Shuttle Computational Grid Generation

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ABSTRACT

The well known Karman-Trefftz conformal transformation, consisting of repeated applications of the same basic formula, were found to be quite successful to body, wing, and simple wing-body cross sections in recent years. In this report, it is intended to extend this grid generation technique to cross sections of more complex forms, and also intended to make the grid generation process more automated.

Computer programs were written for the selection of "hinge points" on cross section with angular shapes, the Karman-Trefftz transformation of arbitrary shapes, and the special transform of hinge point on the imaginary axis.

The present work is served as a feasibility study for the future application of conformal mapping grid generation to complex three dimensional configurations. Examples such as Orbiter vehicle section and a few others were used in the present study. Computer programs are not included in this report.
INTRODUCTION

Conformal mapping is one of the several techniques used in numerical grid generation (1-2). For shapes with corners, the Karman-Trefftz transformation is commonly used to smoothen the angle while mapping the physical configuration into near circle on the computational plane. Grids can be easily generated on computational plane, and inverse transform will map the grids back to the physical plane. On these grids, the appropriate numerical methods can be applied to solve the flow dynamics problems. Just a few are listed in the reference (3-4). Care must be maintained for using those transforms and for complex geometries, difficulties could be encountered.

In the present work, it is intended to extend and automate the transforms for any arbitrary shapes. This should include the hinge points selections, the Karman-Trefftz with arbitrary branch cuts, automatic grid generation on computational plane, and the inverse transforms to the physical plane. Computer programs were written but not included in this report, the results are shown graphically. For these graphics, the vertical axis and horizontal axis are not at exactly the same scale, so, there is a slight distortion for the figures. To show the sequence of transformation, the figures on the same page should be read from left to right and from top to bottom.
LOCATE THE HINGE POINTS

Hinge points are used in the Karman-Trefftz transformation to smoothen the corners of the configurations to be transformed. They're located at these corners or very close to them. If a large number of coordinates are needed to describe the geometries, direct observations to locate the hinge points may be time consuming and less accurate. So, a simple program was written to automate this process of selection.

Refer to figure 1, for an arbitrary shape, a number of complex coordinates (Z (I), I = 1, N) are used to define the shape in the complex plane. Start from one end, Z (1), compute the angles at all the coordinates in sequence, compare with the pre-determined values of maximum deviations from 180°, the location of the hinge points (as well as the Δ used in the transformation) can be selected by the computer. In our scheme, not all the locations exceed the limits are considered as hinge points, only the locations that represent the largest curvature locally are selected as hinge points. The computer program is proved to be successful on Shuttle sections as well as a few other geometries.
\[ \theta \] is computed from the following complex expression

\[
\frac{Z(I+1) - Z(I)}{Z(I-1) - Z(I)}
\]

If the angle computed is negative, add \( 2\pi \) to obtain \( \theta \)

Fig. 1 Selection of hinge points
The Karman-Trefftz transformation is well known in the publications (5). In the present work, it is intended to write a program that can be applied to any hinge point not on the imaginary axis. For each hinge point, formulate the following Karman-Trefftz transforms

\[
\frac{W(I) - 1}{W(I) + 1} = \left( \frac{Z(I) - H}{Z(I) + H} \right)^\delta, \quad I = 1, N
\]

\[
\delta = \frac{\pi}{2\pi - \alpha}
\]

where
- \(Z(I)\) = coordinates used to define the body
- \(W(I)\) = transformed coordinates of the body
- \(\alpha\) = measure of interior angle at hinge point
- \(H\) = coordinate of the hinge point
- \(H^*\) = complex conjugate of \(H\)

Refer to figure 2, at the hinge point, the plane is divided into four quarters but labeled them quarter 0 to quarter 9. Each coordinate of the body is in one particular quarter. The quarter numbers for coordinate \(Z(1)\) and \(Z(N)\) can be obtained easily. The quarter number for other coordinates can be obtained by comparing with the adjacent coordinates. This should be done in sequence from \(Z(1)\) to \(Z(N)\) or from \(Z(N)\) to \(Z(1)\). It can also be done from \(Z(1)\) to hinge point and then from \(Z(N)\) to hinge point. According to the general formulas mentioned in the figure, the Karman-Trefftz transformation can be completed. The difficult job of locating the branch cut is easily solved. Based on this concept, a computer program was written for the transformation. It proves to be successful as can be seen from a test example shown in figure 3. Other examples using this program are shown in the later sections.
1. IF sign of $X_i = \text{sign of } X_{i-1}$
   and sign of $Y_i = \text{sign of } Y_{i-1}$
   \[ ID(i) = ID(i-1) \]

2. IF sign of $X_i = \text{sign of } X_{i-1}$
   and sign of $Y_i = \text{sign of } Y_{i-1}$
   \[ ID(i) = ID(i-1) + (-1) \]

3. IF sign of $X_i = \text{sign of } X_{i-1}$
   and sign of $Y_i = \text{sign of } Y_{i-1}$
   \[ ID(i) = ID(i-1) - (-1) \]

$\theta_B = (-2 + \frac{ID(i)}{2}) \pi + \text{ATAN}(Y/X)$

Fig. 2 Karman-Trefftz Transform
Fig. 3 Test example of Karman-Trefftz Transform from physical plane to computational plane
CONFORMAL MAPPING FOR HINGE POINT ON IMAGINARY AXIS

For hinge point on imaginary axis, the standard Karman-Trefftz transformation fails. But a modified transform formula can be used

\[
\frac{W(I) - i}{W(I) + i} = \left( \frac{Z(I) - H}{Z(I) + H} \right)^\delta
\]

\[
\delta = \frac{\pi}{\theta \alpha}
\]

\( \alpha \) is shown in figure 4. This special transform should be applied to coordinate Z (1), if applied to coordinate Z (N), i in the formula should be replaced by -i. This transform can convert the angle at Z (1) or Z (N) to 90°. At the same time, if it is properly applied, it could lift the other hinge point on the imaginary axis. Special attention should be paid to the location of real axis (refer to figure 4), but within the allowable range, it has minor effect to the transformation as shown in figure 5. In figure 6 and figure 7, an example of rocket section is transformed in sequence from physical plane to computational plane. In this case, both the special transform and the standard Karman-Trefftz transforms are used. In figure 8 and figure 9, similar transformations were applied to an Orbiter vehicle section. Both transforms were done by computer programs. From these examples, it shows that with the suitable combination of both transforms, many problems with complex geometries can be solved.
Fig. 4 Conformal mapping for hinge point on imaginary axis
Fig. 5 Effect of real axis position variations (First figure is before transformation.)
Fig. 6 The sequence to transform a finned rocket section into near circle in computational plane (continue in Fig. 7)
Fig. 7 Continuation of Fig. 6
Fig. 8 Orbiter vehicle section transformation from physical plane to computational plane (continue in Fig. 9)
Fig. 9 Continuation of Fig. 8
A simple program has been written to generate grids on computational plane. The program is written in such a way that there is always a line passing the hinge point. With minor changes, the programs used earlier can be used for inverse transformations. The grids on the computational plane can be mapped to the physical plane. In figure 10, the grids on an Orbiter vehicle section are shown on computational plane as well as on physical plane. In figure 11, that is for rocket section.

In conclusion, the computer programs developed in this study are quite general and can be used to solve complex shapes. But further study is needed to locate or generate hinge points automatically on a smooth but large curvature configuration.
Fig. 10 Grids on an Orbiter vehicle section on computational plane and on physical plane
Fig. 11 Grids on Rocket Section
REFERENCES


