - (65) completely represent the dynamics of the slew maneuver. These equations are nonlinear and coupled including both the rigid-body dynamics and the dynamics of the flexible appendage with kinematic nonlinearities. It is important to note that the nonlinear term $N_2(\underline{\omega})$ is dependent on the rotational velocity and as a result determined by the slew maneuver rate. Thus the basic slew maneuver stretegy has to be developed before this term can be linearized.

(e) Vibration Equations of the Beam with Damping

If damping is included in the derivation of vibration equations of the beam, then the damping effect can be expressed in terms of frictional forces. These are nonconservative, retarding forces and are assumed to be proportional to the generalized velocities. In deriving the vibration equations by means of Lagrange's equations, the following function is introduced

$$F_d = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\,\dot{q}_i\,\dot{q}_j \quad . \tag{66}$$

It also has a positive definite quadratic form similar to the kinetic and potential energy expressions.

With this definition, Lagrange's equations assume the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k}\frac{\partial F_d}{\partial \dot{q}_k} = Q_k - \frac{\partial U}{\partial q_k} \qquad (k=1,2,\ldots\ldots,n) \tag{67}$$

Again, as before

$$\left\{\frac{\partial T}{\partial \dot{\underline{q}}}\right\} = A_1^T\underline{V} + A_2^T\underline{\omega} + A_3\dot{\underline{q}} \tag{55}$$

and

$$\left\{\frac{\partial U}{\partial \underline{q}}\right\} = K\,\underline{q} \tag{58}$$

and it can be seen from (66) that

$$\left\{\frac{\partial F_d}{\partial \dot{\underline{q}}}\right\} = B\,\dot{\underline{q}} \tag{68}$$

where the damping matrix B is symmetrical and is given as

$$B = \begin{bmatrix} b_{11} & b_{12} & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & b_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ b_{n1} & b_{n2} & . & . & b_{nn} \end{bmatrix} \quad . \tag{69}$$

The vibration equations are given as

$$A_1^T\dot{\underline{V}} + A_2^T\dot{\underline{\omega}} + A_3\ddot{\underline{q}} + B\dot{\underline{q}} = -K\underline{q} \quad . \tag{70}$$

The slewing equations (64) and (65) would be modified as

$$I_2\dot{\underline{\omega}} + A_2\ddot{\underline{q}} = \underline{G}(t) + \underline{N}_2(\underline{\omega},\dot{\underline{q}}) \tag{71}$$

$$A_2^T\dot{\underline{\omega}} + A_3\ddot{\underline{q}} + B\dot{\underline{q}} = -K\underline{q} \quad . \tag{72}$$

<u>Nonlinear Term in the Rotational Equations</u>

The nonlinear term $\underline{N}_2$ in the rotational equations (64) and (71) during the slewing maneuver is simplified as

$$\underline{N}_2 = M^{-1}\left(\begin{bmatrix} \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial \theta_1} \\ \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial \theta_2} \\ \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial \theta_3} \end{bmatrix} - \dot{M}\right)\left[I_o\underline{\omega} + A_2\dot{\underline{q}}\right] \tag{73}$$

where

$$\underline{\omega}^T M^{-1}\frac{\partial M}{\partial \theta_1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \tag{74}$$

$$\underline{\omega}^T M^{-1}\frac{\partial M}{\partial \theta_2} = \frac{1}{\cos\theta_2}\Big[(-\omega_1\sin\theta_2\cos^2\theta_3+\omega_2\sin\theta_2\sin\theta_3\cos\theta_3) \quad (\omega_1\sin\theta_2\sin\theta_3\cos\theta_3$$

$$-\omega_2\sin\theta_2\sin^2\theta_3) \quad (\omega_1\cos\theta_2\cos\theta_3-\omega_2\cos\theta_2\sin\theta_3)\Big] \tag{75}$$

$$\underline{\omega}^T M^{-1}\frac{\partial M}{\partial \theta_3} = \frac{1}{\cos\theta_2}\Big[(\omega_2\cos\theta_2) \quad (-\omega_1\cos\theta_2)$$

$$(-\omega_1\sin\theta_2\cos\theta_3 + \omega_2\sin\theta_2\sin\theta_3 + \omega_3\cos\theta_2)\Big] \quad . \tag{76}$$

Since the transformation matrix, M , is a function of $\theta_2$ and $\theta_3$, the time derivative of M can be expressed by the chain rule as

$$\dot{M} = \frac{\partial M}{\partial \theta_2}\dot{\theta}_2 + \frac{\partial M}{\partial \theta_3}\dot{\theta}_3 \tag{77}$$

From equation (5)

$$\frac{\partial M}{\partial \theta_2}\dot{\theta}_2 = \begin{bmatrix} (-\sin\theta_2\cos\theta_3)\dot{\theta}_2 & (\sin\theta_2\sin\theta_3)\dot{\theta}_2 & (\cos\theta_2)\dot{\theta}_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{78}$$

$$\frac{\partial M}{\partial \theta_3}\dot{\theta}_3 = \begin{bmatrix} (-\cos\theta_2\sin\theta_3)\dot{\theta}_3 & (-\cos\theta_2\cos\theta_3)\dot{\theta}_3 & 0 \\ (\cos\theta_3)\dot{\theta}_3 & (-\sin\theta_3)\dot{\theta}_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{79}$$

Substituting these equations (78) and (79) in (77)

$$\dot{M} = \begin{bmatrix} (-\sin\theta_2\cos\theta_3)\dot{\theta}_2+(-\cos\theta_2\sin\theta_3)\dot{\theta}_3 & (\sin\theta_2\sin\theta_3)\dot{\theta}_2+(-\cos\theta_2\cos\theta_3)\dot{\theta}_3 & (\cos\theta_2)\dot{\theta}_2 \\ (\cos\theta_3)\dot{\theta}_3 & (-\sin\theta_3)\dot{\theta}_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{80}$$

From equation (4), this can also be expressed as

$$\dot{M} = \frac{1}{\cos\theta_2}\begin{bmatrix} \begin{gathered}(\sin\theta_2\cos\theta_3)(\omega_1\cos\theta_2\sin\theta_3 \\ +\omega_2\cos\theta_2\cos\theta_3)+(-\cos\theta_2\sin\theta_3) \\ (-\omega_1\sin\theta_2\cos\theta_3+\omega_2\sin\theta_2\sin\theta_3 \\ +\omega_3\cos\theta_2)\end{gathered} & \begin{gathered}(\sin\theta_2\sin\theta_3)(\omega_1\cos\theta_2\sin\theta_3 \\ +\omega_2\cos\theta_2\cos\theta_3)+(-\cos\theta_2\cos\theta_3) \\ (-\omega_1\sin\theta_2\cos\theta_3+\omega_2\sin\theta_2\sin\theta_3 \\ +\omega_3\cos\theta_2)\end{gathered} \\ \begin{gathered}(\cos\theta_3)(-\omega_1\sin\theta_2\cos\theta_3 \\ +\omega_2\sin\theta_2\sin\theta_3+\omega_3\cos\theta_2)\end{gathered} & \begin{gathered}(-\sin\theta_3)(-\omega_1\sin\theta_2\cos\theta_3 \\ +\omega_2\sin\theta_2\sin\theta_3+\omega_3\cos\theta_2)\end{gathered} \\ 0 & 0 \end{bmatrix}$$

$$\left.\begin{matrix} \cos\theta_2(\omega_1\cos\theta_2\sin\theta_3+\omega_2\cos\theta_2\cos\theta_3) \\ \\ \\ 0 \\ \\ 0 \end{matrix}\right] \quad . \tag{81}$$

Also, $M^{-1}$ is given as

$$M^{-1} = \frac{1}{\cos\theta_2}\begin{bmatrix} \cos\theta_3 & \cos\theta_2\sin\theta_3 & -\sin\theta_2\cos\theta_3 \\ -\sin\theta_3 & \cos\theta_2\cos\theta_3 & \sin\theta_2\sin\theta_3 \\ 0 & 0 & \cos\theta_2 \end{bmatrix} \quad . \tag{82}$$

Thus, the nonlinear term $N_2$ can be rewritten as

$$N_2 = A_3(\underline{\omega},\underline{\theta}) \ \left[I_o\,\underline{\omega}+A_2\underline{\dot{q}}\right]$$

Where the term $A_3$ is

$$A_3(\underline{\omega},\underline{\theta}) = M^{-1}\left(\begin{bmatrix} 0 \\ \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial\theta_2} \\ \underline{\omega}^T M^{-1}\dfrac{\partial M}{\partial\theta_3} \end{bmatrix} - \dot{M}\right)$$

$$N_2 = A_3(\underline{\omega},\underline{\theta})I_o\,\underline{\omega} + A_3(\underline{\omega},\underline{\theta})A_2\underline{\dot{q}}$$

$$= A_4(\underline{\omega},\underline{\theta}) + A_5(\underline{\omega},\underline{\theta})\underline{\dot{q}} \tag{83}$$

where $A_4$ depends on the rigid-body slewing and is nonlinear in terms of $\underline{\omega}$ and $\underline{\theta}$. The second term relates the coupling between the rigid-body slewing and the flexible modes.

## 4. NUMERICAL DATA

The analytics developed in the previous section are utilized together with the basic SCOLE data [1] and the three dimensional linear vibration analysis [4] to generate the following numerical data.

$m_1 = 6366.46 slugs.$; $m_2 = 12.42 slugs.$; $\rho = 0.0955 slugs/ft.$; $L = 130 ft.$

$G_\psi = 7.2E+8 lb/ft^2$; $(EI)_x = (EI)_y = (EI) = 4E+7 lb-ft^2$;

$$\underline{r} = \begin{bmatrix} 0.036 \\ -0.036 \\ -0.379 \end{bmatrix}$$

$$\underline{c} = \begin{bmatrix} 0 \\ 0 \\ -65.0 \end{bmatrix}$$

$$I_1 = \begin{bmatrix} 905443.0 & 0.0 & 145393.0 \\ 0.0 & 6789100.0 & 0.0 \\ 145393.0 & 0.0 & 7086601.0 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 18000.0 & -7570.0 & 0.0 \\ -7570.0 & 27407.0 & 0.0 \\ 0 & 0.0 & 27407.0 \end{bmatrix}$$

The three dimensional vibration analysis is given in terms of the first ten modal frequencies and mode shapes in table 1. Here,

$$\phi_{xi}(s) = A_{xi}\sin\frac{\alpha_i s}{L} + B_{xi}\cos\frac{\alpha_i s}{L} + C_{xi}\sinh\frac{\alpha_i s}{L} + D_{xi}\cosh\frac{\alpha_i s}{L}$$

$$\phi_{yi}(s) = A_{yi}\sin\frac{\alpha_i s}{L} + B_{yi}\cos\frac{\alpha_i s}{L} + C_{yi}\sinh\frac{\alpha_i s}{L} + D_{yi}\cosh\frac{\alpha_i s}{L}$$

$$\phi_{\psi i}(s) = A_{\psi i}\sin\alpha_{\psi i}\frac{s}{L} + B_{\psi i}\cos\alpha_{\psi i}\frac{s}{L}$$

$$\alpha_i = \left( \frac{\omega_i^2 \rho L^4}{EI} \right)^{\frac{1}{4}}$$

$$\alpha_{\psi i} = \left( \frac{D L^2 \omega_i^2}{G} \right)^{\frac{1}{2}}$$

Using these data the following matrices are obtained.

$$I_o = \begin{bmatrix} 1216640 & 15167.53 & -115118.9 \\ 15108.34 & 7083005 & -52474.84 \\ -115096 & -52503.9 & 7131493 \end{bmatrix}$$

TABLE 1

FIRST TEN FLEXIBLE MODES OF SCOLE MODEL

| THREE DIMENSIONAL MODE SHAPE CHARACTERISTICS | | |
|---|---|---|
| MODE No. | 1 | 2 |
| FREQ. (Hz.) | 0.27804240E+00 | 0.31357296E+00 |
| $\alpha$ | 0.12012084E+01 | 0.12756518E+01 |
| $A_x$ | 0.16282665E+00 | 0.38855291E-02 |
| $B_x$ | -0.19670286E+00 | -0.14998387E-01 |
| $C_x$ | -0.16983450E+00 | -0.43321018E-02 |
| $D_x$ | 0.19616259E+00 | 0.14985820E-01 |
| $A_y$ | -0.10274618E-01 | 0.14219781E+00 |
| $B_y$ | 0.57579133E-02 | -0.22695797E+00 |
| $C_y$ | 0.11810057E-01 | -0.19283105E+00 |
| $D_y$ | -0.57220462E-02 | 0.22644561E+00 |
| $\alpha_\psi$ | 0.19360955E-01 | 0.21835058E-01 |
| $A_\psi$ | -0.50748354E-01 | 0.31115282E-01 |
| $B_\psi$ | 0.13978018E-04 | -0.75992337E-05 |
| MODE No. | 3 | 4 |
| FREQ. (Hz.) | 0.81300189E+00 | 0.11856099E+01 |
| $\alpha$ | 0.20540387E+01 | 0.24804687E+01 |
| $A_x$ | 0.40868188E-01 | 0.80641794E-01 |
| $B_x$ | -0.61958845E-01 | -0.67233377E-01 |
| $C_x$ | -0.41309992E-01 | -0.80913938E-01 |
| $D_x$ | 0.61880796E-01 | 0.67106316E-01 |
| $A_y$ | -0.22438404E-01 | 0.13728679E+00 |
| $B_y$ | 0.36509234E-01 | -0.11746932E+00 |
| $C_y$ | 0.24390447E-01 | -0.14085209E+00 |
| $D_y$ | -0.36464758E-01 | 0.11725057E+00 |
| $\alpha_\psi$ | 0.56611842E-01 | 0.82557693E-01 |
| $A_\psi$ | 0.92698901E-01 | -0.16158934E-03 |
| $B_\psi$ | -0.87320799E-05 | 0.10437718E-07 |
| MODE No. | 5 | 6 |
| FREQ. (Hz.) | 0.20536300E+01 | 0.49716090E+01 |
| $\alpha$ | 0.32645546E+01 | 0.49716090E+01 |
| $A_x$ | 0.99278129E-01 | 0.45739784E-01 |
| $B_x$ | -0.92344553E-01 | -0.46365581E-01 |
| $C_x$ | -0.99442145E-01 | -0.45763106E-01 |
| $D_x$ | 0.92225801E-01 | 0.46329676E-01 |
| $A_y$ | -0.57396019E-01 | 0.78612940E-01 |
| $B_y$ | 0.53976008E-01 | -0.79952853E-01 |
| $C_y$ | 0.58114853E-01 | -0.78914485E-01 |
| $D_y$ | -0.53906980E-01 | 0.79891039E-01 |
| $\alpha_\psi$ | 0.14300062E+00 | 0.33165303E+00 |
| $A_\psi$ | -0.16588614E-02 | -0.93394833E-05 |
| $B_\psi$ | 0.61861804E-07 | 0.15017211E-09 |

| THREE DIMENSIONAL MODE SHAPE CHARACTERISTICS | | |
|---|---|---|
| MODE No. | 7 | 8 |
| FREQ. (Hz.) | 0.55157833E+01 | 0.12281249E+02 |
| $\alpha$ | 0.53501560E+01 | 0.79833305E+01 |
| $A_x$ | 0.81311804E-01 | 0.44835061E-01 |
| $B_x$ | -0.82056569E-01 | -0.44834914E-01 |
| $C_x$ | -0.81344923E-01 | -0.44840508E-01 |
| $D_x$ | 0.81997259E-01 | 0.44813000E-01 |
| $A_y$ | -0.47145439E-01 | 0.77404756E-01 |
| $B_y$ | 0.47703590E-01 | -0.77465629E-01 |
| $C_y$ | 0.47289807E-01 | -0.77475327E-01 |
| $D_y$ | -0.47669155E-01 | 0.77427782E-01 |
| $\alpha_\psi$ | 0.38408110E+00 | 0.85518143E+00 |
| $A_\psi$ | -0.23855560E-02 | 0.15830371E-05 |
| $B_\psi$ | 0.33122041E-07 | -0.98715017E-11 |
| MODE No. | 9 | 10 |
| FREQ. (Hz.) | 0.12890442E+02 | 0.23679520E+02 |
| $\alpha$ | 0.81789349E+01 | 0.11085347E+02 |
| $A_x$ | 0.78743585E-01 | 0.44348498E-01 |
| $B_x$ | -0.78755259E-01 | -0.44367373E-01 |
| $C_x$ | -0.78752483E-01 | -0.44350511E-01 |
| $D_x$ | 0.78717693E-01 | 0.44351763E-01 |
| $A_y$ | -0.45569244E-01 | 0.76707490E-01 |
| $B_y$ | 0.45609474E-01 | -0.76762782E-01 |
| $C_y$ | 0.45607884E-01 | -0.76733612E-01 |
| $D_y$ | -0.45587726E-01 | 0.76735779E-01 |
| $\alpha_\psi$ | 0.89760145E+00 | 0.16488784E+01 |
| $A_\psi$ | 0.94995483E-03 | -0.51105957E-06 |
| $B_\psi$ | -0.56437766E-08 | 0.16528495E-11 |

$$A_3 = \left[\begin{array}{ccccc}
0.45879E{+}2 & 0.36305E{-}1 & -0.89042E{-}1 & -0.14067E0 & 0.1457E0 \\
0.36305E{-}1 & 0.6211E{+}2 & 0.11263E0 & -0.1471E0 & -0.5518E{-}1 \\
-0.89042E{-}1 & 0.11263E0 & 0.32737E{+}2 & -0.6392E{-}1 & -0.14526E0 \\
-0.14067E0 & -0.1471E0 & -0.6392E{-}1 & 0.2547E{+}3 & 0.1908E0 \\
-0.1457E0 & -0.5518E{-}1 & -0.14526E0 & 0.1908E0 & 0.8103E{+}3 \\
0.1914E{-}1 & 0.19839E{-}1 & 0.7925E{-}2 & -0.4278E{-}1 & -0.2570E{-}1 \\
0.84597E{-}1 & 0.3935E{-}2 & -0.8369E{-}1 & -0.76115E{-}1 & -0.12912E0 \\
-0.6893E{-}2 & -0.7165E{-}2 & -0.2829E{-}2 & 0.1543E{-}1 & 0.9222E{-}2 \\
-0.4269E{-}1 & 0.5969E{-}2 & 0.89767E{-}1 & 0.2859E{-}1 & 0.4611E{-}1 \\
0.4204E{-}2 & 0.41227E{-}2 & 0.1866E{-}2 & -0.9067E{-}2 & -0.5947E{-}2
\end{array}\right.$$

$$\left.\begin{array}{ccccc}
0.1914E{-}1 & 0.84597E{-}1 & -0.6893E{-}2 & -0.4269E{-}1 & 0.4204E{-}2 \\
0.19839E{-}1 & 0.3935E{-}2 & -0.7165E{-}2 & 0.5969E{-}2 & 0.4127E{-}2 \\
0.7925E{-}2 & -0.8369E{-}1 & -0.2829E{-}2 & 0.89767E{-}1 & 0.1866E{-}2 \\
-0.4278E{-}1 & -0.76115E{-}1 & 0.1543E{-}1 & 0.2859E{-}1 & -0.9067E{-}2 \\
-0.2570E{-}1 & -0.12912E0 & 0.9222E{-}2 & 0.4611E{-}1 & -0.5947E{-}2 \\
0.23209E{+}5 & 0.10383E{-}1 & -0.2089E{-}2 & -0.3955E{-}2 & 0.1227E{-}2 \\
0.10383E{-}1 & 0.55561E{+}5 & -0.37286E{-}2 & -0.3859E{-}1 & 0.2397E{-}2 \\
-0.2089E{-}2 & -0.37286E{-}2 & 0.1342962E{+}8 & 0.1421E{-}2 & -0.4427E{-}3 \\
-0.3955E{-}2 & -0.3859E{-}1 & 0.1421E{-}2 & 0.2095672E{+}8 & -0.9108E{-}3 \\
0.1227E{-}2 & 0.2397E{-}2 & -0.4427E{-}3 & -0.9108E{-}3 & 0.8662547E{+}10
\end{array}\right]$$

$$A_2^T = \left[\begin{array}{ccc}
-0.2133821E0 & -0.3687057E{+}3 & -0.7253901E{-}1 \\
0.3808921E{+}3 & -0.3030935E{+}2 & -0.8427658E{-}1 \\
-0.1808478E{+}3 & -0.1318596E{+}3 & -0.125799E0 \\
0.1423380E{+}3 & -0.1135851E{+}1 & -0.2367351E{-}1 \\
-0.2416743E{+}2 & 0.574383E{+}2 & -0.9150328E{-}1 \\
-0.6802273E0 & 0.3104929E2 & -0.3843062E{-}1 \\
0.2784792E{+}2 & 0.6651585E{+}2 & 0.596075E{-}1 \\
0.7842818E{+}1 & -0.1930097E{+}2 & -0.4363533E{-}2 \\
-0.2694455E{+}2 & -0.5544252E{+}2 & -0.4200623E{-}1 \\
-0.9225328E{-}1 & 0.1594045E{+}2 & -0.1626004E{-}1
\end{array}\right]$$

The stiffness matrix $K$ is calculated using equation (57) and the mode shape coefficients given in Table 1. This matrix is a diagonal matrix and is represented in terms of the diagonal elements as

$$K = \begin{bmatrix} k_{1,1} = 0.2820217E0 \\ k_{2,2} = 0.3574692E0 \\ k_{3,3} = 0.2412807E1 \\ k_{4,4} = 0.5285116E1 \\ k_{5,5} = 0.1588654E2 \\ k_{6,6} = 0.8573860E2 \\ k_{7,7} = 0.1146118E3 \\ k_{8,8} = 0.5686101E3 \\ k_{9,9} = 0.6254598E3 \\ k_{10,10} = 0.2114612E4 \end{bmatrix}$$

The damping matrix $B$ used for this analysis is a diagonal matrix and for damping ratio $\zeta = 0.003$, it is calculated to be

$$B = \begin{bmatrix} b_{1,1} = 0.9685964E-3 \\ b_{2,2} = 0.1088608E-2 \\ b_{3,3} = 0.2834016E-2 \\ b_{4,4} = 0.4256808E-2 \\ b_{5,5} = 0.7387177E-2 \\ b_{6,6} = 0.1719014E-1 \\ b_{7,7} = 0.1984237E-1 \\ b_{8,8} = 0.4421234E-1 \\ b_{9,9} = 0.4633434E-1 \\ b_{10,10} = 0.8527647E-1 \end{bmatrix}$$

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## APPENDIX

The following is a summary of transformations between inertial frame and body-fixed frame. Here, $s_i$ and $c_i$ $(i=1,2,3)$ denote $\sin\theta_i$ and $\cos\theta_i$ $(i=1,2,3)$ respectively.

(a) Space-three Angles

$$C = \begin{bmatrix} c_2c_3 & c_2s_3 & -s_2 \\ s_1s_2c_3-s_3c_1 & s_1s_2s_3+c_3c_1 & s_1c_2 \\ c_1s_2c_3+s_3s_1 & c_1s_2s_3-c_3s_1 & c_1c_2 \end{bmatrix}$$

$$M^T = \begin{bmatrix} 1 & 0 & -s_2 \\ 0 & c_1 & s_1c_2 \\ 0 & -s_1 & c_1c_2 \end{bmatrix}$$

(b) Space-two Angles

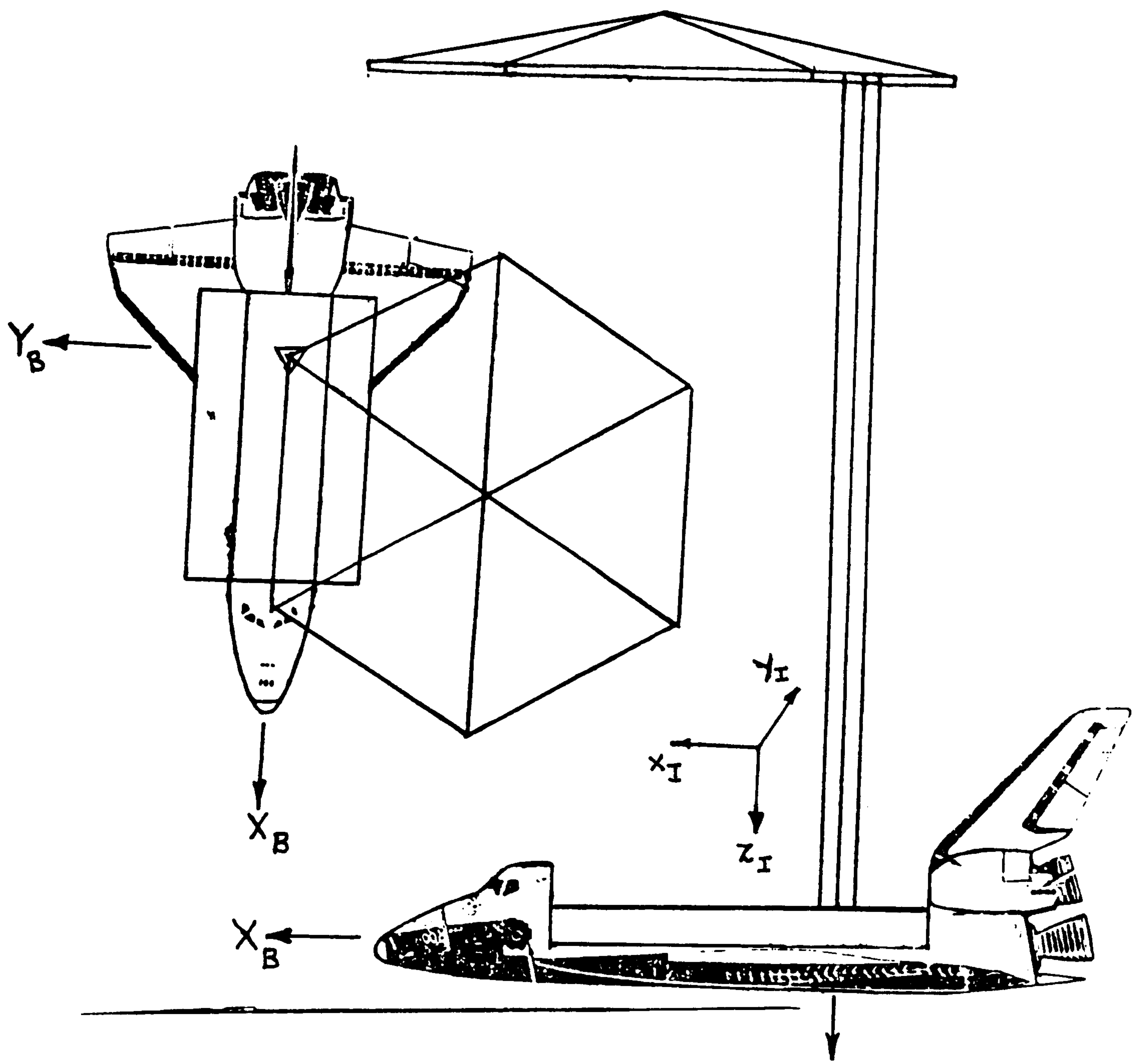
$$C = \begin{bmatrix} c_2 & s_2s_3 & -s_2c_3 \\ s_1s_2 & -s_1c_2s_3+c_3c_1 & s_1c_2c_3+s_3c_1 \\ c_1s_2 & -c_1c_2s_3-c_3s_1 & c_1c_2c_3-s_3s_1 \end{bmatrix}$$

$$M^T = \begin{bmatrix} 1 & 0 & c_2 \\ 0 & c_1 & s_1s_2 \\ 0 & -s_1 & c_1s_2 \end{bmatrix}$$

(c) Body-two Angles

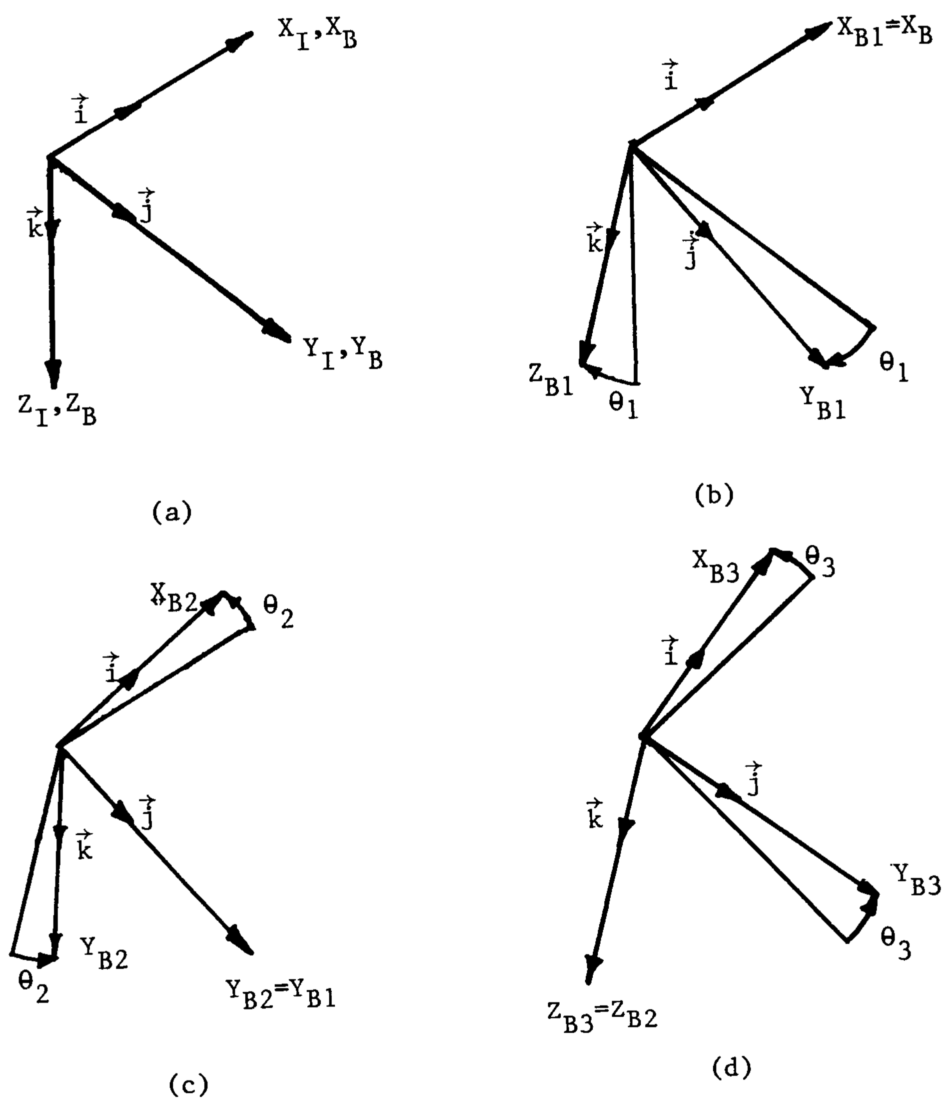
$$C = \begin{bmatrix} c_2 & s_1s_2 & -c_1s_2 \\ s_2s_3 & -s_1c_2s_3+c_3c_1 & c_1c_2s_3+c_3s_1 \\ s_2c_3 & -s_1c_2c_3-s_3c_1 & c_1c_2c_3-s_3s_1 \end{bmatrix}$$

$$M^T = \begin{bmatrix} c_2 & 0 & 1 \\ s_2s_3 & c_3 & 0 \\ s_2c_3 & -s_3 & 0 \end{bmatrix}$$




Spacecraft Control Experiment (SCOLE)

FIGURE 1




(a) Axes in reference position (b) First rotation-about x axis
(c) Second rotation-about y axis (d) Final rotation-about z axis

**FIGURE 2**

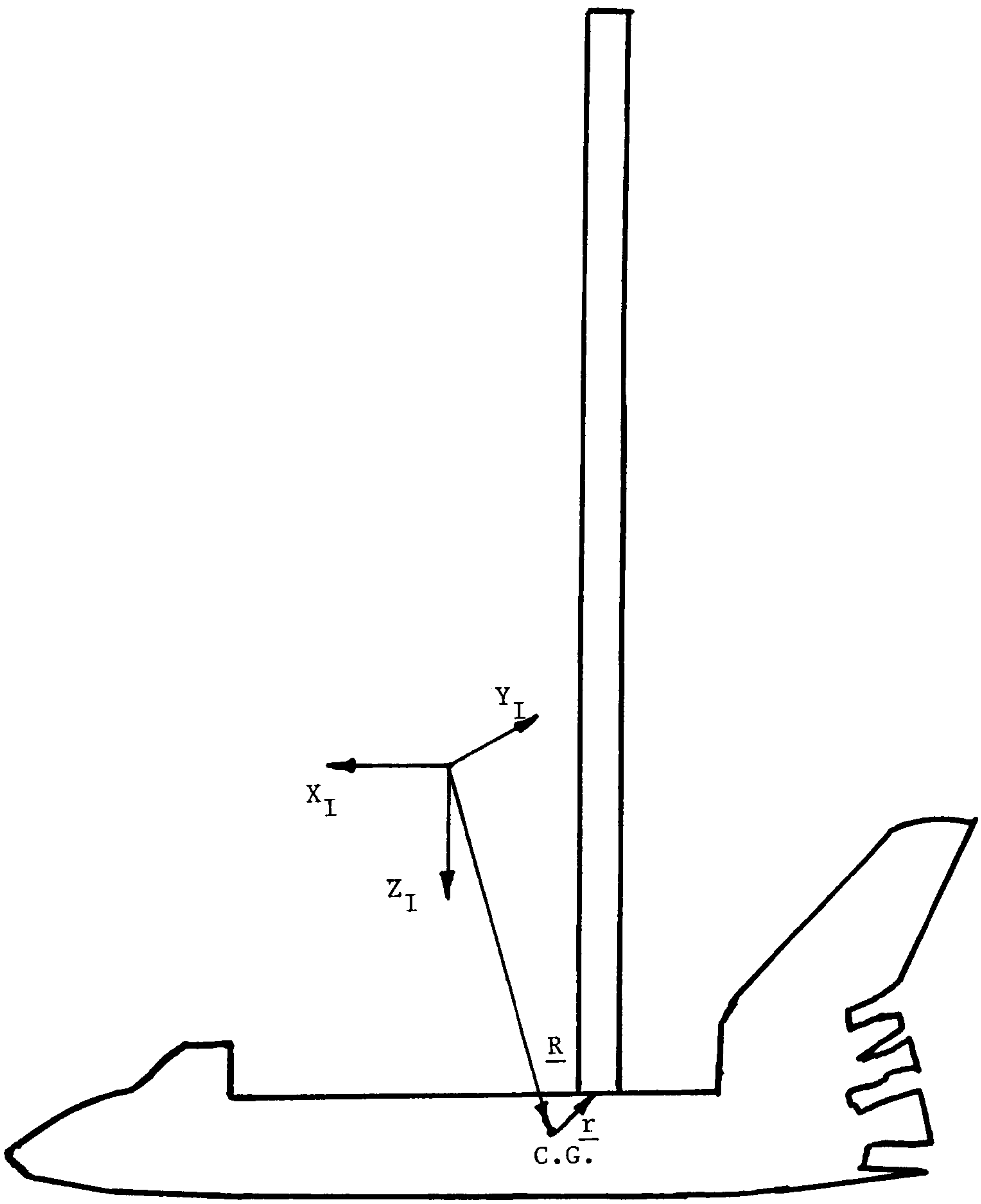



**Figure 3- Position Vectors in Inertial Frame**

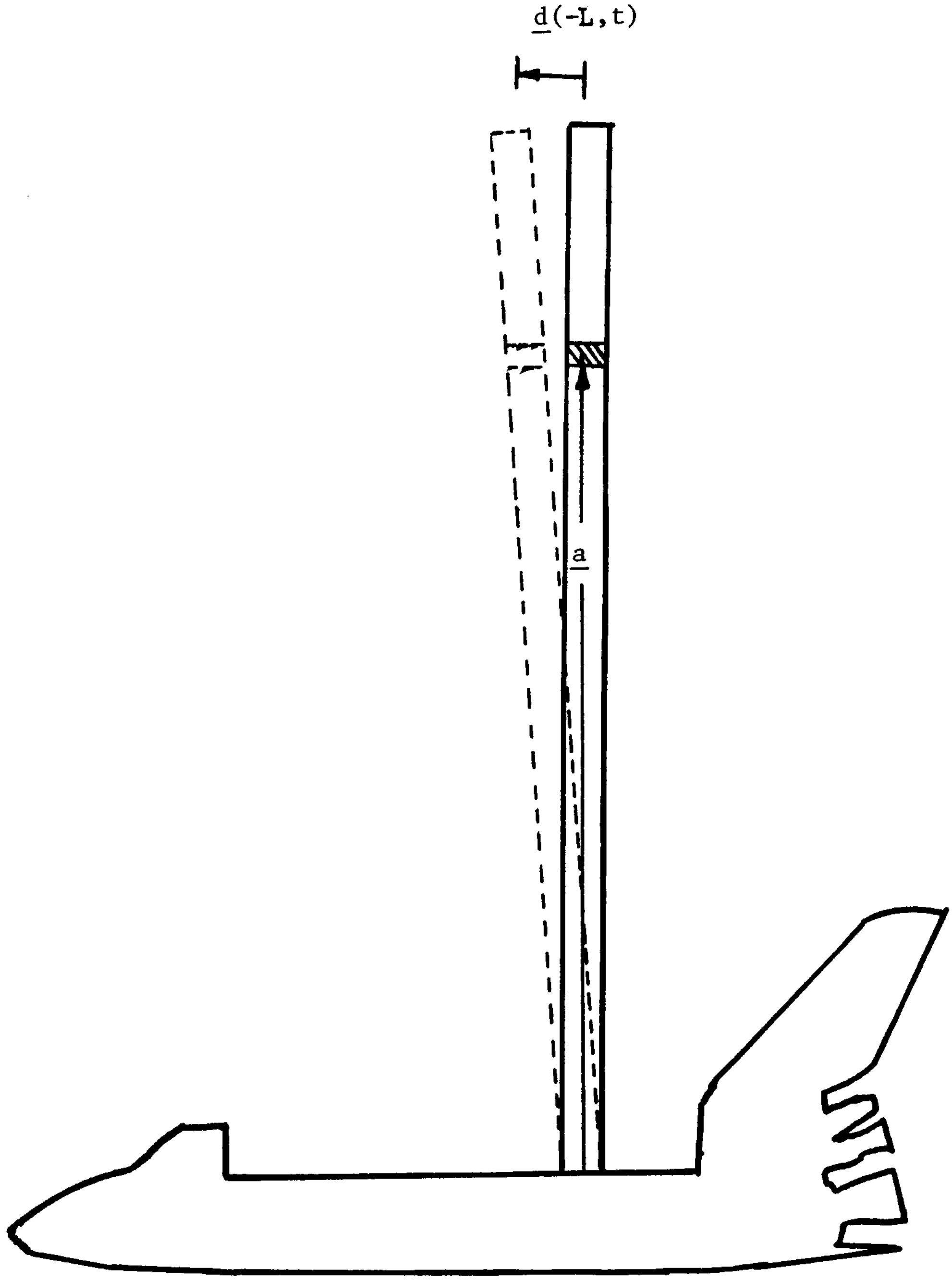



Figure 4- Vectors in Body-fixed Frame

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15. Supplementary Notes

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16. Abstract

This is the first report of a set of two reports on the dynamics and control of slewing maneuvers of NASA Spacecraft Control Laboratory Experiment (SCOLE) article. In this report, the dynamics of slewing maneuvers of SCOLE are developed in terms of an arbitrary maneuver about any given axis. The set of dynamical equations incorporate rigid-body slew maneuver and three-dimensional vibrations of the complete assembly comprising the rigid shuttle, the flexible beam, and the reflector with an offset mass. The analysis also includes kinematic nonlinearities of the entire assembly during the maneuver and the dynamics of the interaction between the rigid shuttle and the flexible appendage. The final set of dynamical equations obtained for slewing maneuvers are highly nonlinear and coupled in terms of the flexible modes and the rigid-body modes.

The equations are further simplified and evaluated numerically to include the first ten flexible modes and the SCOLE data to yield a model for designing control systems to perform slew maneuvers.

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