EXOSAT GUEST OBSERVER PROGRAM

BINARY PARAMETERS OF THE X-RAY PULSAR 4U1626-67

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The pulsing x-ray source, 4U1626-67 ($P_{\text{pulse}} = 7.8\ \text{sec}$) is an accreting neutron star in a binary system with a very low mass companion. My collaborators and I observed this source with the EXOSAT Observatory continuously for 23 hours on March 30-31, 1986, UT. We have completed the study of these data; our results, which are summarized below, are presented in a paper entitled, *4U1626-67: The Binary with the Smallest Known Mass Function*, which has been accepted for publication in *The Astrophysical Journal*.

The source 4U1626-67 is the only x-ray pulsar known to be in a binary system with a very low mass ($M \lesssim 0.1M_\odot$) companion, and hence is unique among the x-ray pulsars as well as among the class of x-ray binaries with low mass ($M \lesssim 1M_\odot$) companions. As such it is a touchstone for theories of low-mass binary evolution.

The optical counterpart of the x-ray source is a faint blue object ($V \approx 18.5$) which also pulses at the 7.68-second x-ray pulse period; the optical pulses are due primarily to the reprocessing of x-rays in the accretion disk around the neutron star. Also present is a much weaker pulsed component of optical emission, which is slightly downshifted in frequency. This component is thought to be due to x-ray reprocessing on the surface of a companion star. The frequency shift implies that the system's orbital period is 41 minutes.

With EXOSAT we searched for shifts in the x-ray pulse period due to orbital motion of the neutron star; none were detected. This null result implies a stringent upper limit on the projected semimajor axis of the orbit of the neutron star of \( \sim 10 \) light-milliseconds for the 41-minute optical period found by Middleditch et al., and a limit of \( \sim 13 \) light-milliseconds for any other plausible period. (For comparison, the 6400 km radius of the Earth is \( \sim 21 \) light-milliseconds.) The corresponding upper limit on the mass function for the 41-minute orbital period is \( 3 \times 10^{-6} M_\odot \), which sets strong constraints to models of the system. For example, we conclude that if the orbital inclination angle, \( i \), equals 90° (orbital plane viewed edge on), then the optical companion star has a mass of \(< 0.02 M_\odot \). On the other hand, we find that a companion star mass \( > 0.06 M_\odot \) is required if gravitational radiation is responsible for driving the mass transfer in this system. Only for \( i < 16° \) can a companion star mass this large be accommodated by our limit on the orbital amplitude. However, the \( a \ priori \) probability of finding a binary at such a low orbital inclination is only \( \sim 0.04 \).

In our paper we derive further theoretical constraints on the binary system parameters under the assumption that the mass transfer in this
system is driven by the emission of gravitational radiation, and also discuss some evolutionary scenarios for the formation of ultracompact binaries such as 4U1626-67. We conclude that a model with a hydrogen-depleted donor star (rather than a main-sequence or pure helium star) may be the most promising for the 4U1626-67 system.

In our paper we also present results on the flaring activity in 4U1626-67 on time scales of $\sim 1000$ s, the energy dependent pulse profiles, and the pulse-period history over the past decade. A search for x-rays scattered from the rotating pulsar beam by matter within the binary system was negative.

This project turned out to be a rewarding international collaboration; all of the collaborators made significant contributions.
4U 1626-67:
THE BINARY WITH THE SMALLEST KNOWN MASS FUNCTION

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ABSTRACT

The pulsing X-ray source 4U1626-67 is an accreting neutron-star in a binary system with a very low-mass companion. We observed the source with EXOSAT continuously for 23 hours on March 30-31, 1986 UT. These observations allow us to set a stringent upper limit on the projected semimajor axis of the orbit of the neutron star of $\sim 10$ $\mu$msec for the 2485-s orbital period found by Middleditch et al. (1981), and a limit of $\sim 13$ $\mu$msec for any other plausible orbital period. The corresponding upper limit on the mass function for the 2485-s orbital period is $1.3 \times 10^{-6} M_\odot$. We conclude that if the orbital inclination angle, $i$, equals 90°, then the optical companion star has a mass of $< 0.02 M_\odot$. However, we find that a companion star mass $> 0.06 M_\odot$ is required if gravitational radiation is responsible for driving the mass transfer in this system. Only for $i < 16^\circ$ can a companion star mass this large be accommodated by our limit on the orbital amplitude. We also present in this work our results on the flaring activity in 4U1626-67 on time scales of $\sim 1000$ s, the energy dependent pulse profiles, and the pulse period history over the past decade. A search for X-rays scattered from the rotating pulsar beam by matter within the binary system was negative. After the observational results are presented, we derive theoretical constraints on the binary system parameters under the assumption that the mass transfer in this system is driven by the emission of gravitational radiation. Finally, we discuss some evolutionary scenarios for the formation of ultracompact binaries such as 4U1626-67.
The X-ray source 4U1626-67 is the only X-ray pulsar known to be in a binary system with a very low-mass companion. X-ray pulsations with a period of 7.68 s were discovered in 1977 during SAS-3 observations of 4U1626-67 (Rappaport et al. 1977). The lack of observable Doppler delays in the pulse arrival times during subsequent observations yielded upper limits on the projected semimajor axis, $a_\times \sin i$, of the orbit of the neutron star of $\lesssim 100-200$ $\mu$-msec for orbital periods, $P_{\text{orb}}$, in the range of 10 minutes $\lesssim P_{\text{orb}} \lesssim 20$ days (Rappaport et al. 1977; Joss, Avni, and Rappaport 1978; Pravdo et al. 1979; Li et al. 1980). For short orbital periods of $\sim 1$ hr, an even more stringent limit for $a_\times \sin i$ of $\lesssim 40$ $\mu$-msec was set by Middleditch et al. (1981) who reanalyzed the data of Pravdo et al. (1979). Based on these observations, Joss, Avni, and Rappaport (1978), Li et al. (1980), and Rappaport (1982) suggested that the binary system was highly compact, with an orbital separation of $\sim 1$ $\AA$-s and a companion mass of $\sim 0.1 \, M_\odot$.

The optical counterpart, KZ TrA, is a faint blue star ($V \approx 18.5$) with little or no reddening ($A_V < 1$: McClintock et al. 1977; Bradt and McClintock 1983; van Paradijs et al. 1986). Its absolute magnitude is $M_V > 4$ for $d < 6$ kpc (see Sect. III), which is faint compared to X-ray burst source binaries that contain a late-type dwarf companion ($M_V = 1.2 \pm 1.1$; van Paradijs 1983). Nevertheless, the dominant source of light is almost certainly X-ray heated gas; a low-mass main-sequence dwarf companion, for example, would be more than 10 magnitudes fainter than the observed counterpart.

Optical photometric pulsations at the X-ray pulse period were discovered by Ilovaisky et al. (1978). Observations by Middleditch et al. (1981) subsequently revealed the presence of two optical pulse periods, one at the frequency of the X-ray pulsations and the other downshifted in frequency by $\sim 0.4$ mHz. They attributed the downshifted peak to X-radiation reprocessed on or near the surface of the companion star; the difference between the two pulse periods can be understood as the difference between the sidereal and synodic rotation periods of the neutron star as seen from the companion, provided that the orbital period is $\sim 2485$ s (Middleditch et al. 1981). From an analysis of these observations, Middleditch et al. derived the following system parameters: $M_\times = 1.8 (+2.9,-1.3) \, M_\odot$, $M_\ast < 0.5 \, M_\odot$, $a_\times \sin i = 0.36 (\pm 0.10) \, \AA$-s, $a = 1.14 (\pm 0.40) \, \AA$-s, and $i = 18^\circ (\pm 18^\circ,-7^\circ)$, where $M_\times$ and $M_\ast$ are the mass of the neutron star and companion star, respectively, $i$ the orbital inclination angle, $a_\times \sin i$ the projected semimajor axis of the orbit of the companion.
star, and $a$ the orbital separation. The presence of the downshifted peak in the optical has subsequently been confirmed by Middleditch (1983).

In addition to periodic pulsations, the X-ray and optical intensity of 4U1626-67 shows quasi-periodic flaring behavior with a characteristic time scale of $\sim 1000$ s (Joss, Avni, and Rappaport 1978; Li et al. 1980; McClintock et al. 1980). The origin of this flaring behavior is not understood.

In this paper we present new results concerning the 4U1626-67 system derived from a 23-hour X-ray observation of this source with EXOSAT. In Sect. II we describe the observations. In Sect. III we summarize the pulse period behavior over the past decade and derive a mass transfer rate for this system of $\sim 2.8 \times 10^{-10}$ $M_\odot$ yr$^{-1}$. In Sect. IV we display a series of energy-dependent pulse profiles with the best statistical precision yet obtained for this source. In Sect. V we present the results of a timing analysis of the 7.7-s pulsations and derive a $3\sigma$ upper limit for the projected semimajor axis of the neutron star orbit of $a_x \sin i < 10 \lambda$-msec. Finally, we discuss the implications of such a small value of $a_x \sin i$ for the nature of the companion star and for the evolution of 4U1626-67.

II. OBSERVATIONS

4U1626-67 was observed with EXOSAT from 12:17, 30 March 1986 to 11:32, 31 March 1986 UT (JD 2446520.012 to JD 2446520.981). Off-source observations were conducted prior to and after the observation of 4U1626-67; they were used to estimate the background count rate during the observation of the source. The observations were conducted using the Medium Energy Experiment (Turner, Smith, and Zimmerman 1981), an array of proportional counters having a collecting area of $\sim 1500$ cm$^2$. Seven of the proportional counters were operational during the observation with the exception of the interval 6:29 to 11:06 on 31 March 1986 when only six of the detectors were functioning. (One of the detectors was turned off during the above interval after it experienced an episode of breakdown for $\sim 100$ seconds.) Both halves of the ME detector bank were coaligned and pointed at 4U1626-67.

We analyzed data accumulated with the instrument data system operating in the HER5 mode, in which the number of counts in 0.25 second time intervals was recorded for each of 32 pulse height intervals. For the purpose of our analysis, we summed the data into 10 broader pulse height intervals which are listed in Table 1. As the data were acquired, they were labelled with the value of the spacecraft clock time. In addition, the data were also
labelled with the UT at which they were received at the ground station at Villafranca, Spain. We utilized the ground-station times in our analysis.

The count rate in the energy range 2.5 - 9 keV is shown, as a function of time, in Figure 1. The variability of 4U1626-67 on time scales ~ 1000 seconds is quite evident. During the 23-hr observation there were ~ 32 flares with enhancements in X-ray intensity ranging between 30% and a factor of 3; each flare had a duration of ~ 500 s. In the interval between the larger flares there were many smaller ones which effectively prevented the intensity from remaining constant for longer than ~ 1000 s. An analysis of the EXOSAT aspect data from the observation indicates that the slow variability in the count rate (by up to ~ 25%) on time scales of ~ 2 \times 10^4 s is due largely to satellite drifting motion and the concomitant changes in source transmission through the detector collimators. On shorter time scales of ~ 100 s, however, variations in collimator transmission can, in general, account for no more than ~ 15% of the peak-to-peak source variations that are observed between the more significant flaring events. Based on this aspect analysis and the fact that our background measurements, both preceding and following the main observation, show no evidence for solar activity, we are confident that almost all of the observed temporal variability on time scales of < 1000 s is intrinsic to 4U1626-67.

Plots similar to that in Figure 1 were constructed for several different energy intervals within the overall range of 1.4 - 13 keV. These indicate that, to within photon counting statistics, the flaring behavior is energy independent.

III. PULSE PERIOD HISTORY

The average pulse period during the EXOSAT observation was obtained as follows. First, time corrections were applied for the motion of EXOSAT around the earth, and for the delays due to the finite propagation time for the telemetry signal from EXOSAT to reach the ground station. The data from the entire observation were then folded modulo different trial pulse periods until the sharpest features in the pulse shape were obtained. This was followed by a more detailed pulse timing analysis (see Sect. V) which yielded a geocentric pulse period of \( P = 7.6659173 \) s. After applying corrections for the motion of the earth around the solar system barycenter we obtained a barycentric pulse period of \( P = 7.6664220 \pm 5 \times 10^{-7} \) s, referenced to an epoch of JD 2445620.50.

All of the pulse periods measured for this source over the past decade, including the
pulse period determined in this work, are plotted in Figure 2. An unweighted fit of a linear function to the pulse periods vs. time yields a mean value for \( \dot{P}/P = -(2.040 \pm 0.024) \times 10^{-4} \text{ yr}^{-1} \) (1σ confidence limit). The fit is significantly improved by the inclusion of a quadratic term; the value for the second derivative in the pulse period can be conveniently expressed as \( \ddot{P}/P = 0.026 \pm 0.005 \text{ yr}^{-1} \) (the inclusion of the quadratic term is justified at the 99.5% confidence limit by an F test). In this latter fit we find \( \dot{P}/P = -(1.810 \pm 0.047) \times 10^{-4} \text{ yr}^{-1} \), referenced to an epoch of JD 2443000 (this value of \( \dot{P}/P \) is in excellent agreement with the values found by Joss, Avni, and Rappaport 1978 and Elsner et al. 1983). Thus, we see that the spin-up time scale is itself decreasing with an e-folding time of \( \sim 40 \text{ yrs} \). It is unclear, however, whether this latter behavior will persist for much longer than the present observational time span of about a decade.

The spin-up rate in 4U1626-67 can be utilized to estimate the mass transfer rate (see also Joss, Avni, and Rappaport 1978). From the theory of the torques exerted on a magnetized neutron star by matter accreted via an accretion disk, we find

\[
\frac{\dot{P}}{P} = -2.5 \times 10^{-5} \left[ \frac{M_x}{M_\odot} \right]^{4/7} R_6^{6/7} R_{g6}^{-2} B_{12}^{2/7} M_{9}^{-6/7} \dot{P} \text{ yr}^{-1} \tag{1}
\]

(Lamb, Pethick, and Pines 1973; Rappaport and Joss 1976; Ghosh and Lamb 1979) where \( R_6 \) and \( R_{g6} \) are the radius at the surface and the radius of gyration of the neutron star, respectively, in units of \( 10^6 \text{ cm} \), \( B_{12} \) is the surface magnetic field (assumed dipolar) in units of \( 10^{12} \text{ G} \), \( M_{9} \) is the mass transfer rate in units of \( 10^{-9} M_\odot \text{ yr}^{-1} \), and \( \dot{P} \) is the rotation period of the neutron star in seconds. For 4U1626-67 we can thus find an expression for \( \dot{M} \) in terms of the unmeasured properties of the neutron star

\[
\dot{M} \approx 1 \times 10^{-9} \left[ \frac{M_x}{M_\odot} \right]^{2/3} R_6^{-1} R_{g6}^{-1/3} B_{12}^{-1/3} M_\odot \text{ yr}^{-1} \tag{2}
\]

For illustrative purposes, we adopt a neutron star mass of \( 1.4 M_\odot \) (Joss and Rappaport 1984) and a surface magnetic field of \( 10^{12} \text{ G} \). The product \( R_6^{-1} R_{g6}^{-7/3} \) can be obtained for a range of neutron star models (see, e.g., Arnett and Bowers 1977) and generally lies in the range of \( \sim 0.18 - 0.64 \). This yields a plausible range for the mass-transfer rate of \( \dot{M} \approx 2.8 \times 10^{-10} M_\odot \text{ yr}^{-1} \).

The mass transfer rate, combined with the measured bolometric high-energy flux of \( F_x \approx 2.4 \times 10^{-9} \text{ ergs cm}^{-2} \text{ s}^{-1} \) (Pravdo et al. 1979), yields a distance estimate of between 3 and 6 kpc to 4U1626-67 (for an assumed time-averaged, isotropic emission; but see Sect. IV).
The X-ray intensity of 4U1626-67 has remained fairly constant over the time scales of months to years. The average 1-20 keV flux during our observation was \( \sim 1.4 \times 10^{-9} \) ergs cm\(^{-2}\) s\(^{-1}\), which is within \( \sim 50\% \) of the several values reported for this source between 1975 and 1979 (see Sect. IV of Pravdo et al. 1979; Elsner et al. 1983).

IV. ENERGY DEPENDENT PULSE PROFILES

Pulse profiles were formed for each of ten pulse height intervals covering the range 1.4-22 keV. The energy-dependent pulse profiles are shown in Figure 3. The width of a pulse-phase bin in these profiles is nearly \( 1/4 \) s which is about equal to the temporal resolution of the original data train. Any features narrower than this will be at least partially smoothed out in the folding process.

For photon energies in the range of \( \sim 2.5-10 \) keV the pulse profile consists of two prominent peaks. The two peaks are separated by \( \sim 73^° \) in pulse phase at the low end of this energy range. This phase separation increases with energy, reaching \( \sim 96^° \) at \( \sim 10 \) keV. At both lower and higher energies the pulse profiles are dominated by a broad minimum whose extent in pulse phase is very similar to that of the two peaks seen in the \( \sim 2.5-10 \) keV band (this property of the pulse profiles was first pointed out by Pravdo et al. 1979). We also note that the pulse profile at each energy in the range \( \sim 2.5-10 \) keV has a point of apparent mirror symmetry (at pulse phase \( \phi = 0.5 \)). To our knowledge, this source is the only X-ray pulsar with sharp features in its pulse profile to exhibit this type of mirror symmetry. This could be indicative of a beam profile at the neutron star that depends only on the angle from the axis of a simple dipole magnetic field. Models for the energy dependent pulse profiles of 4U1626-67 are discussed by Kii et al. (1986).

In the photon energy range \( \sim 2.5-10 \) keV the pulsed fraction (pulsed flux divided by the average flux) is only \( \sim 10\% \). The pulsed fraction in the various pulse height bands is listed in Table 1. Thus, we see that the bulk of the X-radiation received at the earth is unpulsed. We propose that this may occur because we are viewing the orbit of the binary system nearly pole on (Middleditch et al. 1981; Sect. VI), in which case it is plausible that the rotation axis of the neutron star is also pointed in the direction of the earth. To the extent that this latter hypothesis is correct, the amplitude modulation of the X-ray pulse can be reduced to arbitrarily small values regardless of the beam profile or the angle between the magnetic dipole axis and the rotation axis. This picture, however, does not provide a natural explanation for the sharp features in the pulse profiles (especially in the 3-5 keV energy
V. PULSE TIMING ANALYSIS

The pulse timing analysis was carried out with data spanning the energy interval from 2.5-9 keV to maximize the signal to noise (see Table 1). A pulse profile was constructed for each 184-s interval of the data train; this time interval was chosen to be significantly shorter than the reported orbital period of 2485 s (Middleditch et al. 1981; Middleditch 1983). Each pulse profile was then cross-correlated with a master pulse profile constructed from all the data in the 2.5-9 keV interval. A parabolic function was then fit to the highest three points in the cross-correlation function in order to find the maximum with considerably greater precision than the bin width of \( \sim 1/4 \) s used to form the cross-correlation function. The result of this analysis was a pulse-time delay with respect to the pulse arrival time expected for a constant pulse period. In all, 451 such pulse-time delays were obtained over the one-day observation interval. These are plotted in Figure 4. The rms scatter in the time delays is only \( \sim 35 \) msec.

As the neutron star orbits its companion, the pulse-time delays will vary periodically in time. Therefore, to search for orbital motion at the reported orbital period as well as to cover any other possible orbit, we Fourier transformed the pulse-time delays shown in Figure 4; the results of this analysis are shown in Figure 5. For the case where the pulse-time delay data (Fig. 4) are dominated by noise that is uncorrelated from point to point and whose amplitudes are Gaussian distributed, the Fourier amplitudes, \( A \), will be "white" with a distribution given by

\[
p(A) = \frac{2A}{A_o^2} e^{-A^2/A_o^2},
\]

where \( A_o \) is related to the mean amplitude \( <A> \) by \( A_o = 2 <A>/\sqrt{\pi} \). We have verified that the distribution of amplitudes in Figure 5 indeed fits such a distribution quite well for a value of \( A_o \approx 3.4 \) msec. To obtain the limit on \( a_x \sin i \) at a particular orbital period we utilize \( p(A) \) to compute the probability that an actual orbital amplitude \( A \), in the presence of white noise, would have fluctuated as low as the observed amplitude. With this prescription, we find the following 3\( \sigma \) confidence limits on \( a_x \sin i \):
\[ a_x \sin i \leq 10.5 \; \Delta \text{msec} \quad \text{P}_{\text{orb}} = 41.4 \text{ minutes} \ (2485 \text{ s}) \]
\[ a_x \sin i \leq 13 \; \Delta \text{msec} \quad 10 \text{ minutes} \leq \text{P}_{\text{orb}} \leq 10 \text{ hr.} \]

(We note for comparison that the 6400 km radius of the earth corresponds to \(\sim 21 \; \Delta \text{msec}\).)

These limits are about a factor of 4 smaller than the best previous limits set by Middleditch \textit{et al.} (1981) who utilized the data of Pravdo \textit{et al.} (1979).

Finally, we have also carried out a Fourier transform of the entire data set in 0.25-s bins in order to search for a downshifted sideband to the prominent 7.7-s X-ray periodicity (see Figure 6). Such a peak, which is downshifted by the orbital frequency, would be expected if there were scattering of the X-ray beam by any matter distribution that is fixed in the reference frame of the binary system (such as a companion star or a bulge on an accretion disk [Mason 1986]). This effect would be analogous to the downshifted sideband observed in the optical by Middleditch \textit{et al.} (1981) and Middleditch (1983). In the X-ray band, however, there is no statistically significant peak at the appropriate downshifted frequency. This effectively sets an upper limit to the \textit{amplitude} at this frequency of \(\sim 4\%\) that of the prominent 7.7 s peak. We also note that orbital Doppler shifts would produce symmetrically spaced sidebands about the X-ray periodicity. However, since this Fourier technique is generally less sensitive that the 'folding' technique described above, we do not expect to have detected such sidebands in light of the negative results of our search for orbital motion.

VI. DISCUSSION

We have set a stringent upper limit to \(a_x \sin i\) of \(\sim 10 \; \Delta \text{msec}\) at the orbital period of \(\sim 2485 \text{ s}\) found by Middleditch \textit{et al.} (1981) and Middleditch (1983) and a limit of \(\sim 13 \; \Delta \text{msec}\) for any other plausible orbital period. Furthermore, we do not detect any significant pulsations downshifted in frequency by \((2485 \text{ s})^{-1}\) that would be expected from scattering of the X-ray beam (see discussion above). Therefore, we are not able to independently confirm the orbital period of 2485 s found by Middleditch \textit{et al.} (1981) and Middleditch (1983). Nonetheless, this orbital period appears to be quite secure and we adopt it for the purpose of discussion in the remainder of this paper.

The upper limit to the mass function for the 4U1626-67 system is
\[ f(M) = \frac{M_s \sin^3 i}{(1 + M_x/M_s)^2} \leq 1.3 \times 10^{-6} \ M_\odot \ , \tag{4} \]

where we have taken the orbital period to be 2485 s. To our knowledge, this is the smallest known mass function for any binary stellar system. We can use the limit on \( f(M) \) to set constraints on the mass of the companion star as a function of the orbital inclination angle. The results are shown in Figure 7 for various assumed values of the mass of the neutron star. For values of \( i \) near 90°, \( M_\odot < 0.02 \ M_\odot \), while for \( 11^\circ \leq i \leq 36^\circ \), the range of values cited by Middleditch et al. 1981, we find upper limits on \( M_\odot \) to lie in the range of \( \sim 0.03-0.09 \ M_\odot \). Only for \( i < 10^\circ \) can \( M_\odot \) exceed 0.1 \( M_\odot \); however, we note that the \textit{a priori} probability of finding a binary with \( i < 10^\circ \) is only 0.015.

We now utilize the inferred value of \( \dot{M} \) in the 4U1626-67 system to derive theoretical constraints on \( M_\odot \). Consider a binary system consisting of a neutron star of mass \( M_x \) in a circular orbit with a low-mass secondary of mass \( M_\odot \). We assume that the secondary rotates synchronously with the orbit and that its rotational angular momentum is negligible compared to the orbital angular momentum. Systemic angular momentum losses can be conveniently divided into three categories: (1) gravitational radiation reaction; (2) matter that overflows the inner Lagrange point of the Roche potential, a fraction of which, \( 1-\beta \), is subsequently ejected from the binary system with a specific angular momentum \( \alpha \) in units of \( 2\pi a^2/P_\text{orb} \), where \( a \) is the binary separation and \( P_\text{orb} \) is the orbital period (see, e.g., Rappaport, Verbunt, and Joss 1983); and (3) other losses of angular momentum, such as magnetic braking, which are not accompanied by a significant amount of mass loss (see, e.g., Verbunt and Zwaan 1981).

For the low-mass secondaries of interest here, we assume that the only systemic angular momentum losses not associated with significant mass loss are those due to gravitational radiation (see, e.g., Rappaport, Verbunt, and Joss 1983). If other sources of angular momentum losses are not negligible, a larger value of \( \dot{M} \) could be attained for a smaller value of \( M_\odot \); we shall see below that this would lead to better self-consistency in the constraints on \( M_\odot \). There is, however, no direct evidence for such angular momentum losses in low-mass stars.

To provide some analytic insight into the properties of mass transfer within an ultracompact binary system and the resultant binary evolution, we now utilize a simple \( n = 3/2 \) polytropic representation for the mass-radius relation of the secondary. For such a star we have
\[ R_s = 0.0128 (1+X)^{5/3} f \left[ \frac{M_s}{M_\odot} \right]^{1/3} R_\odot \]  
(5)

(see, e.g., Chandrasekhar 1939; Rappaport et al. 1987), where \( X \) is the mass fraction of hydrogen in the secondary (which is assumed to be uniformly distributed through the star), and \( f > 1 \) is the ratio of the stellar radius to the radius of a star of the same mass and composition that is completely degenerate and supported only by the Fermi pressure of the electrons. (This simple expression for \( R_s \) yields values that are larger than the actual radii of low-mass cold stars by \( \sim 10\% \) at \( \sim 0.05 \, M_\odot \); for higher masses, up to \( \sim 0.2 \, M_\odot \), the error in \( R_s \) becomes negligible, while for masses as small as \( \sim 0.02 \, M_\odot \) the error can be as large as \( \sim 22\% \).) If the secondary with this mass-radius relation fills its Roche lobe, there is a simple relation between its mass and the orbital period (see, e.g., Faulkner 1971)

\[ P_{\text{orb}} \approx 0.769 \left( 1+X \right)^{3/2} f^{3/2} \left[ \frac{M_s}{M_\odot} \right]^{-1} \text{ min.} \]  
(6)

An analytic expression for the mass accretion rate, \( \dot{M} \), driven by gravitational radiation reaction, which utilizes the above approximations, has been derived previously by Rappaport et al. (1987; see also Faulkner 1971):

\[ \dot{M} \approx 4.14 \times 10^{-4} \frac{M_x}{M_\odot} \left[ \frac{M_T}{M_\odot} \right]^{-1/3} \left[ \frac{P_{\text{orb}}}{\text{min}} \right]^{-14/3} \left( 1+X \right)^5 f^3 \beta \, M_\odot \text{ yr}^{-1} \]  
(7a)

where \( D(q, \alpha, \beta, \xi) = \frac{2}{3} \frac{(1-\beta)}{3(1+q)} \frac{(1-\beta)\alpha(1+q)+\beta}{q} \),

(7b)

and where \( q \equiv M_x/M_s \) and \( M_T \equiv M_x + M_s \). If we assume that \( M_x \gg M_s \), equation (7) can be written in the following simplified form:

\[ \dot{M} \approx 6.21 \times 10^{-4} \frac{M_x}{[1-1.5 \alpha(1-\beta)]} \left[ \frac{M_T}{M_\odot} \right]^{2/3} \left[ \frac{P_{\text{orb}}}{\text{min}} \right]^{-14/3} \left( 1+X \right)^5 f^3 \beta \, M_\odot \text{ yr}^{-1} \]  
(8)

This expression shows how the mass-transfer rate scales with the parameters of the binary system and the mass-ejection process. Note, in particular, that equation (8) has no explicit dependence on the mass of the secondary.

If we now adopt an orbital period of 41.4 minutes (2485 s) for 4U1626-67, we can
rewrite equations (6) and (8) as:

\[ M_x \approx 0.019 \left(1 + X\right)^{5/2} \beta^{3/2} \frac{M}{M_\odot} \quad ; \quad (9) \]

\[ \dot{M} \approx \frac{2.2 \times 10^{-11}}{\left[1 - 1.5 \alpha(1 - \beta)\right]} \left[\frac{M_x}{1.4M_\odot}\right]^{2/3} \left(1 + X\right)^{5/2} \beta^{3/2} \frac{M}{M_\odot} \text{ yr}^{-1}. \quad (10) \]

If we utilize the limits inferred for \( \dot{M} \) in Sect. III we can rewrite equation (10) in the following form:

\[ 3.0 \leq \left[\frac{M_x}{1.4M_\odot}\right]^{1/3} \left(1 + X\right)^{5/2} \beta^{3/2} \frac{1}{\left[1 - 1.5 \alpha(1 - \beta)\right]^{1/2}} \leq 6.0. \quad (11) \]

Under the assumptions of conservative mass transfer and a mass of 1.4 \( M_\odot \) for the neutron (equation [11] is relatively insensitive to \( M_x \) and to \( \beta \) for \( \beta > 0.6 \) and \( \alpha \approx 1 \)) the quantity \( (1 + X)^{5/2} \beta^{3/2} \) is constrained to lie in the range of 3 - 6. Plugging this constraint into equation (9) we find that \( M_x \) lies in the range of 0.057 - 0.11 \( M_\odot \). The corresponding radius of the secondary would lie in the range of \( \sim 0.07 - 0.09 R_\odot \). We note that for a chemical composition of pure He, the factor \( f \) would lie in the range of \( \sim 2.1 - 3.3 \) (i.e., a radius several times that of a completely degenerate configuration), while for a solar composition, \( f \) would be closer to unity (i.e., \( f < 1.4 \)). For a solar composition, however, only masses less than \( \sim 0.08 \) \( M_\odot \) are acceptable if the star is to avoid significant hydrogen burning, which would lead to a larger value of \( f \) (see, e.g., Nelson, Rappaport, and Joss 1986b).

Thus, we see that a secondary mass of \( \sim 0.06 - 0.11 \) \( M_\odot \) is required to drive the inferred mass transfer rate of \( \sim 2 - 8 \times 10^{-10} \) \( M_\odot \) \text{ yr}^{-1} if the only source of angular momentum loss is gravitational radiation. On the other hand, the constraints on the mass function presented earlier in this Sect. indicate that the mass of the secondary can be as large as \( \sim 0.06 \) \( M_\odot \) only for inclination angles, \( i < 16^\circ \). (The \( a \text{ priori} \) probability of finding a value of \( i \) this small is only \( \sim 0.04 \).)

Finally, we examine the type of secondary that may be expected in the 4U1626-67 system on the basis of evolutionary scenarios put forth to explain ultrashort orbital period binaries (\( P_{\text{orb}} < 80 \text{ minutes} \)). In the standard scenario for the evolution of short orbital period systems (80 minutes < \( P_{\text{orb}} < 10 \text{ hours} \); see, e.g., Paczynski and Sienkiewicz 1981, Rappaport, Joss, and Webbink 1982, and Rappaport, Verbunt, and Joss 1983) the mass donor, at the onset of mass transfer, may be an ordinary solar-type star. As the donor star
loses mass it tends to remain near the hydrogen-burning main sequence and therefore becomes smaller. For the star to continue to fill its Roche lobe, there must be a concomitant decrease in the binary orbital separation which is effected by systemic angular momentum losses, e.g., gravitational radiation. Once the mass of the donor becomes sufficiently small, however, the electron degeneracy in the stellar interior increases significantly, which ultimately causes the stellar radius to increase with further mass loss (see eq.[5]). The orbital period, in turn, begins to increase when $\frac{d \ln R_s}{d \ln M_s} < +1/3$, where the derivative is evaluated along the evolutionary track. Thus, the orbital period eventually reaches a minimum value, which in the standard scenario is around 80 minutes (Paczyński and Sienkiewicz 1981; Rappaport, Joss, and Webbink 1982). This calculated minimum at ~ 80 minutes is apparently reflected in the observed minimum in the orbital period distribution of (all but a few) cataclysmic variables and low-mass X-ray binaries. The type of evolution described above generally proceeds on a time scale that is much shorter than that for nuclear evolution in the core of the mass-donor star; hence the chemical composition of the donor star may be considered fixed at the initial composition.

Two different types of mass-donor stars have been considered to explain systems with shorter orbital periods ($P_{\text{orb}} < 80$ minutes): stars that are hydrogen-depleted (with $X < 0.1$), but which nonetheless generate significant nuclear luminosity through hydrogen burning (Iben and Tutukov 1984; Nelson, Rappaport, and Joss 1986a; Pylyser and Savonije 1987), and helium-burning stars (Iben and Tutukov 1985, 1987; Savonije, de Kool, and van den Heuvel 1986). Nelson et al. (1986a) showed that binaries with a mass donor that is hydrogen depleted when mass transfer commences can evolve to ultrashort orbital periods. To reach an orbital period of ~40 minutes, the initial donor star must have $X < 0.02$ (for an assumed uniform chemical composition). At the minimum orbital period the theoretically derived mass transfer rates are in good agreement with that deduced for 4U1626-67 in Sect. III. Iben and Tutukov (1984) and Pylyser and Savonije (1987) discuss scenarios wherein a low-mass hydrogen-depleted secondary can be formed. They consider binaries, with secondaries which are slightly more massive than the sun, that commence mass transfer just before the secondary ascends the giant branch. For such systems, magnetic braking (Verbunt and Zwaan 1981) still dominates the evolution and causes the orbit to contract; the evolution then proceeds much as described above for the standard scenario, except that the core of the mass-donor star is now hydrogen depleted, which results in a short minimum orbital period.

The other proposed type of secondary in an ultracompact binary is a helium-burning star
or remnant thereof. The progenitor of such a star would have been substantially more massive than the sun, and in a relatively wide orbit about a neutron star. When such a star evolves up the giant branch it can engulf the neutron star in a common envelope (see, e.g., Paczyński 1976; Meyer and Meyer-Hofmeister 1979), after which a spiral-in phase can ensue, leaving the neutron star in orbit with the helium-burning core as its close companion. The radius of the helium-burning star decreases with mass loss until its mass becomes too small to sustain nuclear burning, at which point electron degeneracy begins to set in. The minimum period for this type of system can be as short as $\sim 10$ minutes (Iben and Tutukov 1985, 1987; Savonije, de Kool, and van den Heuvel 1986). On its way to shorter periods, the binary at 40 minutes has a relatively massive donor with $M_\odot > 0.5 M_\odot$; the luminosity of the donor star at this point is greater than $30 L_\odot$ (Savonije et al. 1986). Observations of the optical counterpart of 4U1626-67 imply a much lower luminosity (McCrimmon et al. 1987) and thereby exclude a helium-burning donor star that is this massive. About a hundred million years after reaching the minimum orbital period, the binary again attains a period of $\sim 41$ minutes, now with a mass-donor star of $0.034 M_\odot$ and a mass transfer rate $\dot{M} = 9 \times 10^{-11} M_\odot \text{yr}^{-1}$ (Savonije 1986). This value for the mass transfer rate is probably too small to be consistent with that inferred for 4U1626-67.

We conclude that the model with a hydrogen-depleted donor star may be the most promising for the 4U1626-67 system. In any such model, the evolution to an orbital period of $\sim 40$ minutes requires at least $\sim 5 \times 10^8 \text{yr}$. In this regard, we note that observations of radio pulsars indicate that the magnetic fields of neutron stars decay with an e-folding time of $\sim 10^7 \text{yr}$ (see, e.g., Lyne, Manchester, and Taylor 1985). Thus, the question arises as to how it is possible that we still see this source as an X-ray pulsar; the sharp energy-dependent features in the pulse profiles of 4U1626-67, as well as the relatively hard X-ray spectrum, indicate a strong magnetic field (see White, Swank, and Holt 1983, and references therein). One solution that has been proposed is that the neutron star in 4U1626-67 is, in fact, relatively young. (This would also be consistent with its apparent 'spin-up age' of only $\sim 5000 \text{years}$ [see Sect. III].) In this explanation 4U1626-67 started out as a binary in which the accreting star was a massive O-Ne-Mg degenerate dwarf that was ultimately driven over the Chandrasekhar limit. At that point, which occurred a relatively short time ago, the white dwarf collapsed to a neutron star (see, e.g., Taam and van den Heuvel 1986; Nomoto 1986). A potential problem with the hypothesis of a young neutron star is simply the improbability of observing a system, that will undoubtedly persist as an X-ray source for at least $\sim 10^9 \text{yr}$, so early in its existence. On the other hand, about 1% ($\sim 10^7 \text{yr}/10^9 \text{yr}$) of all low-mass
X-ray binaries might be expected to be in this early evolutionary phase. This statistic is consistent with the fact that 4U1626-67 is the only low-mass X-ray binary to exhibit X-ray pulsations.

Another solution to the age problem of the neutron star in 4U1626-67 is that the neutron star is indeed old, but somehow has retained a relatively strong magnetic field. An indication that old neutron stars may retain a relatively strong field is given by the radio pulsar PSR 0655+64. The white dwarf in this binary system is estimated to be $\sim 5 \times 10^8$ yr old, which is taken to be a lower limit on the age of the neutron star (Kulkarni 1986), whereas the surface field strength of the neutron star is still $\sim 10^{10}$ G (Damashek et al. 1982). However, we note that even for a surface magnetic field of $\sim 10^{10}$ G in 4U1626-67, the inferred value of $M$ from the accretion torque model (see eq.[2]) would be about 5 times higher than for $10^{12}$ G, yielding implausibly high values for $M_\odot$ in the range of $\sim 0.14 - 0.24 M_\odot$ (see eqs.[9 and 10]).

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Table 1

Source Properties vs. Pulse Height Interval

<table>
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<tr>
<th>Channel Number</th>
<th>Pulse Height (keV)</th>
<th>Source Count Rate (s⁻¹)</th>
<th>Background Rate (s⁻¹)</th>
<th>Pulsed Fraction</th>
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<tr>
<td>1</td>
<td>1.4 - 1.9</td>
<td>4.47</td>
<td>2.53</td>
<td>0.17</td>
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<tr>
<td>2</td>
<td>1.9 - 2.4</td>
<td>7.20</td>
<td>1.54</td>
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</tr>
<tr>
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<td>2.4 - 2.9</td>
<td>7.07</td>
<td>1.36</td>
<td>0.08</td>
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<tr>
<td>4</td>
<td>2.9 - 3.9</td>
<td>12.33</td>
<td>2.43</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>3.9 - 5.0</td>
<td>9.93</td>
<td>2.40</td>
<td>0.11</td>
</tr>
<tr>
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<td>5.0 - 6.1</td>
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<td>6.1 - 7.3</td>
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<tr>
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REFERENCES

FIGURE CAPTIONS

Figure 1: 4U1626-67 counting rate in the 2.5 - 9 keV band averaged over 46 s time bins. The intervals of low counting rate at the beginning and end of the observation are times when the EXOSAT detectors were pointed away from the source for background determination. One of the seven detectors was turned off after time JD 2446520.77.

Figure 2: Pulse period history of 4U1626-67. Individual pulse period determinations were taken from Joss, Avni, and Rappaport (1978); Pravdo et al. (1979); McClintock et al. (1980); Elsner et al. (1983); Nagase et al. (1984); present work. In general, the uncertainties in the pulse periods are smaller than the plotted points. The solid curve is a best-fit quadratic function (see text).

Figure 3: Pulse profiles of 4U1626-67 in ten pulse height intervals covering the range 1.4 - 22 keV. Non-source background has been subtracted. Typical ±2σ error bars due to Poisson counting statistics are indicated.

Figure 4: Pulse time delays for 4U1626-67 with respect to the pulse arrival times expected for a constant pulse period. The rms scatter in the pulse time delays is only 35 msec. No evidence for periodic orbital motion is seen.

Figure 5: Fourier transform of the pulse time delays from 4U1626-67 shown in Figure 4. The ordinate (amplitude in msec) can be directly interpreted in terms of $a_x \sin i$ (in $\Delta t$-msec) for an assumed circular orbit. The mean amplitude in this plot is only 3.0 msec. These results can be used to set stringent limits on $a_x \sin i$ for orbital periods in the range of $\sim$ 10 minutes to $\sim$ 10 hours.

Figure 6: Fourier power spectrum of 4U1626-67 summed over the first 8 harmonics. The axis label $\Delta \nu$ corresponds to the frequency shift from the center of the harmonics. A downshifted sideband due to X-ray scattering from matter fixed in the binary frame of reference would appear at $\Delta \nu = -0.402$ mHz, for an orbital period of 2485 s. The apparent correlation of the amplitudes over several frequency bins results from 'padding' the data set.
to the nearest power of 2 and applying a window function (one half cycle of a sinusoid) to the data train before the Fourier transform was performed.

Figure 7: Upper limits to the mass of the companion star in 4U1626-67 as a function of assumed orbital inclination angle. The limits were derived from the measured upper limit to the mass function (equation [4]) for three assumed values of the mass of the neutron star: 1.0, 1.4, 1.8 $M_\odot$. 
Figure 3
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