

# Initial Dynamic Load Estimates During Configuration Design

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*Daniel Schiff*

Dynamic analysis of aerospace systems is required to insure that the structures will maintain their integrity and provide predictable and acceptable mechanical performance throughout the mission profile of specified acceleration environments. This analysis includes the structural response to shock and vibration and evaluates the maximum deflections and material stresses and the potential for the occurrence of elastic instability, fatigue and fracture. The required computations are often performed by means of finite element analysis (FEA) computer programs in which the structure is simulated by a finite element model which may contain thousands of elements. The formulation of a finite element model can be time consuming, and substantial additional modeling effort may be necessary if the structure requires significant changes after initial analysis. This paper presents rapid methods for obtaining rough estimates of the structural response to shock and vibration, for the purpose of providing guidance during the initial mechanical design configuration stage.

## INTRODUCTION

Structures are often made up of simple components such as beams, rings, arcs, plates and shells. The natural frequencies of such a structure cannot usually be found from the frequencies of these components. However, the stiffness, damping and mass of these components, the stiffness and damping of the connections between components, and the type of attachment of the structures to mounting surfaces will determine the natural frequencies of the structure. Estimates of natural frequencies can only be made for simple structures without developing an FEA (finite element analysis) model and utilizing an FEA computer program. But even rough estimates of natural frequency can provide a relatively rapid means of comparing maximum acceleration, stress and fatigue in different design approaches, and identifying potential problem areas in a structure. This type of information can help to avoid excessive modification of the FEA model when a comprehensive computer analysis is done.

The methods presented here for evaluating natural frequencies of simple structures will typically have a frequency error on the low side, which will result in a conservative (larger than actual) estimate of stress.

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## Composite Beams

Beams may be made of two or more layers of different materials adhered to one another, with each layer running the length of the beam. In this section, the layers are assumed to have constant, rectangular cross sections. The layers may be oriented so that the direction of vibration is parallel to the layer interfaces or normal to the layer interfaces.

### Vibration Parallel to Layer Interfaces

Figure 1 shows the case where the direction of vibration is parallel to the layer interfaces. The term  $EI/L$  is referred to as the stiffness of a beam. Defining  $EI$  as the stiffness factor of a beam, the stiffness factor of the composite beam of Figure 1 is

$$EI = \sum_{i=1}^n E_i I_i = (1/12) \sum_{i=1}^n E_i b_i h_i^3 \quad \text{1bf.in}^2 \quad (1)$$

where  $E_i$  = modulus of elasticity of layer  $i$ , 1bf/in<sup>2</sup>,  
 $b_i$  = width of layer  $i$ , inch,  
 $h_i$  = height of layer  $i$ , inch,  
 $I_i$  = area moment of inertia of layer  $i$  about neutral (Z) axis, in<sup>4</sup>.

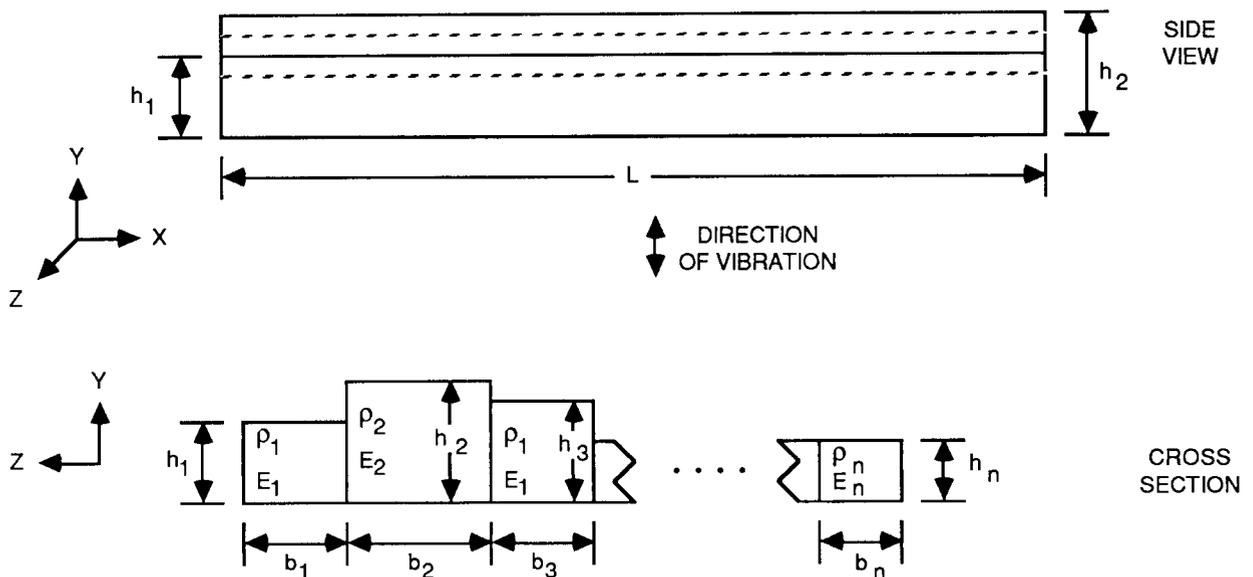


Fig. 1 Layered Beam, Vibration Parallel To Layer Interfaces

The weight per unit length of the composite beam is

$$W = \sum_{i=1}^n \rho_i b_i h_i \text{ lbf/in} \quad (2)$$

where  $\rho_i$  = weight density of layer  $i$ , lbf/in<sup>3</sup>

Equations (1) and (2) may be used with slender beam frequency formulas when the composite beam is uniformly loaded and the neutral axis through the beam cross section, parallel to the Z axis, remains undeflected (no bending along the Z axis).

### Vibration Normal to Layer Interface

Figure 2 shows the case where the vibration is normal to the layer interfaces. The stiffness factor of the composite beam of Figure 2 is

$$EI = \sum_{i=1}^n b_i h_i E_i [(\bar{Y} - Y_i)^2 + (1/12)h_i^2] \text{ lbf.in}^2 \quad (3)$$

where  $Y_i = \sum_{j=0}^{i-1} h_j + (1/2)h_i$  inch, where  $h_0 = 0$ ,

and  $\bar{Y} = \frac{\sum_{i=1}^n b_i h_i E_i Y_i}{\sum_{i=1}^n b_i h_i E_i}$  inch

See Equation 1 for definitions.

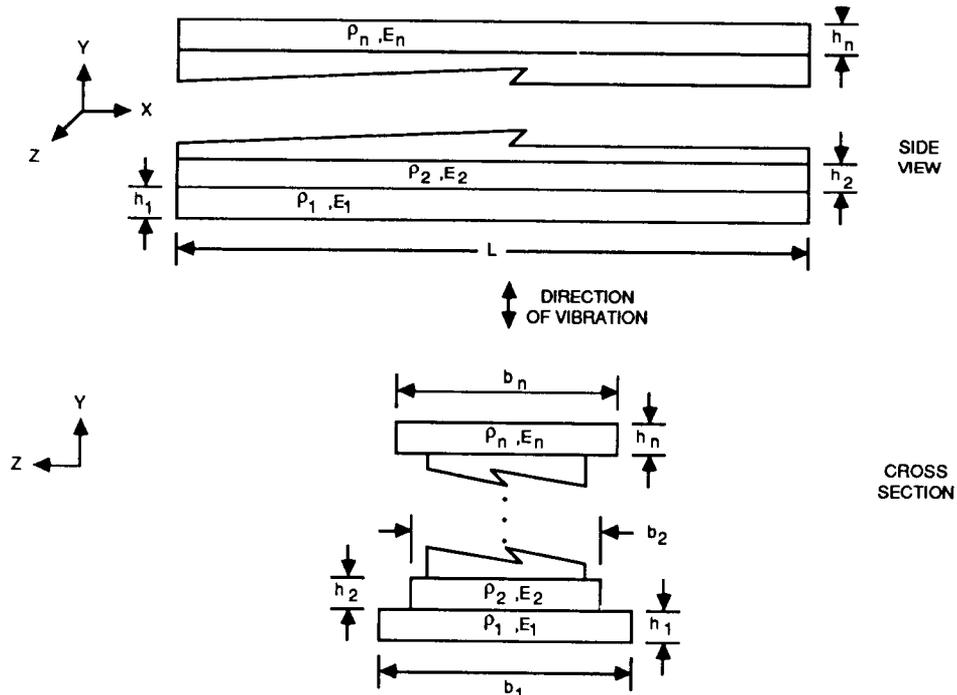


Fig. 2 Layered Beam, Vibration Normal to Layer Interfaces

The weight per unit length of the composite beam,  $W$ , is given by Equation (2). Equations (2) and (3) may be used with slender beam frequency formulas to obtain approximate values of natural frequency. Accuracy is improved when the layer widths,  $b_i$ , approach equality with one another.

### Stepped Beams

Stepped beams have two or more different cross sections along their span, resulting in two or more different moments of inertia. Figure 3 shows two examples of stepped cantilever beams. In Figure 3a the beam has two different cross sections and the average moment of inertia for the beam is<sup>[1]</sup>

$$I_A = L^3 I_1 I_2 / [3(a^2 b + b^2 a + b^3/3) I_1 + a^3 I_2] \text{ in}^4 \quad (4)$$

In Figure 3b the beam has three different cross sections and the average moment of inertia for the beam is

$$I_A = L^3 I_1 I_2 I_3 / \left\{ 3[(a+b)^2 c + (a+b)c^2 + c^3/3] I_1 I_2 + 3(a^2 b + ab^2 + b^3/3) I_1 I_3 + a^3 I_2 I_3 \right\} \text{ in}^4 \quad (5)$$

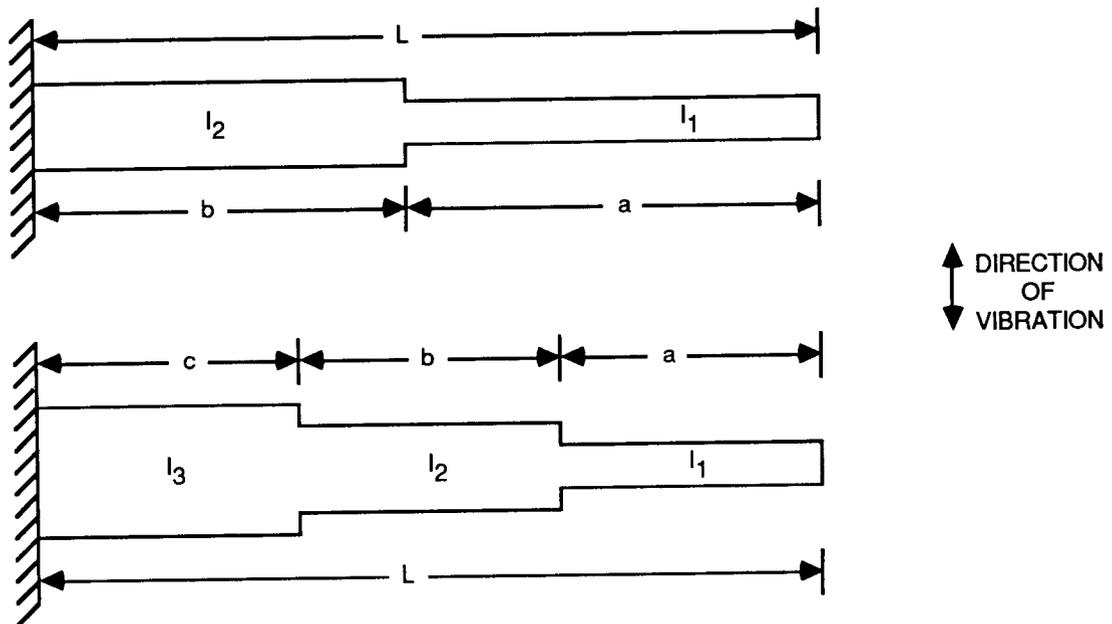


Fig. 3 Stepped Cantilever Beams With a) Two Different Cross Sections, and b) Three Different Cross Sections

In general, for a beam with n different cross sections, an approximate value for the average moment of inertia is

$$I_A = (1/L) \sum_{i=1}^n x_i I_i \text{ in}^4 \quad (6)$$

where  $x_i$  = spanwise length of cross section i, inch,  
 $I_i$  = moment of inertia of cross section i,  $\text{in}^4$ ,  
 $L = \sum x_i$  = full span of beam, inch.

Equations (4), (5) and (6) may be used in standard frequency formulas to obtain values of natural flexural frequencies of stepped beams. Equations (4) and (5) are only for cantilever beams and should provide accurate results for slender beams. Equation (6) may be used for any end support conditions, and will usually yield a natural frequency roughly 5% to 10% lower than the correct value.

#### Slender Right Angles and U Bends

Figure 4 shows a right angle and a U bend with intermediate supports. The ends, E, may have any combination of pinned, P, or clamped, C, boundary conditions. The intermediate supports, S, prevent transverse motion (perpendicular to the beam axis) at the support but allow the beam to move parallel to its own axis and to rotate about any axis. The fundamental natural frequency for vibration in the plane of the figures (in-plane vibration) is<sup>[2]</sup>

$$F = (\lambda/2\pi R^2) \cdot (EI_y g/W)^{1/2} \text{ Hz} \quad (7)$$

where  $\lambda$  = dimensionless frequency parameter in Table 1,  
 $R$  = radius of curvature shown in Figure 4, inches,  
 $E$  = modulus of elasticity of beam material,  $\text{lbf/in}^2$ ,  
 $I_y$  = area moment of inertia about axis perpendicular to the plane of the figures,  $\text{in}^4$ ,  
 $g$  = gravitational acceleration at surface of earth =  $386 \text{ in/sec}^2$ ,  
 $W$  = weight per unit length of beam,  $\text{lbf/in}$ .

The fundamental natural frequency for vibration perpendicular to the plane of the figures (out-of-plane vibration) is

$$F = (\lambda/2\pi R^2) \cdot (GI_p g/W)^{1/2} \text{ Hz} \quad (8)$$

where  $R$ ,  $E$ ,  $g$  and  $W$  are as defined for Equation (7),

and  $\lambda$  = dimensionless frequency parameter in Table 2,  
 $G$  = shear modulus =  $E/2(1+\nu)$   $\text{lbf/in}^2$ ,  
 $\nu$  = Poisson's ratio, dimensionless,  
 $I_p$  = polar area moment of inertia =  $I_x + I_y$ ,  $\text{in}^4$ ,  
 $I_x$  = area moment of inertia about axis in the plane of the figure and perpendicular to the local beam axis,  $\text{in}^4$ ,  
 $I_y$  is as defined for Equation 7.

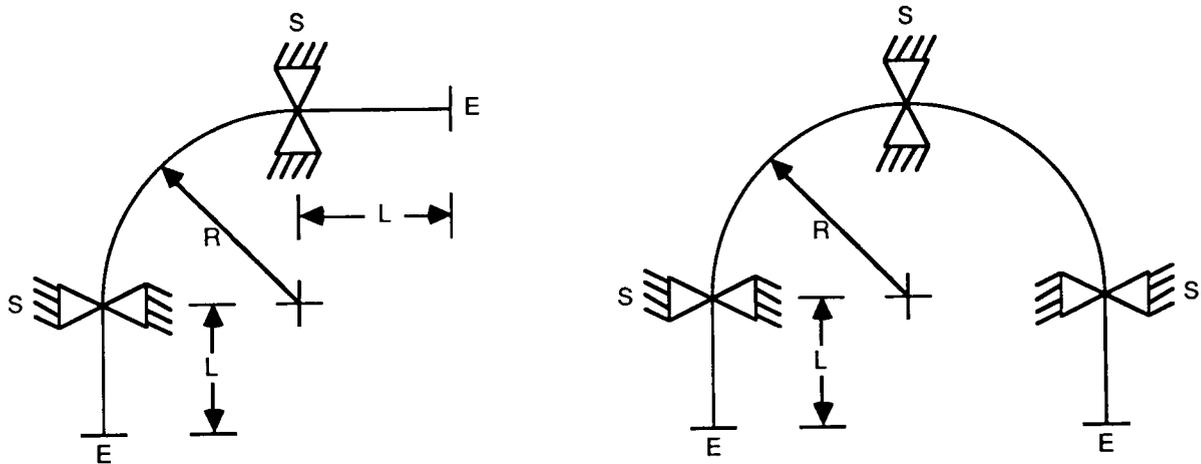


Fig. 4 Slender a) Right Angle Bend, and b) U Bend

Table 1 In-Plane Vibration of Right Angles and U Bends

$\lambda$ FOR USE IN EQUATION (7)						
L/R	RIGHT ANGLES (FIGURE 4a)			U BENDS (FIGURE 4b)		
	P-P	P-C	C-C	P-P	P-C	C-C
0	22.8	22.8	22.8	4.5	4.5	4.5
0.4	18.3	18.5	19.0	3.7	3.8	3.8
0.8	14.5	15.5	16.8	3.4	3.5	3.5
1.2	8.3	8.3	11.8	3.2	3.3	3.3
1.6	5.0	5.0	7.5	2.8	2.9	3.1
2.0	3.5	3.5	5.0	2.4	2.6	2.9

- NOTES: 1. L = length of legs, inch  
 2. R = radius of curvature, inch  
 3. P = pinned end condition  
 4. C = clamped end condition  
 5. Vibration in the plane of Figures 4a and 4b

Table 2 Out-of-Plane Vibration of Right Angles and U Bends

$\lambda$ FOR USE IN EQUATION (8)						
L/R	RIGHT ANGLES (FIGURE 4a)			U BENDS (FIGURE 4b)		
	P-P	P-C	C-C	P-P	P-C	C-C
0	9.5	9.5	9.5	5.8	5.9	5.9
0.4	7.5	7.6	7.8	5.1	5.3	5.3
0.8	6.0	6.4	6.8	4.7	4.8	4.9
1.2	5.0	5.4	5.9	4.3	4.4	4.6
1.6	3.5	4.1	5.0	3.6	3.8	4.2
2.0	2.6	3.0	3.8	2.7	2.9	3.6

- NOTES: 1. See notes 1-4 of Table 1  
 2. Vibration perpendicular to the plane of Figures 4a and 4b  
 3.  $\lambda$  values for  $\nu = 0.3$

Equations (7) and (8) do not take into account shear deformation, cross sectional distortion due to torsion, or coupling of rotation and displacement. The rotary inertia of the beam twisting about its own axis is not included in Equation (7) but is included in Equation (8). However, the values of  $\lambda$  given in Table 2 are only valid for circular beams or tubes with a value of  $\nu = 0.3$ .

Simple Frames

The simple frames shown in Figure 5 are also called portal frames in structural applications or bents in electronic applications. The following formulas provide approximate values for the fundamental natural frequencies in the specified vibration modes. [3]

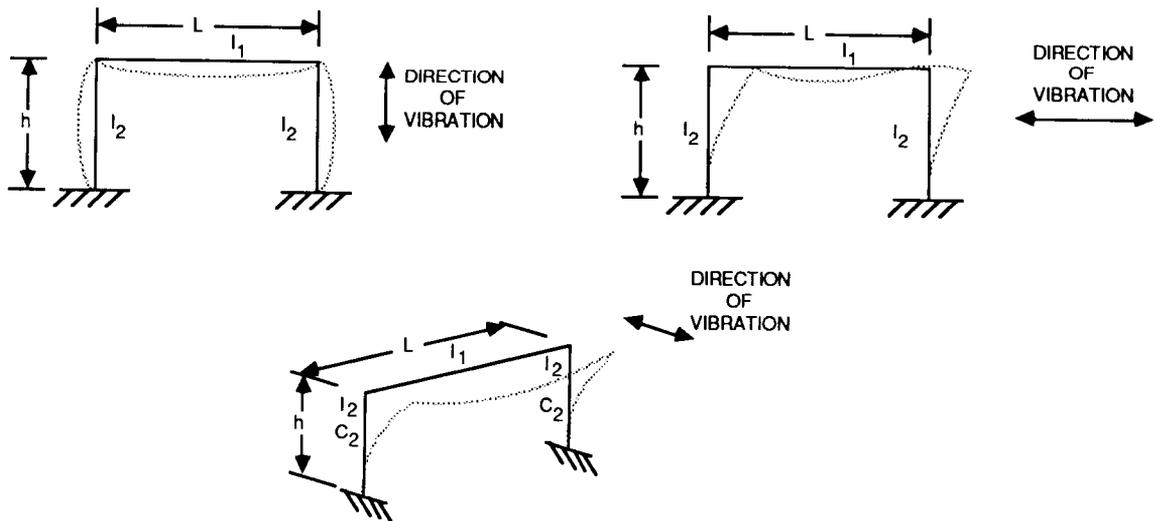


Fig. 5 A Simple Frame in a) In-Plane Vertical Vibration, b) In-Plane Lateral Vibration, and c) Out-of-Plane Transverse Vibration

For in-plane vertical vibration with legs hinged at the supports, Figure 5a,

$$F = (1/2\pi) \left\{ 48EI_1g/WL^3[1-2.25/(2K+3)] \right\}^{1/2} \text{ Hz} \quad (9)$$

where  $E$  = modulus of elasticity of frame material, lbf/in<sup>2</sup>,  
 $I_1$  = area moment of inertia of top of frame about neutral axis, in<sup>4</sup>,  
 $g$  = gravitational acceleration at surface of earth = 386 in/sec<sup>2</sup>,  
 $W$  = total weight of frame, lbf,  
 $L$  = length of top (span of frame), inch,  
 $K$  =  $hI_1/LI_2$ , dimensionless,  
 $h$  = height of frame (length of leg), inch,  
 $I_2$  = area moment of inertia of frame legs about neutral axis, in<sup>4</sup>.

For in-plane vertical vibration with legs fixed at the supports, Figure 5a,

$$F = (1/2\pi) \left\{ 48EI_1g/WL^3[1-3/(2K+4)] \right\}^{1/2} \text{ Hz} \quad (10)$$

For in-plane lateral vibration with legs fixed at the supports, Figure 5b,

$$F = (1/2\pi) \left\{ 24EI_2g/Wh^3[1+3/(6K+1)] \right\}^{1/2} \text{ Hz} \quad (11)$$

For out-of-plane transverse vibration with legs fixed at the supports, Figure 5c,

$$F = (g^{1/2}/2\pi) \left\{ (W/2)[L^3/24EI_1 + h^3/3EI_2 - L^4GC_2/32EI_1(2hEI_1+LGC_2)] \right\}^{-1/2} \text{ Hz} \quad (12)$$

where  $C_2$  = torsional constant, in<sup>4</sup>.

The approximate fundamental natural frequency for a rigid body of mass  $M_0$  supported by  $n$  slender, uniform legs of length  $L$ , all in the same plane, clamped at their feet and at the rigid body, as shown in Figure 6, for vibration in the plane of the legs, is given by<sup>[4]</sup>

$$F = (1/2\pi) [(12\sum E_i I_i)/L^3(M_0+0.37\sum M_i)]^{1/2} \text{ Hz} \quad (13)$$

where  $M_0$  = rigid body mass, lbs.mass,  
 $M_i$  = mass of leg  $i$ , lbs.mass,  
 $E_i$  = modulus of elasticity of leg  $i$ , lbf/in<sup>2</sup>,  
 $I_i$  = area moment of inertia of leg  $i$  about its neutral axis, in<sup>4</sup>,  
 $\sum$  = sum over all legs,  $i = 1, 2, 3, \dots, n$ ,  
 $n$  = number of legs  $\geq 2$ .

### Housings

Housings may be analyzed to estimate their fundamental natural frequencies. These frequencies may include flexural vibration along one or more axes of the structure, torsional vibration, and coupled modes of vibration. The frequencies will depend on the geometry and material properties of the

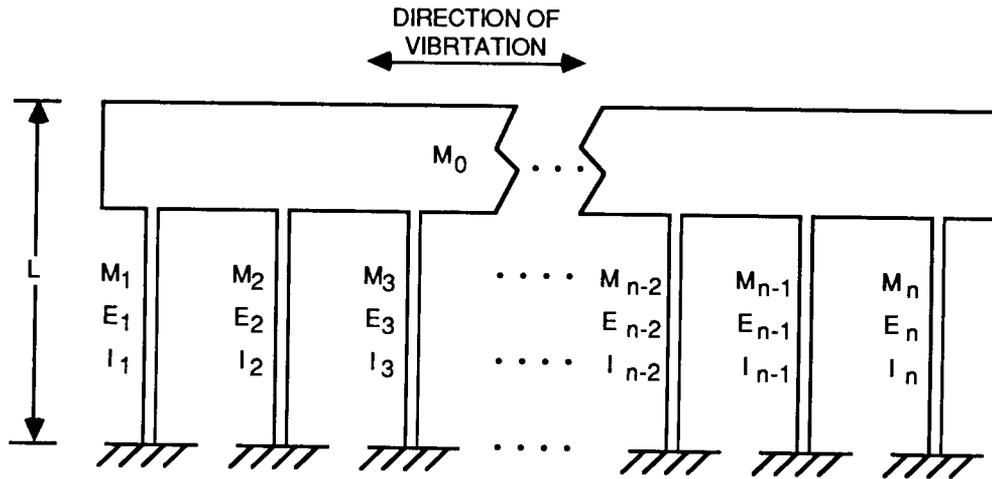


Fig. 6 Rigid Body on Slender Legs

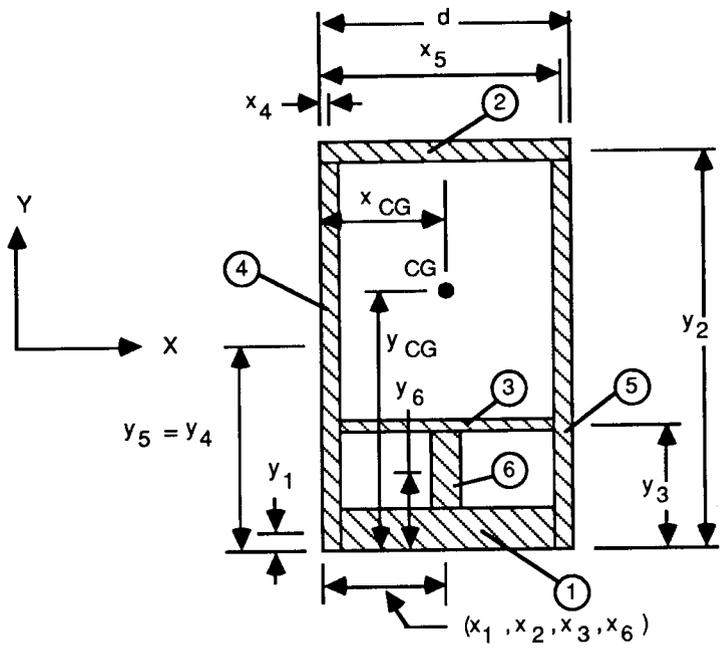
structure[5], the attachment efficiency factor between parts of the structure and the connection of the structure to mounting surfaces (the boundary conditions).

#### Flexure

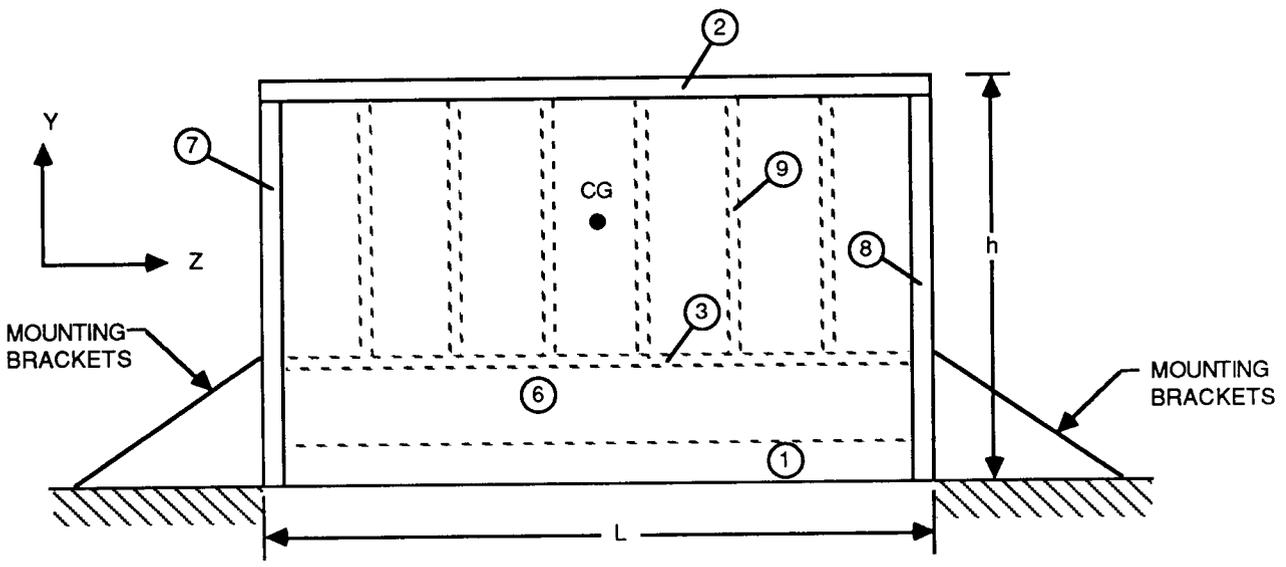
Figure 7 shows a housing composed of several structural elements. It is mounted by means of brackets attached at each end of the longer dimension, near the bottom (Figure 7b). It will be analyzed as a simply supported beam which can vibrate in flexure in the X direction and in the Y direction, and which can vibrate in torsion about the Z axis. Evaluation of the flexural frequencies requires an estimate of the stiffness factors of the structure,  $E_x I_x$  and  $E_y I_y$ . These are determined as follows.

$$E_x I_x = \sum_{i=1}^6 \eta_i E_i [A_i (\bar{X} - X_i)^2 + I_{x,i}] \quad \text{lb} \cdot \text{in}^2 \quad (14)$$

where  $\eta_i$  = attachment efficiency factor for element  $i$ , dimensionless,  
 $E_i$  = modulus of elasticity of element  $i$ ,  $\text{lb} \cdot \text{f} / \text{in}^2$ ,  
 $A_i$  = cross sectional area of element  $i$  in the  $x$ - $y$  plane,  $\text{in}^2$ ,  
 $X_i$  = distance from left edge of structure to neutral axis (or mid-point) of element  $i$ , inch,  
 $I_{x,i}$  = area moment of inertia of element  $i$  about the neutral axis parallel to the Y direction at  $X_i$ ,  $\text{in}^4$ ,



A) SIDE VIEW CROSS SECTION



B) FRONT VIEW

Fig. 7 Housing

and

$$\bar{X} = \frac{\sum_{i=1}^6 \eta_i A_i E_i X_i}{\sum_{i=1}^6 \eta_i A_i E_i} \text{ inch.} \quad (15)$$

$$E_y I_y = \sum_{i=1}^6 \eta_i E_i [A_i (\bar{Y} - Y_i)^2 + I_{y,i}] \text{ lbf.in}^2 \quad (16)$$

where  $\eta_i, E_i$  and  $A_i$  are defined following Equation (14),

and  $Y_i$  = distance from bottom of structure to neutral axis (or mid-point) of element  $i$ , inch,  
 $I_{y,i}$  = area moment of inertia of element  $i$  about the neutral axis parallel to the X axis at  $y_i$ , in<sup>4</sup>,

and

$$\bar{Y} = \frac{\sum_{i=1}^6 \eta_i A_i E_i Y_i}{\sum_{i=1}^6 \eta_i A_i E_i} \text{ inch} \quad (17)$$

Note that structural elements 7, 8 and 9 are not included in Equations (14) through (17) because they are not subjected to bending but are either fixed to the mounting brackets or displaced parallel to their own plane. The fundamental natural frequencies for flexural vibration in the X and Y directions are

$$F_x = (\pi/2L^2)(E_x I_x g/W)^{1/2} \text{ Hz} \quad (18)$$

and

$$F_y = (\pi/2L^2)(E_y I_y g/W)^{1/2} \text{ Hz}$$

where  $L$  = length of housing, Figure 7b, inches,  
 $g$  = acceleration of gravity at surface of earth = 386 in/sec<sup>2</sup>,  
 $W$  = weight of housing per unit length, lbf/inch (Total weight =  $WL$ )  
 $E_x I_x$  and  $E_y I_y$  are found from Equations (14) and (16).

If the ends of the housing, structural elements 7 and 8, were mated with and fixed to mounting surfaces, then the constant in Equation (18) would be  $(22.373/\pi)$  instead of  $\pi$ .

#### Torsion

Acceleration in the X direction will produce torsion as well as bending since the housing is supported near its bottom and the center of gravity (CG) is located above the support (Figure 7b). The axis of rotation for torsion will be at the bottom of the housing parallel to the Z axis. See Figure 8. Since the CG is not on the axis of rotation, the torsional natural frequency of the housing will be coupled to the flexural natural frequency,  $F_x$ .

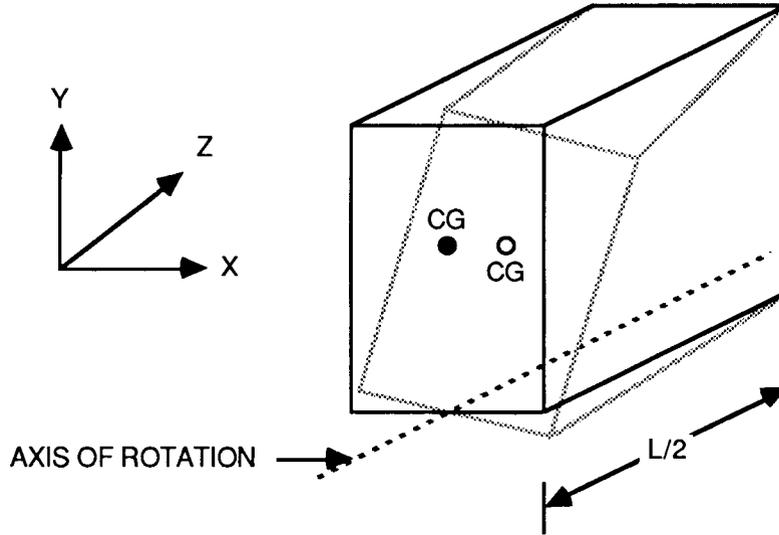


Fig. 8 Torsional Deflection, With One-Half of Housing Shown

The frequency formula for torsional vibration is

$$F_{\theta} = (\lambda/2\pi L) \cdot (CGg/\mu I_p)^{1/2} \text{ Hz} \quad (19)$$

where  $C$  = torsional constant of beam cross section,  $\text{in}^4$   
 $G$  = shear modulus of beam material,  $\text{lb/in}^2$   
 $\mu$  = weight density of beam material,  $\text{lb/in}^3$   
 $I_p$  = polar area moment of inertia of beam cross section about the beam axis of torsion,  $\text{in}^4$ .

The polar mass moment of inertia,  $J$ , about the axis of rotation is an important factor in the present case. The half-housing shown in Figure 8 may be analyzed with the frequency constant  $\lambda$  found by solving the transcendental equation which includes the parameter  $J$ :

$$\cot \lambda = (Jg/\mu LC)\lambda \quad (20)$$

For the half-housing,  $L$  must be replaced by  $L/2$  in Equations (19) and (20), and  $J$  must be the polar mass moment of inertia about the axis of rotation for the half-housing. The same results may be obtained by using the formula

$$F_{\theta} = (1/2\pi) \cdot (4CG/LJ)^{1/2} \text{ Hz} \quad (21)$$

instead of Equation (19) when  $(\mu L I_p/gJ)^{1/2} \ll 1$ . In Equation (21),  $L$  is the full length of the housing and  $J$  is the polar mass moment of inertia about the axis of rotation for the full housing.  $C$  and  $G$  have the same values as in Equations (19) and (20). Using the relation  $\mu = Mg/LA \text{ lb/in}^3$ , where  $M$  is the total mass of the housing and  $A$  is the housing cross sectional area in the X-Y plane, and equating Equations (19) and (21) yields:

$$I_p/A = \lambda^2 J/M \text{ in}^2 \quad (22)$$

where the radius of gyration is  $(I_p/A)^{1/2}$ . In arriving at Equation (22), L was replaced by L/2 in Equation (19).

The value of the polar area moment of inertia about the axis of rotation,  $I_p$ , may be estimated by

$$I_p = (E_x I_x + E_y I_y)/E_a \text{ in}^4 \quad (23)$$

where  $E_a$  is an appropriate average value for the structure. In the case where the structure is composed primarily of a single structural material,  $E_a$  is the modulus of elasticity for that material.

The value of J is given by

$$J = \sum_{i=1}^9 [J_{z,i} + m_i(x_i^2 + y_i^2)] \text{ lbmass.in}^2 \quad (24)$$

where  $J_{z,i}$  = polar mass moment of inertia about axis through the neutral axis of element i and parallel to the axis of rotation,  $\text{lbmass.in}^2$ ,

$m_i$  = mass of element i,  $\text{lbmass}$ ,  
 $x_i, y_i$  previously defined.

A more rapid but less accurate approximation for J is

$$J = (M/12)(4h^2 + d^2) \text{ lbmass.in}^2 \quad (25)$$

where M is the total mass of the housing. (See Figure 7.)

If J is calculated for the entire housing, then only half of its value must be used in Equation (20).

The torsional constant C is much more difficult to accurately estimate, even for a simple structure. The following rough approximation may be used:<sup>[6]</sup>

$$C \cong [1/2]I_p \text{ in}^4 \quad (26)$$

#### Coupled Modes

As pointed out previously, acceleration in the X direction produces both bending at a frequency  $F_x$  and torsion at a frequency  $F_\theta$ . These vibration modes will be coupled to produce a fundamental natural mode of the structure which can be approximated by Dunkerley's method<sup>[7]</sup>

$$F_c = (F_x^{-2} + F_\theta^{-2})^{-1/2} \text{ Hz} \quad (27)$$

where  $F_x$  and  $F_\theta$  are found from Equations (18) and (19). The coupling of modes of vibration always results in a natural frequency lower than the coupled frequencies, and a consequent increase in deflection and stress. In the example given in the preceding two sections, this coupling of modes may be

avoided by mounting the housing so that the center of gravity (CG) lies on the mounting plane. Torsional modes may still occur but they will not be coupled with the bending mode. Figure 9 is an example of a CG mount.

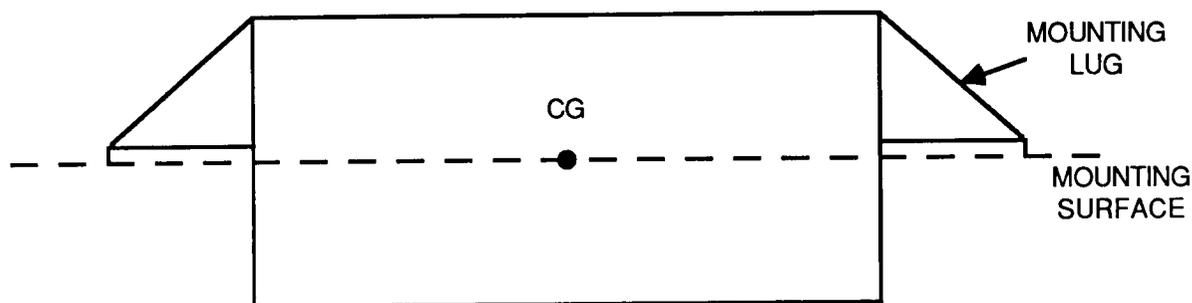


Fig. 9 A CG Mount

#### Other Housing Configurations

There are other housing geometries and mounting configurations where the housing may be modeled as a beam or a plate with boundary conditions which approximate the mounting attachments. In these cases the standard frequency formulas may be used to estimate the fundamental natural frequency of the housing. The flexural stiffness factors,  $EI$ , and the torsional frequency parameters  $C$ ,  $I_p$  and  $J$  must be estimated as in Section 5.3.

The examples shown in Figure 10 could be analyzed as follows.

Figure 10a.

- 1) For  $L > h, t$ : A beam with simply supported ends.  
 Flexural vibration in X and Z directions.  
 Torsional vibration about the axis through centroid, parallel to Y axis.  
 Flexural and torsional modes not coupled.
- 2) For  $L \cong h > t$ : A plate with two opposite sides simply supported, other two sides free.  
 Flexural vibration in Z direction.

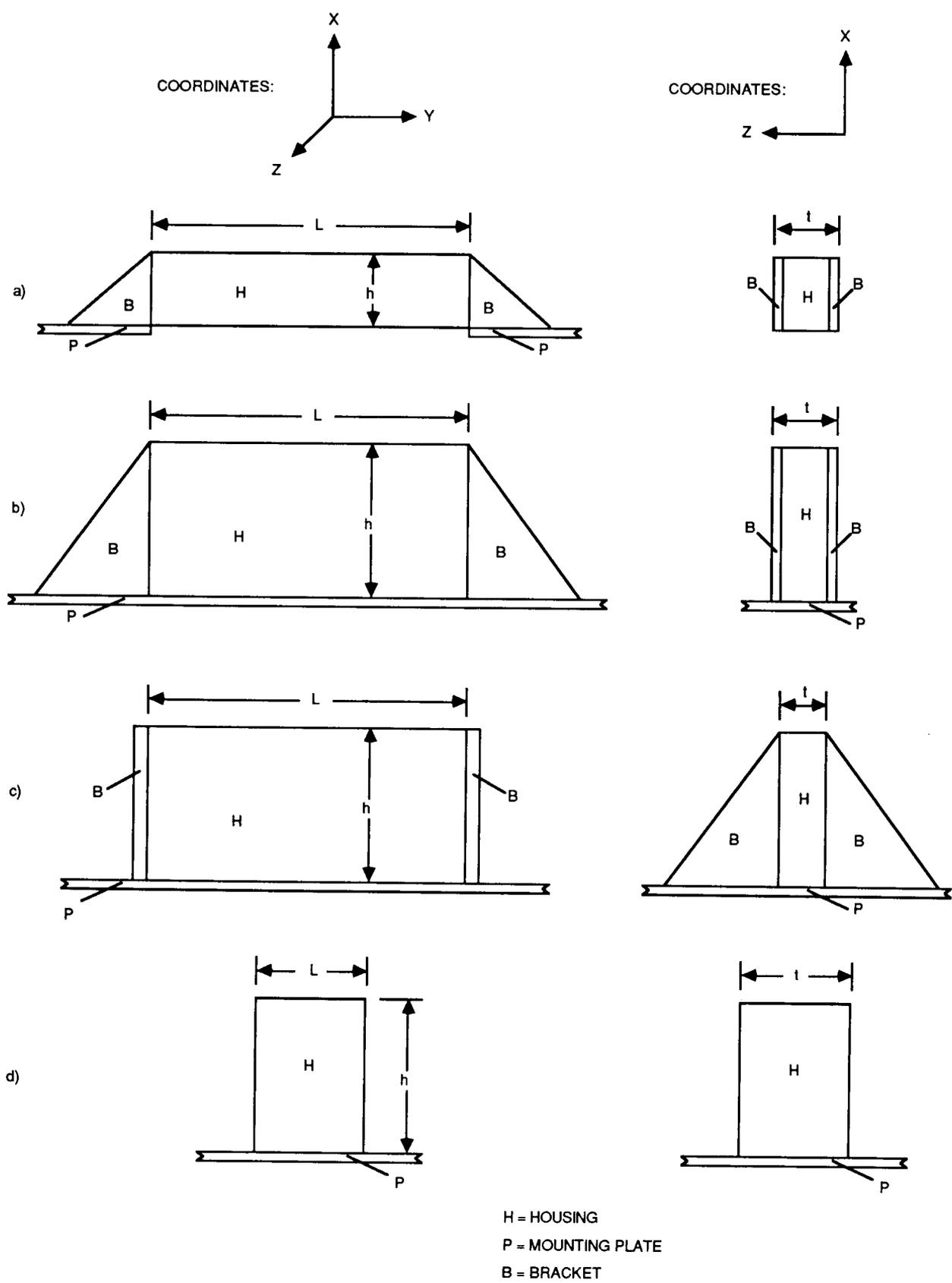


Fig. 10 Examples of Housing and Mounting Configurations

- 3) For  $L \cong t > h$ : A plate with four corner supports.  
Flexural vibration in X direction.

Figure 10b.

- For  $L \cong h > t$ : A plate with three simply supported sides.  
Flexural vibration in Z direction.

Figure 10c. Same as 10b.

Figure 10d.

- 1) For  $h > L, t$ : A cantilever beam.  
Flexural vibration in Y and Z directions.  
Torsional vibration about the axis through centroid, parallel to X axis.  
Flexural and torsional modes coupled if CG is not at the mid-point of the L and the t dimensions.
  
- 2) For  $L \cong h > t$ : A plate simply supported on one side with other three sides free.  
Flexural vibration in Z direction.  
Torsional vibration about the axis through centroid parallel to X axis.  
Flexural and torsional modes coupled if CG is not at the mid-point of the L dimension.

#### Lumped Elements

One type of model that may be used to represent structures is the lumped element model. In this approach, parts of the structures are treated as masses and other parts as springs. Rigid, heavy components may be treated as masses, while flexible, light weight components may be treated as springs. The spring elements may be some of the same structural components that make up the masses, even though they are treated as massless in the analysis. The combined weight of the masses must add up to the total weight of the complete structure.

Figure 11 shows a three degree-of-freedom structure composed of a transformer mounted on a bracket which is attached to a PWB mounted in a housing. The bottom of the housing is fixed to a mounting plate. When the acceleration is parallel to the mounting plate and normal to the PWB, the housing will vibrate as a cantilever, the PWB will vibrate as a loaded plate, and the bracket will vibrate in the direction shown. Figure 12 is a lumped element model of the structure shown in Figure 11. The values of the spring constants, K, may be determined by use of the following equation:

$$K = (W/g) \cdot (2\pi F)^2 \text{ lbf/inch} \quad (28)$$

where  $W$  = weight of element, lbf,  
 $g$  = acceleration of gravity at surface of earth = 386 in/sec<sup>2</sup>,  
 $F$  = fundamental natural frequency of the element, Hz.

The frequency formulas for finding the values of  $F$  in Equation (28) are given by Equation (9) for the bracket and by standard formulas for the PWB and the cantilever housing. The static deflections, for a one-g acceleration, are

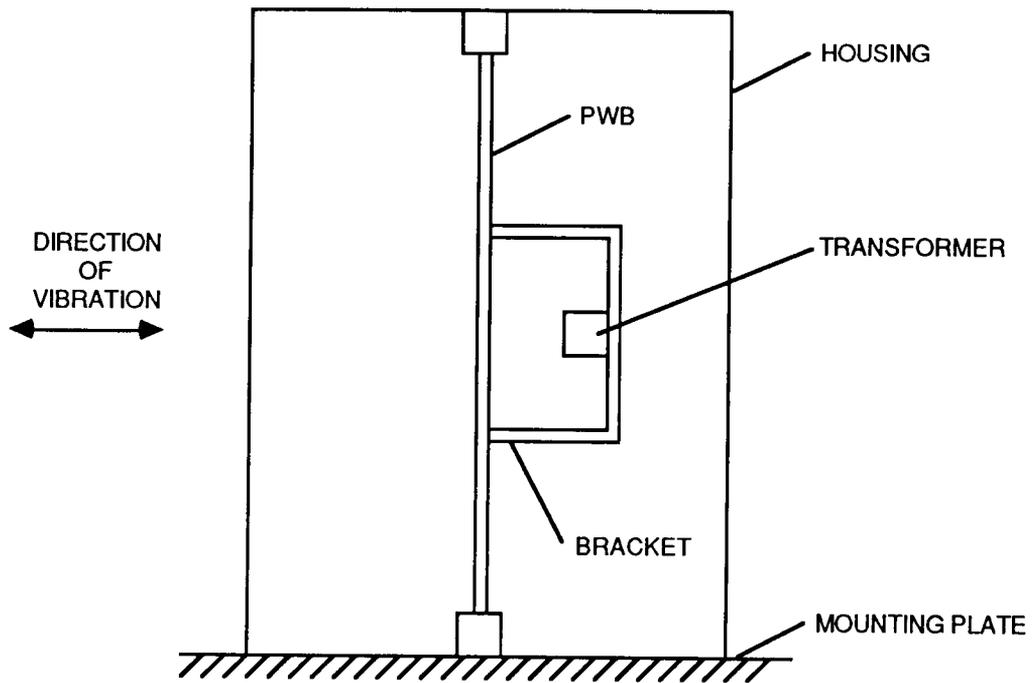


Fig. 11 Example of Three Degree-of-Freedom Structure

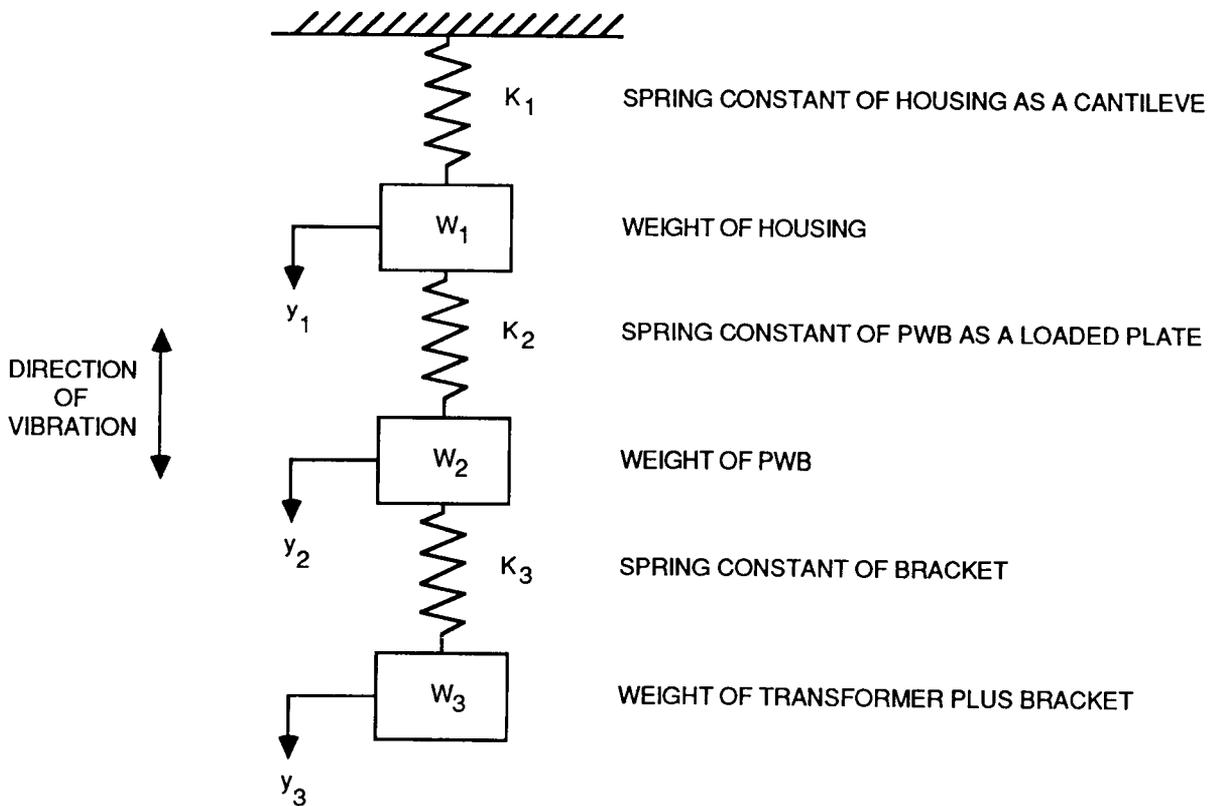


Fig. 12 Lumped Element Model of Three Degree-of-Freedom Structure

$$y_1 = (W_1+W_2+W_3)/K_1 \text{ inch} \quad (29)$$

$$y_2 = y_1+(W_2+W_3)/K_2 \text{ inch}$$

$$y_3 = y_1+y_2+W_3/K_3 \text{ inch}$$

The fundamental natural frequency of the structure shown in Figures 11 and 12 is [8]

$$F = (1/2\pi) \left[ g \frac{\sum_{i=1}^3 W_i y_i}{\sum_{i=1}^3 W_i y_i^2} \right]^{1/2} \text{ Hz} \quad (30)$$

For a structure with n degrees of freedom, which can be represented by n spring/mass elements in series,

$$y_i = \sum_{j=1}^{i-1} y_j + \sum_{j=i}^n W_j / K_i \text{ inch} \quad (31)$$

and

$$F = (1/2\pi) \left[ g \frac{\sum_{i=1}^n W_i y_i}{\sum_{i=1}^n W_i y_i^2} \right]^{1/2} \text{ Hz} \quad (32)$$

#### Accelerations and Stresses

The natural frequency of a structure will determine what will be its acceleration response in a shock or vibration environment. Two different structural designs with two different natural frequencies will generally experience different accelerations, resulting in different stresses when exposed to the same shock or random vibration spectrum. The estimates of natural frequency allow comparison of structural designs in terms of dynamic loads due to the specified acceleration environments.

Figure 13 shows the shock acceleration response spectrum for an NSI ordinance, one-inch separation nut. The acceleration response is the three-sigma peak value (exceeded only 0.28% of the time) in units of the gravitational acceleration at the earth's surface, and it peaks at 15,000g and 10,000Hz. A structure with a natural frequency of 200Hz will experience a 215g shock, while one with a natural frequency of 250Hz will experience a 310g shock. The acceleration ratio is  $310/215 = 1.44$ .

Figure 14 shows a typical random vibration power spectral density (PSD) to which a space structure would be exposed during launch. The three-sigma peak acceleration response is approximated by

$$G_{pk} = 3 \left[ (\pi/2) \cdot \text{PSD} \cdot Q_0 \cdot F_0 \right]^{1/2} g \quad (33)$$

where  $Q_0$  = transmissibility at the structure's natural frequency,  $F_0$ . Assuming the same value of  $Q_0$  for the two structures with natural frequencies of 200Hz and 250Hz, the acceleration ratio will be 1.12.

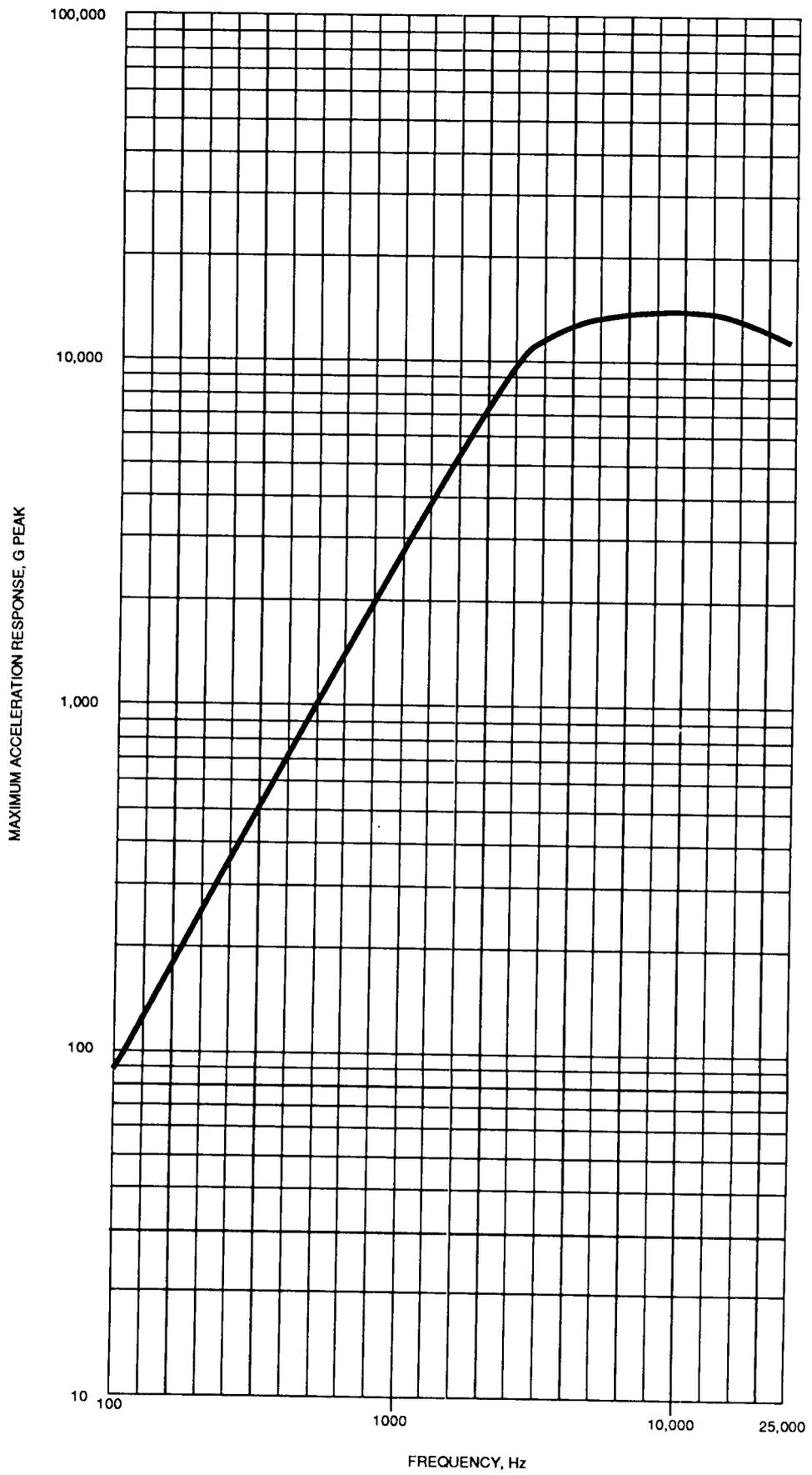


Fig. 13 Separation Nut Source Shock Spectrum (Reference 9)

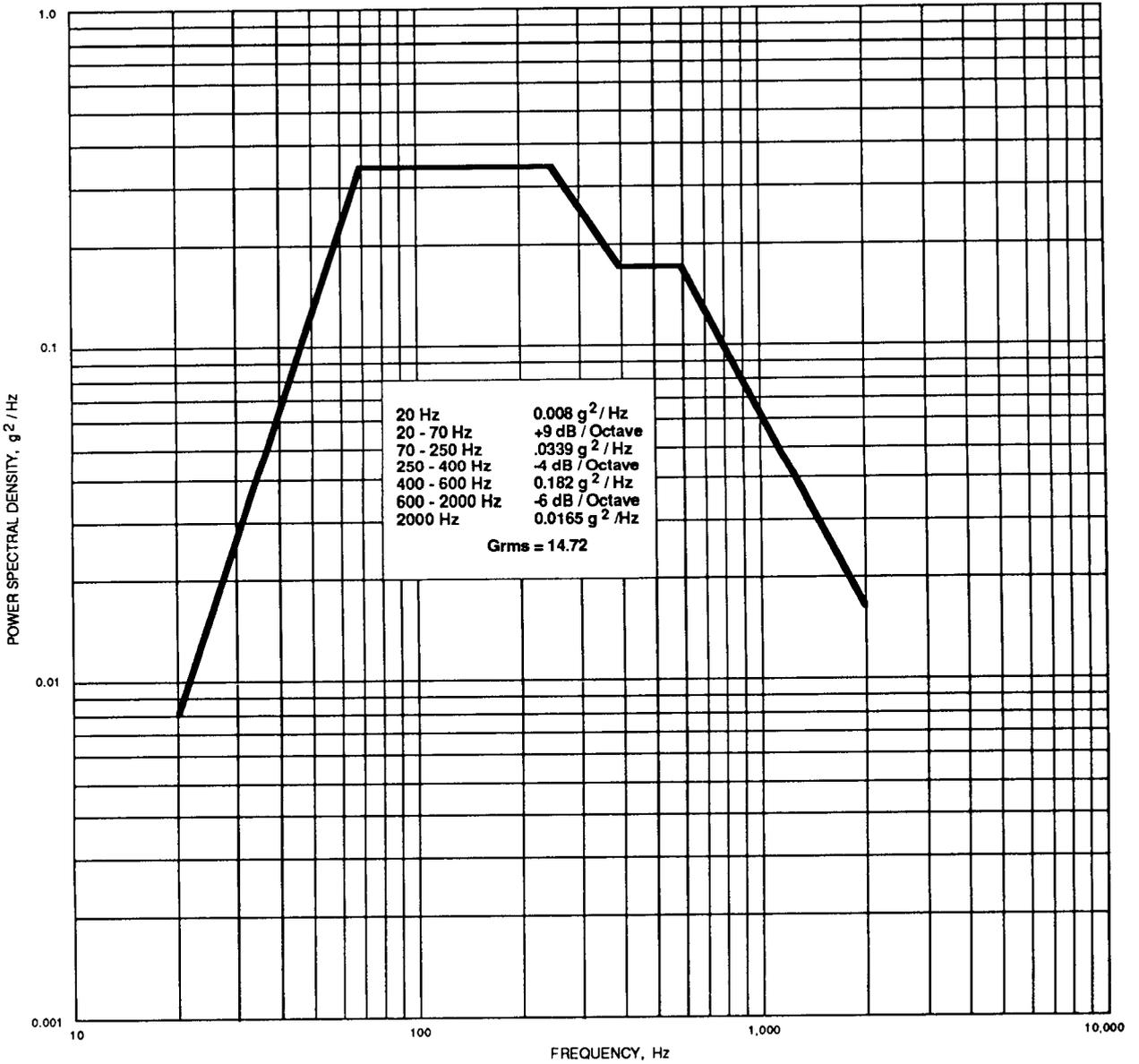


Fig. 14 A Typical Random Vibration Power Spectral Density (PSD)

In the above example, it is seen that the accelerations, and consequently the dynamic loads, between different structural designs may be compared when estimates of natural frequency can be made.

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