

Electro-Mechanical Vibratory System

Mario Paz

The problem of predicting the phase angle of two self-synchronized rotors starting from rest, is presented in this paper. It is shown that with insufficient power the rotors may not reach the final operating speed of the motors and stay locked at one of the lower natural frequencies of the vibrating system, thus producing large amplitude and failure of the equipment.

INTRODUCTION

Mechanical vibrators are widely used for producing the motion required in some manufacturing processes such as packing cement mixtures, screening, vibration transportation, vibration pile drivers, etc. The principal component of the mechanical vibratory is an unbalanced mass rotated by an electric motor. In some of these applications, rectilinear vibratory motion is required. This motion may be attained using two synchronized motors having eccentric masses rotating in opposite directions. Synchronization of the rotating masses means coordinating the rotation to maintain a definite phase angle between the rotation of the two motors. Such synchronization may be accomplished by using kinematic coupling such as toothed or chain drivers between the two rotors. However, some years ago, it became apparent the synchronization could be maintained without resorting to any kind of kinematic coupling. It was observed that the angular velocities and phases of the motors automatically kept a definite phase rotation without the use of coupling. Huygens [1] first observed a case of self-synchronization in the seventeenth century. He noticed that two pendulum clocks, regardless of their initial phase angle, came into synchronization if they were mounted on a common resilient support such as a flexible beam. He realized that this phenomenon of self-synchronization could be explained by the small imperceptible motion in the beam transmitted by the pendulums.

At the present time, self-synchronization is finding wider applications in the design of vibrating machines [2]. The use of self-synchronization makes forced synchronization

unnecessary. The condition of self-synchronization, as well as the phase angle maintained by the rotors, has been investigated initially by Blekman [3] for one directional motion, by Paz [4,5] for two directional motions and subsequently by others [6,7,8]. Paz investigated analytically and experimentally the problem of predicting the stable phase angle and the direction of the resultant force of any two rotors with parallel axis of rotation. He showed that the phase angle between two rotors, as well as the condition necessary for a stable solution could be predicted with the application of Hamilton's Principle.

These developments gave solutions prescribing the conditions for self-synchronization, but did not focus in the initial transient portion of the motion until it reaches the final steady-state condition. In this transient state, the system may have to transverse some of its natural frequencies at which resonance may occur, locking the motion at a speed lower than the operating speed of the motors, thus, resulting in large amplitudes of motion and failure of the equipment. The electro-mechanical analysis of vibrating equipment during the initial transient motion, is presented in this paper.

CHARACTERISTICS OF INDUCTION MOTOR

The torque capacity of an induction motor is primarily a function of its rotating speed. Fig.1 shows a typical torque-speed curve for this type of motor. The figure also shows an assumed torque-speed curve required to maintain rotation as demanded by the load. In this case, the system will operate in the transient state until it reaches the steady-state condition at which the torque demanded by the load equals the torque of the motor, point P in Fig.1.

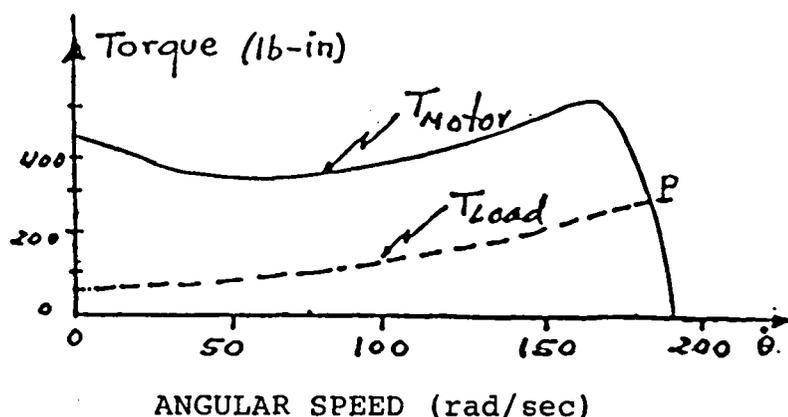


Fig. 1 Speed-Torque Curve For Typical Induction Motor

At any speed $\dot{\theta}$ (rad/sec), the torque differential ΔT between that produced by the motor and the required by the load is available to accelerate the motor. Consequently, applying Newton's second law for a rotating mass, this differential torque is

$$\Delta T = J \cdot \frac{d\dot{\theta}}{dt} \quad (1)$$

in which J is the mass moment of inertia of the rotating parts of the motor and θ the angular velocity of the rotor.

MECHANICAL VIBRATING SYSTEM

The equations of motions of a mechanical vibrating system may be obtained by direct application of Newton's Law of motion or alternatively by using an energy method such as the application of Lagrange's equation. In any case, the equations of motion for a linear system may, in general, be expressed in matrix notation as

$$[M]\{\ddot{Y}\} + [C]\{\dot{Y}\} + [K]\{Y\} = \{F(t)\} \quad (2)$$

where $[M]$, $[C]$ and $[K]$ are respectively the mass, damping and stiffness matrices; and $\{Y\}$, $\{\dot{Y}\}$, $\{\ddot{Y}\}$ the displacement, velocity and acceleration vectors of the generalized coordinates.

In the solution of Eq.(2), it is convenient to use modal superposition method to transform these equations to a system of uncoupled equation of the form

$$\ddot{z}_i + 2\zeta_i \omega_i \dot{z}_i + \omega_i^2 z_i = P_i(t) \quad (i = 1, 2, \dots, N) \quad (3)$$

where z_i = modal displacement
 \dot{z}_i = modal velocity
 \ddot{z}_i = modal acceleration
 ω_i = natural frequency
 ζ_i = modal damping ratio

and $P_i(t)$ = modal force

which is given by

$$P_i(t) = \sum_{j=1}^N F_j(t) \phi_{ji} \quad (4)$$

where ϕ_{ji} is the j -component of i -th eigenvector.

The solution of the uncoupled equation, Eq.(3) during the transient motion may be obtained by numerical integration in terms of the modal coordinates z_i .

The motion in function of the generalized coordinates is then given by the transformation

$$\{Y\} = [\bar{\Phi}]\{Z\} \quad (5)$$

where $[\bar{\Phi}]$ is the modal matrix of the system.

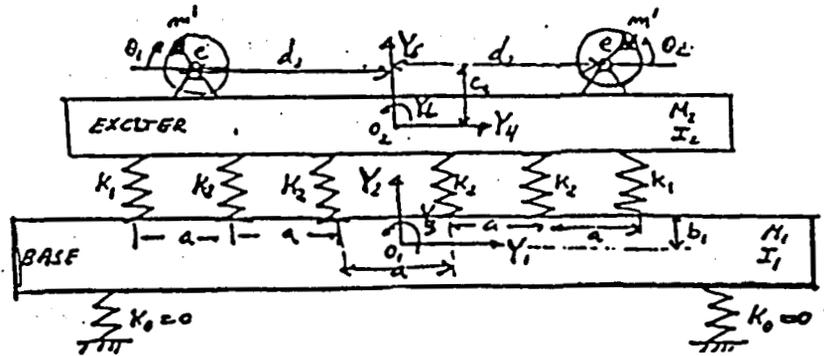


Fig.2 Mathematical model for the example of a vibrating system excited by two self-synchronizing rotors.

TORQUE DEMANDED BY THE LOAD

The torque demanded by the load may be determined from the power consumed by the exciting forces of the rotors. The forces $F_1(t)$ and $F_2(t)$, developed by the two rotors exciting the system are given by

$$\begin{aligned} F_1(t) &= m'e \dot{\theta}_1^2 \\ F_2(t) &= m'e \dot{\theta}_2^2 \end{aligned} \quad (6)$$

where m' and e are the eccentric mass and its corresponding eccentricity; $\dot{\theta}_1$ and $\dot{\theta}_2$ the angular velocities of the rotors.

The power demanded by the load may be calculated as the product of the rotor forces times the corresponding linear velocities of the points of application of these forces. It is convenient to express this power separately for the two force components in the plane of the rotating forces as

$$\begin{aligned} F_{11}(t) &= F_1(t) \sin \theta_1(t) \\ F_{12}(t) &= F_1(t) \cos \theta_1(t) \end{aligned} \quad (7)$$

and

$$\begin{aligned} F_{21}(t) &= F_2(t) \sin \theta_2(t) \\ F_{22}(t) &= F_2(t) \cos \theta_2(t) \end{aligned} \quad (8)$$

where F_{ij} is the j -component of the i -th motor; θ_1 and θ_2 the angular displacements of the rotors.

The power demanded by the load is given by

$$\begin{aligned} P_{11}(t) &= F_{11}(t) \cdot \dot{X}_{11}(t) \\ P_{12}(t) &= F_{12}(t) \cdot \dot{X}_{12}(t) \end{aligned} \quad (9)$$

and

$$\begin{aligned} P_{21}(t) &= F_{21}(t) \cdot \dot{X}_{21}(t) \\ P_{22}(t) &= F_{22}(t) \cdot \dot{X}_{22}(t) \end{aligned} \quad (10)$$

where P_{ij} = Power of j force component of the i -th motor
 \dot{X}_{ij} = Linear velocity of the force component j of the i -th motor.

Finally, the torque T_L demanded by the load is obtained by simply dividing the power components in Eqs.(9) and (10) by the corresponding angular velocity of the rotors. Consequently, the total torque demanded by the loads is given by

$$T_L = \sum_{k=1}^2 \frac{1}{\theta_k} [P_{k1}(t) + P_{k2}(t)] \quad (11)$$

ANGULAR VELOCITY OF THE ROTORS

The angular velocity for the rotors is determined by numerical integration of Eq.(1) which expressed in finite differences form may be written as

$$\Delta \dot{\theta}(t) = \frac{\Delta T(t)}{J} \Delta t \quad (12)$$

where $\Delta \dot{\theta}(t)$ is the incremental angular velocity of the rotors attained in time step Δt and $\Delta T(t)$ is the torque available at accelerate the rotors. As illustrated in Fig.1, this torque is given by the difference between the torque of the motor $T_M(t)$ and the torque $T_L(t)$ demanded by the load; thus for each motor, the differential torque is given by

$$\Delta T(t) = T_M(t) - T_L(t) \quad (13)$$

The angular velocity for each rotor is then calculated, starting with initial velocity zero ($\dot{\theta}(0)=0$), through the recurrence formula

$$\dot{\theta}(t+\Delta t) = \dot{\theta}(t) + \Delta\dot{\theta}(t) \quad (14)$$

The angular displacement of the rotors, θ_1 , and θ_2 is calculated as

$$\begin{aligned} \theta_1(t+\Delta t) &= \theta_1(t) + \dot{\theta}_1(t) \times \Delta t \\ \theta_2(t+\Delta t) &= \theta_2(t) + \dot{\theta}_2(t) \times \Delta t \end{aligned} \quad (15)$$

starting with the initial displacements of the rotors $\theta_1(t=0)$ and $\theta_2(t=0)$.

The phase angle $\alpha(t)$ between the rotors is then given by the difference between the angular displacements of the rotors. Hence:

$$\alpha(t) = \theta_1(t) - \theta_2(t)$$

NUMERICAL EXAMPLE

Figure 2 shows the mathematical model of a vibrating conveyor, excited by two motors with eccentric masses rotating in opposite directions. The differential equations of motion for the six degrees of freedom in this model were obtained using Lagrange's equation. The numerical values for the components of this system are indicated in Table 1. Discrete values of the torque-speed function for the motors, provided by the manufacture, are given in Table 2.

A time step $\Delta t = 0.01$ sec. was selected and damping in the system was assumed equal to 5% of the critical damping in each mode.

TABLE 1 NUMERICAL VALUES VIBRATING SYSTEM OF FIG.2

BASE:

$$\text{Mass: } M_1 = 31.863 (\text{lb-sec}^2/\text{in})$$

$$\begin{aligned} \text{Mass moment of inertia:} \\ I_1 = 42,772 (\text{lb-sec}^2\text{-in}) \end{aligned}$$

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$$\text{Mass: } M_2 = 15.236 (\text{lb-sec}^2/\text{in})$$

$$\begin{aligned} \text{Mass moment of inertia:} \\ I_2 = 4,578 (\text{lb-sec}^2\text{-in}) \end{aligned}$$

Springs constant:

$$\begin{aligned}
 K_1(\text{axial}) &= 3k & (k = 9,400 \text{ (lb/in)}) \\
 K_2(\text{axial}) &= 9k \\
 K_1(\text{transverse}) &= 30k \\
 K_2(\text{transverse}) &= 90k
 \end{aligned}$$

Distance:

$$\begin{aligned}
 a &= 9.0 \text{ in} \\
 b_1 &= 29.0 \text{ in} \\
 b_2 &= 13.25 \text{ in} \\
 c_2 &= 27.0 \text{ in} \\
 d_2 &= 8.0 \text{ in}
 \end{aligned}$$

Eccentric rotor:

$$m'e = 0.1425 \text{ (lb}\cdot\text{sec}^2)$$

TABLE 2 SPEED-TORQUE VALUES

Speed (rad/sec)	0	47.12	162.3	184.3	188.5
Torque (lb-in)	428.4	321.3	616.75	142.8	0

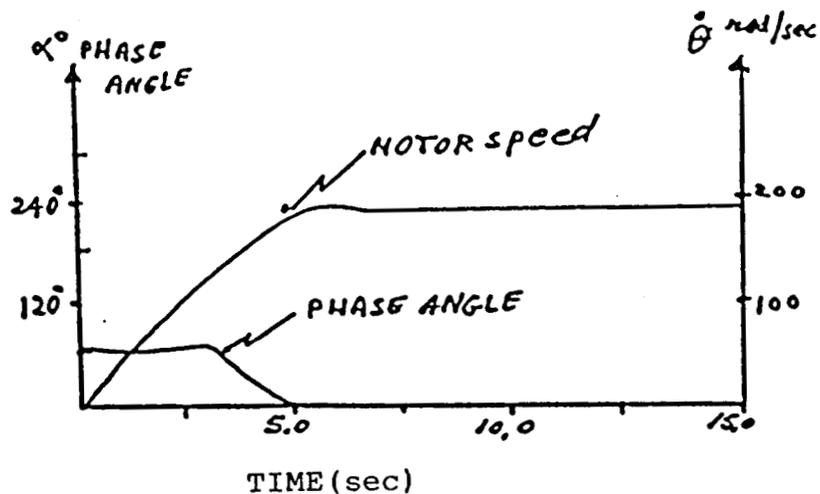


Fig.3 Motor speed and phase angle of the rotors - Motors having full torque.

The analysis of the system modeled as shown in Fig. 2 provided the following three non-zero values for the natural frequencies:

$\omega_1 = 114.7$ rad/sec, $\omega_2 = 195.7$ rad/sec and $\omega_3 = 783.6$ rad/sec.

The solution output includes the motion of the system and of the rotors, as a function of time, during the initial 15 seconds of the transient state. The solution also includes, the torques of the motors and the torques demanded by the load as well as the phase angle between the rotors in the transient state.

For the motors having the speed-torque characteristic depicted in Fig.1 (Numerical values in Table 2), Fig.3 shows the plots of the phase angle between rotors and the angular speed of the motors during the initial 40 seconds motion, while Fig.4 shows these plots assuming that the motors can provide only one-half of the torque indicated in Table 2. The initial phase angle was 60 degrees due to the physical arrangement of the rotors under the effect of gravity.

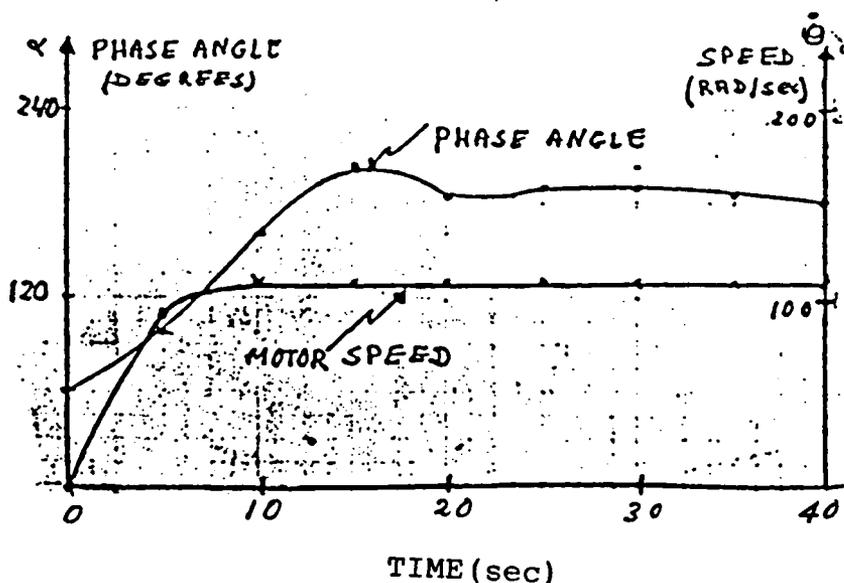


Fig.4 Motor speed and phase angle of the rotors - Motors limited to 50% of the full torque.

DISCUSSION

It may be observed in Fig.3 that for the motors having the speed-torque characteristics of Fig.1, the motors rapidly, in about five seconds reach their operating speed in the neighborhood of $\dot{\theta} = 185$ rad/sec and that rotors attain self-synchronization with a phase angle $\alpha = 0$ degrees. On the other hand, Fig.4 shows that the motors do not have

sufficient power to reach the operating speed and become locked in the neighborhood of resonant speed $\omega_r = 114.7$ (rad/sec) attaining out of phase self-synchronization of the rotors ($\alpha = 180^\circ$). In the first case, the self-synchronized machine will perform as intended in the design, while in the second case the system will vibrate erratically at the resonant condition with large amplitudes resulting in failure of the equipment.

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