SOUND PROPAGATION OVER UNEVEN GROUND
AND IRREGULAR TOPOGRAPHY

Semiannual Report, August 1987 - January 1988

by

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Submitted to
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665

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January, 1988
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INTRODUCTION

The goal of this research is to develop theoretical, computational, and experimental techniques for predicting the effects of irregular topography on long range sound propagation in the atmosphere. Irregular topography here is understood to imply a ground surface that (1) is not idealizable as being perfectly flat or (2) that is not idealizable as having a constant specific acoustic impedance. The interest of this study focuses on circumstances where the propagation is similar to what might be expected for noise from low-altitude air vehicles flying over suburban or rural terrain, such that rays from the source arrive at angles close to grazing incidence.

The objectives of the project, the experimental facility, and the early progress up through August 1987 have been described in the five previous semiannual reports [1,2,3,4,5]. The present report discusses those activities and developments that have resulted during the period, August 1987 through January 1988.

PERSONNEL

In addition to A. D. Pierce, Yves H. Berthelot, and G. L. Main, (the three co-principal investigators on the project), a visiting scholar, Professor Ji-xun Zhou of the Acoustics Institute of the Academy of Sciences of China (Beijing), has been working on the project since May 1987. Last fall, the graduate student working on the project, James A. Kearns, successfully defended his thesis proposal (a requirement of the Graduate School of Georgia Tech). During the past reporting period, Dr. Berthelot, Professor Zhou, and James Kearns have been mainly concerned with the experimental phases of the project, while Allan Pierce and Dr. Main have been working primarily on the theoretical aspects.

Allan Pierce and Yves Berthelot visited NASA Langley Research Center on February 23, 1987, and discussed complementary NASA and Georgia Tech research activities with the NASA technical officer, Dr. John Preisser, with William Willshire, and with their colleagues. These discussions resulted in an extension of the original grant for a period of one more year (February 1988-January 1989).
EXPERIMENTAL VALIDATION OF THE MAE THEORY

As indicated in the previous technical report [5] on NAG-1-566, a reliable method established by Embleton, Piercy and Daigle [6] can now be used to measure in situ the acoustic impedance of the diffracting surface used in our scale model experiments. A comparison between theory and experiments was presented in the last technical report [5] only for data collected over the high impedance plywood ridge. During the past few months, a special effort has been made to analyze thoroughly the more interesting and realistic case of the ridge covered with a low impedance material (carpet) with a scaling such that it simulates a grass covered hill. The comparison between the results predicted by the MAE theory and the experimental data has been presented by Prof. Zhou at the 114th meeting of the Acoustical Society of America, held in Miami, last November. The viewgraphs used for the presentation and the corresponding text of the presentation are appended to the present report (see p. 41.) The general conclusion of the study is that the asymptotic results of the MAE theory (sound field on the diffracting surface, or above it, or far behind it) are in excellent agreement with experimental data, and they are computationally simple to evaluate.

Figures 1-3 show some typical results for situations where the ridge is covered with a carpet of effective flow resistivity $\sigma = 1600$ cgs-Rayls. As indicated in the previous report [5], the acoustic impedance of the carpet is then determined from the semi-empirical single parameter model of Delany and Bazley [7]. Figure 1 shows the sound pressure level (relative to the free field expected value) as a function of height above the apex of the ridge, at 5, 10, and 15 kHz. The experimental data (dots) follow the predictions of geometrical acoustics (solid curve). For comparison, the predictions corresponding to the case of a very high impedance surface (plywood) are shown on Fig. 1.

Figure 2 shows the insertion loss of the ridge covered with the same carpet along the surface of the ridge, as a function of a arclength $\xi$ nondimensionalized by the frequency and the radius of curvature of the ridge. The dimensionless parameter $q$ is a measure of the reciprocal of the acoustic impedance normalized by the frequency, the radius of curvature of the ridge, and the characteristic impedance of air. (See Reference [3]). Experimental data compare well with the predictions of the MAE theory and one can also conclude that the method used to estimate the acoustic impedance of the diffracting surface is indeed very reasonable, even though the thickness of the carpet is only 6 mm (1/4 of an inch).
An interesting result which is predicted correctly by the MAE theory is that, unlike for the case of a plywood ridge (high acoustic impedance), the insertion loss on a curved surface of low impedance (such as carpet) is frequency dependent.

Figure 3 shows the insertion loss as a function of nondimensionalized height ($\psi$) for situations where the sound field is measured far behind (2 meters) the ridge. The dots are representative of experimental values while the solid line is obtained from the knife-edge diffraction formula, which is an asymptotic version of the more general Fock-Van der Pol-Bremmer diffraction integral [3]. The theory predicts that the knife-edge diffraction approximation is adequate in the so-called penumbra region (i.e., the transition region between the bright zone and the shadow zone). It can be seen from Fig. 3 that, in the penumbra region ($-1 < \psi < +1$) the agreement between theory and experiment is excellent. Two interesting results not previously reported in the literature on the subject come out of the analysis: (1) the insertion loss along the (slightly bent) axis $\psi = 0$ is correctly predicted by the theory to be 6 dB; and (2), the insertion loss far behind the ridge is does not depend on the acoustic impedance of the diffracting surface.
Figure 1. Sound pressure level as a function of height above the apex of the ridge, at three different frequencies (5 kHz, 10 kHz, and 15 kHz). Shown are predicted (solid line) and measured (dots) SPL-versus-height curves for sound propagating over carpet (effective flow resistivity $\sigma = 1600$). The dashed line is the prediction for the plywood surface without the layer of carpet.
\[ n = 0.5 + 1.58 \]

Figure 2. Insertion loss on the ridge covered with carpet for four different frequencies (5 kHz, 10 kHz, 15 kHz, and 20 kHz). Shown are the predicted (solid lines) and the measured (dots) SPL-versus-distance $\xi$ corresponding to a dimensionless arc length from the apex of the ridge. From these data, it is inferred that the insertion loss of a diffracting surface of low impedance is frequency dependent.
Figure 3. Insertion loss in the penumbra region as a function of dimensionless height, 2 meters behind the ridge, for four different frequencies (10 kHz, 15 kHz, 20 kHz, and 25 kHz). The penumbra region is centered around a dimensionless height of zero.
REFERENCES


EXPERIMENTS ON THE APPLICABILITY OF MAE THEORY

The following pages reprint a paper written and published by Yves H. Berthelot, Allan D. Pierce, and J. A. Kearns, during reporting period. The proper citation for this paper is as follows:

AIAA-87-2668
Experiments on the Applicability of MAE Techniques for Predicting Sound Diffraction by Irregular Terrains
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AIAA 11th Aeroacoustics Conference
October 19-21, 1987/Palo Alto, California

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Abstract

The sound field diffracted by a single smooth hill of finite impedance is studied both analytically, within the context of the theory of Matched Asymptotic Expansions (MAE), and experimentally, under laboratory scale modeling conditions. Special attention is given to the sound field on the diffracting surface and throughout the transition region between the illuminated and the shadow zones. The MAE theory yields integral equations that are amenable to numerical computations. Experimental results are obtained with a spark source producing a pulse of 42 μs duration and about 130 Pa at 1 m. The insertion loss of the hill is inferred from measurements of the acoustic signals at two locations in the field, with subsequent Fourier analysis on an IBM PC/AT. In general, experimental results support the predictions of the MAE theory, and provide a basis for the analysis of more complicated geometries.

Introduction

At first glance, it may seem that the propagation of sound outdoors is quite simple to analyze. However, this superficial attitude changes rapidly when one starts thinking about the challenges arising in real life situations. For instance, the irregular topography of the terrain may cause scattering and diffraction of an incident sound wave. The nature of the terrain, i.e., the acoustic impedance of the boundary between the terrain and the surrounding medium, may vary with location. At grazing angles of incidence, the sound wave may see a non-local reaction of the terrain. All the above mentioned difficulties occur in practice, even if one neglects the additional complications due to atmospheric conditions (temperature gradients, wind gradients, turbulence, etc...). The method of Matched Asymptotic Expansions (MAE) is nevertheless a powerful mathematical tool in the analysis of this class of problems, provided that the topography varies slowly in terms of an acoustic wavelength. In this paper, the diffraction of sound by a single curved ridge of finite impedance is analyzed with the hope that it may eventually serve as a building block for the analysis of more complicated topographies. Another reason for focusing on the diffraction of sound by a single ridge is that there is a direct analogy between sound propagation over a flat surface in a curved atmosphere (curvature due to temperature and wind gradients) and sound propagation over a curved surface in a flat atmosphere (with constant properties). Since it is much easier to control surface curvature than temperature and wind gradients, there is a lot to be gained in modeling one by the other. It is therefore crucial to have a good understanding of the diffraction of sound by a curved ridge under controlled laboratory conditions. The first part of the paper deals with the theory of plane wave diffraction by curved surfaces of finite impedance, and the following section gives a description of the experimental setup used to obtain the results shown in the last section. These results include essentially the measurement of the insertion loss of the ridge as a function of frequency and local radius of curvature, both on the diffracting surface and throughout the transition region between the illuminated and the shadow zones, i.e., in the so-called penumbra region.

Theory

The theory of wave diffraction by curved surfaces can be traced back to the early 1900's when scientists were studying the transmission of radio waves beyond the line of sight and into the electromagnetic shadow of the earth. In the 1940's and 1950's, V. A. Fock published a number of important papers on the theory of the diffraction of electromagnetic waves. The following discussion, which has
been presented earlier, is largely a translation of Fock's work to the equivalent acoustical problem under investigation. The derivation, however, is based on the method of Matched Asymptotic Expansions (MAE) and it yields a uniformly valid solution of the problem.

Outer solution

Let a plane wave of constant frequency reflect from a curved surface of finite impedance $Z_s$, whose radius of curvature $R$ is not necessarily constant, but remains nevertheless always very large compared to the acoustic wavelength. (See Fig. 1). Reasonably far from the surface, in the so-called outer region, the details of the actual wave diffraction are overshadowed by the superposition of the direct and the reflected waves, so that the field is accurately predicted by geometrical ray acoustics. Simple geometrical considerations lead to an expression for the field pressure in the form:

$$p = P_1 e^{ikz} + P_1 [A(0)/A(l)]^{1/2} \Re e^{ikx} e^{i	heta}, \quad (1)$$

where $P_1 e^{ikz}$ represents the direct wave, and $\Re$ is the reflection coefficient at the point $z_0, y_0$ on the surface. $A(l)$ is a measure of the ray tube area after the wave travels a distance $l$ from the reflection point. The resulting field pressure can be expressed explicitly in terms of the field point coordinates $(x, y)$, the acoustic wavenumber $k$, the radius of curvature $R$ at the apex of the surface, and the surface impedance $Z_s$, in the following manner:

$$p = P_1 e^{ikz} F(x, y, Z_s, R, k), \quad (2a)$$

where

$$F = 1 + \left[ \frac{Q - \frac{1}{3} x}{3Q} \right]^{1/2} \left[ \frac{Q - \frac{1}{3} x - \frac{k^2}{R} R_s}{Q - \frac{1}{3} x + \frac{k^2}{R} R_s} \right] e^{i\theta}, \quad (2b)$$

with

$$Q = \left( \frac{4}{9} x^2 + \frac{2}{3} R y \right)^{1/2}, \quad (2c)$$

and

$$\psi = (2k/R^2) (Q^3 - \frac{8}{27} x^2 - \frac{2}{3} R x y). \quad (2d)$$

The next step is to introduce the following scaled parameters $L_s = R/(kR)^{1/2}, L_r = R/(kR)^{1/2},$ and $Z_{s,\text{refl}} = \rho \epsilon (kR)^{1/2},$ so that the functional dependence of the field pressure is reduced from five dimensional variables to a combination of only three nondimensional variables. In other words, one writes the acoustic pressure in the form:

$$p \approx P_1 e^{ikz} F \left( \frac{x}{L_s}, \frac{y}{L_r}, \frac{z}{Z_{s,\text{refl}}} \right). \quad (3)$$

Since a smooth transition between the solutions far from the surface (outer region) and close to the surface (inner region) is required, Eq. (3) is expected to provide an adequate outer boundary condition of the inner solution.

Inner solution

In order to get a good picture of the acoustic field close to the surface, one has to solve a wave diffraction problem, instead of a simplified geometrical acoustics reflection problem. To develop the inner solution, the top of the surface is approximated by a parabola $y = -x^2/2R$, and the Helmholtz equation is expressed in parabolic cylinder coordinates $u$ and $v$, such that

$$z = u[1 + (v/R)] \quad (4a)$$

$$y = v[1 + (v/2R)] - u^2/2R \quad (4b)$$

so $v = 0$ corresponds to the diffracting surface. The impedance boundary condition is also expressed as

$$\frac{\partial p}{\partial n} + Qp = 0 \quad \text{on the surface}, \quad (5)$$

where $Q = \rho k \epsilon / Z_s$. The next step is to change from the cartesian coordinates $x, y$ to the parabolic cylinder coordinates $\xi = u/(2^{1/2} L_s)$ and $\eta = 2^{1/2} v/L_s$. It can be shown that the new boundary value problem is now:
\[ \frac{\partial G}{\partial \xi} + \frac{\partial^2 G}{\partial \eta^2} + \eta G = 0, \]  
(6)

with
\[ \frac{\partial G}{\partial \eta} + qG = 0, \text{ at } \eta = 0, \]  
(7)

where \( q = \frac{i(kR/2)^{1/3}}{\rho c Z_p} \) is a nondimensional surface admittance. The unknown function \( G(\xi, \eta, \theta) \) is related to the field pressure \( p \) by
\[ p = e^{ikR} e^{i\theta/3} G(\xi, \eta, \theta). \]  
(8)

The outer boundary condition of the inner problem is that Eq. (8) matches Eq. (2) at large positive \( y \) or large negative \( x \).

The general solution of the above posed boundary value problem can be developed by Fourier transform and complex variable techniques, with the result
\[ G(\xi, \eta, \theta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{i\xi x} \left[ v(\alpha - \eta) - \frac{v'(\alpha) - qv(\alpha)}{w'_1(\alpha) - qw(\alpha)} w_1(\alpha - \eta) \right] d\alpha, \]  
(9)

where \( v(\xi) \) and \( w_1(\xi) \) are the Fock functions defined by:
\[ v(\xi) = \pi^{1/2} A_1(\xi), \]  
(10a)
\[ w_1(\xi) = 2\pi^{1/2} e^{x^{1/2}} A_1(x) \]  
(10b)

Eq. (9) is simply related to what Logan\footnote{Logan} terms Fock's form of the van der Pol-Bremmer diffraction formula.

**Limiting cases**

Deep into the shadow zone, the diffraction integral [Eq. (9)] can be evaluated by a contour deformation and it becomes \( 2i\pi \) times the sum of those residues corresponding to poles in the first quadrant, each such term giving rise to a creeping wave. The residues series representation in the deep shadow zone is given in detail in Ref. 5.

Another limiting case of interest is the acoustic pressure on the surface of the ridge, at \( \eta = 0 \). It can be rewritten in the form
\[ p = P e^{ikR} G(\xi, 0, \theta), \]  
(11)

where \( s \) denotes the arc distance along the surface from the apex of the ridge down the shaded side. In this case, one can use the wronskian identity \( w_1' v - w_1 v' = 1 \) to simplify Eq. (9) as follows:
\[ G(\xi, 0, \theta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{ix}(H(X) e^{-i(1/2)x} - 1 + \frac{1}{2} \theta A_0(X)) \]  
(12)

where \( X = (2/\pi)^{1/2} \eta^{1/4} (\xi - \eta^{1/2}) \) and \( A_0(X) \) is the diffraction integral, which is simply related to Fresnel integrals, and which is characteristic of the diffraction by sharp edges. The above analysis yields analytical expressions which are valid throughout the field and which are amenable to numerical computations. The following sections are devoted to experimental results obtained under laboratory conditions, in order to test the applicability of MAE techniques for predicting sound diffraction over curved surfaces.

**Experimental setup**

Laboratory scale experiments are being conducted on a 4.88 m by 2.44 m (16 ft by 8 ft) bench made of 2 cm thick CDX plywood. The diffracting surface is an arc of cylinder with a radius of curvature of about 2.5 m. The sound source is produced by an electric spark across two electrodes separated by a 1 mm gap, and excited by a voltage of 2 kV. The acoustic signature is a repeatable transient of 42 ± 2 ms duration, with an amplitude of about 130 Pa at 1 m. There are two major reasons for using a transient broadband source instead of a continuous single frequency signal. First, undesirable echoes from the non-anechoic room can be eliminated by properly gating the signals of interest; and, second, the frequency dependence of the in-
Insertion loss of the ridge can be obtained directly over the whole frequency range by means of Fourier transform techniques in a single measurement. Although the spark source is much more repeatable than anticipated, the experimental procedure does not rely on the consistency between consecutive sparks. Instead, two B&K 4136 1/4" microphones capture the signal at a reference position in the free field (unaffected by the ridge) and at a variable field point. The ratio of the Fourier transforms of the field and reference pressures is independent of the spark signature to within 0.5 dB between 8 and 40 kHz. Consequently, the ratio of the Fourier transforms, corrected for the difference in spherical spreading, the difference in absorption over the distances travelled by the signals, and the difference of sensitivities of the two microphones capturing those signals, can be interpreted as a measure of the insertion loss as a function of frequency, at a given field point.

The data acquisition system comprises the two microphones and their preamplifiers, an analog-to-digital converter (RC-electronics ISC-16), and an IBM PC/XT with a 64 kilobyte memory buffer. All the Fourier analysis is performed on the computer by a standard FFT algorithm, with no window applied to the signals, because it was found that spectral leakage was not significant. The signals are sampled at a 250 kHz sampling rate, digitized with 12 bits resolution, and digitally filtered with a cut-off frequency of 75 kHz.

Results

The experimental setup described above is used to analyze the sound field diffracted by the ridge, both on the surface and throughout the transition between the bright zone and the shadow zone. For the results reported below, the diffracting surface is a 2 cm thick plywood with very high impedance $Z_s$. The model of Delany and Bazley is used to determine the impedance of the plywood from the value of the effective flow resistance ($\sigma$), which is determined from a best fit technique described by Embleton et al. Figure 2 shows a set of predicted sound spectra for various values of the effective flow resistivity of the plywood, together with some experimental data (circles). The reference sound pressure level is that which would exist at the receiver in the free space. Source and receiver heights are 2 cm and 0.3 cm respectively, and the horizontal separation between the source and the receiver is 100 cm. It can be inferred from Fig. 2 that the diffracting surface used in our experiment has an effective flow resistivity of about $0.8 \times 10^6$ rayls-cgs. This value is sufficiently high to justify the approximation made further in this study, that the diffracting surface is perfectly rigid, i.e., that $Z_s \to \infty$ and $q \to 0$.

The first test of the MAE theory presented above is shown in Fig. 3 which depicts the insertion loss measured on the surface of the ridge as a function of the nondimensional...
sional parameter $\xi = (s/R)(kR/2)^{1/3}$, where $s$ is the arc length measured from the top of the apex of the ridge, $R$ is the radius of curvature of the ridge, and $k$ is the wavenumber. The solid line denotes the predicted dependence (see Eq. 12) for the case of a rigid surface; the symbols represent data measured at 5, 15, and 25 kHz. The data is in good agreement with the theory for positive values of $\xi$, (i.e., in the shadow zone). For negative values of $\xi$, however, the data deviates from the prediction, especially at higher frequencies. A possible explanation of this discrepancy is that the receiving microphone was not perfectly flush with the surface. The prediction obtained from the MAE theory is nevertheless quite adequate and it indicates that the scaling used in the nondimensionalization of the problem is correct.

A second test of the MAE theory is shown in Fig. 4 which depicts the insertion loss measured at the apex of the ridge, as a function of the nondimensional height $\psi = (2kR)(3y/3R)^{3/2}$, where $y$ is the dimensional height above the ridge. The solid curve represents the predicted insertion loss (derived from Eq. 2), and the squares and circles denote the experimental data at 5 and 15 kHz, respectively. Both theory and experiment show clearly the oscillation in the insertion loss due to the interference between the direct wave and the reflected wave. Note that, although Eq. 2 is strictly valid only in the outer region (large $\psi$), the agreement between theory and experiment is quite acceptable very close to the ridge, as $\psi \rightarrow 0$.

A third test of the MAE theory is to measure the insertion loss in the transition region between the bright zone and the deep shadow zone, (i.e., in the penumbra region), and compare the results with the knife-edge diffraction pattern predicted by Eq. 13. This is the purpose of Fig. 5, which shows the insertion loss as a function of nondimensional height $\psi = y(k/\pi z)^{3/2}$, where $y$ is the dimensional height above the plywood, and $z = 2.09$ m is the horizontal distance between the apex of the ridge and the measuring microphone. The solid line is the result of a computation based on Eq. 13, and the squares, the empty circles, and the black circles represent data points at 5, 15, and 25 kHz, respectively. Again, there is fairly good agreement between theory and experiment, and the nondimensionalization appears to be adequate. The result of a similar experiment is shown in Fig. 6, where the only difference with Fig. 5 is the value of $z$, the distance behind the apex of the ridge. In Fig. 6, $z = 0.64$ m, and, consequently, the approximation of large $\eta$ in Eq. 13 is not as good as for Fig. 5, where $z = 2.09$ m. Also, for fairly small values of $z$, the impedance of the diffracting surface is expected to have an effect on the insertion loss, and the assumption of a perfectly reflective surface, for
the theory shown in Fig. 6, contributes to the discrepancy between the theory and the experimental data. Nevertheless, the proper trend is predicted by the MAE theory and the nondimensionalization appears to be satisfactory.

Figure 6. Insertion loss 0.64 m behind the ridge.

Summary and Conclusions

The diffraction of sound by a smooth hill has been studied both analytically (within the theory of Matched Asymptotic Expansions) and experimentally (under laboratory scale modeling conditions.) The insertion loss at any point in the field is expressed in terms of an integral called the Fock-van der Pol-Bremmer integral (Eq. 9) which involves Airy function of complex argument. The applicability of the MAE theory has been checked against experimental results in some limiting cases, (on the diffracting surface, along a vertical axis at the apex of the hill, and behind the hill in the penumbra region.) The experimental results support the predictions of the MAE theory, and provide a basis for the analysis of more complicated topographies.

Acknowledgements

The authors thank John Preisser and William Willshire, Jr. for helpful discussions during the course of this research. The work reported here was supported by NASA, Langley Research Center.

References

VIEWGRAPHS USED IN PRESENTATION OF 
THE AIAA-87-2668 PAPER

The following pages reproduce the viewgraphs that were used by Yves Berthelot in his presentation at the 11th aeroacoustics conference in Sunnyvale, Ca (October 19-21,1987) of the paper


The text of this paper appears in the preceding section.
Experiments on the Applicability of MAE Techniques for Predicting Sound Diffraction by Irregular Terrains

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OUTLINE

0. Introduction
1. Theoretical model
2. Experimental method
3. Surface impedance at grazing angles
4. Experimental results
5. Conclusions
MOTIVATION: PREDICT THE SOUND FIELD OF LOW-ALTITUDE FLYING AIRCRAFTS.

irregular topography

METHOD: MATCHED ASYMPTOTIC EXPANSIONS (MAE)

PROTOTYPE PROBLEM: SINGLE CURVED RIDGE
Different regimes near the top of a curved ridge
when a plane wave is incident from the left
In the vicinity of the top of the ridge:

\[ p \approx p_i e^{i\theta} F(x, y, Z, R, k) \]

With scaling:

\[ L_x = \frac{R}{(kR)^{1/3}} ; \quad L_y = \frac{R}{(kR)^{2/3}} ; \quad Z_{char} = \frac{\rho c}{(kR)^{1/3}} \]

\[ p \approx p_i e^{i\theta} F\left(\frac{x}{L_x}, \frac{y}{L_y}, \frac{Z}{Z_{char}}\right) \]
INNER SOLUTION: wave diffraction

\[
\begin{cases}
\nabla^2 p + k^2 p = 0 \\
\frac{\partial p}{\partial n} + Q p = 0 \quad \text{on the surface (parabola)}
\end{cases}
\]

with \( Q = i k \rho e / Z \)

Change to parabolic cylinder coordinates \( \xi, \eta \)

\[
\begin{cases}
in \frac{\partial G}{\partial \xi} + \frac{\partial^2 G}{\partial \eta^2} + \eta G = 0 \\
\frac{\partial G}{\partial \eta} + q G = 0
\end{cases}
\]

at \( \eta = 0 \)

with:

\[
p = e^{i\rho} e^{ik^3/3} G(\xi, \eta, q)
\]

\[
q = i \left( \frac{kR}{2} \right)^{1/3} \frac{\rho c}{Z}
\]
GENERAL SOLUTION

MATCHING PRINCIPLE: AT LARGE \( \eta \) THE INNER SOLUTION MUST MATCH THE OUTER SOLUTION.

\[ p = P_i \ e^{i \omega a} \ e^{i \sigma / \beta} \ G(\xi, \eta, \vartheta) \]

\[ G(\xi, \eta, \vartheta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \left[ v(\alpha - \eta) - \frac{v'(\alpha) - qv(\alpha)}{w_1'(\alpha) - qw_1(\alpha)} w_1(\alpha - \eta) \right] e^{-\alpha} \ d\alpha \]

\[ v(\chi) = \chi^{1/2} Ai(\chi) \]

\[ w_1(\chi) = e^{i \pi / 6} 2\chi^{1/3} \ Ai(\chi e^{i \pi / 3}) \]

FOCK - VAN DER POL - BREMMER DIFFRACTION INTEGRAL
LIMITING CASES

1) Shadow zone \((\xi > \eta^{1/2})\)

\[ G(\xi, \eta, q) = 2i\pi \sum \text{(residues at poles of integrand)} \]
\[ = \sum \text{(creeping waves)} \]

It reduces to the results of Keller (1956) and Hayek et al. (1978).

2) Penumbra \((\xi \sim \eta^{1/2})\)

\[ G \approx e^{-\xi^2/2} e^{i\eta} - e^{i(2/3)\eta^{2/3}} \left[ H(X)e^{-i(\eta/3)X^2} + \frac{1+i}{2} A_D(X) \right] \]

**KNIFE-EDGE DIFFRACTION PATTERN**

\(X = \left(\frac{1}{\pi X}\right)^{1/2} \cdot y\)
EXPERIMENTAL METHOD

\[ p_1(t) \]

\[ p_2(t) \]

**reference pressure**

**field pressure**

\[ R \gg \lambda \]

**spark**

**THE RATIO** \( \frac{p_2(\omega)}{p_1(\omega)} \) **IS INDEPENDENT OF THE SPARK SIGNATURE.**

**IT IS THEREFORE A MEASURE OF THE INSERTION LOSS OF THE RIDGE.**
Measurements of Interest

- For two different surfaces
  (a) Hard (1/4" thick plywood)
  (b) Soft (1/4" thick carpet)

- Measure the insertion loss
  (a) vertically, at the apex
  (b) vertically, behind the ridge
  (c) vertically, far behind the ridge
  (d) on the surface of the ridge
3. Surface Impedance


\[ \sigma = \text{effective flow resistivity} \]

Then use the "1-parameter" model of Delany-Bazley:

\[ \frac{Z}{\rho c} = [1 + (\frac{f}{\sigma})^{-0.75}] + i [11.9 (\frac{f}{\sigma})^{-0.73}] \]
(Plywood)
Scaling

• Experimental results:

- Frequency range 4.0 - 20 kHz
- Grazing angles 1° - 15°

Surface can be treated as locally reacting with an effective flow resistivity $\sigma$

- $\sigma = 1,600$ cgs-Rayls (for carpet)
- $\sigma = 80,000$ cgs-Rayls (for plywood)

• Scaling: 400 - 2,000 Hz

- $\sigma_{carpet} = 160$ cgs-Rayls $\rightarrow$ grass\(^(*)\)
- $\sigma_{plywood} = 8,000$ cgs-Rayls $\rightarrow$ hard packed earth\(^(**)\)

INSERTION LOSS
ABOVE THE APEX

(Geometrical Acoustics)

\[
\text{RELATIVE SOUND LEVEL} \quad \text{dB}
\]

\[
\text{HEIGHT (m)}
\]

Theory (plywood)

--- Theory (copper)
CONCLUSION

• Laboratory scale experiments indicate that the MAE method is adequate to predict the sound field diffracted by a curved boundary of finite impedance.

• The Fock-Van der Pol-Bremmer diffraction integral can be efficiently computed at any field point without having to solve for the whole field (e.g., finite elements). It also reduces to some simple form for some special regions of the field (e.g., outer region, far behind the ridge, deep in the shadow zone, and on the diffracting surface).

• The MAE method can be used as a “building block” for the analysis of more complicated geometries (e.g., undulating terrain).
THE EFFECT OF FINITE SURFACE ACOUSTIC IMPEDANCE

The following pages reprint a paper that has been presented at the 114th meeting of the Acoustical Society of America, held in Miami, November 16-20, 1987. Professor Zhou gave the paper. The proper citation for this paper is as follows.

The effect of finite surface acoustic impedance on sound field near a smooth diffraction ridge

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Allan D. Pierce

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I. Introduction
II. Summary of the MAE diffraction theory by a smooth ridge
III. Experimental apparatus
IV. Experimental results
V. Some conclusion
(I) Introduction

Sound Propagation over undulating terrain

BRIGHT ZONE

**Geometrical Acoustics**

```
R(s), Z(s)
kR \gg 1
```

Simplified Diffraction Model
(II) The summary of the MAE diffraction theory by a smooth ridge

(2.1) Outer ray resolution
( in the vicinity of ridge's top )

\[ p \approx P_i e^{ikx} \left\{ 1 + \left[ \frac{Q - \frac{2}{3} x}{3Q} \right]^{1/2} \left[ \frac{Q - \frac{2}{3} x - \frac{e^c}{\xi^2} R}{Q - \frac{2}{3} x + \frac{e^c}{\xi^2} R} \right] e^{i\psi} \right\} \]

\[ Q = \left[ (4/9)x^2 + (2/3)R_y \right]^{1/2} \]

\[ \psi = (2k/R^2)[Q^3 - (8/27)x^3 - (2/3)Rx_y] \]
(2.2) General inner wave resolution

\[ p = P_i e^{ikx} e^{it^3/3} G(\xi, \eta, q) \]

\[ G(\xi, \eta, q) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \left[ v(\alpha - \eta) - \frac{v'(\alpha) - qv(\alpha)}{w_1'(\alpha) - qw_1(\alpha)} w_1(\alpha - \eta) \right] e^{i\alpha t} d\alpha \]

\[ v(\chi) = \pi^{1/2} Ai(\chi) \]

\[ w_1(\chi) = e^{i\pi/6} 2^{1/2} \pi^{1/2} Ai(\chi e^{i\pi/3}) \]

\[ q = i(kR/2)^{1/3} pc/Z_s \]

FOCK - VAN DER POL - BREMMER DIFFRACTION INTEGRAL
(2.3) The sound pressure on the surface of a ridge

\[ G(\xi, 0, q) = \pi^{-1/2} \int_0^\infty \frac{e^{-i\beta \xi/2} e^{-\beta \xi^{3/2}}}{w_2'(\beta) - q w_2(\beta)} \, d\beta \\
+ \pi^{-1/2} \int_0^\infty \frac{e^{i\alpha \xi}}{w_1'(\alpha) - q w_1(\alpha)} \, d\alpha \]

\[ w_1(z) = e^{i\pi/6} \frac{2^{1/2}}{\sqrt{2}} \text{Ai}(ze^{i2\pi/3}) \]
\[ w_2(z) = e^{-i\pi/6} \frac{2^{1/2}}{\sqrt{2}} \text{Ai}(ze^{-i2\pi/3}) \]
\[ w_1'(z) = e^{i\pi/6} \frac{2^{1/2}}{\sqrt{2}} \text{Ai}'(ze^{i2\pi/3}) \]
\[ w_2'(z) = e^{-i\pi/6} \frac{2^{1/2}}{\sqrt{2}} \text{Ai}'(ze^{-i2\pi/3}) \]
(2.4) The sound pressure in the transition region

\[ P = e^{-i\xi^2/3}e^{i\eta} \left[ H(\bar{X})e^{-i(\pi/2)}x^2 - \frac{1+i}{2}A_D(\bar{X}) \right] \]

where \( \bar{X} = (2/\pi)^{1/2} \eta^{1/4} (\xi - \eta^{1/2}) \)

\[ \xi = (u/R) \left( kR/2 \right)^{1/3} \]
\[ \eta = (v/R) \left( k^2 R^2 / 2 \right)^{1/3} \]
\[ u = \frac{1}{\sqrt{2}} \left[ -(2Ry + R^2) + \sqrt{(2Ry + R^2)^2 + 4R^2 x^2} \right]^{1/2} \]
\[ v = R \left[ (x/u) - 1 \right] \]

\[ P_{\text{total}} = \frac{1}{2} P_{\text{inc}} \quad \text{at} \quad \xi = \eta^{1/2} \]
(III) Experimental apparatus

![Block diagram of the experiment. Spark source (S) includes power supply (P), resistor (R), and capacitor (C): signals from reference microphone ($M_1$) and field microphone ($M_2$) are amplified (A), digitized (A/D), and processed by the IBM PC/XT.](image-url)
(IV) Experimental results

(4.1) Surface acoustic impedance

\[ \frac{p}{p_0} = \left( \frac{e^{ik_1r_1}}{k_1r_1} \right) + R_p \left( \frac{e^{ik_2r_2}}{k_1r_2} \right) + \frac{(1 - R_p)F(w)e^{ik_2r_2}}{k_1r_2} \]

\[ F(w) = 1 + i w^{1/2} e^{-w} \text{erfc}(-i \sqrt{w}) \quad w = i \frac{2k_1r_2}{(1 - R_p)^2} \left( \frac{Z_1}{Z_2} \right)^2 \left( 1 - \frac{k_1^2}{k_2^2} \cos^2 \phi \right) \]

Earlier work by Embleton, Piercy and Daigle

Impedance model of Delany and Bazley

\[ \frac{R_2}{\rho_1 c_1} = 1 + 9.08(f/\sigma)^{-0.75} \]
\[ \frac{X_2}{\rho_1 c_1} = 11.9(f/\sigma)^{-0.73} \]
\[ \frac{\alpha_2}{k_1} = 1 + 10.8(f/\sigma)^{-0.70} \]
\[ \frac{\beta_2}{k_1} = 10.3(f/\sigma)^{-0.59} \]
The effective flow resistivity, extracted from the transmission spectra, is about 1600 (for carpet).

Slide 9
Sound pressure level as a function of height for a given frequency (x = 0)
Relative sound level as a function of frequency for a given height (X = 0)
Insertion loss along the ridge covered with carpet

\[ \xi = \frac{s}{R} \left( \frac{kR}{2} \right)^{1/3} \]
Insertion loss along the ridge surface

\[ \xi = \frac{2(kR)}{R^2}^{1/2} \]
Insertion loss in the penumbra area as a function of frequency for a given vertical height \((x = \pm n)\)
Insertion loss in the penumbra area as a function of dimensionless height (X = 2m)
Insertion loss in the penumbra area as a function of dimensionless height for different horizontal distances ($f = 10$ kHz).
Insertion loss in the penumbra as a function of dimensionless height for different horizontal distances ($f = 20$ kHz).
(V) Some conclusions

1. The comparison of the MAE diffraction theory with experiment from a smooth ridge is excellent.
2. For small grazing angles and the experimental frequency range, the carpet backed with the plywood can be treated as a local reaction impedance.
3. The insertion loss along a finite impedance ridge is more sensitive to the frequency than the insertion loss along a rigid ridge.
4. In the penumbra, far behind the ridge, the diffraction of a smooth ridge with a finite impedance is the same as a rigid knife-edge diffraction.
THESIS PROPOSAL OF JAMES A. KEARNS

The following pages reprint the thesis proposal successfully defended by James A. Kearns on November 12, 1987. The thesis committee was made of Drs. Berthelot (Chairman), Pierce, Main, Jarzynski, Zhou, Long, and Benkeser. (The thesis proposal is a requirement of the Graduate School of the Georgia Institute of Technology which is an intermediate step toward the completion of the doctoral degree.)
AN INVESTIGATION OF THE PROPAGATION OF SOUND

OVER A CURVED SURFACE OF FINITE IMPEDANCE

A Thesis Proposal

Submitted to

The Graduate Committee of the School of Mechanical Engineering

by

James A. Kearns

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

School of Mechanical Engineering
Georgia Institute of Technology

October, 1987
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INTRODUCTION

The principal goal of this research is to gather and analyze experimental data that is useful in understanding the effects of irregular topography on long range sound propagation in the atmosphere. The experiments are focused on situations where the rays associated with the propagating sound wave are impinging upon the topography at angles near grazing incidence. Such a situation may be visualized as that when a low flying aircraft produces an acoustical disturbance over the nearby terrain. An analogous situation arises when wind or temperature gradients result in curved propagation paths in the atmosphere. In either situation, the interaction between the impinging sound field and the surface topography is expected to be quite complex. In light of this expectation, a much simpler model will be examined. The model chosen is that of a single, cylindrical ridge lying on a flat table top. The expectation is that a complex topography may be theoretically reconstructed as a set of adjoining single ridges. A spark generator provides an acoustical point source. Experiments will be conducted to describe the field on, above, and behind the ridge. The data will be compared with numerical predictions derived from the theory as presented by Pierce.1

This proposal consists of three major parts. The first of these is a synopsis of the relevant theory and its history. This includes a survey of influential papers since 1945. The second part is a review of previous and relevant experimental work including studies of diffraction over cones and other bodies of revolution. Finally, the third part contains descriptions of the proposed experimental and computational work. The proposed computational work is presented immediately after the section on the history of the theory. Likewise, the proposed experimental work is presented immediately following the review of previous experimental work. A summary of the objectives of this work is presented at the end of the text.

Dr. Ji-xun Zhou has been working and will continue to work closely with the author. He has assisted in all aspects of the project including the capturing of data and the computation of theoretical results.
SURVEY OF RELATED THEORETICAL WORK

The study of high frequency diffraction over curved surfaces dates at least as far back as the most recent turn of the century. At that time there was a good deal of interest in studying the way in which radio waves propagated over the surface of the earth. A typical model of this phenomenon was that of a vertical electric dipole in the presence of a metallic sphere. In all cases, the radius of the sphere was assumed to be much greater than the wavelength of propagation. Unfortunately, under such circumstances the eigenfunction expansion of the solution in the shadow zone is slowly convergent. However, Watson (1918), who drew upon the work of Poincaré (1910) and Nicholson (1910), managed to express the expansion in terms of a residue series. From this residue series, an associated contour integral was constructed. Watson showed by deforming the contour in a particular way that a new, faster converging residue series can be derived. This process is known as Watson's transformation and is still used to evaluate the solution deep in the shadow zone. At points near the geometrical shadow boundary, this series solution is often slow to converge. White (1922) extended the work of Watson for cases when the observer is outside the shadow zone.

In the late 1930's, van der Pol and Bremmer (1936-38) published several notable papers on the propagation of radio waves over the surface of the earth. Shortly thereafter, Fock (1945) published a paper in which he put forth the concept of the local field in the penumbra region. Fock considered the case of a high frequency wave incident upon a perfectly conducting surface of continuously varying curvature. By way of a physical argument, Fock claimed that the surface current distribution in and near the shadow boundary depends upon the local curvature and upon the incident field. This dependence was expressed in the form of a "universal" function

\[ G(\xi) = e^{i\xi^2/2} \pi^{-1/2} \int_\Gamma \frac{e^{i\alpha \xi}}{w''(\alpha)} d\alpha \]  

(1)

where 1) \( \xi \) is a representative distance of a given point on the surface to the geometrical shadow boundary, and 2) \( \Gamma \) is the contour which runs from infinity to the origin along the path \( Arg z = 2\pi/3 \) and from the origin to infinity along the real axis. \( w''(\alpha) \) is an Airy function and is a bounded solution to \( w''''(\alpha) = \alpha w(\alpha) \). The function \( G(\xi) \) decreases exponentially with \( \xi \) for large positive values of \( \xi \) in the shadow zone and asymptotically approaches 2 for large negative values of \( \xi \) in the illuminated region. The characteristic width of the penumbra was given as \( d = \left( \frac{\pi}{2} R_0^2 \right)^{1/3} \), where \( R_0 \) is the radius of curvature of the surface shadow boundary in the plane of incidence. In the following year, 1946, Fock derived the same result directly from the reduced wave equation.
In addition, Fock extended the solution to the region in the "neighborhood" of the surface and included bodies which are "good" conductors in the sense of the Leontovich boundary condition.

Approximately one decade after the publication of Fock's work, Keller (1956) published the "Geometrical Theory of Diffraction." Keller's theory was based upon a heuristic application of Fermat's principle. In order to account for diffraction effects, Keller added a modified form of the principle which states that "the diffracted rays connecting the points P and Q are those curves which render stationary the Fermat integral among all curves connecting P and Q, and having a point or arc on the diffracting obstacle." (Note: The original wording was "... arc on the cylinder." since this was the geometry originally examined.) This statement can be applied to any body of arbitrary shape. The resulting solution is in terms of unknown "diffraction coefficients", which are analogous to reflection coefficients, and which are found by comparison to well known exact solutions of various canonical problems. These coefficients are functions of the incident field, the wavelength, and the surface properties and geometry at the point of diffraction. These coefficients follow the spirit of Fock's notion of the local field in the penumbra. Keller's theory formalized the concept of "creeping" waves, so-called by Franz and Depperman (1952), for an arbitrarily shaped body. The "creeping" wave hypothesis is that a surface wave is produced at the point of diffraction and that this wave proceeds to travel into the shadow zone along a geodesic of the surface. This wave continuously sheds rays tangential to every point along the geodesic in accord with the modified Fermat's principle of diffraction. Keller's theory is a high frequency approximation although good results have been reported at wavelengths on the order of the dimensions of the diffracting obstacle. This theory is not valid near the geometric shadow boundary or on the surface of the diffracting obstacle.

In 1959, Goodrich examined Fock's theory in light of the work of Keller, Franz and Depperman, and Kazarinoff and Ritt (1959). Goodrich showed that by changing Fock's choice of coordinate transformation that Fock's theory will yield properly the creeping wave behavior discussed above. Also, Goodrich noted that Fock's theory is essentially a 2-dimensional one in that it does not account for ray convergence on the surface of a 3-dimensional body. This difficulty was overcome in the derivation of Hong (1967).

A respected analysis of previously published diffraction theories was made by Logan (1959). Subsequently, Logan and Yee (1962) presented a thorough mathematical treatment of the interrelationships between the relevant theories for diffraction from a convex body. More recently, Ivanov (1971) examined plane wave diffraction from an ideally reflecting cylinder. He proceeded to match an asymptotic solution valid in the shadow zone to an asymptotic solution valid in the illuminated region. Ivanov's matched solution was continuous but not necessarily smooth at the shadow boundary. Later, Pathak (1979) found his own uniform, asymptotic solution. His solution is for a perfectly conducting surface and is valid everywhere except at the surface. Pathak's solution is given in terms of well tabulated functions similar to those of Fock.
The current theory of interest is that as presented by Pierce (1986). His solution employs the method of Matched Asymptotic Expansions to derive a uniform solution that 1) is valid on much of the surface, and 2) reduces asymptotically to the geometrical acoustics solution in the illuminated region. A diagram of the situation and the associated regions of interest is shown in Fig. (1). The general solution has the form

\[ p = P e^{i k u} e^{i \xi^2/3} G(\xi, \eta, q) \]  

(2)

and

\[ G(\xi, \eta, q) = \pi^{-1/2} \int_{-\infty}^{\infty} \left[ v(\alpha - \eta) - \frac{v'(\alpha) - q v(\alpha)}{w'_1(\alpha) - q w_1(\alpha)} w_1(\alpha - \eta) \right] e^{i \alpha t} d\alpha \]  

(3)

where \( v(\zeta) \) and \( w_1(\zeta) \) are "Fock functions" and simply related to Airy functions of complex argument, \( \xi \) and \( \eta \) are nondimensionalized and transformed coordinates, and \( q \) is a nondimensionalized surface admittance. The variable \( u \) is proportional to \( \xi \). The above integral solution (3) is termed by Pierce the Fock-van-der-Pol-Bremmer function and is "trivially related to what Logan (1959) terms Fock's form of the van der Pol-Bremmer diffraction formula."

In the limit of an acoustically ridge surface, which is analogous to a perfectly conducting surface for cases of electromagnetic propagation, the above integral function, \( G \), reduces to

\[ G(\xi, 0, 0) = \pi^{-1/2} \int_{-\infty}^{\infty} \frac{e^{i \alpha t}}{w'_1(\alpha)} d\alpha \]  

(4)

on the surface of the ridge. This result is nearly identical to that derived by Fock (Eq. 1). Fock includes the factor \( e^{i \xi^2/3} \) as part of the function \( G \) while Pierce separates it from \( G \) as in Eq. (2). The differing contour paths are the only real discrepancy.

Proposed Computational Work

The proposed computational work consists of finding solutions to Eq. (3) for various ranges of the parameters \( \xi \) and \( \eta \). Algorithms for many of the ranges of interest have been previously derived by Pierce and Main (1985). These algorithms will be used and their solutions compared with the experimental data. New or improved versions of these algorithms will be applied to those cases in which significant discrepancies between theory and experiment are found. A program for the evaluation of the Airy function for arbitrary complex argument has already been written by Pierce. This program will be utilized as a base for the other computations. Any necessary refinements to this program will be carried out. The results will be compared against experimental data for certain regions of space on, above, and behind the ridge. The particular regions are outlined in the section on proposed experimental work.
1. The different regimes behind and near the top of a curved surface when a plane wave is incident.
SURVEY OF RELEVANT EXPERIMENTAL WORK

A survey of related experimental studies is presented in this section. This includes experimental studies of the diffraction of waves incident upon cones and other bodies of revolution as well as experiments on wave propagation over and on curved surfaces. A significant portion of the relevant experimental work has been carried out in the context of electro-magnetic wave propagation. An effort is made to include this work although an emphasis is placed upon work performed in the realm of acoustics. To the best of the author's knowledge, the survey includes most of the relevant experimental work from the past two decades. The goal of this survey is to summarize the recent experimental work.

Cones and other bodies of revolution offer simple 3-dimensional geometries which 1) are easy to find or construct and, thus, allow for experimental testing of theories such as the Geometrical Theory of Diffraction (GTD), and 2) readily admit numerical solutions due to symmetry considerations. For this reason, many recent papers have reported the use of cones, spheres, and ellipsoids in their model experiments. Keller (1961) made a comparison between theoretical and experimental results for the backscattering cross-section produced by the electromagnetic irradiation of a perfectly conducting cone. The experimental results were obtained by Keys and Primih (1959) for six different cones with half-angles ranging from 4 to 20 degrees. The data is in good agreement with predictions based upon Keller's Geometrical Theory of Diffraction. Bechtel (1965) carried out a series of experiments to measure the radar cross-section (RCS) of cones with half-angles of 4, 15, and 90 degrees. The size of the base, a, was also varied between 0.98 and 2.87 wavelengths in diameter. Again, Bechtel compared his results with predictions based upon Keller's GTD. For $ka$ on the order of 8 or 9, Bechtel found good agreement between the theory and experiment except for cases when the cone is observed within 30-40 degrees of nose-on with a vertically polarized wave (i.e. wavefront parallel to the axis of the cone). For smaller cone bases on the order of $ka = 3$, the predicted RCS matched the observed RCS to within 5 dB but the predicted shape or form of the RCS was not well observed, particularly within 30 degrees of nose-on. Another set of reported data on cone diffraction was gathered by Bargeliotes et al. (1975). Mittra and Safavi-naini (1978) compared these experimental results with theory which considered the potential field produced by point sources on the surface of a cone. The array of point sources represented the surface currents as derived asymptotically by Fock and others. The comparison showed good agreement when the diffracted rays had traveled a significant angular distance around the cone. No results for grazing angles of incidence were found in their report. An extensive study of acoustic scattering from various bodies was made by Lang (1980).
He measured forward and back scattering as well as creeping wave type phenomena from smooth, rigid objects such as baffles, cylinders, cones, and cone variants.

Experimental data on high frequency diffraction and/or reflection from other bodies of revolution has been reported by a number of sources. Neubauer (1967) measured the travel time from source to receiver of a sonic pulse diffracted around a circular aluminum cylinder. The measurements were made in water. He positioned a narrow-beam source such that the central ray arrived tangential to the cylinder surface and normal to the axis of the cylinder. An array of five equally spaced hydrophones measured the amplitude and arrival time of the pulse. The hydrophones were deep in the shadow zone. Neubauer found that his results were in excellent agreement with those predicted by the "creeping wave" theory of Franz and Keller. Neubauer also reported that a slit of less than a wavelength in width and positioned along the grazing line allowed a "...large part of the wave to pass." Further, he placed a baffle against the cylinder surface to block passage through the water and, subsequently, found little or no transmission. Thus, he concluded that the wave was transmitted on the water side of the cylinder boundary. Foxwell (1970) and Blake and Wilson (1977) carried out related experiments. Foxwell measured the diffracted field on the shadowed surface of a rigid sphere. A point source located on the surface formed the illuminated pole. At high wavenumbers, Foxwell observed interference effects predicted by creeping wave theory. Blake and Wilson performed an analogous experiment using a highly eccentric prolate spheroid. Their sound source was located at a distance from the surface equal to the length of the spheroid and along the major axis. Measurements revealed an illuminated spot at the antipole and the existence of the creeping wave interference pattern near the antipole. In addition, Blake and Wilson report that 1) the measured levels at the antipole "decrease roughly as the reciprocal of frequency", 2) that the shadow zone measurements are "well approximated by Keller's Geometric Theory of Diffraction", and 3) that deviations on the order of 3 dB from the theory "are observed at high frequencies and at coordinates off the antipole". Lang (1980) examined acoustic scattering from the surface of a thin prolate spheroid and used the results to deduce characteristics of acoustic scattering from any smoothly curved surface. Lang concluded that the "backscattered pressure from smoothly curved bodies is determined almost exclusively by specular effects, even at wavelengths that are relatively large compared to the appropriate dimensions of the scatterers." He also observed "evidence of surface fields very close to smoothly curved scatterers" that he "identified with Franz-type creeping waves." Finally, Lang derived the Freedman theory of echo formation and compared the predictions of this theory with experimental results. He found that the theory "produces a reasonably good model of scattering from baffles but ... exhibits serious errors in cases involving three-dimensional bodies."

More recently, Almgren (1986) measured the insertion loss above and behind convex and concave cylinders. A distant spark source provided a near planar incident wave. Almgren was interested in examining the analogous relationship between sound propagation over a curved surface and that in a medium with a vertical sound speed gradient. He found that "it is reasonable
to simulate the effect of refraction due to a sound speed gradient ... with a curved ground scale model.” Only small errors on the order of 3 dB or less were found for all measurements including those at grazing incidence. These errors were relative to the theory as presented in the works of Pekeris (1946), Pridmore and Brown (1962), and Pierce (1981). Berry and Daigle (1987) also were interested in the refraction of sound in a stratified atmosphere. Like Almgren, they chose to examine the analogous case of diffraction over a convex curved surface. A tone burst mechanism was used to produce a narrow band point source. Their measurements were confined to listener positions deep in the shadow zone and in the penumbra region while the point source was placed either on or slightly above the surface of the ridge. Measurements were made over a frequency range of 0.3 to 10 kHz. For listener positions below the shadow boundary, they compared their results to predictions based upon an extended version of the creeping wave theory. For listener positions above the shadow boundary, they compared their results to predictions based upon geometrical theory. At the shadow boundary, they reported that the two theories agreed to within \( \frac{1}{2} \) dB. However, the experimental results differed from the theoretical results by as much as 5 dB when they were compared for listener positions on or near the shadow boundary. Comparisons outside of this region were good.
The intent of the proposed work is to investigate the acoustic scattering at large angles of incidence from a curved surface, with particular attention paid to transition phenomena. A laboratory and experiments have been designed in order to make such an investigation. The experiments can be grouped into four classes. They are:

1) Preliminary experiments to determine “free field” propagation properties such as the sound speed, source directionality, attenuation, and non-linear steepening of waveforms;
2) Experiments to determine the specific acoustic impedance of two or more surfaces;
3) Experiments to determine the insertion loss on, above, and behind the model curved surface; and,
4) Experiments to determine the effect of echoes from one ridge on the insertion loss near another ridge for a series of two ridges aligned back to back.

The majority of the proposed work consists of carrying out the above experiments and comparing the acquired data to the results predicted by the theory presented earlier.

Experimental Facility

The following is a brief description of the laboratory and some common components of the experimental methods.

Four tables were made to be used for the scale model experiments. Each table is 1.2 meter wide by 2.4 meter long and 0.9 meter high. These four tables were bolted together, forming one table 4.9 meter long by 2.4 meter wide. The table top is CDX plywood, 2 cm thick. A curved surface to be mounted on the table was constructed of exterior plywood and used as a laboratory scale model of a topographical ridge. The contour of the surface has the shape of an arc of a circle with an approximate radius of curvature of 2.5 m. The complete laboratory including the table with the curved surface (ridge) mounted upon it is depicted in Fig. (2).

A sound source is produced by an electric spark generator. The design of the spark generator is a modification of a design by Dr. Mendel Kleiner at the Chalmers Institute of Technology. The spark gap is approximately 1 to 2 millimeters wide. A gap voltage of 2 kV is sufficient for discharge. Typical peak acoustic pressures at a distance of 1 meter are on the order of 120 to 130 Pascals. Typical rise time and characteristic pulse duration are of the order of 10 μs and 42 μs, respectively. The resulting sound source is broadband (2 – 40 kHz) with the peak level residing in the range of 20 – 25 kHz.
The data acquisition system is composed of two Brüel & Kjær quarter-inch condenser microphones, two amplifiers, an analog-to-digital converter, and an IBM personal computer. The amplified analog signal of each microphone is converted to digital form by an integrated hardware and software system produced by RC Electronics Inc. and called “Computerscope ISC-16.” This system consists of a 16 channel A/D board which is inserted into the IBM PC, an external instrument interface, and the scope driver software. The system has a 64 kilobyte memory buffer and is capable of gathering data at an aggregate rate of 500 kHz.

General Procedure

The general experimental procedure is similar for all of the experiments described here. Most of the experiments use two microphones, one at a reference point in the free field, and the other at a desired field point. The two corresponding analog voltage signals, which are output from the microphones, are amplified, sampled at intervals of 4.0 μs, and digitized to 12 bit precision. The sampling process is triggered such that the source waveform arrives at each microphone within the limited time window afforded by the finite memory buffer. A constant is automatically added to each data sample such that the mean of the data sample is approximately zero. Since the voltage increment registered by a microphone is opposite in sign to that of the corresponding pressure increment, the sign of the digitized data is reversed. Then the data sets are filtered by a lowpass digital filter with an upper cutoff frequency of 75 kHz. Waveform portions of interest, and representative of what would be received if there were no undesired reflections contaminating the data, are selected. Each portion is centered in a field of zeroes, and thus two new abbreviated waveform windows are created. These windows have the same duration (usually, 500 times 4.0 μs), but usually not the same time beginnings. The replacement of the extra data points by zeroes is in accord with the expectation that an acoustic pulse has negligible residual effect on the ambient pressure, and that the acoustic pressure before and after each waveform should ideally be zero. The digital Fourier transform of each window is computed and interpreted as if the value of the acoustic pressure were identically zero before the start and after the end of each window.

Preliminary Experiments

A set of preliminary experiments have been performed in order to establish 1) the ambient speed of sound, 2) the directionality of the source, 3) the magnitude of propagation losses due to spreading and absorption, and 4) the extent of non-linear effects. The experiments were performed over the flat table top, with a fixed source height equal to the height of the ridge, and with a common reference microphone position. Results show that the ratio of the Fourier transform of the field signal to that of the reference signal is generally independent of the particular source signal. Also, the source is within ± 1 dB of being omnidirectional over a range of −12 to 24 de-
2. Schematic of the interior of the laboratory room used in the study, showing the sequence of data processing associated with a single firing of the spark source. Reference and field microphones are shown stationed in front and behind of the ridge, respectively. A triggering device activates the data capturing system simultaneous to the firing of the spark source. The analog signals of the two microphones are sampled at a rate of 250 kHz, digitized to 12 bit precision, stored in a 64 kilobyte memory buffer, and low pass filtered. Interesting portions of the total data field are then input to a discrete Fourier transform program. The output of this program is interpreted as the Fourier components of the original waveform portions.
degrees from the horizontal. The local speed of sound was deduced after measuring the arrival time of a pressure wave at 11 different downstream positions. The measurements were made at equal intervals over a downstream range of 1 to 3.5 meters.

The same set of 11 data points were used to deduce the free field dependence of the ratio of the Fourier transform of the pressure waveform at the field microphone to that at the reference microphone. The results show that free field propagation losses can be approximated by those attributed to spherical spreading and absorption in air. Finally, the shape of the pressure waves were observed for signs of wave steepening while the magnitude of their Fourier transforms were observed for redistribution of acoustic energy from lower frequencies into higher frequencies.

**Specific Acoustic Impedance Measurement**

Several experimental methods were applied to determine the acoustic impedance of both the bare plywood table top and the table top covered with a low-cut commercial carpet. Later, the acoustic impedance of the surface of the ridge will be assumed to be roughly the same as that of the table top. The goal of these experiments is to develop and/or test a procedure for acquiring a reasonable estimate of the acoustic impedance of a surface at grazing angles of incidence.

The first method is referred to as the “Direct” method. In this method, the pressure pulse arrives at a single microphone along two paths, one direct (distance \( R_1 \)) and one reflected (distance \( R_2 \)). The paths are chosen such that the difference in travel time is sufficient for the pulses to arrive at distinct intervals in time. After the propagation losses and phase shifts are properly corrected, the incident and reflected pulses are deduced at the point of reflection. The ratio of the Fourier transforms of these pulses yields the reflection coefficient and, with additional calculation, the impedance. Plane wave reflection is assumed at the surface since \( kR \gg 1 \). In cases of grazing incidence, the direct and reflected pulses may not have distinct arrivals due to the small difference in path length. Then, a second microphone is employed to measure the direct pulse at a point where the arrivals are still distinct. This measurement is extrapolated to the position of the original microphone. In the past, Davies and Mulholland (1979), and Cramond and Don (1984), have used methods similar to this one.

A second method employed is referred to as the “Standing Wave Ratio” (SWR) method. The ratio of the Fourier transform of the direct plus reflected pulses to the Fourier transform of the extrapolated direct pulse yields an interference pattern oscillating about unity. The values at the maxima and minima of the pattern are used to approximate 1) the magnitude of the reflection coefficient at the mid-point between each maximum/minimum pair, and 2) the phase of the reflection coefficient at each maximum and minimum.

A third method employed is based upon an empirical model for impedance proposed by Delany and Bazley (1970). This model relates the real and imaginary parts of the surface impedance...
3. Typical results for determination of flow resistivity $\sigma$ of carpet material on plywood. Shown are the predicted and measured SPL versus frequency curves for sound propagating over the flat plywood table top covered with a thin commercial carpet. In this context, the SPL refers to the sound level at the reflection point relative to the free field sound level extrapolated to that point. The source and receiver heights are 2 cm and 0.3 cm, respectively; the horizontal separation distance between source and receiver is 80 cm. From the data shown it is inferred that $\sigma$ is approximately 1600 rayls cgs.
to the ratio of frequency to an effective flow resistance parameter. This relation is expressed in the power law formula

\[
\text{Re}\{Z_2\} = R_2/\rho_1 c_1 = 1 + 9.08(f/\sigma)^{-0.75} \\
\text{Im}\{Z_2\} = X_2/\rho_1 c_1 = 11.9(f/\sigma)^{-0.73}
\]

(5a) (5b)

where \( f \) is the frequency, \( \rho_1 c_1 \) is the characteristic impedance of air, \( Z_2 \) is the specific acoustic impedance of the reflecting surface, and \( \sigma \) is an effective flow resistance parameter. The method consists of comparing analytical curves, based upon the theory of reflection of a spherical wave from an impedance plane, with experimental data of the SPL above the flat table top for particular source and receiver positions. The analytical curves are computed using an algorithm presented by Chien and Soroka (1975). The SPL is considered to be a function of the source and receiver positions, the characteristic impedance of air, the impedance of the surface, and frequency. Thus, the effective flow resistance parameter, \( \sigma \), can be inferred from a comparison of analytical to experimental data once the other parameters are measured, calculated or picked. An initial result for carpet-on-plywood is shown in Fig. (3). Notice that despite the thinness of the carpet, \( \approx 6 \text{mm} \), the experimental curves are well fitted by analytical curves for certain values of \( \sigma \). Similar results were reported previously by Embleton, Piercy, and Daigle (1983) as well as by Berry and Daigle (1987), all of whom used the same approach.

**Insertion Loss Measurements**

A set of experiments have been designed to measure the effects of the presence of the ridge on the sound field. These effects are expressed in terms of the insertion loss. In the following experiments, the insertion loss is taken as the ratio (in dB) of the Fourier transforms of the pressure at a point in the field with the ridge present to the pressure expected at that point with the ridge absent. The expected pressure at a point in the field is extrapolated from a reference pressure measured well above and in front of the ridge.

The experiments are of a 2-dimensional nature in that all measurements are made in a vertical plane which is perpendicular to the axis of revolution of the cylindrical ridge and contains the point source. All of the experiments are performed for both the bare plywood and carpet-on-plywood surfaces and perhaps one other acoustically soft surface. The acoustic impedance of the surface of the ridge is assumed equivalent to that measured on the surface of the table for the same surface material or covering.

Where possible, the incremental distance between successive measurements will be on the order of the smallest length scale in the direction of measurement. The length scales

\[
L_x = R/(kR)^{1/2} \quad L_y = R/(kR)^{2/2}
\]
were deduced from the theory. At 40 kHz, $L_x$ and $L_y$ are approximately 25 cm and 2.5 cm, respectively.

The first measurements were made of the insertion loss along the bare plywood surface of the ridge. The field microphone measured the combined direct and reflected pressure at 10 cm intervals over the span of the ridge. Additional measurements were made over two different vertical paths originating behind the ridge. The horizontal position of the first of these two paths was roughly 64 cm downstream from the apex of the ridge, while the second path was located roughly 210 cm downstream from the apex (Fig. 4). The measurements over each of the vertical paths were made at intervals of 1-2 cm.

A similar set of experiments will be performed over the ridge with a carpeted surface, and perhaps one other soft surface, depending upon the results obtained with the carpet. In general, the measurements over each of the vertical paths will be made at intervals of 1-2 cm and will extend to heights at least twice that of the ridge or roughly 65 cm above the top surface of the table. This will be necessary in order to observe oscillations in the geometrical acoustic region at low frequencies ($\approx 5$ kHz).

After all of the above measurements of insertion loss due to a single ridge have been conducted, a second ridge will be placed in series with the first and, experiments will be performed in order to deduce the impact of echoes from the second ridge upon the insertion loss as measured for the single ridge. The measurements themselves will be similar to those made for the single ridge. It is expected that echo effects near the back surface of the first ridge could be significant since in this region certain echoed arrivals from the second ridge may dominate rays diffracted from the first ridge.

SUMMARY

The objective of this proposed work is to produce experimental and numerical data portraying key aspects of high frequency acoustic diffraction over a curved surface. The experimental work consists of measuring the impedance of the curved surface as well as the insertion loss on, above, and behind the curved surface. Several different surfaces will be used in order to observe the effects of various surface impedances on the results. The data is to be presented in such a way as to describe well the transition from shadow to light. In addition, corresponding numerical data is to be produced from the theory as presented by Dr. A.D. Pierce. Hopefully, a comparison of the numerical data with the experimental data will yield insights into 1) the behavior of the diffracted field in the transition region and near a caustic such as that along the curved surface, 2) the importance of echoes relative to diffracted rays, and 3) the limitations of various theoretical assumptions such as that of a locally reacting surface.
2.09 m BEHIND THE RIDGE

4. Plot of the insertion loss at approximately 209 cm behind the bare plywood ridge as a function of the dimensionless height parameter $\psi$. The point $\psi = 0$ corresponds to a point on the shadow boundary ($y = 0$) for a given value of frequency and $z$. Positive (negative) values of $\psi$ correspond to positive (negative) values of the height above the shadow boundary, $y$, for given values of frequency and $z$. The experimental data correspond to frequencies of 5 kHz ($\square$), 15 kHz ($\circ$), and 25 kHz ($\ast$). The source was at a net distance of 243.1 cm from the apex of the ridge. The solid curve represents the insertion loss as predicted by the theory for knife-edge diffraction from a rigid surface.
ENDNOTES


BIBLIOGRAPHY

ALMGREN, M. [1986], Scale model simulation of sound propagation considering sound speed gradients and acoustic boundary layers at a rigid surface, Report F86-05, Chalmers University of Technology, Göteborg, Sweden.
ANSI STANDARD of the ACOUSTICAL SOCIETY of AMERICA [1978], Method for the Calculation of the Absorption of Sound by the Atmosphere, American Institute of Physics, ANSI S1.26-1978.


MATHEW, J. and R.J. ALFREDSON [1982], An Improved Model for Predicting the Reflection of Acoustical Transients from Fibrous Absorptive Surfaces, J. Sound Vib. 84, 296-300.


NICHOLSON, J.W. [1910], On the Bending of Light Waves Round a Large Sphere, I, Phil. Mag. 19, 516-537.


Proposal.

PEKERIS, C.L. [1947], The Field of a Microwave Dipole Antenna in the Vicinity of the Horizon, Part 2, J. Appl. Phys. 18, 1025-1027.
USLENHI, P. [1964], Radar Cross Section of Imperfectly Conducting Bodies at Small Wavelengths, Alta Frequenza 33, 541-546.
PAPERS AND PUBLICATIONS ASSOCIATED WITH THIS PROJECT


of Finite Impedance, in Advances in Computer Methods for Partial Differential Equations
• VI, R. Vichnevetsky and R.S. Stepleman eds., Inter. Assoc. for Math. and Comp. in Sim.
(IMACS), New Brunswick, New Jersey.

[12] PIERCE, A.D. and PEI-TAI CHEN [1987], Echoes from Hills. paper presented at the 113th
mtg. of Acoust. Soc. Am., Indianapolis, Indiana (abs. in J. Acoust. Soc. Am. Suppl. 1, 81,
S97).

Finite Surface Acoustic Impedance on Sound Fields near a Smooth Diffracting Ridge, to be
ABSTRACTS OF TWO FORTHCOMING PRESENTATIONS

The following pages reproduce two abstracts that have been submitted for presentation at two different conferences. The first one will be presented by Yves Berthelot at INTERNOISE '88, to be held in Avignon, France, August 30-September 1, 1988. The second one will be presented by James A. Kearns at the upcoming 115th meeting of the Acoustical Society of America, to be held in Seattle, WA, May 16-20, 1988.
Les Effets de Diffraction Associés avec le Rayonnement d'une Source Sonore Proche d'une Surface ni Parfaitement Plane ni Parfaitement Réfléchissante. Yves H. Berthelot, Allan D. Pierce, Ji-Xun Zhou, and James A. Kearns, School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia, 30332-0405, USA.

Lorsqu'une source sonore est proche d'une surface ni parfaitement plane ni parfaitement rigide, un champ sonore très complexe s'établit. Pour essayer de mieux comprendre cette classe de problèmes, des expériences de modélisation sont en cours d'études pour vérifier la théorie des expansions asymptotiques (Matched Asymptotic Expansions) appliquées à la théorie de Fock-van der Pol-Bremmer pour la diffraction des ondes par une surface courbe d'impédance finie. En particulier, on présente des résultats (théoriques, numériques, et expérimentaux) modélisant le champ acoustique diffracté par une ou deux collines recouvertes soit par de la terre sèche soit par de l'herbe. On s'intéresse particulièrement à la région de pénombre acoustique et au phénomène d'échos. [Recherche financée par NASA-Langley].

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Computational studies of the diffraction integral occurring in the MAE theory of sound propagation over hills and valleys. James A. Kearns, Ji-xun Zhou, Yves H. Berthelot, and Allan D. Pierce (School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332)

An important class of problems pertaining to outdoor sound propagation is that of the diffraction which occurs when the ground is neither perfectly flat nor perfectly rigid. Such problems are encountered in the study of long range propagation of sound over hills and valleys. It has been shown previously [J. Acoust. Soc. Am., Suppl. 1, 79, S30-31] that the theory of matched asymptotic expansions allows one to express the diffracted field in terms of a complex integral involving Airy functions of complex argument. In some limiting cases, the diffraction integral reduces to some computationally very efficient forms: an equation based on geometrical acoustics in the illuminated region, a creeping wave series in the shadow zone, and a knife-edge Fresnel diffraction integral far behind the ridge. In the present paper, the transition between these different regimes is investigated numerically by computing the general integral, and particular attention is given to the matching with the creeping waves series solution in the penumbra region. Computational results are compared with data obtained in laboratory scaled experiments. [Work supported by NASA Langley Research Center.]

Technical committee: Physical acoustics or Noise (Outdoor sound propagation)

If possible, I would like to see this paper presented in the same session as any paper by Gilles Daigle on outdoor sound propagation.

Subject classification number: 43.50.V or 43.85.B

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Special facility: N/A