Design Procedures for Fiber Composite Box Beams

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February 1988
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SUMMARY

Step-by-step procedures are described which can be used for the preliminary design of fiber composite box beams subjected to combined loadings. These procedures include a collection of approximate closed-form equations so that all the required calculations can be performed using pocket calculators. Included is an illustrative example of a tapered cantilever box beam subjected to combined loads. The box beam is designed to satisfy strength, displacement, buckling, and frequency requirements.

INTRODUCTION

The design of fiber composite structural components requires analysis methods and procedures which relate the structural response of the component to the specified loading and environmental conditions. Subsequently, the structural response is compared to given design criteria for strength, displacement, buckling, vibration frequencies, etc. in order to ascertain that the component will meet all the design requirements and will perform satisfactorily.

An important class of structural components that can readily be made using fiber composites are box beams. Box beams are generally used to span long distances and to resist combined loads. Box beams are the main structural components in aircraft wings. They are made using thin flat/curved laminates, are designed to resist the loads primarily through membrane action and are designed to have constant or tapered cross sections. In addition, the laminate thickness for the covers and sides can be different and varied along the span. In a previous paper (ref. 1) step-by-step procedures were described for the preliminary design of composite panels subjected to combined loadings. These procedures have since been extended for the preliminary design of composite box beams. The objective of this paper is to describe these extended procedures.

These procedures include a collection of simple equations to expedite the various calculations performed during the preliminary design phase. These procedures are demonstrated by applying them to a preliminary design of a tapered cantilever box beam. The box beam is subjected to combined loads at the free end. It is designed to meet strength, displacement, buckling, and frequency

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requirements. The various steps in these procedures are described in detail with ample explanatory notes so that they can be used to aid in the preliminary design of built-up composite structural components in general.

SAMPLE DESIGN

It is necessary to have as complete a definition of the specific design as is possible in order to initiate the preliminary design phase. For the illustrative example described herein, this definition consists of the following.

1. Structural Component:
   Cantilever, 3-bay box beam (schematics figs. 1 and 2).

2. Specified Loads:
   Free end static loads (fig. 1).
   6600 lb vertical; 3300 lb lateral; 100 000 lb in twist moment.

3. Displacement Limits:
   Tip displacements less than 1.5-percent of length; angle of twist less than 1°.

4. Frequencies:
   Flap greater than 100 cycle/sec, edge greater than 150 cycle/sec; twist greater than 450 cycle/sec.
   Local panel frequencies to be greater than box beam global frequencies.

5. Safety Factor:
   2.0 times specified load.

6. Composite System:
   As graphite fiber in epoxy matrix (AS/E) about 0.6 fiber volume ratio.

7. Design Procedure/Requirements:
   Box beam not to exceed displacement limits.
   Laminates in various bays not to exceed ply fiber-controlled strengths at design loads or ply matrix controlled strengths at specified loads.
   Composite panels in each bay not to exceed combined stress buckling.

8. General philosophy on preliminary design of composite box beams:
   Size covers for only the vertical load and add plies for the combined loads (lateral and twist moment).
Size side walls for only the lateral load and add plies for the combined loads (vertical and twist moment).

STEP-BY-STEP DESIGN PROCEDURE

Once the design is defined to the extent just outlined, we are ready to design the composite laminates for the covers and the walls of the box beam by following the step-by-step design procedure.

Step 1: Identify Design Variables

Number of plies, ply orientation and stacking sequence for the composite covers and side walls for the three different bays.

Step 2: Establish Design Loads

Safety factor times specified loads (fig. 1):

\[ N_{cXX} = 2 \times \text{vertical load (6600 lb)} = 13 \text{ 200 lb} \]
\[ N_{cYY} = 2 \times \text{lateral load (3300 lb)} = 6600 \text{ lb} \]
\[ M_{cXX} = 2 \times \text{twist moment (100 000 lb in.)} = 200 000 \text{ lb-in.} \]

Step 3:

Obtain composite material properties (ply and ±\( \theta \) angleply) for AS/E from table I and figures 3 and 4.

Step 4:

Select laminate configurations for box beam covers and side walls in each of the three bays.

(a) Calculate in-plane membrane loads at the bulkhead locations (figs. 1 and 2): These loads are calculated by dividing the moment at that section by the respective depth and width.

At span station 0:

covers:

\[ N_{cXX} = P_z x L/(h \times w) \]
\[ = 13 \text{ 200 lb x 60 in.}/(10 \text{ in. x 20 in.}) \]
\[ = 3960 \text{ lb/in.} \]
walls:
\[ N_{cyy} = \frac{P_y \cdot 2}{(h \cdot w)} \]
\[ = \frac{6600 \text{ lb} \times 60 \text{ in.}}{20 \text{ in.} \times 10 \text{ in.}} \]
\[ = 1980 \text{ lb/in.} \]

covers:
\[ N_{cxy} = \pm M_x / [(w \cdot h) \text{covers} + (w \cdot h) \text{side walls}] \]
\[ + \frac{P_y}{W_{\text{covers}}} \]
\[ = \pm 200 \, 000 \text{ lb-in.} / [(20 \text{ in.} \times 10 \text{ in.}) \]
\[ + (10 \text{ in.} \times 20 \text{ in.})] \]
\[ + 6600 \text{ lb/(2 x 20 in.)} \]
\[ = -335 \text{ lb/in. (top cover)} \]
\[ 665 \text{ lb/in. (bottom cover)} \]

covers:
\[ N_{czz} = \pm M_x / [(w \cdot h) \text{covers} + (w \cdot h) \text{side walls}] \]
\[ + \frac{P_y}{h_{\text{side walls}}} \]
\[ = \pm 200 \, 000 \text{ lb-in.} / [(20 \text{ in.} \times 10 \text{ in.}) \]
\[ + (10 \text{ in.} \times 20 \text{ in.})] \]
\[ + 13 \, 200 \text{ lb/(2 x 10 in.)} \]
\[ = 160 \text{ lb/in. (front wall)} \]
\[ 1160 \text{ lb/in. (back wall)} \]

Repeating the calculations for the other span stations and summarizing in table form, we have the results shown in table 2.

(b) Design bottom cover (pressure surface, figs. 1 and 2). This surface is in tension. We need to use the longitudinal tensile strength. Number of 0° plies \( N_{00} = \) Design load \( (N_{cxx}) \) / longitudinal tensile strength \((S_{llt} = 220 \, 000 \text{ psi}) \times \) ply thickness \((t_0 = 0.005 \text{ in.})\).

\[ N_{00} = \frac{N_{cxx}}{S_{llt} \cdot t_0} = \frac{+3960 \text{ lb/in.}}{220 \, 000 \text{ lb/sq in.} \times 0.005 \text{ in.}} = 3.6 \approx 4. \]

Number of ±45° plies \( N_{0\pm 45} = \) Design load \( (N_{cxy}) \) \times one-half the ratio of the ply longitudinal modulus \((E_{ll1} = 18.5 \text{ mpsi})\) to ±45° composite shear modulus \((G_{12} = 5.8 \text{ mpsi})\) / longitudinal compressive strength \((S_{ll1c} = 180 \, 000 \text{ psi}) \times \) ply thickness \((t_0 = 0.005 \text{ in.})\).
\[ N_{\perp 45} = \frac{N_{cxy} \times (1/2)(E_{11}/G_{12})}{s_{ll}c \times t_L} = \frac{665 \text{ lb/in} (1/2)(18.5/5.8)}{180,000 \text{ lb/sq in.} \times 0.005 \text{ in.}} = 1.16 \approx 2 \]

The number of plies is rounded up since plies are available in fixed thicknesses. Number of 90° plies \( N_{90} \): Note there is no \( N_{cyy} \) design load and therefore no 90° plies are needed. However, we will use two plies for laminate integrity and improved buckling resistance. Therefore, \( N_{90} = 2 \)

Thus, the laminate configuration for the bottom cover is eight plies as follows: 4 at 0°, 2 at ±45° and 2 at 90°. Note that this is not a symmetric laminate.

(c) Design top cover (suction surface figs. 1 and 2). This surface is in compression. We need to use the longitudinal compression strength to determine the number of zero plies.

\[ N_{\perp 0} = \frac{N_{cxy}}{s_{ll}c t_L} = \frac{3960 \text{ lb/in.}}{180,000 \text{ lb/sq in.} \times 0.005 \text{ in.}} = 4.4 \approx 4 \]

The number of ±45° plies is now determined from the ratio:

\[ N_{\perp 45} = \frac{N_{cxy}(\text{Top})}{N_{cxy}(\text{Bottom})} \times 1.16 \text{ plies} \]

\[ = \frac{335}{665} \times 1.16 = 0.58 \approx 1 \rightarrow \text{use two plies for a balanced laminate} \]

\[ N_{90} = 2 \ (\text{same reason as for bottom cover}) \]

Therefore, the top cover is an eight-ply laminate same as bottom cover.

(d) Design side wall (leading edge (front), figures 1 and 2, and loads from table step 4.a)

\[ N_{\perp 0} = \frac{N_{cxx}}{s_{ll}t x t_L} = \frac{1980 \text{ lb/in.}}{220,000 \text{ lb/sq in.} \times 0.005 \text{ in.}} = 1.8 \approx 2 \]

\[ N_{\perp 45} = \frac{N_{cxy}(1/2)(E_{11}/G_{12})}{s_{ll}c x t_L} \]

\[ = \frac{160 \text{ lb/in.}(1/2)(18.5/5.8)}{180,000 \text{ lb/in.}^2 \times 0.005 \text{ in.}} = 0.3 \text{ plies} \rightarrow \text{use two plies} \]

\[ N_{90} = 2 \ (\text{same reason as for covers}) \]

(e) Design side wall (trailing edge (back) figures 1 and 2 and loads from table step 4.a)
Therefore the laminate configuration for the sidewall is six plies as follows: 2 at 0°, 2 at ±45°, and 2 at 90°. Note that this also is not a symmetric laminate with respect to bending.

(f) Select plies at the other span stations. Examining the loads in table step 4.a, we see that: (1) \(N_{cxx}\) is smaller at all the other span stations, and (2) \(N_{cxy}\) is maximum at the 60-in. span station. At this point we can either calculate the number of plies at each of the span sections or calculate the number of plies we need for the maximum \(N_{cxy}\) at the 60 in. station and use a uniform laminate through the box beam. Using a uniform laminate simplifies the fabrication procedure but it will increase the weight. Assuming that the weight is not critical, we will calculate the number of ±45° plies we need at maximum \(N_{cxy}\) and use this number throughout the box beam.

The number of ±45 plies at maximum \(N_{cxy}\) (60-in. station) is

\[N_{\pm45} = \frac{N_{cxy} E_{11}/2G_{12}}{S_{11} c_t} = \frac{3320 \text{ lb in.} \times (1/2)(18.5/5.8)}{180 \text{ 000 lb/sq in.} \times 0.005 \text{ in.}} = 5.8 \approx 6\]

Therefore, the laminate configuration for the box beam is 10 plies as follows: 4 at ±45°, 4 at 0°, and 2 at 90°. Using the conventional notation, this is expressed:

\[\{\pm45/0_2/90\}_s\]

(g) Select the minimum number of plies at the various span stations to meet strength requirements. Repeating the calculations in steps 4.b, 4.c, and 4.d for the other span stations and summarizing the results we have the results listed in table 3.

Remarks:

(1) The maximum number of 0° plies required is four while that for ±45 is six. These ply combinations result in an unsymmetric laminate.

(2) The total number of plies required varies from six in the walls to nine in the covers.

(3) For fabrication convenience, assume constant number of plies throughout the box beam.
Since the laminate is relatively thin, panel buckling will control the design.

Decide on a laminate configuration and check the panel buckling load.

A reasonable symmetric laminate configuration is the 10-ply laminate, 0.050 in. thick, as follows: four plies at 0°, four plies at ±45°, and two plies at 90° as was already mentioned. Using conventional notation this laminate configuration is designated:

\[ [±45°/0/90]_s \]

Step 5:

Calculate the composite stresses at the four-span stations. These stresses are calculated by dividing the in-plane load at that station with the laminate thickness (\(N_{cxx}/t_c\), etc.). Summarizing the results in tabular form, we have the results listed in table 4.

The composite stresses summarized above will be used to check the buckling stresses and the ply stresses as described below.

Step 6:

Calculate the buckling stresses of the panels at each bay. In order to expedite these calculations, we use the midbay panel dimensions and apply the stresses summarized above. This approach is reasonable for preliminary design. However, it needs to be checked with finite element analysis for more accurate results.

Tabulating the results we have the summary shown in table 5.

We calculate the buckling stresses by using the approximate interaction equation (ref. 1):

\[
\frac{\sigma_{cxx}}{\sigma_{cxx}^{(cr)}} + \left(\frac{\sigma_{cxy}}{\sigma_{cxy}^{(cr)}}\right)^2 \leq 1
\]

where

\[
\sigma_{cxx}^{(cr)} \approx \frac{\pi^2 t_c^2 E}{12b^2(1 - v_{cxy}v_{cyx})} \left(\frac{a}{b} + \frac{b}{a}\right)^2
\]

\[
\sigma_{cxy}^{(cr)} \approx \frac{7\pi^2 t_c^2 E}{12b^2(1 - v_{cxy}v_{cyx})} \left(1 \leq \frac{a}{b} \leq 2\right)
\]

\[
E = 3\sqrt{\frac{E_{cxx}E_{cyy}G_{cxy}}{E_{cxx}}} \]

7
The moduli and Poisson's ratios for a \([\pm 45/02/90]_s\) AS/E angleplied laminate are (ref. 1):

\[
\begin{align*}
E_{cxx} &= 9.6 \text{ mpsi}; \\
E_{cyy} &= 6.5 \text{ mpsi}; \\
G_{cxy} &= 2.3 \text{ mpsi}; \\
\nu_{cxy} &= 0.33; \\
\nu_{cyx} &= 0.22.
\end{align*}
\]

Substituting these values, we calculate

\[
E = 3\sqrt{4 \times 9.6 \times 6.5 \times 2.3 \text{ mpsi}} = 8.31 \text{ mpsi}
\]

First we check the buckling stresses of the top cover at the first midbay (0 to 20)

\[
\sigma_{cxx}^{(cr)} = \frac{\pi^2 (0.05)^2 \times 8.31 \times 10^6}{12(18.4)^2(1 - 0.33 \times 0.22)} \left(\frac{20}{18.4} + \frac{18.4}{20}\right)^2 = 218 \text{ psi}
\]

\[
\sigma_{cxy}^{(cr)} = \frac{7\pi^2 (0.05)^2 \times 8.31 \times 10^6}{12(18.4)^2(1 - 0.33 \times 0.22)} = 380.9 \text{ psi}
\]

Using these buckling stresses in the combined stress interaction equation above, we calculate:

\[
\begin{align*}
\left(\begin{array}{c}
-79200 \\
-218
\end{array}\right) + \left(\begin{array}{c}
-6700 \\
-380.9
\end{array}\right)^2 &\leq 1 \\
672.7 &> 1
\end{align*}
\]

indicating that the panel will buckle.

Remark: These panel thicknesses are too low to resist buckling due to the applied load stresses in the top cover and back side wall panels since all these panels are subjected to combined compressive and shear stresses. On the other hand, the panels in the bottom cover and front side wall may not buckle because these panels are subjected to combined tensile and shear stresses. The most critical case is the bottom cover at the third midbay (40 to 60). The calculated individual buckling stresses for this panel are:

\[
\sigma_{cxx}^{(cr)} = 714 \text{ psi}; \sigma_{cxy}^{(cr)} = 942 \text{ psi}
\]

and combined in the interaction equation:

\[
\begin{align*}
\left(\begin{array}{c}
59260 \\
-714
\end{array}\right) + \left(\begin{array}{c}
27400 \\
942
\end{array}\right)^2 &\leq 1 \\
763 &> 1
\end{align*}
\]

763 >> 1 and this panel will also buckle.

The conclusion from the above calculations is that the panel thickness sized for strength is too thin to resist buckling.
Step 7:

At this point, we can consider several alternatives to increase the panel buckling resistance. The obvious ones are: (1) increase the panel thickness, (2) reduce the panel edge dimensions by using inner walls and additional bulkheads, and (3) use combinations of these.

First we check alternative (1) - increase panel thickness. Calculate panel thickness to resist buckling stress. Since the buckling stress varies with the thickness squared and assuming panel thickness in multiples of \(\pm 45/0/90\) s we calculate a thickness for the compressive stress:

\[
t_c \approx \left( \frac{79 \times 2000}{218} \right)^{1/3} 0.36 \text{ in.}; \text{ use } 0.45 \text{ in. } \approx 0.50 \text{ in.}
\]

for combined stress.

This results in a \([\pm 45/0/90]_{100}\) symmetric laminate with 100 plies. This many plies will substantially increase the material and fabrication costs.

Check buckling stresses for the same panel.

\[
\sigma_{cXX} = (0.05/0.50) \times 79 \times 200 \text{ psi } = 7920 \text{ psi}
\]

\[
\sigma_{cXY} = (0.05/0.50) \times 670 \text{ psi } = 670 \text{ psi}
\]

\[
\sigma_{cXX}^{(cr)} = (0.50/0.05)^2 218 \text{ psi } = 21800 \text{ psi}
\]

\[
\sigma_{cXY}^{(cr)} = (0.5/0.05)^2 380.9 \text{ psi } = 38090 \text{ psi}
\]

\[
\frac{\sigma_{cXX}}{\sigma_{cXY}^{(cr)}} + \left( \frac{\sigma_{cXY}}{\sigma_{cXX}^{(cr)}} \right)^2 \leq 1
\]

\[
\frac{7920}{21800} + \left( \frac{670}{38090} \right)^2 = 0.363 < 1 \quad \text{O.K.}
\]

Therefore the panel satisfies the combined stress buckling interaction equation with a margin of safety \(1 - 0.363 = 0.64\).

Alternative (2) will increase the buckling stresses but will not reduce the stresses due to applied loads. Alternative (3) on the other hand will increase the buckling stresses and also reduce the stresses due to applied loads.

Check buckling stresses by adding an inner vertical wall through the box beam center. Since this will reduce the panel edge dimension by 2 it will increase the buckling stress by 4. Therefore we assume 60-ply laminate as follows:
\([\pm 45/0_2/90]_6s\)

with 0.3 in. thickness.

The stresses in the panel at the first bay are:

\[
\sigma_{cxx} = (0.05/0.3) \times 79200 \text{ psi} = 13200 \text{ psi}
\]

\[
\sigma_{cxy} = (0.05/0.3) \times 6700 \text{ psi} = 1117 \text{ psi}
\]

The corresponding buckling stresses are

\[
\sigma_{cxx}^{(cr)} = 4(0.3/0.05)^2 \times 218 \text{ psi} = 31392 \text{ psi}
\]

\[
\sigma_{cxy}^{(cr)} = 4(0.3/0.05)^2 \times 380.9 \text{ psi} = 54850 \text{ psi}
\]

The combined stress buckling interaction equation is

\[
\frac{13200}{31392} + \left(\frac{1117}{54850}\right)^2 = 0.42 < 1 \quad \text{O.K.}
\]

and the margin of safety MOS = 0.58.

This is a lighter weight design (by 30 percent) compared to that of alternative (1).

**Step 8: Summarize Design**

The designed box-beam, therefore is a 60-ply \([\pm 45/0_2/90]_6s\) laminate 0.3 in. thick with an inner vertical wall through the box-beam center. The panel geometry at mid bays and the respective stresses at the bulkheads are summarized in table VI.

**Step 9: Check Tip Displacements**

(a) **Vertical displacement.** - This displacement is calculated from (neglecting box beam weight)

\[
\omega = \frac{p_z \ell^3}{3E_{cxx} I_{cyy}}
\]
where $I_{cyy}$ is calculated at midspan as follows:

$$I_{cyy} = \left[ 2b \left( \frac{h}{2} \right)^2 t_c + 3 \left( \frac{1}{12} \right) h^3 t_c \right] \text{in.}^4$$

$$= \left[ 2 \times 15 \left( \frac{7.5}{2} \right)^2 (0.3) + 3 \frac{1}{12} (7.5)^3 (0.3) \right] \text{in.}^4$$

$$= (126.6 + 31.6) \text{in.}^4$$

$$= 158.2 \text{ in.}^4$$

The values of the variables required in the above equation are

- $P_z = 13200 \text{ lb}$
- $L = 60 \text{ in.}$
- $E_{cxx} = 9.6 \text{ mpsi}$
- $I_{cyy} = 158.2 \text{ in.}^4$

$$w = \frac{13200 \text{ lb} \times 60 \times 60 \times 60 \text{ in}^3}{3 \times 9600000 \left( \text{lb/in}^2 \right) \times 158.2 \text{ in}^4}$$

$$w = 0.63 \text{ in.} < 0.90 \left( 1.5\% 60 \text{ in.} \right) \text{ in.} \text{ O.K.}$$

$$MOS = \frac{0.90}{0.63} - 1.0 = 0.43$$

(b) Lateral displacement. - This displacement is calculated from

$$v = \frac{p_y a^3}{3E_{cxx} I_{czz}}$$

where, again, $I_{czz}$ is calculated at midspan as follows:

$$I_{czz} = \left[ 2h \left( \frac{b}{2} \right)^2 t_c + 2 \left( \frac{1}{12} \right) b^3 t_c \right] \text{in.}^4$$

$$= \left[ 2(7.5) \left( \frac{15}{2} \right)^2 (0.3) + 2 \left( \frac{1}{12} \right) (15)^3 (0.3) \right] \text{in.}^4$$

$$= [253.1 + 168.8] \text{in.}^4$$

$$= 421.9 \text{ in.}^4$$
The values of the variables required to calculate the lateral displacement are

\[ P_y = 6600 \text{ lb} \]
\[ \ell = 60 \text{ in.} \]
\[ E_{cxx} = 9.6 \text{ mpsi} \]
\[ I_{czz} = 421.9 \text{ in.}^4 \]

\[ v = \frac{6600 \text{ lb} \times 60 \times 60 \times 60 \text{ in.}^3}{3 \times 9 \times 600 \times 000 \text{ (lb/in.}^2) \times 421.9 \text{ in.}^4} \]

\[ v = 0.12 \text{ in.} < 0.90 \text{ in.} \text{ O.K.} \]

\[ \text{MOS} = \frac{0.90}{0.12} - 1 = 6.5 \]

(c) Angle of Twist. - This angle is calculated from

\[ \theta \approx \frac{M_x \ell}{JG} \]

where

\[ J = I_{cxx} + I_{cyy} \]

and

\[ G = G_{cxy} \]

The values of the variables required to calculate the twist angle are

\[ M_x = 200 \times 000 \text{ in.-lb} \]
\[ \ell = 60 \text{ in.} \]
\[ J = (158.2 + 421.9) \text{ in.}^4 = 580.1 \text{ in.}^4 \]
\[ G = 2.3 \text{ mpsi} \]

\[ \theta = \frac{200 \times 000 \text{ in.-lb} \times 60 \text{ in.}}{580.1 \text{ in.}^4 \times 2300 \times 000 \text{ 1b/in.}^2} \text{ rad} \]

\[ = 0.008994 \text{ rad} \]

\[ \theta = 0.52^\circ < 1.0^\circ \text{ O.K.} \]

\[ \text{MOS} = \frac{1.0}{0.52} - 1 = 0.92 \]
Step 10:

Calculate the first flap-wise, edge-wise frequencies, and the first torsional frequency.

(a) The flap-wise (vertical) frequency is calculated from

\[ \omega_z \approx \frac{1}{2\pi} \left( \frac{1.9}{\lambda} \right)^2 \left[ \frac{E_{cxx} I_{cxx}}{M} \right]^{1/2} \text{ cyc/sec} \]

where \( M \) is the mass per unit length and \( I_{cxx} \) is the moment of inertia, both calculated at midspan (2-covers, 3-walls and 4-bulkheads)

\[
\begin{align*}
M &= \left[ 2b + 3h + \frac{1}{\lambda} \sum_{i=1}^{4} (b \times h)_i \right] \frac{t_c p/g}{\text{lb}/\text{in.}^2} = \left[ 30 + 22.5 + \frac{1}{60} (478) \right] \frac{0.3 \times 0.06}{386.4} \text{ lb/sec}^2 \\
&= 0.0028 \text{ lb sec}^2/\text{in.} \times \frac{1}{\text{in.}}
\end{align*}
\]

The values of the variables required to calculate this frequency are

\( \lambda = 60 \text{ in.} \)

\( E_{cxx} = 9.6 \text{ mpsi} \)

\( I_{cxx} = 158.2 \text{ in.}^4 \)

\( M = 0.0028 \text{ lb sec}^2/\text{in.} \times \frac{1}{\text{in.}} \)

\[
\begin{align*}
\omega_z &= \frac{(1.9 \times 1.9)}{2\pi (60 \text{ in.} \times 60 \text{ in.})} \left[ \frac{9600 000(\text{lb/in.}^2)158.2 \text{ in.}^4}{0.0028 \text{ lb/sec}^2/\text{in.} \times \frac{1}{\text{in.}}} \right]^{1/2} \text{ cyc/sec} \\
&= 117.5 \text{ cyc/sec} > 100 \text{ cyc/sec} \text{ O.K.}
\end{align*}
\]

\[
MOS = \frac{117.7}{100} - 1 = 0.18
\]

(b) The edge-wise lateral frequency is calculated from

\[ \omega_y = \frac{1}{2\pi} \left( \frac{1.9}{\lambda} \right)^2 \left[ \frac{E_{cxx} I_{cyy}}{M} \right]^{1/2} \]

This equation differs from \( \omega_z \) only in \( I_{cyy} \). We can expedite the calculation.
\[ \omega_y \approx \left( \frac{I_{cyy}}{I_{cxx}} \right)^{1/2} \]  
\[ \approx \left( \frac{421.9}{158.2} \right)^{1/2} \times 117.5 \text{ cyc/sec} \]  
\[ \omega_y \approx 191.9 \text{ cyc/sec} > 500 \text{ cyc/sec} \quad \text{O.K.} \]

\[ \text{MOS} = \frac{191.9}{150} - 1 = 0.23 \]

(c) The torsional frequency is calculated from

\[ \omega_t \approx \frac{1}{4}\pi\left(\frac{G}{\rho}\right)^{1/2} \]

The values for the variables are

\[ \ell = 60 \text{ in.} \]
\[ G = 2.3 \text{ mpsi} \]
\[ g = 386.4 \text{ in/sec}^2 \]
\[ \rho = 0.06 \text{ lb/in.}^3 \]

\[ \omega_t \approx \frac{1.0}{4 \times 60} \left[ \frac{2300000(\text{lb/in.}^2) \times 386.4 \text{ in./sec}^2}{0.06 \text{ lb/in.}^3} \right]^{1/2} \]

\[ \omega_t \approx 507.1 \text{ cyc/sec} > 450 \text{ cyc/sec} \quad \text{O.K.} \]

\[ \text{MOS} = \frac{507.1}{450} - 1 = 0.13 \]

Step II: Check Local Panel Vibration

We calculate the first frequency for the first bay panel (0-20 span) assuming a rectangular panel with midside dimensions. This frequency is given by

\[ \omega = \frac{\pi^2 c}{2a^2} \left( \frac{g}{12p(1 - \nu_{cyx}^2 \nu_{cxy})} \right)^{1/2} \left[ (1 + 2\nu_{cyx}^2 \nu_{cxy}^4)E_{cxy} + E_{uy}C^4 \right]^{1/2} + 4C^2(1 - \nu_{cyx}^2 \nu_{cxy})G_{cxy} \]
where

t_c \text{ panel thickness (in.)}

a \text{ panel x-edge dimension (in.)}

g \text{ gravity acceleration (in./sec}^2\text{)}

\rho \text{ composite laminate density (lb/in.}^3\text{)}

C \text{ a/b where b is the panel y-edge dimension (in.)}

where \( E_{cxx}, E_{cyy}, \) and \( G_{cxy} \) are the composite laminate moduli and where \( \nu_{cxy} \) and \( \nu_{cyx} \) are composite laminate Poisson's ratios.

The values for the variables in the frequency calculation are

\[
t_c = 0.3 \text{ in.} \quad E_{cxx} = 9.6 \text{ mpsi} \\
a = 20 \text{ in.} \quad E_{cyy} = 6.5 \text{ mpsi} \\
g = 386.4 \text{ in./sec}^2 \quad G_{cxy} = 2.3 \text{ mpsi} \\
\rho = 0.06 \text{ lb/in.}^3 \quad \nu_{cxy} = 0.33 \\
c = (20/9.2) = 2.17 \quad \nu_{cyx} = 0.22
\]

Substituting these values in the frequency equation

\[
\omega = \frac{\pi(0.3)}{2 \times 20 \times 20} \left[ \frac{386.4}{12 \times 0.06 \times (1 - 0.33 \times 0.22)} \right]^{1/2} \left[ (1 + 2 \times 0.22 \times 2.17^2) \times 9.6 \right.
\]

\[
+ 4(2.17)^2(1 - 0.33 \times 0.22) \times 2.3 + 6.5 \times (2.17)^4 \right]^{1/2} \times 1000 \text{ cyc/sec}
\]

\[\omega = 414.4 \text{ cyc/sec}\]

This frequency is greater than the first two (\( \omega_x \) and \( \omega_y \)) frequencies of the box beam. Therefore, no local vibration will occur prior to the first box beam modes. However it could occur prior to the first torsional mode.

Step 12: Check Ply Stresses

The ply stresses are determined through the use of the ply stress influence coefficients as described in detail in Ref. 1. The ply stress influence coefficient for the laminate selected are shown in table VII.

The ply stress \( \sigma_{l11} \) is calculated as follows (include only nonzero coefficients)

\[
\sigma_{l11} = 1.98 \sigma_{cxx} - 0.56 \sigma_{cyy} + 9.35 \Delta T + 627 \text{ M}
\]
Examining the ply stress influence coefficients we see that \( \sigma_{11} \) for the +45\(^\circ\) and \( \sigma_{22} \) for the -45\(^\circ\) ply have relatively large values. From the panel composite stresses summary in Step 8 we check the bottom cover in the bays 2 and 3 as follows: (neglecting temperature and moisture).

(a) +45\(^\circ\)-Ply (3-bay bottom cover)

\[
\sigma_{11} = 0.70 \sigma_{cxx} + 4.0 \sigma_{cxy}
= 0.70 \times 9877 + 4.0 \times 4567
= 25182 \text{ psi} < 220000 \text{ psi} \quad \text{O.K.}
\]

\[
\text{MOS} = \frac{220000}{25182} \quad -1 = 7.74
\]

(b) -45\(^\circ\)-Ply (1-bay bottom cover)

\[
\sigma_{22} = 0.55 \sigma_{cxx} + 0.19 \sigma_{cxy}
= 0.55 \times 13200 + 0.19 \times 2217
= 7681 \text{ psi} < 8000 \text{ psi} \quad \text{O.K.}
\]

\[
\text{MOS} = \frac{8000}{7681} \quad -1 = 0.04
\]

(c) Check the above ply stresses by including residual and moisture stresses. For residual stresses \( \Delta T = -300 \text{ °F} \) which is the difference between the cure and room temperatures. For the moisture stresses assume \( M = 1 \) percent by weight.

+45\(^\circ\)-Ply:

\[
\sigma_{11} = 0.70 \sigma_{cxx} + 4.00 \sigma_{cxy} + 21.02 \Delta T + 1413 M
= 0.70 \times 9877 + 4.0 \times 4567 + 21.02 (-300) + 1413 (1)
= 6914 + 18268 - 6306 + 1413
= 20289 \text{ psi} < 220000 \text{ psi} \quad \text{O.K.}
\]

\[
\text{MOS} = \frac{220000}{20289} \quad -1 = 9.8
\]

-45\(^\circ\)-Ply:

\[
\sigma_{22} = 0.55 \sigma_{cxx} + 0.19 \sigma_{cxy} - 1876 \Delta T - 1263 M
= 0.55 \times 13200 + 0.19 \times 2217 - 18.76 (-300) - 1263 (1)
= 7620 + 421 + 5628 - 1263
= 12046 \text{ psi} > 8000 \text{ psi} \quad \text{N.G.}
\]
This last calculation indicates that the -45°-Ply will crack in transverse tension due to combined design mechanical and environmental loads. The last calculation also illustrates the significance of residual stresses in composites. Since this is a matrix failure mode, we check the ply stress at specified mechanical loads. Recall that the design loads are two times the specified loads.

\[
\sigma_{22} = \frac{1}{2} (7620 + 421) + 5628 - 1263
\]

\[
= 8386 \text{ psi} > 8000 \text{ psi}
\]

\[
\text{MOS} = \frac{8000}{8386} - 1 = -0.05
\]

This may be considered acceptable in the absence of cyclic loads.

Step 13: Summarize Design Results

(a) Laminate configuration [±45/02/90]6s 0.3 in. thick

(b) Box-beam design - uniform laminate thickness, two intermediate bulkheads, and one inner wall located at the box beam center (see fig. 2)

(c) Box-beam weight = 66 lb (composite volume times density)

(d) Tip displacements MOS

<table>
<thead>
<tr>
<th>Displacement</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>0.43</td>
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<tr>
<td>v</td>
<td>6.50</td>
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<tr>
<td>θ</td>
<td>0.92</td>
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</table>

(e) Buckling load MOS = 0.58

(f) Vibration frequencies MOS

<table>
<thead>
<tr>
<th>Box beam frequency</th>
<th>MOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_z )</td>
<td>0.18</td>
</tr>
<tr>
<td>( \omega_y )</td>
<td>0.23</td>
</tr>
<tr>
<td>( \omega_t )</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Panel 1-bay
top cover 1.16

(g) Ply stresses

+45°-Ply (3-bay-bottom cover)  
MOS
longitudinal stress
mechanical loads 7.7  
mechanical and environmental 9.8

-45°-Ply (1-bay-bottom cover)
transverse stress
mechanical loads 0.04  
mechanical and environmental -0.34

CONCLUDING REMARKS

Step-by-step design procedures are described which can be used for the preliminary design of composite box beams. The various calculations in these procedures are arranged so that they can be performed using a pocket calculator. The sample calculations are for the design of a cantilevered composite box beam subjected to end loads. The composite laminate is selected to satisfy design requirements for local buckling, tip displacements, beam and panel vibrations, and ply stresses including thermal and hygral (moisture) stresses. The procedures and the sample calculations illustrated can be used for the preliminary design of composite built-up structures in general.

REFERENCES

TABLE I. - TYPICAL PROPERTIES OF UNIDIRECTIONAL COMPOSITES AT ROOM TEMPERATURE

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Units</th>
<th>Boron/epoxy</th>
<th>Boron/polyamide</th>
<th>S-glass/epoxy</th>
<th>Modmor 1/epoxy</th>
<th>Modmor 1/polyamide</th>
<th>Thornel 300/epoxy</th>
<th>Kevlar 49/epoxy</th>
<th>Graphite AS/epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber volume ratio</td>
<td>( k_f )</td>
<td>---------</td>
<td>0.50</td>
<td>0.49</td>
<td>0.72</td>
<td>0.45</td>
<td>0.45</td>
<td>0.70</td>
<td>0.54</td>
<td>0.60</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>lb/in³</td>
<td>0.073</td>
<td>0.072</td>
<td>0.077</td>
<td>0.056</td>
<td>0.056</td>
<td>0.058</td>
<td>0.049</td>
<td>0.057</td>
</tr>
<tr>
<td>Longitudinal thermal coefficient</td>
<td>( \alpha_{11} )</td>
<td>10⁻⁶ in/°F</td>
<td>3.4</td>
<td>2.7</td>
<td>2.1</td>
<td>--------</td>
<td>0.0</td>
<td>0.01</td>
<td>-1.60</td>
<td>0.40</td>
</tr>
<tr>
<td>Transverse thermal coefficient</td>
<td>( \alpha_{22} )</td>
<td>10⁻⁶ in/°F</td>
<td>16.9</td>
<td>15.8</td>
<td>9.3</td>
<td>18.5</td>
<td>14.1</td>
<td>12.5</td>
<td>31.3</td>
<td>16.4</td>
</tr>
<tr>
<td>Longitudinal modulus</td>
<td>( E_{11} )</td>
<td>10⁶ psi</td>
<td>29.2</td>
<td>32.1</td>
<td>8.8</td>
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<td>31.3</td>
<td>21.0</td>
<td>12.2</td>
<td>16.0</td>
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<td>Transverse modulus</td>
<td>( E_{22} )</td>
<td>10⁶ psi</td>
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<td>3.6</td>
<td>1.03</td>
<td>0.72</td>
<td>1.5</td>
<td>0.70</td>
<td>2.2</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>( G_{12} )</td>
<td>10⁶ psi</td>
<td>0.78</td>
<td>1.11</td>
<td>1.74</td>
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<td>0.65</td>
<td>1.0</td>
<td>0.41</td>
<td>0.72</td>
</tr>
<tr>
<td>Major Poisson's ratio</td>
<td>( \nu_{12} )</td>
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<td>0.17</td>
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<td>0.23</td>
<td>0.10</td>
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<td>0.28</td>
<td>0.32</td>
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<td>Minor Poisson's ratio</td>
<td>( \nu_{21} )</td>
<td>---------</td>
<td>0.02</td>
<td>0.02</td>
<td>0.09</td>
<td>--------</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.34</td>
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<tr>
<td>Longitudinal tensile strength</td>
<td>( S_{11T} )</td>
<td>psi</td>
<td>199000</td>
<td>151000</td>
<td>187000</td>
<td>122000</td>
<td>117000</td>
<td>218000</td>
<td>172000</td>
<td>220000</td>
</tr>
<tr>
<td>Longitudinal compressive strength</td>
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<td>psi</td>
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<td>158000</td>
<td>119000</td>
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<td>180000</td>
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<td>Transverse tensile strength</td>
<td>( S_{22T} )</td>
<td>psi</td>
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<td>1600</td>
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<td>9400</td>
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<tr>
<td>Intralaminar shear strength</td>
<td>( S_{12S} )</td>
<td>psi</td>
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<td>6500</td>
<td>8900</td>
<td>3150</td>
<td>9800</td>
<td>4000</td>
<td>10000</td>
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<td>Longitudinal moisture coefficient</td>
<td>( \beta_{11} )</td>
<td>10⁻² in</td>
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<td>0.003</td>
<td>0.014</td>
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<td>10⁻² in</td>
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<td>0.168</td>
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<td>Glass transition temperature (estimate)</td>
<td>( T_{GD} )</td>
<td>°F</td>
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TABLE II. - FORCES IN COVERS AND SIDEWALLS

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<tr>
<th>Span station, in.</th>
<th>Length, in.</th>
<th>Covers Depth, in.</th>
<th>Width, in.</th>
<th>( N_{CXX}, ) kis</th>
<th>( N_{CXY}, ) kis</th>
<th>Side Walls Depth, in.</th>
<th>Width, in.</th>
<th>( N_{CYY}, ) kis</th>
<th>( M_{CXX}, ) kis</th>
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<td>bot</td>
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<td>3809</td>
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<td>10</td>
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<td>----</td>
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<td>+2330</td>
<td>10</td>
<td>5</td>
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</table>

Span station, in. | Length, in. | Covers Depth, in. | Width, in. | \( N_{CXX}, \) kis | \( N_{CXY}, \) kis | Side Walls Depth, in. | Width, in. | \( N_{CYY}, \) kis | \( M_{CXX}, \) kis |
<table>
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<td>-2330</td>
<td>+2330</td>
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<td>5</td>
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</tbody>
</table>

19
TABLE III. - PLIES IN COVERS AND SIDEWALLS

<table>
<thead>
<tr>
<th>Span station</th>
<th>Plies in the covers</th>
<th>Plies in the walls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0°</td>
<td>±45°</td>
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<tr>
<td>1. 0</td>
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<td>3. 40</td>
<td>3</td>
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<tr>
<td>4. 60</td>
<td>-</td>
<td>4</td>
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TABLE IV. - PLY STRESSES IN COVERS AND WALLS

<table>
<thead>
<tr>
<th>Span station</th>
<th>Covers (psi)</th>
<th>Side walls (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Bottom</td>
</tr>
<tr>
<td></td>
<td>σcxx</td>
<td>σcxy</td>
</tr>
<tr>
<td>1. 0</td>
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<td>-7 600</td>
</tr>
<tr>
<td>2. 20</td>
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<td>-10 460</td>
</tr>
<tr>
<td>3. 40</td>
<td>-59 260</td>
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</tr>
<tr>
<td>4. 60</td>
<td>-59 260</td>
<td>46 600</td>
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</tbody>
</table>

TABLE V. - BUCKLING STRESSES

<table>
<thead>
<tr>
<th>Midbay Panel y</th>
<th>Bay/span station</th>
<th>Covers</th>
<th>Walls</th>
<th>Covers</th>
<th>Walls</th>
<th>Covers</th>
<th>Walls</th>
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</thead>
<tbody>
<tr>
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<td>Stresses, psi</td>
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<td>σcxy</td>
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<td>Bottom cover</td>
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### TABLE VI. - FINAL DESIGN STRESSES

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<th>Bay/span station</th>
<th>1 (0-20)</th>
<th>2 (20-40)</th>
<th>3 (40-60)</th>
<th>Covers</th>
<th>Walls</th>
<th>Covers</th>
<th>Walls</th>
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<td>20</td>
<td>20</td>
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### TABLE VII. - COMBINED HYGROTHERMOMECHANICAL LOAD STRESS ASSESSMENT

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<th>Ply/ply stress</th>
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<td>-45°-ply</td>
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FIGURE 1. - COMPOSITE BOX BEAM GEOMETRY AND SPECIFIED LOADING CONDITIONS (ALL DIMENSIONS IN INCHES; LOADS IN POUNDS; TWIST MOMENT IN INCH-POUNDS).

FIGURE 2. - COMPOSITE BOX BEAM SURFACE AND DESIGN LOAD NOMENCLATURE.
Figure 3. - Elastic properties of AS-graphite-fiber/epoxy (AS/E) ±0 laminates.

Figure 4. - Reduced stiffnesses of AS graphite-fiber/epoxy (AS/E) ±0 laminates.
Design Procedures for Fiber Composite Box Beams

Christos C. Chamis and Pappu L. N. Murthy

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

Christos C. Chamis, NASA Lewis Research Center; Pappu L.N. Murthy, Cleveland State University, Civil Engineering Department, Cleveland, Ohio 44115.

Step-by-step procedures are described which can be used for the preliminary design of fiber composite box beams subjected to combined loadings. These procedures include a collection of approximate closed-form equations so that all the required calculations can be performed using pocket calculators. Included is an illustrative example of a tapered cantilever box beam subjected to combined loads. The box beam is designed to satisfy strength, displacement, buckling, and frequency requirements.