INVENTORY BEHAVIOR AT REMOTE SITES

Final Report

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Abstract

An operations research study is conducted concerning inventory behavior on the Space Station. Historical data from the Space Shuttle is used. The results demonstrate a high logistics burden if Space Shuttle reliability technology were to be applied without modification to Space Station design (which it will not be). Effects of rapid resupply and on board repair capabilities on inventory behavior are investigated.

Introduction

"Adde parvum parvo magnus acervus erit."
"Add little to little, and there will be a great pile."

Ovid, ca. 50 A. D.

Intuition is linear and works best on small systems. Large systems with additive effects are not easy to understand. They produce surprises, like Ovid’s pile. Large systems with nonlinear components and additive effects are worse; they produce shocks. These systems cannot be intuitively understood. They must be mathematically understood, or not understood at all.

Inventory systems are nonlinear in important ways, and they are additive as well. This report describes an operations research study of local inventory required to support remote site operations. The study attempts to quantify inventory requirements for generic remote site operations in such a way as to anticipate the order of inventory demands early enough to permit their incorporation into the design process.

As a test case, historical data from Space Shuttle operations is used. This data was very kindly provided by Rockwell International, which deserves much of the credit for the usefulness of these results. In some ways, Space Shuttle data is not appropriate for remote site modeling, in that the Shuttle was designed for ground maintenance after each 1 week mission, whereas the Space Station will never receive ground maintenance, and has a 30 year design life. However, Space Shuttle module reliability was probably increased to the limits of cost effectiveness for 1970 era technology in an attempt to reduce maintenance costs and increase vehicle availability. Also, it is tempting to rely on proven Space Shuttle technology when designing the Space Station. The analysis does not show how Space Station inventory will behave. It does show how Space Station inventory would behave if Space Station technology had the same overall failure rate and repair time distribution as Shuttle technology, which it most
certainly will not.

More significant than the actual numbers is the demonstration that inventory stockouts (that is, exhaustion of inventory for some classes of modules) will always be a factor. If there is a module failure, and no modules of that type are in inventory, there are three courses of action: 1) ignore the failure, 2) repair an already failed module, or 3) get a replacement module by extempore resupply. If the failure is in a redundant system, option (1) may be attractive. If not, (2) or (3) will be the options of choice. The results of this analysis strongly suggest that (2) and (3) will both be desirable during Space Station operations, as they are at any remote site operation.

It is discovered on analysis of Space Shuttle operations records that systems comparable to the Space Shuttle require unexpectedly large inventory for long missions. It is further discovered that, even with a fairly large spares inventory, shortages will occur at a significant rate. Makeup of these shortages, either through repair and improvisation, or through special equipment delivery, may be a significant problem in space station and planetary expedition logistics.

The mathematics, the data, the analyses, and the conclusions are described separately to permit replication by the interested reader. A reader not interested in replication or time consuming development of mathematical intuition in this area may skip to the results section.

Mathematical Approach

The techniques used are elementary. Elementary statistical methods are to be preferred over advanced methods on grounds of clarity if they correctly model all significant aspects of a situation. The approach described below is correct in the sense that it provides an estimate of stockout probability distribution parameters which is moderately reliable and can be shown on theoretical grounds to be insensitive to actual item failure distributions, which are not ordinarily known. I make no claims to the method's optimality if actual item failure distributions are known exactly.

The statistical method used to set inventory stock levels and predict number of stockouts is developed below:

1) Independent actors, each performing the same act with an idiosyncratic distribution of times between performance, will as a whole perform the act with a negative exponential distribution of times between
performance. For example, each person in a dialing area makes telephone calls on an individual basis. The time between calls for any individual is a random variable, and the distribution with which this variable is associated will, in general, vary from one person to the next. That is, each person's time between calls will have an associated and unique, or idiosyncratic, distribution. If calls made by all people in the dialing area are observed (by monitoring call arrivals at the telephone exchange), it will be observed that the time between arrival of new calls will behave like a random variable with a negative exponential distribution. The same argument holds true for arrival of customers at a bank, arrival of cars at a gas station, or "arrival" of equipment failures. The density function for a negative exponential distribution is:

\[ f(\lambda, t) := \lambda \cdot \exp(-\lambda t) \]

"\( \lambda \)" is the average arrival rate, in events/unit time. "\( t \)" is the time between events. It is obtained by adding up individual arrival rates for all independent actors. The average time between arrival is, of course, \( 1/\lambda \); the variance in time between arrivals is \( 1/(\lambda^2) \). Thus, high rates of arrival give a smaller variance in interarrival time than do large rates. That is, the process varies more at low arrival rates than at high.

2) Any process with density function \( f(\lambda, t) \) of interarrival time will have a Poisson distributed number of events per unit time. That is, if instead of observing time between events, one counts events per unit time, one will find that the number of events per unit time is Poisson distributed if and only if the time between events has a negative exponential distribution \( f(x) \).

The Poisson distribution is as follows:

\[ P(\lambda, N) := \exp(-\lambda) \cdot (\lambda^N)/N! \]

"\( \lambda \)" is defined as before (and is indeed identical to the \( \lambda \) for \( f(\lambda, t) \)). \( \lambda \) is also the mean and variance of \( P(\lambda, N) \). Note that variance of \( P(\lambda, N) \) increases with increased \( \lambda \), whereas that of \( f(\lambda, t) \) decreased under the same condition. The higher the arrival rate, the more variance, hence the more spread in observed number of arrivals over several observations. "\( N \)" is the number of events. Note that \( N \) is a cardinal number, that is a member of the set \( \{0, 1, 2, \ldots\} \). \( N \) cannot be 3.5, or -2, as these cannot count events. This will cause certain minor problems later, and has the immediate consequence that \( P(\lambda, N) \) is not a density function, but rather a distribution. In general, the probability of \( N \) failures per unit time will
be given by $P(\Lambda, N)$. The above can be re-written recursively as:

$$P(\Lambda, 0) := \exp(-\Lambda)$$

$$P(\Lambda, N) := P(\Lambda, N-1) \cdot \frac{\Lambda}{N}$$

The behavior of the function $P(\Lambda, N)$ as expressed recursively is clear. If $\Lambda > 1$, it $P(\Lambda, N)$ increases until $N > \Lambda$, then decreases. If $\Lambda < 1$, $P(\Lambda, N)$ always decreases. Note that $P(\Lambda, N)$ is always positive and finite, as one would expect a probability to be.

3) In planning for inventory, one might be interested in having just enough inventory to avoid stockout. That is, one wants to have enough in inventory so that one never runs out, and at the same time wants nothing superfluous in inventory so that one does not waste money. This is impossible, as will be shown.

If failures are Poisson distributed, then the probability of having $N$ failures or less is given by the cumulative Poisson distribution,

$$C(\Lambda, N) := \sum_{0}^{N} P(\Lambda, N)$$

Obviously, the probability of covering all failures when there are $N$ items in inventory and $\Lambda$ is constant is just $C(\Lambda, N)$. By the same token, the probability of stockout must be $1 - C(\Lambda, N)$.

The behavior of $1 - C(\Lambda, N)$ is therefore of interest. Let us re-write:

$$1 - C(\Lambda, N) = \sum_{N+1}^{\infty} P(\Lambda, N)$$

and we have seen above that $P(\Lambda, N)$ will be finite (although possibly very small) for all cardinal numbers $N$.

From this it is apparent that $1 - C(\Lambda, N)$ will always be finite, no matter how large $N$, and that (accordingly) there is always some probability of stockout, no matter how large $N$, the number of replacement units in inventory.

4) Fortunately, the above is not merely an impossibility proof. It also leads to an algorithm for sizing inventory. If one is satisfied with a fairly small desired probability of stockout per mission, $P_{sd}$, then, if $\Lambda$ per mission is known, one need only specify an $N$ large enough such that:

$$1 - C(\Lambda, N) < P_{sd} \text{ and } 1 - C(\Lambda, N-1) > P_{sd}$$
N is the smallest number of items which can be kept in store if Psd is to be met. N is uniquely specified by this algorithm, which may be written as N(Psd). The actual probability of stockout is, of course,

\[ P_{sa} := 1 - C(\Lambda, N(Psd)) \]

\( \Lambda \) for the mission is \( \Lambda \) per unit time multiplied by number of units of time per mission. Note that, in general, \( P_{sa} < Psd \). This is a result of \( N \) being a cardinal number, and is the inconvenience mentioned above. The probability of 1 stockout is \( P(\Lambda, N+1) \), the probability of 2 stockouts is \( P(\Lambda, N+2) \) and so on. The expected number of stockouts for the entire mission is thus \( E(s) := \sum_{n=N}^{\infty} P(\Lambda, n) \). Ordinarily only a few terms of this sum must be calculated before \( P(\Lambda, n) \) becomes negligible.

5) One must somehow specify Psd. For inventories containing only one kind of item this is simple. One defines a cost function \( Ma(N(Psd)) \) and minimizes it. For inventories containing several noninterchangeable classes of items, each with its own \( P_{sa J} \) (\( J \) being an index describing the kind), but supporting a single system (such as the Space Station), this procedure is not satisfactory. The cost function is properly a function of system reliability, which is a function of subsystem reliability, which is in turn a function of the number of working modules within each subsystem, which is decreased by each stockout. While the \( P_{sa J} \) must obviously meet certain criteria, because these criteria are dependent on the criticality of the subsystem supported by units of kind \( J \), there emerges an additional, system wide, criteria: the total number of stockouts during a mission (TSM). If \( J \) is over about 10, one would expect system degradation and a TSM nonlinearly related to the \( P_{sa J} \).

6) By the reasoning in (1), the random variable representing time between stockouts for the inventory as a whole should have a negative exponential distribution. Total\( \Lambda \) should have a value equal to the sum of the hazard rates over all classes in inventory. The hazard rate for each kind in inventory over the entire mission is the expected value of stockouts the kind, \( E(s_J) \). That is,

\[ \text{Total}\Lambda := \sum_{all J} E(s_J) \]

7) Given the distribution types, and Total\( \Lambda \), one can calculate probability of having \( N \) or fewer stockouts per mission by the cumulative Poisson as above:

\[ C(\text{Total}\Lambda, N) \]
This can be used to calculate a range of stockouts most likely to be seen. We assume that things which happen less than 5% of the time are often not seen; thus, the smallest \( N \) seen would be \( N_{\text{small}} \) such that

\[
C(\text{Total}\Lambda, N_{\text{small}} - 1) < 0.05 \\
\text{and} \\
C(\text{Total}\Lambda, N_{\text{small}}) > 0.05
\]

Similarly, the largest \( N \) seen would be \( N_{\text{large}} \) such that

\[
C(\text{Total}\Lambda, N_{\text{large}} - 1) < 0.95 \\
\text{and} \\
C(\text{Total}\Lambda, N_{\text{large}}) > 0.95
\]

These are approximations, of course, and merely show that one has at least a 5% chance of seeing \( N_{\text{small}} \) or less and at most a 5% chance of seeing \( N_{\text{large}} \) or more.

That is a narrative description of the technique. Perhaps it should be illustrated graphically.

The inventory of one kind of unit could be sized by drawing its Cumulative Poisson \( C(\Lambda_j, N) \), and using the curve to translate \( P_s \) into minimum \( N \) as in Graph A, which looks like a flight of stairs. It is obvious that increasing the number of units in stock from 3 to 4 will reduce \( P_s \) to below the 5% minimum specified.

By the same token, given \( \text{Total}\Lambda \) for the completed inventory, stockouts could be determined by drawing its Cumulative Poisson \( C(\text{Total}\Lambda, N) \), as in Graph B, and using the curve to translate probabilities of 0.05 and 0.95 into numbers of stockouts as shown. Again, it is obvious that the 5% limits of observation lie somewhere around 33 and 55.

Buried in the above analysis is the assumption that stockout hazard rates are constant. In fact, they increase as stock on hand is consumed. In the normal Space Station case, inventory will not be severely depleted during the mission. Perhaps half of it will be used up, and perhaps 1.5% of the inventory classes by module kind will have stockout. One could therefore confidently predict an increase in stockouts during the latter part of a mission. Unfortunately, there seems to be no easy way to solve analytically for stockout distribution as a function of time, and simulation would be necessary to determine this distribution. The distribution given is a valid average distribution for the mission; one would expect the lower limit to be more...
**List of function values.**

<table>
<thead>
<tr>
<th>CumPoisson(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000000</td>
</tr>
<tr>
<td>1.0000000</td>
</tr>
<tr>
<td>2.0000000</td>
</tr>
<tr>
<td>3.0000000</td>
</tr>
<tr>
<td>4.0000000</td>
</tr>
<tr>
<td>5.0000000</td>
</tr>
<tr>
<td>6.0000000</td>
</tr>
</tbody>
</table>

\[ \lambda = 1.67 \text{ failures/mission} \]

(6 modules, 0.7779 hrs MTBF)

- 3 modules: \( \approx 8.9\% \) chance of stockout.
- 4 modules: \( \approx 2.8\% \) chance of stockout.
- 5 modules: \( \approx 0.74\% \) chance of stockout.

**Graph A**
\( \lambda : 43.4 \text{ stockouts/mission} \)

\[
\begin{array}{c|c}
\text{CumPoisson} & \text{CumPoisson}(x) \\
30.000000 & 0.020759638 \\
31.000000 & 0.030820003 \\
32.000000 & 0.044546850 \\
33.000000 & 0.062973483 \\
34.000000 & 0.085230079 \\
35.000000 & 0.111246892 \\
36.000000 & 0.140376007 \\
37.000000 & 0.172560927 \\
38.000000 & 0.208586094 \\
39.000000 & 0.248400715 \\
40.000000 & 0.292074638 \\
41.000000 & 0.339546256 \\
42.000000 & 0.390309518 \\
43.000000 & 0.444546850 \\
44.000000 & 0.502228511 \\
45.000000 & 0.563360927 \\
46.000000 & 0.628734832 \\
47.000000 & 0.700230079 \\
48.000000 & 0.777260759 \\
49.000000 & 0.859309518 \\
50.000000 & 0.946222222 \\
51.000000 & 1.03820003 \\
52.000000 & 1.136288927 \\
53.000000 & 1.240376007 \\
54.000000 & 1.350507596 \\
55.000000 & 1.467044547 \\
56.000000 & 1.590215472 \\
57.000000 & 1.719222222 \\
58.000000 & 1.853260759 \\
59.000000 & 1.992622222 \\
60.000000 & 2.137348322 \\
61.000000 & 2.28780003 \\
62.000000 & 2.443946850 \\
63.000000 & 2.605973483 \\
64.000000 & 2.774730079 \\
65.000000 & 2.950309518 \\
66.000000 & 3.132822222 \\
67.000000 & 3.322507596 \\
68.000000 & 3.519222222 \\
69.000000 & 3.723348322 \\
70.000000 & 3.934236653 \\
71.000000 & 4.151507596 \\
72.000000 & 4.375904346 \\
73.000000 & 4.607607596 \\
74.000000 & 4.846822222 \\
75.000000 & 5.093348322 \\
76.000000 & 5.347300799 \\
77.000000 & 5.608822222 \\
78.000000 & 5.878822222 \\
79.000000 & 6.157507596 \\
80.000000 & 6.444946850 \\
81.000000 & 6.741075963 \\
82.000000 & 7.045822222 \\
83.000000 & 7.359222222 \\
84.000000 & 7.681348322 \\
85.000000 & 8.012154721 \\
86.000000 & 8.352607596 \\
87.000000 & 8.703095186 \\
88.000000 & 9.064346850 \\
89.000000 & 9.444946850 \\
90.000000 & 9.847154721 \\
\end{array}
\]

Graph B
typical of the mission's early part and the upper limit more typical of the mission's later part. Again, simulation is necessary if the distribution is to be numerically specified as a function of time.

Data

Two kinds of data can be used. The first set is for initial sizing studies, and is approximate. A uniform MTBR (Mean Time Between Removals) of 8,000 hours, MTTReplace (Mean Time To Replace) of 4 hours, MTTRepair (Mean Time To Repair) of 80 hours, and average commonality of 2 items per type are reasonable starting values.

Historical data on Shuttle subsystem characteristics was provided by Rockwell International. The data set used was the Logistics Master Control File, Report No. 9100, as of 07/13/87. Only QPEI / WUC, MDR, and MTBF data were used. QPEI / WUC (Quantity Per End Item) is the number of the item per individual OV. MDR (Maintenance Demand Rate) is number of unexpected removals per each 1000 hours of OV operation. Therefore, MTBR (Mean Time Between Removals) = 1000/MDR. MTTRepair (Mean Time To Repair) is the actual time on the repair bench required to restore a removed module to fully tested and operational status. RTAT is the number of hours including handling required for module repair. Generally, RTAT >> MTTRepair. Since handling in the sense of transport to repair facilities will not be a major factor at a remote site, RTAT was not used in the analysis. MTTReplace (Mean Time To Replace) was arbitrarily set to 4 hours.

There are 2763 classes of modules in Report No. 9100. Since STOCK2 accepts only 400 module types, some data selection and trimming was necessary.

The first decision was to use only data with MTTR specified. Of 2763 classes, only 1702 (62%) specified MTTR. A t test (see discussion and Table, below) conducted on hazard rate and population supported per class showed no reason to reject the hypothesis of equal means, so the subset was accepted as representative. This subset will be called "MTTR Subset".

The second decision was to select a random subset of MTTR specified data. This involved writing a program (RAND) to generate 400 random numbers sorted in increasing order over a range equal to the cardinality of MTTR Subset. These numbers were treated as indices specifying a second subset of MTTR Subset, called Subset A (which accordingly has 400 classes). A t test (see discussion and Table below) conducted on hazard rate, population supported per class, and mean time to repair showed no reason to reject the hypothesis of equal means, so the subset was accepted as representative. Subset A was
actually used in analyses. It would be desirable to conduct these analyses with a Subset B, C, etc., but time did not permit this.

Statistical characteristics of the data are as follows:

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Reciprocal Sample Average Hazard Rate</th>
<th>Sample Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete</td>
<td>8291 hr</td>
<td>8286 hr</td>
</tr>
<tr>
<td>MTTR&gt;0</td>
<td>8264 hr</td>
<td>8264 hr</td>
</tr>
<tr>
<td>Not given</td>
<td>8333 hr</td>
<td>8321 hr</td>
</tr>
<tr>
<td>Subset A</td>
<td>7231 hr</td>
<td>7231 hr</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Average Sample MTT Repair</th>
<th>Sample Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete</td>
<td>51 hr</td>
<td>253 hr</td>
</tr>
<tr>
<td>MTTR&gt;0</td>
<td>82 hr</td>
<td>318 hr</td>
</tr>
<tr>
<td>Not given</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Subset A</td>
<td>93.3 hr</td>
<td>348 hr</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Average Sample Population Per Class</th>
<th>Sample Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete</td>
<td>1.74</td>
<td>2.62</td>
</tr>
<tr>
<td>MTTR&gt;0</td>
<td>1.85</td>
<td>2.91</td>
</tr>
<tr>
<td>Not given</td>
<td>1.55</td>
<td>2.07</td>
</tr>
<tr>
<td>Subset A</td>
<td>1.91</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Note: Total population for any data set can be obtained by multiplying Average Population per Class by Number of Classes.
<table>
<thead>
<tr>
<th></th>
<th>Hazard Rate</th>
<th>MTT Repair</th>
<th>Population Per Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete and MTTR Subset</td>
<td>t = 4.72E-4</td>
<td>t = 3.41</td>
<td>t = 1.27</td>
</tr>
<tr>
<td></td>
<td>Nu = 5.9E23</td>
<td>Nu = 3.0E3</td>
<td>Nu = 1.2E8</td>
</tr>
<tr>
<td>MTTR Subset and Subset A</td>
<td>t = 0.825</td>
<td>t = 0.594</td>
<td>t = 0.360</td>
</tr>
<tr>
<td></td>
<td>Nu = 9.5E21</td>
<td>Nu = 570</td>
<td>Nu = 1.5E6</td>
</tr>
</tbody>
</table>

For the Nu values given, the t random variable is distributed normally for all practical purposes. A t<1.282 implies acceptance of the null hypotheses (that data set means are equal) on the 10% level.

The final table, listing t statistics, suggests that the MTTR Subset is very similar to the complete data set except, of course, in MTTRRepair. The MTTR Subset and Subset A appear alike for all data types. Accordingly, the behavior of inventory for Subset A should statistically resemble that of the MTTR subset for all data types, and should statistically resemble that of the complete data set for all data types except MTTRRepair.

Subset A is stored under file name STOCK2.STK on floppy disk, and will be included with STOCK2. Use STOCK2's export feature to change STOCK2.STK into STOCK2.ASC, which can be imported by a database program as a standard ASCII file with comma as delimiter.

Software and Computer

The software and hardware used are commonly available; those readers wishing to check or extend results may obtain a copy of the software and data on request to the author, if said request is accompanied by a 5 \( \frac{1}{4} \) inch MS-DOS floppy diskette, formatted for a 360k drive. The program is written in Turbo Pascal, and it is advisable to use a database program, such as Borland's Reflex, for data analysis. It should run on any MS-DOS machine.

Stock2R.com does not require a mathematics coprocessor, Stock2X.com does. The program is written in Turbo Pascal, by Borland International, and you will need a compiler for this language should you wish to modify and re-compile the program. Please use STK2R or STK2X to install the program if its messages are not properly displayed on your terminal screen. Instructions for the STK2 programs are in document INSTALL.TXT.

A program to generate random numbers uniformly distributed over a specified range is included as RAND and
RANDX. Again, the X suffix means a mathematics co-processor is necessary. Installation is by RND and RNDX. The source file is provided, but requires sort procedures from Borland's Turbo Toolbox to compile.

A random subset of 400 module descriptions from the Space Shuttle's operating history is on the diskette as STOCK2.STK. It is suggested that this information (which includes output data) be imported to STOCK2 by the "typed file" initialization option. Detailed analysis of the results should by choice be attempted with the help of a good database program, as 400 items are too many for the crude display facilities in STOCK2. After alteration by the database program (Reflex by Borland may be a good choice) the data can be re-imported to STOCK2. Use comma as the delimiter.

Although hazard rates scale linearly, confidence limits do not. Accordingly, a program, called LIMITS.com, has been provided to scale the results of STOCK2 up or down. To obtain confidence intervals for a target installation, such as the Space Station, scale up the global hazard rates from Stock2 by:

\[
\text{TargetLambda} = \frac{\text{Stock2Lambda} \times \text{TargetModuleName} / \text{Stock2ModuleName}}{\text{Stock2Lambda} + \text{Stock2ModuleName} / \text{Stock2ModuleName}}
\]

with Stock2Lambda and Stock2ModuleName being the values from Stock2, and TargetLambda and TargetModuleName being the values of the target installation. For example, if Stock2 gives a repair Lambda of 3.16 for 400 classes of modules with an average of 1.91 modules / class and the Space Station is assumed to have 2578 modules, then the repair hazard rate for the Space Station is:

\[
3.16 \times 2578 / (400 \times 1.91) = 10.7
\]

To obtain Space Station confidence intervals, run LIMITS and specify a hazard rate of 10.7. Use the same scale factor for inventory.

**Methods of Analysis**

Item demand will change as the population of supported modules ages. If MTBF only changes by a few percent within an inventory period, this can be treated by setting MTBF to the average for the period in question. Calculations involving multiple periods use the appropriate average MTBF for each period. For the purposes of this report, a constant MTBF is the appropriate choice.
A rough feel for this class of inventory problem can be acquired using Borland's Eureka, and interactive equation solver described under "Software and Computer." The algorithm above can be implemented quite directly, and run for simple cases. In particular, it is possible to graphically illustrate the relation between the Poisson and Cumulative Poisson distributions for various values of hazard rate, and to develop one's intuition for stocking levels and stockout probabilities.

In general, one finds that (a) stock levels increase nonlinearly as probability of stockout decreases and that (b) number of modules in inventory divided by number of modules in use tends to decline as population supported increases. The results of such investigations are not reported here, as they are not based on historical data.

When one wishes to use non-aggregated historical data, the sheer mass of computation involved overtaxes Eureka, and a specialized program must be used. This program permits direct entry of equipment characteristics, and executes the mathematical approach described above to determine replacement and stockout hazard rates for each kind of equipment and globally, for the entire inventory. This program, STOCK2, was used to conduct the following preliminary analyses:

1. Inventory behavior under varying uniform stockout probabilities.

Specified probability of stockout for all parts types are set to a uniform value for each run. Inventory size, repair event hazard rate, and 90% confidence interval for number of repair events are calculated.

2. Effect of provision for module repair / cannibalization on inventory behavior and crew size.

Probability of exhaustion is set independently for each part type so as to minimize stockout probability for part types with large repair times, and inventory level is kept constant by increasing stockout probability for part types with small repair times. The net result is to keep inventory size approximately constant while decreasing time required to repair or cannibalize failed modules. After establishing a set of probabilities at average specified probability of stockout of 0.1, probabilities were linearly transformed to obtain other average specified stockout probabilities, and the effect of this on repair time and inventory stock level were recorded.

3. Effect of mission length on inventory behavior.

Longer and shorter mission times are considered with fixed average specified stockout probability. The
shortest time is the length of a Space Shuttle mission; the longest is about that of a round trip Mars mission. The behavior of inventory at an average specified stockout probability of 0.01 is considered.

4. Effect of commonality on inventory behavior.
Greater commonality was simulated by multiplying population supported in Subset A by 5 and by 10, then scaling all results up to the same Space Station number of modules.

There was not sufficient time to conduct these analyses thoroughly. It is likely that only the stronger effects were discovered, and that many weak effects remain as yet unknown.

Results

General:

Space Station is assumed to have 2578 ORUs at 1.91 ORU/Class, for a total of 1350 classes.

On at most 5% of the missions values will exceed the Upper Limit; on at least 5% of the missions values will be below the Lower Limit; the two limits thus form an approximate 90% confidence range. An exact 90% confidence range cannot be specified for events, as they cannot be subdivided.

1. Inventory behavior under varying uniform stockout probabilities.

<table>
<thead>
<tr>
<th>Specified Probability of Stockout</th>
<th>Inventory Size</th>
<th>Stockout Hazard Rate</th>
<th>Number of Stockouts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower Limit</td>
</tr>
<tr>
<td>1.00</td>
<td>0</td>
<td>853</td>
<td>806</td>
</tr>
<tr>
<td>0.316</td>
<td>967</td>
<td>155</td>
<td>135</td>
</tr>
<tr>
<td>0.100</td>
<td>1550</td>
<td>51</td>
<td>40</td>
</tr>
<tr>
<td>0.0316</td>
<td>2110</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>0.0100</td>
<td>2800</td>
<td>5.2</td>
<td>2</td>
</tr>
</tbody>
</table>
Graph C: No Repair
Log $P(\text{Stockout})$ vs. Inventory Size

Graph D: No Repair
Log $P(\text{Stockout})$ vs. Stockout Hazard
2. Effect of provision for module repair /
cannibalization on inventory behavior and crewsize.

<table>
<thead>
<tr>
<th>Specified Probability of Stockout</th>
<th>Inventory Size</th>
<th>Stockout Hazard Rate</th>
<th>Number of Stockouts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower Limit</td>
</tr>
<tr>
<td>1.00</td>
<td>0</td>
<td>853</td>
<td>806</td>
</tr>
<tr>
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<tr>
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</table>

<table>
<thead>
<tr>
<th>Specified Probability of Stockout</th>
<th>Average Crew on Repair</th>
<th>Repair Hazard Rate</th>
<th>Number of Repair Hours</th>
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<tbody>
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<td>87700</td>
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<td>1100</td>
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<tr>
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<td>388</td>
</tr>
<tr>
<td>0.0100</td>
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<td>121</td>
<td>103</td>
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</tbody>
</table>
**Graph E: Repair**

Log $P(\text{Stockout})$ vs. Inventory Size

**Graph F: Repair**

Log $P(\text{Stockout})$ vs. Crew on Repair
3. Effect of mission length on inventory behavior.

These calculations use data optimized for repair, as given in STOCK3.STK. Average specified probability of stockout is slightly less than 0.1.

<table>
<thead>
<tr>
<th>Mission Length (Days)</th>
<th>Inventory Size</th>
<th>Stockout Hazard Rate</th>
<th>Number of Stockouts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Limit</td>
<td>Upper Limit</td>
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<tr>
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<td>6</td>
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<tr>
<td>14</td>
<td>542</td>
<td>15.7</td>
<td>9</td>
</tr>
<tr>
<td>90</td>
<td>1930</td>
<td>44.0</td>
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</tr>
<tr>
<td>270</td>
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<td>72.0</td>
<td>58</td>
</tr>
<tr>
<td>900</td>
<td>12100</td>
<td>97.5</td>
<td>82</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Mission Length (Days)</th>
<th>Average Hours Crew on Repair</th>
<th>Repair Hazard Rate</th>
<th>Number of Repair Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower Limit</td>
<td>Upper Limit</td>
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<tr>
<td>7</td>
<td>9.5</td>
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<tr>
<td>14</td>
<td>5.9</td>
<td>658</td>
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<tr>
<td>90</td>
<td>1.6</td>
<td>1110</td>
<td>1060</td>
</tr>
<tr>
<td>270</td>
<td>0.9</td>
<td>1910</td>
<td>1830</td>
</tr>
<tr>
<td>900</td>
<td>0.4</td>
<td>2660</td>
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</tbody>
</table>

4. Effect of commonality on inventory behavior.

<table>
<thead>
<tr>
<th>Commonality</th>
<th>Inventory Size</th>
<th>Replacement Event Hazard Rate</th>
<th>Stockout Event Hazard Rate</th>
</tr>
</thead>
<tbody>
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<td>1.91</td>
<td>1930</td>
<td>855</td>
<td>44</td>
</tr>
<tr>
<td>9.55</td>
<td>1350</td>
<td>852</td>
<td>15</td>
</tr>
<tr>
<td>19.1</td>
<td>1270</td>
<td>852</td>
<td>13</td>
</tr>
</tbody>
</table>
Discussion

As can be seen, there is significant intuitive content to the mathematical treatment:

1) Hazard rates are additive.
2) Inventory levels are nonlinear with protection. High protection from stockouts requires a big inventory. Perfect protection requires an infinite inventory.
3) Moderate protection can be accomplished with a much smaller inventory.
4) If there are a large number of separate inventories, the hazards from "moderate protection" add up to significant numbers of observed stockouts.

Mathematical technique, in this case, becomes most interesting when applied to the interpretation of historical data. The intuitive content requires scaling; are we talking about an inventory of 100 or 1,000,000? Will we run out of stock for 1 or 100 items? Historical data for the Space Station is nonexistent; it has not been built. Let us suppose, however, that the Space Station was to be built using the same general order of reliability as the Space Shuttle. This is unlikely, but makes an interesting test of the mathematics, and gives us some idea of the changes in Shuttle era technology required to make a logistically tractable Space Station.

Inventory behavior under varying uniform stockout probabilities is as expected. Stockout hazard rate decreases nonlinearly as specified probability of exhaustion increases. Inventory size increases exponentially with specified probability of exhaustion. In practice, an inventory size of 2800 supporting 2578 running might be chosen to ensure that there will usually be fewer than 9 uncorrected failures per 90 day mission. This is about one item in inventory for each item running.

The effect of provision for module repair and cannibalization is to trade inventory volume for labor. If one is willing to devote the labor of 1.6 people in orbit, inventory can be reduced by 886 modules, from 2800 modules to 1914 modules, and number of uncorrected failures reduced from a maximum of 9 to a maximum of 0. This requires a distortion of inventory policy, however, in that one must stock more of the high repair time modules and fewer of the low repair time modules to obtain the low average repair time which makes on orbit repairs feasible. If only 0.52 people work on repairs, inventory size is reduced only by 411 modules, and assigning 0.17 people actually increases inventory by 351 modules.

Increased mission length, or time between resupply, tends to increase inventory. Accordingly, a doubling of the Space Station’s mission from 90 to 180 days would be
expected to increase its required inventory. Unfortunately, not enough points could be taken in the available time to quantify this effect reliably. The algorithm would have to be rewritten so as to keep stockouts/unit time constant (assuming repair) or to keep total stockouts constant (assuming no repair or on demand resupply). However, it is clear that increasing mission length while keeping specified probability of stockout constant will produce significant increases in inventory levels. The 900 day mission requires 4.7 items in inventory for every item running.

The mission length data provides a reality check of the data and mathematics. The predicted removal hazard rate for the Shuttle is 136 removals/mission (90% confidence interval of 117 to 155), whereas Rockwell’s data indicate that there are actually 224.5 removals/mission during ground reconditioning. Since not all of the actual removals are for unexpected faults, the prediction seems reasonable.

Specified probability of stockout (SPS) was kept constant for all mission lengths. The effect of SPS is to set a global upper limit on number of stockouts. During the short mission, many of the more reliable modules will not fail, so that the actual probability of exhaustion for that module time will be much less than the SPS for that type, and the global limit will not be approached. As the mission grows longer, the difference between SPS and actual probability of exhaustion diminishes, and the global upper limit is more closely approached. For example, the results listed show a global stockout hazard rate increasing from 10.6 to 97.5. The interesting side effect is that the increase is not linear. That is, the number of stockouts per day becomes less as the mission gets longer, and fewer people are required to repair failures. This suggests that failure repair may be a more attractive option as missions grow longer. Alternatively, by devoting more crew to repair activities, inventories shown might be substantially reduced.

The effect of commonality on inventory behavior is to decrease inventory. While only three points were taken, they suggest that inventory size can be reduced by a maximum of about a half by increasing commonality. It is not clear to what extent commonality can be increased.

Operationally, the above shows conclusively that the Space Station (and other remote site bases) cannot be built under the same design philosophy as the Space Shuttle. The Space Station has a 30 year design life, and does not visit a refitting base. It is the refitting base. Accordingly, it must be kept operational on a continuing basis.
Achieving this will clearly require significant attention to module reliability, repairability, and on demand resupply. Modules must be more reliable to reduce inventory requirements; they must be repairable to minimize the effect of the inevitable stockouts; and rapid resupply of nonrepairable modules must be available on a routine basis. This is not a matter of opinion or engineering judgement. It is implicit in the mathematics.

Learning to do this is a subgoal of the Space Station project, which is (in large part) intended to provide experience in remote site base operation. Proper attention to support factors of the same general type as those considered in this report will result in a Space Station which can be supported, and which will require less ground support as it matures. The lessons learned in Space Station operation will later be applied to design of a Lunar Base, and lessons from the Lunar Base, in turn, applied to a Manned Mars Mission. In this way, logistics problems will be solved incrementally, in a timely and effective manner, and the manned exploration of the Solar System will continue at a steady pace, as foreseen in the Report of the President's Commission on Space and in the Ride report.

Further Work

First, analysis (3) should be carried out more rigorously, providing some convenient way of predicting behavior inventory levels for long missions when those for short missions are known.

The repair hour hazards developed above are really averages. Repair hour hazard is a function of time; it will be higher towards the end of the mission, when inventory is depleted, then it will be at mission start. The only feasible ways of determining the repair hour hazard distribution appears to be simulation. In extreme cases, the repair activity might absorb all available crew labor towards mission end. It would be useful to predict and avoid that.

The existence of high failure rate parts should be investigated further. Preliminary examination suggests that there are surprisingly few such parts in the STOCK2.STK data set, and that reliability improvement would require significant MTBF increase over many part types. For that matter, the effect of increasing MTBF is not properly considered above. It will, obviously, decrease inventory size, but will probably do so nonlinearly. It would be interesting to attempt to characterize the effect in the same way that commonality has been considered above, so as to determine the probable contribution of increased MTBF to reducing the logistics
STOCK2 could be extended to accept more than 400 data classes. It would be interesting to process the entire data set and compare the results with those of Subset A.

Application of STOCK2 to data sets from other equipment, such as planetary probes and modern commercial aircraft, would provide some badly needed perspective on inventory policy. Data from Antarctic bases, extended Oceanographic expeditions, and industrial plants in countries without a local industrial infrastructure might provide a look at successful inventory policy. Application of STOCK2 to the Hubble Space Telescope might prove particularly interesting.

Failure while in inventory has not been explicitly treated, as it is believed to be of negligible importance. If it is important, some fraction of the modules in inventory must be added to the population of modules supported. For example, if MTBF in inventory is twice that in service, than half of the modules in inventory must be counted as in service. This operation produces a convergent series, and one would expect the convergence to be rapid. Note also that if some of the modules are in active standby, possibly with automatic switchover, it may be desirable to count them as inventory rather than as running. The effect is to reduce the number of "running" modules to the minimum required for station operation. The risk, of course, is that one may find the inventory exhausted with only one "necessary" module in a redundant system running and half the mission to go. Obviously, some difficult and real decisions about what is "necessary" are called for before any such reduction can be considered.

Specified probability of stockout is the only independent variable in the current algorithm. It would be convenient for some kinds of analyses to modify it so as to accept number in stock as the independent variable, and in others to modify it so as to accept number of stockouts as the independent variable.

Optimization of the inventory under the following conditions would be of interest, and has not been addressed above:
- Partial resupply (perhaps by small ELV). Total resupply is treated above as a shortened mission.
- Constrained availability of Criticality 1 modules.
- Repairability of only a subset of module types.

Characteristics of the inventory of replacement components used in repair have not been considered.
In addition to the above, there are related questions concerning ground based logistics, which must have the modules needed for restock available when needed, to include rapid resupply missions. There is also the question of proper techniques, tools, inventory, and so on, used for on orbit module repair. The scope of these questions is beyond what any one person could accomplish, and are suggested as tasks for an organization.

Acknowledgements

The North American Rockwell company was extremely helpful in providing recent shuttle data. Many people at JSC provided assistance in obtaining data, and listened to earlier versions of this work with considerable patience. Roy Glanville, who provided the data for the first rough calculations, is especially to be thanked.

The Borland International Company, which made most of the support software which permitted this project to be carried as far as it has been during about 2 month's work, is also to be congratulated on the power of their products, as is Symantec, who made the data base and text editing program used to enter and partially analyze the data, and to write this report.

Bibliography

Data used is Rockwell International’s Logistic Master Control File Baseline Report, also called Report 9100, dated 07/13/87. Permission to see the original data may be requested from the logistics branch of Rockwell’s plant in Downey, CA.