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MAGNETIC FIELDS IN THE OVERSHOOT ZONE: THE GREAT ESCAPE

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ABSTRACT

In order that magnetic flux be confined within the solar interior for times comparable to the solar cycle period it has been suggested that the bulk of the solar toroidal field is stored in the convectively stable overshoot region situated beneath the convection zone proper. Such a magnetic field though is still buoyant and is therefore subject to Rayleigh-Taylor type instabilities. In this paper we consider the model problem of an isolated region of magnetic field embedded in a convectively stable atmosphere. The fully nonlinear evolution of the two-dimensional interchange modes is studied, thereby shedding some light on one of the processes responsible for the escape of flux from the solar interior.

1. INTRODUCTION

Despite considerable strides in recent years in both dynamo theory and the study of magnetoconvection, several problems remain regarding certain key features of the sun's magnetic cycle. One of these concerns the location of the sun's primary toroidal field and the means by which this can escape from the interior, eventually to appear at the surface to form sunspots or smaller bipolar active regions. The inherent buoyancy of isolated magnetic flux tubes, together with the unstable convective motions, will carry tubes of equipartition strength through the entire convection zone on a timescale of about a month (see, for example, Parker, 1975; Moreno-Insertis, 1983). This presents certain difficulties for keeping the requisite amount of magnetic flux within the convection zone for timescales comparable to that of the solar cycle. To overcome this difficulty it has been suggested (Spiegel and Weiss, 1980) that the bulk of the toroidal field is contained in the convectively stable overshoot region situated beneath the convection zone proper. Of course such a field alters the density stratification, causing the atmosphere to become top-heavy — in consequence, Rayleigh-Taylor instabilities can ensue.

In this paper we consider the instabilities which can result when an isolated region of magnetic field is embedded in a convectively stable atmosphere. It should be noted that this is a very different situation to that of an isolated flux tube in a field-free gas. In particular, our initial static state possesses a continuous temperature distribution and is a *bona fide* solution of the hydrostatic equation (see Section 2) — for an isolated flux tube in thermal equilibrium with its surroundings, on the other hand, there is *no* static state and the tube is forced to rise. Our calculation is both fully compressible and nonlinear but, for simplicity, we shall here only consider the two-dimensional interchange modes (no bending of the field lines) and shall ignore the effects of rotation. The account given is, of necessity, rather brief — a much more detailed version, which also treats rotational effects, is currently in preparation.

2. MATHEMATICAL FORMULATION

We consider a plane parallel layer of conducting fluid of thickness d. Within the layer the magnetic field $\mathbf{B}$ and gravity $\mathbf{g}$ are constrained to be mutually orthogonal for all times. The fluid flow is in the xz plane only, perpendicular to the magnetic field. The computational domain can be thought of as a section of a meridional cut near the equator. The field is then toroidal and the flows are axisymmetric. To simplify the problem the thermal conductivity K, the shear viscosity μ and the magnetic diffusivity η are assumed to be constants; furthermore the plasma obeys the perfect gas law with constant specific heats. By choosing the layer depth d and the sound crossing time $d/\sqrt{p/\rho}$ as the units of length and time respectively the evolution equations can be expressed in dimensionless form as follows:

$$p = \rho T, \tag{1}$$

$$\partial_t \rho + \nabla \cdot \rho \mathbf{u} = 0, \tag{2}$$

$$\partial_t B + \nabla \cdot (B\mathbf{u}) = \tau C_k \nabla^2 B, \tag{3}$$

$$\partial_t \rho \mathbf{u} + \nabla \cdot \rho \mathbf{u}\mathbf{u} = -\nabla(p + \frac{B^2}{2\beta}) + \theta(m+1)\rho \hat{\mathbf{z}} + C_k \sigma(\nabla^2 \mathbf{u} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{u})), \tag{4}$$

$$\rho(\partial_t T + \mathbf{u} \cdot \nabla T) + (\gamma - 1)p\nabla \cdot \mathbf{u} = \gamma C_k \nabla^2 T + (\gamma - 1)C_k(\sigma \partial_i u_j \phi_{ij} + \tau(\nabla B)^2/\beta), \tag{5}$$

where $\phi_{ij} = \partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\nabla \cdot \mathbf{u}$.
The six dimensionless parameters which appear explicitly are defined by:

$$\sigma = \frac{\mu C_p}{K}, \qquad \tau = \frac{\eta \rho_0 C_p}{K}, \qquad \beta = \frac{\mu_0 p_0}{B_0^2},$$
$$m = (\frac{gd}{R\Delta} - 1), \qquad \theta = \frac{\Delta d}{T_0}, \qquad C_k = \frac{K}{\rho_0 C_p d \sqrt{RT_0}},$$

where Δ is the initial temperature gradient.

The formulation of the problem is completed by imposing boundary conditions at the top and bottom and by requiring that all variables be periodic in the horizontal direction. The boundary conditions at $z = 0$ (top) and $z = 1$ (bottom) are chosen so as to minimise their effects in the interior; they are:

$$T = 1 \quad \text{at} \quad z = 0 \quad \text{and} \quad T = 1 + \theta \quad \text{at} \quad z = 1,$$

$$\rho w = \partial_z u = \partial_z B = 0 \qquad \text{at} \quad z = 0, 1.$$

If the magnetic field has a top hat profile then the equations admit a piecewise-polytropic static solution. Small random disturbances (typically about 1%) of this equilibrium are used as initial conditions for the fully nonlinear calculations. The stability of the static solution to infinitesimal disturbances is determined by linear theory.

3. THE LINEAR REGIME

The linearised versions of Eqs. (1) to (5) are separable in x and t with solutions of the form $f(x,z,t) = \hat{f}(z)e^{st}\cos\ell x$ (or $\sin\ell x$). We are then left with a set of coupled ordinary differential equations (involving only z) with the growth rate s as the eigenvalue. For small values of τ instability always occurs via an exchange of stabilities. We find that there is always one dominant unstable mode and that this has grown appreciably before any of the harmonics lose stability.

As expected, increasing the strength of the magnetic field (reducing β) leads to the instability becoming more vigorous (see figure 1). The figure also shows that as the field is increased then the mode of maximum growth rate moves to larger wavenumbers (smaller cell widths). In the absence of all diffusion the growth rate is maximised for infinitesimally thin cells ($\ell \rightarrow \infty$); however, dissipation acts most effectively on such cells and therefore a balance is struck at a finite cell size. For small β the instability is sufficiently strong that fairly small cells can overcome the dissipative effects and grow most rapidly. This behaviour is analogous to that which occurs in the standard Rayleigh-Taylor instability of two superposed viscous fluids (Chandrasekhar, 1961, p.446). A more surprising feature is that the dependence of the mode of maximum growth rate on the depth of the magnetic layer is very weak (see figure 2). This would appear to be because the instability occurs primarily at the upper interface of the magnetic field and that the depth of the field below that is not of fundamental importance.

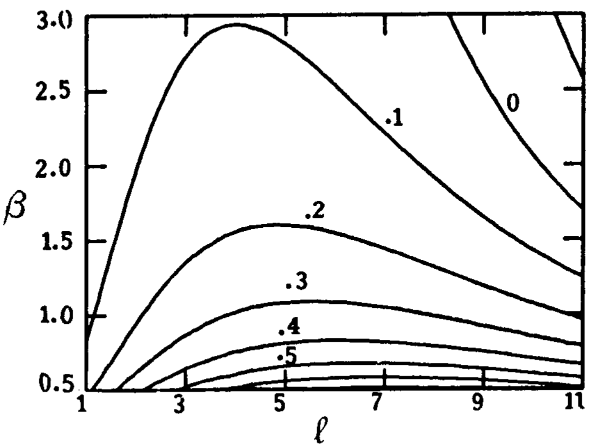

Figure 1 Contours of the growth rate for β versus ℓ. $\sigma = .1$, $\tau = .05$, $C_k = .05$, $m = 1.6$, $\theta = .1$

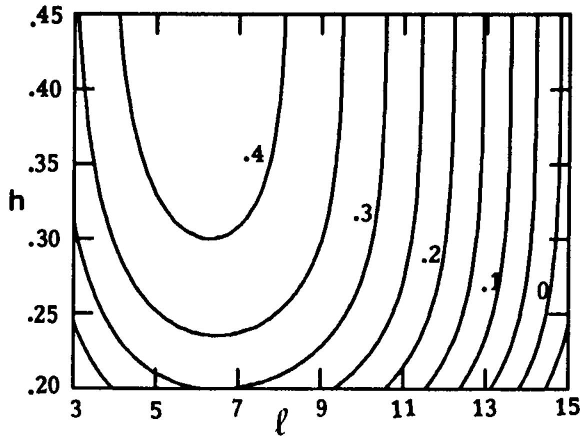

Figure 2 Contours of the growth rate for h, the depth of the magnetic region, versus ℓ. $\sigma = \tau = .1$, $C_k = .05$, $m = 1.6$, $\theta = 1$

4. THE NONLINEAR REGIME

The evolution of a uniform magnetic field confined initially to the region $0.6 < z < 0.8$ is shown in figure 3. The instability due to magnetic buoyancy leads to an initial break-up of the layer and, eventually, to the spreading of magnetic field over the entire domain in a time (a few Alfvén crossing times) short compared to the diffusion time.

In the initial phases of the instability the magnetic layer's upper interface forms the familiar mushroom-shaped structures associated with Rayleigh-Taylor instabilities (see, for instance, Daly, 1967). In all cases considered the density difference between the magnetic and field-free regions was such that mushrooms were formed rather than bubbles and spikes. Associated with the wings of the mushrooms are regions of intense vorticity which, once most of the available potential energy has been released, play a key rôle in the subsequent evolution of the layer. The characteristic wrap-around structures visible in figures 3(c) to 3(f) are typical of flows generated by random distributions of vortex pairs. Some of the vortex-vortex interactions are strong enough to do, albeit locally, negative buoyancy work and to produce the downward displacement of regions of high magnetic field even though these regions are lighter than their surroundings and would have, in the absence of any other flow, risen upwards.

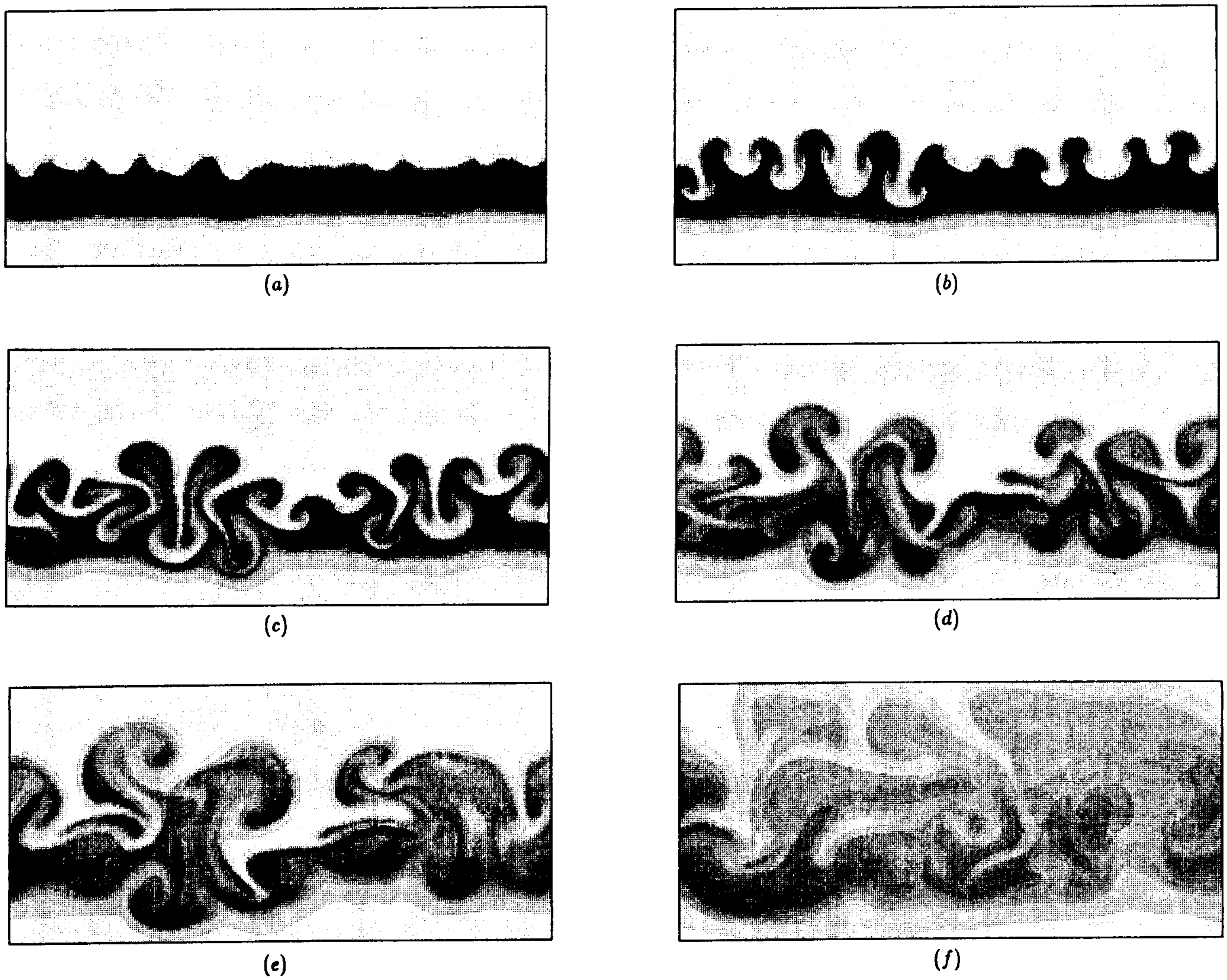

Figure 3 Time evolution of an unstable magnetic layer. $\sigma = .1$, $\tau = .01$, $C_k = .01$, $m = 1.6$, $\theta = 2$, $\beta = 1$. The simulation times for the frames are respectively: (a) 1.9, (b) 3.5, (c) 5.1, (d) 6.7, (e) 8.3 and (f) 13.1.

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