Passively damped joints, in which the conventional adhesives are replaced by high damping viscoelastic materials, have the potential of being effective practical means for passive vibration control. However, this potential cannot be materialized unless the associated structural penalties are reduced to acceptable limits. The paper describes a rational methodology for the development of advanced joining concepts for structural and mechanical systems, capable of providing enhanced dissipation of vibrational energy without serious penalties in strength, stiffness or weight characteristics. One such configuration is that of a rhombic-type joint, that provides a beneficial deformation coupling between the direction of load transfer and less critical offset directions. A comprehensive parametric study has been carried out in order to establish design guidelines for favorable tradeoffs between damping benefits and the associated stiffness, strength and weight penalties in a rhombic joint. The results are compared with the corresponding tradeoffs for a double-lap joint made of the same materials.

INTRODUCTION

Artificial damping devices are the most powerful means of energy dissipation available to the designer for passive vibration control. They may include damping layers applied over large areas or local dampers attached to problem components. Layer treatments usually rely upon material damping mechanisms [1], whereas common types of local damping treatments are dynamic absorbers, dashpots [2], inertial [3], friction [4], tuned and broad-band viscoelastic [3] dampers. Although the dominant contribution of joints and supports to structural damping is well recognized, past investigations have been focused on friction associated with interfacial slip [5]. Despite an early analysis of the damping benefits achievable by incorporating viscoelastic materials in structural supports of beams and plates [6], no systematic research efforts have pursued this approach so far. Elastomeric materials, like rubber or synthetic rubberlike products are occasionally used in mechanical couplings or bridge bearings to allow higher flexibility of such connections, along with a certain reduction in vibration levels. Only limited applications of elastomeric bearings for vibration control can be found so far, mainly in base-isolation systems for earthquake protection of buildings and bridges [7].
Renewed interest in utilizing viscoelastic materials in structural joints has been spawned recently by technological developments that have increased the need for effective vibration control on one hand, and have improved the engineering properties of viscoelastic materials on the other hand. Both the damping and the general structural analysis of viscoelastic materials have been placed on a sound mathematical basis, especially in regard with the linear viscoelastic behavior. The most popular modeling approaches are based on the complex modulus concept [8] and numerical algorithms either in the transform or time domains [9]. General analysis methods of viscoelastic damping have been proposed recently by using a "fractional calculus" [10] or an integro-differential formulation of the equations of motion.

Passively damped joints, in which high damping viscoelastic materials are incorporated, usually as a replacement to conventional adhesives, have the potential to be an effective means of passive vibration control over a broad frequency band. This potential is indicated by recent theoretical and experimental investigations of passively damped joints based on the double [11,12] or single [13] lap configurations. It cannot materialize, however, unless a "designed-in" approach is adopted for the development of joint configurations that provide favorable tradeoffs between damping enhancement and associated structural penalties. Passively damped lap joints dissipate mechanical energy when worked in the axial direction, due to shear deformation of the viscoelastic layers. Consequently, their damping properties in the 0.1-100 Hz frequency range can be as much as one order of magnitude higher than those of similar joints with conventional elastic adhesives [11]. However, the associated penalty in axial stiffness is about 80%, even if the designed-in approach is adopted. The addition of elastic connection elements between the members of the joint can reduce the stiffness penalty to only 60%, but then the damping enhancement may drop to less than one-half of that achievable without the elastic links. It is difficult to attain favorable tradeoffs between damping and stiffness in passively damped lap joints since high damping requires high shear deformation of the adhesive layers which, in turn, requires large relative displacements between the adherends along the axial direction. Moreover, the lap configuration implies also an adverse relationship between the damping and strength characteristics of the joint since the shear deformation of the viscoelastic layers is part of the load transfer process through the joint.

This paper presents a theoretical performance analysis of a different configuration for passively damped joints, that can possibly be used as an alternative to the conventional double-lap configuration. It is based on a rhomb-like geometry where the viscoelastic adhesive is enclosed by the elastic joint members in an arrangement as shown in Fig. 1. The tradeoffs achievable with such a configuration between damping benefits and associated structural penalties may be superior to those of double-lap joints since large shear deformation of the adhesive can be obtained by displacement coupling between the x and y directions (Fig. 1). The paper includes numerical results from parametric studies of a rhombic joint configuration and a comparative evaluation with a double lap passively damped joint (Fig. 2).
FIG. 1 - PHYSICAL MODEL OF RHOMBIC JOINT

FIG. 2 - PHYSICAL MODEL OF DOUBLE-LAP JOINT
ANALYSIS METHOD

The major objective of this section is to describe a rational methodology for predicting the effects of structural interactions between various constituents of a passively damped rhombic joint on some of its performance characteristics like weight, damping, strength and stiffness in the load transfer direction. The model employed for this purpose is focused, therefore, on elastic stress analysis rather than the viscoelastic behavior of the adhesive. Besides elastic strength and stiffness, it predicts only the relative energy dissipation with respect to variations in geometric parameters and stiffness ratio between the adhesive and the adherend. The adhesive loss factor is not included explicitly in the model, but it is not expected to change the predicted effects of the above design parameters on the overall joint damping. For a given viscoelastic material, it may be regarded as a factor independent of the joint configuration that determines the actual level of dissipated energy when multiplied by the corresponding values of relative energy dissipation provided by this analysis. A similar approach has been employed in Ref. [14] for a double lap joint and has been validated, subsequently, by comparison with an equivalent viscoelastic investigation [12].

The underlying assumptions of such an approach are that the joint is subjected to oscillatory loading and the viscoelastic adhesive behaves similarly to a Voigt solid at any one frequency, so that its constitutive description can rely on the complex modulus concept [8]:

$$G_a = G_1 (1 + i\eta)$$  \hspace{1cm} (1)

If inertia effects are ignored, these assumptions lead to a quasi-static analysis, in which the material properties may change from one frequency [11] or temperature [15] to an other, but the form of the governing elasticity equations remains unchanged. Effectiveness investigations of constrained layer damping treatments are commonly confined to quasi-static models, which are sufficiently accurate for low frequency vibrations [1].

In accordance with the above assumptions, the total mechanical energy dissipated per cycle by the viscoelastic adhesive can be evaluated as follows [1]

$$D = \pi G_1 \eta \int_{V_a} |\gamma|^2 \, dV_a$$  \hspace{1cm} (2)

If the adhesive is assumed to be the only source of energy dissipation in a passively damped joint, the parameter $D$ is a direct measure of the overall joint damping. Eq. (2) shows that for a given viscoelastic material, at given frequency and temperature conditions, the damping characteristics are determined by the magnitude of shear strains induced in the adhesive. This observation is consistent with the well known finding that the major mechanism of viscoelastic energy dissipation is cyclic shear, rather than extensional, deformation [16].

The above discussion supports the approach of employing a fully elastic model in order to evaluate the effect of structural interactions between the constituents of a passively damped joint on the amount of energy dissipated by its viscoelastic adhesive. For a certain loss factor of the viscoelastic material, the joint damping may be considered to be proportional to the elastic distortional (octahedral) energy.
stored in the adhesive during a loading cycle. If the plane stress assumption is adopted for the stress analysis of a rhombic joint, this octahedral energy is expressed as [17]:

\[ U_d = \frac{1}{12\varepsilon_1} \int_{V_a} \left[ (\sigma_{xx} - \sigma_{yy})^2 + \sigma_{yy}^2 + \sigma_{xx}^2 + 6\sigma_{xy}^2 \right] \, dV \]  (3)

where the stress components in Eq. (3) correspond to the amplitude value of the external oscillatory load on the joint. The damping assessment in the present work relies, therefore, on the approximation

\[ D \approx C \cdot U_d \]  (4)

where "C" is a proportionality constant that depends on the loss factor of the viscoelastic adhesive.

A plane stress quasi-static analysis has been conducted on a rhombic joint configuration by using an "in-house" boundary element program. Consequently, only the rhombic frame formed by the elastic members of the joint has to be discretized, whereas a continuum solution for the stress and deformation fields in the adhesive can be obtained from the integral equations of the boundary element method [18]. Displacement continuity conditions are imposed in all directions along the adhesive-adherend interfaces, so that no debonding and microslip effects are included in this investigation. The boundary element program had been previously validated by application to several test cases.

The stress and deformation results provided by this program for the loading case shown in Fig. 1 have been utilized to predict the following three performance characteristics of a rhombic joint:

1. Damping - by calculating \( U_d \) from Eq. (3).

2. Stiffness - by calculating the ratio between the applied load and the corresponding elastic deformation.

\[ K = \frac{P}{u_y} \]  (A)

3. Strength - by calculating the minimum value of \( P \) at which shear-induced debonding may occur at the adhesive-adherend interface.

Each of the above properties, along with weight characteristics, have been evaluated over a broad range of design parameters in an effort to identify design configurations that yield favorable tradeoffs between damping benefits on one hand and stiffness, strength and weight penalties on the other hand. Selected numerical results are presented in the following section and compared with those corresponding to a double lap joint.

NUMERICAL RESULTS

An extensive parametric study has been conducted on the joint model shown in Fig. 1, by following the procedure outlined in the previous section. Its major results are depicted, in non-dimensional form, in Figs. 3-5. Constant values have been selected in all these figures for the following design parameters:
Dimensions: \( a = 3.0 \text{ in.}, \ t = 0.25 \text{ in.} \)

Material Properties: \( E_m = 10^7 \text{ psi}, \ \nu_m = 0.3, \ \nu_a = 0.45 \)
\[ \mu_m = 0.1 \text{ lb/in}^3, \ \mu_a = 0.036 \text{ in/in}^3 \]

The normalizing factors used for the selected performance characteristics are listed below:

1. **Damping** - the total strain energy, \( U \), stored in the joint for the corresponding design configuration.

2. **Stiffness** - an arbitrary value of 1,250,000 lb/in, that may correspond to the extensional stiffness of a 6 x 3 x 0.25 in. aluminum prismatic bar.

3. **Strength** - an arbitrary value of 30,000 lb for \( P \), that may correspond to a shear bond strength of 2,000 psi and a uniform shear stress distribution over a bond area of 15 in\(^2\).

4. **Weight** - an arbitrary value of 0.225 lb, that may correspond to the weight of a 6 x 1.5 x 0.25 prismatic bar made of aluminum.

All the calculations covering the parameter ranges shown in Figs. 3-5 have revealed that the octahedral strain energy, \( U_d \), is an approximately constant percentage, of about 90%, of the total strain energy stored in the adhesive of a rhombic joint. The damping performance of the joint, which is measured by \( U_d \) in this analysis, is determined, therefore, by the distribution of strain energy between the adhesive and the adherends.

The effect of the stiffness level of the adhesive on the joint damping and stiffness characteristics, is illustrated in Fig. 3 for \( \theta = 20^\circ \) and \( c/a = 0.1 \). This effect is much more pronounced on damping than on stiffness, especially for values of \( G_a/E_m \) below 0.001, which may represent most viscoelastic adhesives available today [11]. It indicates the importance of selecting stiffer adhesives not only to improve the stiffness of the joint, but also its damping by increasing the strain energy share of the adhesive.

Figure 4 shows the effect of the geometric parameter \( c/a \) on all the performance characteristics selected for this investigation, for \( \theta = 20^\circ \) and \( G_a/E_m = 0.005 \). This parameter does not appear to have a significant effect on the strength of the joint, but it has opposite effects on damping and weight on one hand and stiffness on the other hand. A light weight slender member, that may correspond to \( c/a = 0.1 \) for example, will provide higher damping but lower stiffness, whereas a "bulker" member, that may correspond to \( c/a = 0.2 \), can provide a better tradeoff between damping and stiffness, but at the expense of higher weight.

The opening angle of the rhomb, \( \theta \), has a significant effect on the joint properties, as depicted in Fig. 5 for \( c/a = 0.1 \) and \( G_a/E_m = 0.005 \). While damping considerations would demand for a value of \( \theta \) about 30-50 degrees, this range should be avoided from the strength and stiffness standpoints. A small value of \( \theta \), about 10-20 degrees, appears to be preferable for overall performance optimization of such a joint configuration.
FIG. 3 - RELATIVE PERFORMANCE CHARACTERISTICS OF RHOMBIC JOINT AS FUNCTIONS OF MATERIAL MODULI OF ITS CONSTITUENTS

FIG. 4 - EFFECT OF MEMBER DIMENSIONS ON RELATIVE PERFORMANCE CHARACTERISTICS OF RHOMBIC JOINTS
The above performance characteristics of a rhombic joint have been compared with the corresponding properties of a passively damped joint of double lap configuration, whose physical model is shown in Fig. 2. A plane strain finite element analysis has been conducted on the double lap joint in order to predict its damping, stiffness and strength characteristics, by following the same procedure as for the rhombic joint. The results of this comparison are summarized, in non-dimensional form, in Figs. 6-8. The same constant parameters and normalizing factors used for Figs. 3-5 have been retained in Figs. 6-8, both for the rhombic and double lap configurations. An adhesive layer thickness of 0.02 in. has been assumed for the double lap joint.

Figure 6 illustrates the stiffness penalties of the two joint configurations as functions of the adhesive stiffness, for $c/a = 0.2$ and $\theta = 20^\circ$. The "stiffness penalty" parameter is defined as follows:

$$ SP = \frac{K - K_o}{K_o} $$

(6)

where the reference stiffness $K_o$ is selected as the joint stiffness in the particular case when $G_a/E_m = 0.5$, both for the rhombic and double lap configurations. The rhombic joint is evidently superior to the double lap in the small values range of the $G_a/E_m$ ratio, that covers most adhesives available today, but this conclusion is reversed in the less practical case of $G_a/E_m > 0.0005$.

A similar comparison between the two joint configurations is shown in Fig. 7 in regard with damping. The "damping benefit" parameter displayed in this figure is defined as follows:

$$ DB = \frac{U_d - U_{do}}{U_{do}} $$

(7)

where $U_{do}$ is the octahedral strain energy in the adhesive in the particular case when $G_a/E_m = 0.5$, both for the rhombic and double lap configurations. The use of soft adhesives is desirable in double lap joints from the damping standpoint, but not in rhombic joints where the associated damping benefit is less significant. Stiff adhesives appear to provide better damping performance in rhombic, rather than double lap, joints.

Strenght and weight comparisons between rhombic and double lap joints are illustrated in Fig. 8 for $G_a/E_m = 0.005$ and $\theta = 20^\circ$. The predicted strength of the rhombic joint is about one order of magnitude higher than that of the double lap joint. This indicates the practical potential of achieving significant improvements in the load transfer efficiency through structural joints by using rhombic configurations, which are not associated with high free-edge stress concentrations. Because of their different geometry, the weight of a double lap joint appears to be more sensitive than the weight of a rhombic joint to variations of the $(c/a)$ parameter.

**CONCLUSIONS**

Passively damped joints based on a rhombic configuration have the potential of providing better tradeoffs between their damping, stiffness, strength and weight characteristics than the conventional double lap configuration. The strength properties, in particular, appear to be significantly higher, which indicates the
FIG. 5 - EFFECT OF OPENING ANGLE ON RELATIVE PERFORMANCE CHARACTERISTICS OF RHOMBIC JOINTS

FIG. 6 - COMPARISON BETWEEN STIFFNESS PENALTIES FOR RHOMBIC AND DOUBLE-LAP JOINT CONFIGURATIONS
FIG. 7 - COMPARISON BETWEEN DAMPING BENEFITS FOR RHOMBIC AND DOUBLE-LAP JOINT CONFIGURATIONS

FIG. 8 - STRENGTH AND WEIGHT COMPARISONS BETWEEN RHOMBIC AND DOUBLE-LAP JOINT CONFIGURATIONS
possibility of enhancing load transfer efficiencies by employing rhombic joints. Proper matching between the stiffness levels of the adhesive and the adherend appears to be a dominant factor in establishing the resultant damping-stiffness tradeoff of the joint.

This paper illustrates the importance of modeling and understanding structural interactions among the various components of passively damped joints. It presents a simple, but useful methodology for predicting overall performance parameters of such joints and conducting a systematic quantitative analysis for their enhancement. Although this analysis procedure does not include explicitly the viscoelastic behavior of the adhesive, it is an expedient tool for preliminary design studies of passively damped joints.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a</td>
<td>Length of joint member (Figs. 1 and 2)</td>
</tr>
<tr>
<td>c</td>
<td>Width of joint member (Figs. 1 and 2)</td>
</tr>
<tr>
<td>D</td>
<td>Energy dissipated per cycle</td>
</tr>
<tr>
<td>E_m</td>
<td>Young modulus of joint member</td>
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<tr>
<td>G_a</td>
<td>Adhesive shear modulus</td>
</tr>
<tr>
<td>G_l</td>
<td>Storage shear modulus of adhesive</td>
</tr>
<tr>
<td>i</td>
<td>Imaginary unit (i = \sqrt{-1})</td>
</tr>
<tr>
<td>K</td>
<td>Joint stiffness along loading direction</td>
</tr>
<tr>
<td>P</td>
<td>Amplitude of external oscillatory load</td>
</tr>
<tr>
<td>t</td>
<td>Thickness of joint member (Figs. 1 and 2)</td>
</tr>
<tr>
<td>u_y(A)</td>
<td>Displacement of point A (Fig. 1) along y-direction</td>
</tr>
<tr>
<td>U_d</td>
<td>Distortional strain energy in adhesive</td>
</tr>
<tr>
<td>U</td>
<td>Total strain energy in the joint</td>
</tr>
<tr>
<td>V_a</td>
<td>Volume of adhesive material</td>
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<tr>
<td>\gamma</td>
<td>Shear strain in adhesive</td>
</tr>
<tr>
<td>\eta</td>
<td>Adhesive loss factor</td>
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<tr>
<td>\nu_m, \nu_a</td>
<td>Poisson ratio of member and adhesive, respectively</td>
</tr>
<tr>
<td>\rho_m, \rho_a</td>
<td>Mass density of member and adhesive, respectively</td>
</tr>
<tr>
<td>\theta</td>
<td>Opening angle of rhombic joint (Fig. 1)</td>
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<tr>
<td>\sigma_{xx}, \sigma_{yy}, \sigma_{xy}</td>
<td>Cartesian stress components</td>
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REFERENCES


