LASER DOPPLER VELOCIMETRY PRIMER

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1.0 INTRODUCTION

Advanced research in experimental fluid dynamics requires a familiarity with sophisticated measurement techniques. In some cases, the development and application of new techniques is required for difficult measurements. Optical methods and in particular, the laser Doppler velocimeter (LDV) are now recognized as the most reliable means for performing measurements in complex turbulent flows. As such, the experimental fluid dynamicist should be familiar with the principles of operation of the method and the details associated with its application. Thus, the goals of this primer are to efficiently transmit the basic concepts of the LDV method to potential users and to provide references that describe the specific problem areas in greater detail.

Aerodynamic Research Branch personnel and associated contractors have made significant contributions to the development of the LDV technology. These developments were driven by the need to obtain inviscid and turbulent boundary layer flow measurements in relatively large transonic facilities. Although the measurements were challenging and taxed the existing technology, the data obtained have proven to be reliable and have withstood the careful scrutiny of flow modellers trying to explain poor agreement with the data. As the researchers who developed the technology and who have the experience required in performing the measurements are promoted or moved on to the other activities, the technology is, in part, lost to the Branch. Hence, the present work is an effort to retain some of the essential information within the Fluid Dynamics Research Branch.

Although the optical methods have provided a means for obtaining turbulence data which was previously unattainable, the simple material probes should not be rejected totally. In relatively simple flows where only the mean flow speed profiles are required and the flow direction is known, a pitot probe is probably the most efficient means for obtaining the data. Turbulence measurements in incompressible nonreversing flows can be made with good accuracy and relatively easily using hot-wire or hot-film anemometers. Combined measurements with material probes and the LDV are also recommended as a means for obtaining complete data efficiently and with sufficient redundancy to evaluate the results.

Since its inception in 1964, the laser Doppler velocimeter has been revised, developed, and tested by many researchers and it’s capabilities are reasonably well-understood. The method has the advantage of performing instantaneous, non-intrusive velocity measurements of small particles borne by the flows. To be able to measure the flow without
disturbing it is especially important in recirculating flows. The flow direction can be resolved and three-dimensional velocity measurements are becoming more common. Finally, the measurements are relatively independent of the fluid properties and the method does not require frequent in situ calibrations.

However, because of the range of disciplines involved including optics, electronics, light scattering, and signal processing, the implementation of the method requires more than just a knowledge of where the switches are and which way to turn them. Serious measurement errors can occur if the measurement techniques are not understood. Thus, in the following sections, the fundamental principles will be described, setup procedures and the operation of the instrument outlined, and some of the required measurement characteristics presented.

This primer will concentrate primarily on the many practical aspects of operating and maintaining a LDV system and forego detailed explanation of the theory for which abundant resources are available. Where it is deemed necessary, however, the theoretical basis for decisions concerning operation of LDV systems will be included.
2.0 THE NATURE OF LIGHT

In order to explain a broad range of optical phenomena, and primarily, those phenomena central to the understanding of laser Doppler velocimetry and interferometry, light must be treated with classical wave theory. This treatment is referred to as 'physical optics' and comprises those phenomena bearing on the nature of light. Typically, large-scale (> light wavelength) effects can be explained by the treatment of light as rays and the use of geometrical optics. However, the phenomena of interference and diffraction which causes deviation from rectilinear motion of light rays that is not explainable by refraction or reflection must be treated by the wave theory.

Light waves are typical of a broad range of waves known as electromagnetic waves. On the electromagnetic spectrum, light waves are intermediate between long radio waves, which can be thought of as oscillating and propagating electric fields and short x-rays, which can be considered as energetic particles. The wavelength of light is sufficiently short so that it can be observed as rectilinear motion, as though the light is propagated as a stream of particles, and yet long enough for us to observe many interference and diffraction effects.

Compared to some other wave phenomena, light waves do not transport any material properties as, for example, sound waves which propagate air pressure and velocity or water waves that propagate water level and velocity. Electromagnetic waves are waves both of electric and magnetic fields. The two are, however, closely linked - in free space, the ratio of the field strengths is fixed, and the field directions are mutually perpendicular while each is perpendicular to the line of propagation of the wave. The electric field is generally easier to detect, and thus, the field E is the variable used to describe the phenomena. The instantaneous direction of the E vector is referred to as the polarization direction of the light wave.

To describe a wave, four quantities are required - its wavelength, frequency, velocity and amplitude. The wavelength is the distance between two successive crests (or other corresponding parts of the wave profile). The visible portion of the spectrum extends from wavelengths of .4 microns (approaching the ultraviolet) to about .75 microns (approaching the infrared). The frequency is the number of waves passing a given point per second and is expressed in cycles per second (light frequencies are on the order of $10^{14}$ Hz). Wave velocity is the velocity at which the wave profile moves forward; it is equal to the frequency multiplied by the wavelength. Amplitude is a measure of the magnitude of the vibrations and is defined as the height of a wave crest. Light has the added requirement of specifying the polarization, that is, it is a vector quantity.

2.1 Summary of Wave Theory

Without going into the details of wave theory, a simple wave propagating in one direction (along the z axis) at a velocity $v$ is described as

$$\psi(x, t) = f(x - vt)$$ (2.1)
This represents the most general form of the one-dimensional wave function. We then select the shape of the wave, \( f(x) \) and substitute \((x - vt)\) for \(x\) and the resulting expression describes a moving wave having the desired profile. For example, the wave to be described may be sinusoidal (figure 2.1) so the expression would be

\[
\psi(x, t) = A \sin(k(x - vt))
\] (2.2)

In this expression, \(A\) is the maximum amplitude or simply the amplitude of the wave and \(k\) is the wave or propagation number, \(k = \frac{2\pi}{\lambda}\). The spatial period of the wave is known as the wavelength. An increase or decrease in \(x\) by an amount \(\lambda\) leaves the wave function unaltered. The value \(kx\) is in radians.

The temporal period, \(\tau\), is defined as the amount of time it takes for one complete wave to pass a stationary observer. Specifying the repetitive nature of the wave in time, we have

\[
\psi(x, t) = \psi(x, t \pm \tau)
\] (2.3)

and

\[
\sin k(x - vt) = \sin k[x - v(t \pm \tau)] = \sin[k(x - vt) \pm 2\pi]
\] (2.4)

Therefore

\[
|kvt| = 2\pi
\] (2.5)

so

\[
\frac{2\pi}{\lambda} \nu \tau = 2\pi
\] (2.6)

and

\[
\tau = \frac{\lambda}{\nu}
\] (2.7)

The period is the number of units of time per wave and the inverse of the period is the frequency

\[
\nu = \frac{1}{\tau}
\] (2.8)

Angular frequency is given by

\[
\omega = \frac{2\pi}{\tau} \text{ (radians/sec)}
\] (2.9)

The entire argument of the harmonic wave function is known as the phase, \(\phi\),

\[
\phi = (kx - \omega t)
\] (2.10)

In the following expression,

\[
\psi(x, t) = A \sin(kx - \omega t + \epsilon)
\] (2.11)
Figure 2.1  A Progressive Wave at Three Different Times
 is the initial phase or epoch angle. The epoch angle is just the constant contribution to the phase arising at the generator and is independent of how far in space, or how long in time, the wave has travelled.

### 2.2 Complex Representation

Because it is cumbersome to deal with trigonometric manipulations, the complex number representation of waves offers an alternative that is mathematically simpler to work with. The complex notation is used extensively in optics. As a brief review of complex notation and manipulations, let

\[ z = x + iy \]  

(2.12)

where \( i = \sqrt{-1} \). The real and imaginary parts of \( z \) are respectively \( x \) and \( y \) where both \( x \) and \( y \) are real numbers. In terms of polar coordinates \((r, \theta)\), \( x \) and \( y \) are written

\[ x = r \cos \theta \quad y = r \sin \theta \]  

(2.13)

and thus

\[ z = x + iy = r(\cos \theta + i \sin \theta) \]  

(2.14)

The very useful Euler formula is

\[ e^{i\theta} = \cos \theta + i \sin \theta \]  

(2.15)

and

\[ z = re^{i\theta} = r(\cos \theta + i \sin \theta) \]  

(2.16)

where \( r \) is the magnitude of \( z \) and \( \theta \) is the phase angle. The complex conjugate of \( z \) denoted at \( z^* \) is

\[ z^* = re^{-i\theta} = r(\cos \theta - i \sin \theta) \]  

(2.17)

The modulus of a complex quantity is given by

\[ |z| = (zz^*)^{1/2} \]  

(2.18)

Any complex number can be represented by the sum of its real part \( Re(z) \) and its imaginary part \( Im(z) \). In the polar form \( Re(z) = r \cos \theta \) and \( Im(z) = r \sin \theta \). It is customary to choose the real part in which case the harmonic wave is written

\[ \psi(x,t) = Ae^{i(\omega t - kx + \epsilon)} = A \cos(\omega t - kx + \epsilon) \]  

(2.19)

For convenience, the wave function can be written as

\[ \psi(x,t) = Ae^{i(\omega t - kx + \epsilon)} = Ae^{i\phi} \]  

(2.20)
2.3 Wave Descriptions

Only plane and spherical waves will be discussed since they are the most common to optical applications.

**Plane Waves**

Plane waves exist at a given time, when all the surfaces upon which a disturbance has constant phase form a set of planes, each generally perpendicular to a propagation direction. The mathematical expression for a plane wave is given with respect to a given vector $\mathbf{k}$ that is perpendicular to the plane. The vector $\mathbf{k}$, whose magnitude is the propagation number $k$ is called the wave propagation vector. A vector that is in the plane is perpendicular to the propagation vector if the vector dot product (see figure 2.2)

$$\mathbf{r}' \cdot \mathbf{k} = \text{constant} = a$$

The vector $\mathbf{r}$ taking on all possible values sweeps out the plane. Thus, a concise equation of a plane perpendicular to $\mathbf{k}'$ is

$$\mathbf{k} \cdot \mathbf{r}' = \text{constant} = a$$

We can thus construct a set of planes over which $\psi(\mathbf{r})$ varies sinusoidally in space; that is

$$\psi(\mathbf{r}) = A \sin(k \cdot \mathbf{r})$$

or

$$\psi(\mathbf{r}) = A \exp(i\mathbf{k} \cdot \mathbf{r})$$

Over every plane $\psi(\mathbf{r})$ is constant defined by $\mathbf{k} \cdot \mathbf{r} = \text{constant}$, figure 2.3.

To introduce motion into the planes, the time dependence is described analogous to the one dimensional wave as

$$\psi(r, t) = A \exp(i(k \cdot \mathbf{r} \pm \omega t))$$

with $A$, $k$, and $\omega$ are constants as before. At any given time, the surfaces joining all points of equal phase are known as wavefronts or wave surfaces.

**Spherical Waves**

If we consider a point source of light, the radiation emanating from it spreads radially and uniformly in all directions. This type of source is said to be isotropic and the resulting wavefronts are concentric spheres. The expression for the spherical wavefronts is

$$\psi(r, t) = \frac{f(r - vt)}{r}$$

(2.26)
Figure 2.2  A Plane Wave Moving in the $k$th Direction
Figure 2.3  Wavefronts for a Harmonic Plane Wave
Equation 2.26 represents a spherical wave progressing radially outward (figure 2.4a) from the origin, at a constant speed \( v \) and having an arbitrary form \( f \). The expression for a harmonic spherical wave is

\[
\psi(r, t) = \left( \frac{\alpha}{r} \right) \cos k' (r \pm vt) \tag{2.27}
\]

or

\[
\psi(r, t) = \left( \frac{\alpha}{r} \right) \exp ik (r \pm vt) \tag{2.28}
\]

where \( \alpha \) is the source strength. Notice that the amplitude of the spherical wave is a function of the radial distance \( r \) and \( r^{-1} \) serves as the attenuation factor. The spherical wave increases in radius with distance from the source, thereby changing its profile as it expands. Far enough from the source, a portion of the spherical wave will resemble a plane wave, figure 2.4b.

### 2.4 Electromagnetic Waves

Although the descriptions given here apply to all electromagnetic waves, we will be primarily concerned with light which corresponds to radiation in the narrow band of frequencies from about \( 3.84 \times 10^{14} \) Hz to \( 7.69 \times 10^{14} \) Hz. It is important to note that light waves are exclusively a transverse wave motion. That is, the vibrations are always perpendicular to the direction of motion of the waves. Maxwell's theory describing the dynamical electromagnetic fields required that the vibrations of light be strictly transverse and gave a definite connection between light and electricity. The theory need not be covered here but is only mentioned to connect the electric and magnetic field theory to electromagnetic radiation. That is to say, the concepts developed to describe electric and magnetic fields appear to describe many of the phenomena observed in electromagnetic radiation.

The electric field \( \vec{E} \), is a vector quantity. In the discussions of diffraction and interference we will use the idealized description of the electric field. That is, we will assume that the wave is monochromatic (consists of a single wave-length) and linearly polarized. Laser light approaches these ideal conditions which is why it is such an important light source. For these idealized conditions, the field of the linearly polarized light wave traveling in the direction of \( \vec{k} \) can be written

\[
\vec{E}(r, t) = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \epsilon) \tag{2.29}
\]

The intensity or irradiance is then

\[
I = \varepsilon_0 c < \vec{E}^2 > \tag{2.30}
\]

where \( \varepsilon_0 \) is the electric permittivity of the surrounding medium, \( c \) is the speed of light (\( c = 3 \times 10^8 \) m/sec). In this case, \( \vec{E} \) can be considered as the optical field and the
Figure 2.4a  Spherical Wavefront
Figure 2.4b  The Flattening of Spherical Waves with Distance
brackets $<>$ indicate that the $\vec{E}$ field is generally assumed stationary in classical optics considerations. If we are only concerned with the relative irradiences within the same medium, then the expression is simply

$$I = <\vec{E}^2>$$ \hfill (2.31)

Recall that $\vec{E}$ is a complex quantity so

$$\vec{E}^2 = (\vec{E}_0 e^{i\alpha})(\vec{E}_0 e^{i\alpha})^*$$ \hfill (2.32)

where * implies the complex conjugate.

### 2.5 Coherence

In speaking of light as a wave phenomena, we wrote expressions for the waves as if the light source was ideally monochromatic and the waves perfectly plane or spherical. However, even laser light only approaches these conditions. The frequency and amplitude of the wave varies slowly (relative to the oscillation, $10^{14}$ Hz). The time over which the wave train maintains its average frequency is the coherence time $\Delta t$ and is given by the inverse of the light source frequency bandwidth.

If the light source was ideally monochromatic $\Delta \nu$ would be zero and the coherence time or length given by $c\Delta t$ would be infinite. Over an interval of time much shorter than $\Delta t$, an actual wave behaves essentially as if it was monochromatic. Coherence time is the temporal interval over which we can reasonably predict the phase of the light wave at a given point in space.

When we speak of temporal coherence, we are actually referring to the time $\Delta t$ of the light source over which the wave remains a constant frequency. The length $c\Delta t$ is often referred to as the coherence length of the source and this length can be from $10^{-5}$ meters for mercury lamps to 1 meter for some lasers. In interferometry and specifically, for laser Doppler velocimetry, we match the paths of the beams so that the difference in the path length is less than the coherence length. This will ensure the formation of interference fringes of sufficient visibility.

In general, the time dependence of a light wave varies over a given wavefront. The degree of this wavefront variation is referred to as the spatial coherence. It arises because of the finite extent of light sources. Suppose we have a classical broad band source of monochromatic light and we consider two point radiators on it separated by a lateral distance which is large compared to $\lambda$. These two point sources will presumably behave independently and there will be a lack of correlation existing between the phases of the two emitted disturbances. Spatial coherence is closely related to the concept of the wavefront. If two laterally displaced points reside on the same wavefront at a given time, the fields at those points are said to be spatially coherent.

Young's experiment (figure 2.5) could be used to demonstrate the spatial coherence of a source. If a single source was used, the wavelets issuing from two apertures $S_1$ and $S_2$ will
maintain their relative phase and they will be precisely correlated and therefore coherent. A well-defined spatial array of stable interference fringes will result at the viewing screen. At the other extreme, if the two pinholes are illuminated by separate sources far enough apart, no correlation will exist, no fringes will be observable on the screen and the fields $S_1$ and $S_2$ are then said to be incoherent. The generation of fringes (and fringe visibility) is a convenient measure of the coherence. The loss of visibility with separation between points on the source is used to measure the spatial coherence.

The quality of the fringes produced by an interferometer system can be described quantitatively using the visibility which is given by

$$V(r) = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$  \hspace{1cm} (2.33)

$I_{\text{max}}$ and $I_{\text{min}}$ are the irradiances corresponding to the maximum and adjacent minimum in the fringe system, figure 2.6.

In figure 2.6, a sequence of fringe systems are shown wherein the circular primary source is constant in size but the separation, $a$, between $S_1$ and $S_2$ is increased. Note that the fringe spacing is inversely proportional to $a$. With increasing aperture separation $a$, the visibility of the fringe pattern decreases. These results would be similarly observed on the LDV signals. Since the signal-to-noise is proportional to the square of the signal visibility signals, high visibility is important to the reliability of the measurements.

### 2.6 Interference

The wave theory of the electromagnetic nature of light provides a basis from which to proceed to the phenomena of optical interference. Because the relationship describing the optical disturbances are linear, the principle of superposition applies so that the resultant electric field (or optical field) intensity $\vec{E}$ at a point in space where two or more light waves overlap, is equal to the vector sum of the individual constituent disturbances. That is, for fields $\vec{E}_1$, $\vec{E}_2$, ... the resultant field is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$$  \hspace{1cm} (2.34)

Consider the simple case of two point sources $S_1$ and $S_2$ emitting monochromatic waves of the same frequency at a separation, $a$, much greater than the wavelength, $\lambda$. A point $P$ on an observation plane is taken far from the sources so that the waves at $P$ can be considered as plane (figure 2.7). Assuming that the light is linearly polarized, the expressions for the two waves are

$$\vec{E}_1(\vec{r}, t) = \vec{E}_{01} \cos(\vec{k}_1 \cdot \vec{r} - \omega t + \epsilon_1)$$
$$\vec{E}_2(\vec{r}, t) = \vec{E}_{02} \cos(\vec{k}_2 \cdot \vec{r} - \omega t + \epsilon_2)$$  \hspace{1cm} (2.35)

The irradiance is

$$I = \langle \vec{E}^2 \rangle = \langle \vec{E} \cdot \vec{E} \rangle$$  \hspace{1cm} (2.36)
where \( S' \) and \( S'' \) are point sources lying along the extended slit source, \( d \).

\( P' \) and \( P'' \) are the corresponding image points of the above lying on the image of the source, \( b \).

\( \Sigma_a \) is the aperture plane containing apertures \( S_1 \) and \( S_2 \) a distance \( a \), apart.

\( \Sigma_o \) is the observation plane located a distance \( s \), from the aperture plane.

Figure 2.5  Young's Experiment with an Extended Slit Source
Figure 2.6 Double-Beam Interference Showing Visibility Variations
and

\[ E^2 = \bar{E} \cdot \bar{E} = (\bar{E}_1 + \bar{E}_2) \cdot (\bar{E}_1 + \bar{E}_2) = E_1^2 + E_2^2 + 2 E_1 \cdot E_2 \]  \hspace{1cm} (2.37)

Taking the time average of both sides, the irradiances become

\[ I = I_1 + I_2 + I_{12} \]  \hspace{1cm} (2.38)

where

\[ \begin{align*}
I_1 &= E_1^2 \\
I_2 &= E_2^2 \\
I_{12} &= 2 \bar{E}_1 \cdot \bar{E}_2
\end{align*} \hspace{1cm} (2.39)\]

The term \( \bar{E}_1 \cdot \bar{E}_2 \) is known as the interference term. In the present expression it can be evaluated as

\[ \bar{E}_1 \cdot \bar{E}_2 = E_{01} \cdot E_{02} \cos(\bar{k}_1 \cdot \bar{r} - \omega t + \epsilon_1) \cos(\bar{k}_2 \cdot \bar{r} - \omega t + \epsilon_2) \]  \hspace{1cm} (2.40)

Taking the time average over a period much longer than the wave period \((T \gg \tau)\)

\[ < \bar{E}_1 \cdot \bar{E}_2 > = \frac{1}{2} E_{01} \cdot E_{02} \cos(\bar{k}_1 \cdot \bar{r} + \epsilon_1 - \bar{k}_2 \cdot \bar{r} + \epsilon_2) \]  \hspace{1cm} (2.41)

or simply

\[ I_{12} = E_{01} E_{02} \cos \delta \]  \hspace{1cm} (2.42)

where \( \delta = (\bar{k}_1 \cdot \bar{r} - \bar{k}_2 \cdot \bar{r} + \epsilon_1 - \epsilon_2) \) is the phase difference arising from the combined path length and epoch angle difference. The most common practical case is where the polarization vectors are parallel. In that case the interference term reduces to a scalar

\[ I_{12} = E_{01} E_{02} \cos \delta \]  \hspace{1cm} (2.43)

or

\[ I_{12} = 2\sqrt{|I_1 I_2|} \cos \delta \]  \hspace{1cm} (2.44)

and the total irradiance is

\[ I = I_1 + I_2 + 2\sqrt{|I_1 I_2|} \cos \delta \]  \hspace{1cm} (2.45)

At various points in space, the resultant irradiance will be greater, less than, or equal to \( I_1 + I_2 \) depending on the value of \( \delta \). A maximum irradiance occurs when \( \cos \delta = 1 \), so that

\[ I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1 I_2} \]  \hspace{1cm} (2.46)
Figure 2.7 Waves from Two Point Sources Overlapping in Space
when

\[ \delta = 0, \pm 2\pi, \pm 4\pi, \ldots \]

In this case the phase difference between the two waves is an integer multiple of \(2\pi\), and the disturbances are said to be in-phase. One speaks of this as total constructive interference. The minimum in the irradiance results when the waves are 180 degrees out-of-phase, troughs overlap crests, \(\cos \delta = -1\), and

\[ I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1 I_2} \]  
(2.47)

This, of course, occurs when \(\delta = \pm \pi, \pm 3\pi, \pm 5\pi, \ldots\), and it is referred to as total destructive interference. These special cases are shown schematically in figure 2.8.

Another somewhat special, yet very important case arises when the amplitudes of both waves reaching \(P\) in figure 2.7 are equal, i.e., \(E_{01} = E_{02}\). Since the irradiance contributions from both sources are then equal, let \(I_1 = I_2 = I_0\). Equation 2.45 can now be written as

\[ I = 2I_0(1 + \cos \delta) = 4I_0 \cos^2 \frac{\delta}{2} \]  
(2.48)

from which it follows that \(I_{\text{min}} = 0\) and \(I_{\text{max}} = 4I_0\).

### 2.7 Conditions of Interference

If the interference pattern is to be observable, the phase difference \((\epsilon_1 - \epsilon_2)\) between the two sources must remain fairly constant in time. Such sources are said to be coherent. Two overlapping beams arising from separate emitters will interfere, but the resultant pattern will not be sustained long enough to be readily observable.

If two beams are to interfere to produce a stable pattern, they must have very nearly the same frequency. A significant frequency difference would result in a rapidly varying, time dependent, phase difference which in turn would cause \(I_{12}\) to average to zero during the detection interval.

The clearest patterns will exist when the interfering waves have equal or very nearly equal amplitudes. The central regions of the dark and light fringes will then correspond to complete destructive and constructive interference, respectively, yielding maximum contrast.

We have assumed that the two overlapping optical disturbance vectors were linearly polarized and parallel. Nonetheless, the formulae apply as well to more complicated situations; indeed the treatment is applicable regardless of the polarization state of the waves. To appreciate this, recall that any polarization state can be synthesized out of two orthogonal \(P\)-states. For natural (unpolarized) light these \(P\)-states are mutually incoherent, but that represents no particular difficulty.

Suppose that every wave has its propagation vector in the same plane so that we can label the constituent orthogonal \(P\)-states with respect to that plane, e.g., \(E_{\parallel}\) and \(E_{\perp}\),
Figure 2.8 Superposition of Two Waves Showing Interference
which are parallel and perpendicular to the plane, respectively (figure 2.9a). Thus, any plane wave, whether polarized or not, can be written in the form \((E_{\parallel 1} + E_{\perp 1})\). Imagine then, that the waves \((E_{\parallel 1} + E_{\perp 1})\) and \((E_{\parallel 2} + E_{\perp 2})\) emitted from two identical coherent sources superimpose in some region of space. The resulting flux-density distribution would consist of two independent, precisely overlapping, interference patterns, \(< (E_{\parallel 1} + E_{\parallel 2})^2 >\) and \(< (E_{\perp 1} + E_{\perp 2})^2 >\). Therefore, while we derived the equations of the previous section specifically for linear light, they are applicable, as well for any polarization state including natural light.

Notice that even though \(E_{\perp 1}\) and \(E_{\perp 2}\) are always parallel to each other \(E_{\parallel 1}\) and \(E_{\parallel 2}\), which are in the reference plane, need not be. They will only be parallel when the two beams are themselves parallel (i.e., \(k_1 - k_2\)). The inherent vector nature of the interference process as manifest in the dot-product representation of \(I_{12}\) cannot therefore be ignored. There are a great many practical situations in which the beams approach being parallel and for these the scalar theory will do rather nicely. Even so, the cases in figures 2.9b and 2.9c are included as an urge to caution. They depict the imminent overlapping of two coherent linearly polarized waves. In figure 2.9b the optical vectors are parallel, even though the beams are not, and interference would nonetheless result. In figure 2.9c, the optical vectors are perpendicular and \(I_{12} = 0\), which would be the case here even if the beams were parallel.

Fresnel and Arago made an extensive study of the conditions under which the interference of polarized light occurs and their conclusions summarize some of the above considerations. The Fresnel-Arago laws are as follows:

1. Two orthogonal, coherent \(P\)-states cannot interfere in the sense that \(I_{12} = 0\) and no fringes results.
2. Two parallel, coherent \(P\)-states will interfere in the same way as will natural light.
3. Two constituent orthogonal \(P\)-states of natural light cannot interfere to form a readily observable fringe pattern even if rotated into alignment. This last point is understandable since these \(P\)-states are incoherent.

2.8 Lasers and Laser Light

The advent of the laser has made interferometry a practical and useful tool for fluid dynamics research. Lasers allow the production of high energy density, single frequency, coherent light waves. A laser is comprised of an optical cavity, a lasing medium, and an external energy source. The principle behind the laser, from which it derives its name, is Light Amplification By Stimulated Emission of Radiation. The lasing medium, which may be gaseous, liquid, or solid, is driven into an excited electronic state by the external energy source. As the electrons relax to intermediate energy levels or fall to their ground state, they release this additional energy in the form of photons. The energy of the emitted photon is characteristic of the electron transitions of the medium. This photon emission will occur randomly from excited atoms unless the atom is stimulated to electron transition while it is in the excited state. An external photon of the same energy as the transition
Figure 2.9  Interference of Polarized Light
level which interacts with the excited atom will stimulate the atom to emit not one, but two similar photons which travel in-phase (read coherently) in the same direction as the incident photon. Those photons which travel along the axis of the laser cavity are continually redirected back through the medium to stimulate additional excited atoms resulting in a many-fold amplification of in-phase photons. To propagate this effect, a sufficient number of atoms must exist in the excited state. This condition is referred to as a ‘population inversion’ within the medium and is produced and/or maintained by the external energy source.

The optical cavity usually refers to the space between a set of reflectors (mirrors). These reflectors are precisely aligned with respect to each other such that the cavity is resonant at particular optical wavelengths. In the case of continuous wave (CW) lasers, one reflector is highly reflective (> 99.9%) while the other, at the output end of the cavity, is only partially reflective (~ 90% or more). The partially reflective end allows a small percentage of the light to leak past the output mirror producing a continuous beam. The remaining light continues to produce the lasing effect within the cavity. In the case of pulsed lasers, the output reflector is removed from the optical path (this is done with an electro-optic device) allowing all of the light in the cavity to exit the laser. The cavity is then closed and the process repeated.

LDV systems almost exclusively use CW lasers. For systems which do not require high power, the laser of choice is the helium-neon (He-Ne) gas laser. As the name implies, the optical cavity is filled with a mixture of helium and neon gases which are ‘pumped’ into population inversion by means of electrical discharge maintained through the mixture. This device lases principally at a wavelength of 0.6328 microns, with the emitted light appearing red. He-Ne lasers are available with output beam powers in the range 1-50mW and beam diameters of 0.5-1.0mm.

For most aerodynamic LDV applications, however, more power is required than is available from a He-Ne laser. In those cases, the more suitable laser is the argon-ion (Ar+) gas laser. This laser operates at total beam powers up to several tens of watts. In addition, the laser can be configured to produce up to 12 discrete wavelengths simultaneously from 0.351 to 0.528 microns. The beams are coaxially emitted from the laser and can be later separated from each other using prisms or special mirrors. There are two principal wavelengths (or ‘lines’) of the Ar+ laser that are most frequently used for LDV work. They are the green line at 0.5145 microns and the blue line at 0.4880 microns. The combined power at these two wavelengths is nearly 3/4 the total beam power available from the laser. As will be explained later, an LDV is often configured into a ‘two-component’ system which operates very much like two independent LDV systems. The Ar+ laser is ideally suited to a two-component LDV in which a different wavelength beam is associated with each of the components.

There are, of course, a multitude of other laser types many of which would also be suitable for LDV applications. They range from helium-cadmium and krypton-ion gas lasers to the newer and more compact semiconductor diode lasers. Differences between lasers include wavelength, beam power and quality, physical size, power requirements, and price.
Although suitable, and in some applications more desirable, there use in aerodynamic research LDV systems is not prevalent enough to warrant further discussion.

The beam properties of lasers are of utmost importance to LDV applications where adequate interference and spatial resolution are critical. These properties include size, shape, power, quality, polarization state, and coherence length. All of these properties are functions of the type of laser from which they are generated although some can be altered with optical components. Both the He-Ne and the Ar+ lasers discussed above generate beams with similar properties.

Within a laser cavity there are several paths between the reflectors which satisfy the resonance and gain requirements of the lasing action. These paths are referred to as modes of which two types are distinguished, transverse and longitudinal. Ideally, a laser operates on only one mode and much effort by laser manufacturers is directed toward achieving this goal. In practice, however, a laser cavity supports several modes with resulting competition between modes affecting the beam shape, output power, and coherence. The lowest order fundamental mode is the TEM$_{00}$ (Transverse Electric and Magnetic) mode. The beam produced by a laser operating solely in this mode has a gaussian intensity profile given by

$$I(r) = I_0 \exp(-2r^2/\omega^2)$$

where $I_0$ is the peak beam intensity found at the center of the beam, $r$ is the radial distance from the center, and $\omega$ is the diameter of the beam at that point. This is the preferred operating mode for LDV, is easily obtained with He-Ne lasers, and usually obtained from Ar+ lasers although some have been known to exhibit the TEM$_{01}$ (doughnut mode) when operated at higher powers.

A TEM$_{00}$ laser beam will have a gaussian cross-section at any plane taken normal to the beam axis or propagation direction. Since a gaussian profile extends theoretically (although in reality the beam is limited by the bore of the laser cavity) to infinity it is necessary to establish a convention for designating the effective diameter of such a beam. When quoting a beam diameter, it is usually measured to the $1/e^2$ intensity point. At this point the intensity has fallen to 13.5% of the peak intensity. Keep in mind that the beam contains energy which extends out beyond the $1/e^2$ intensity point. The waist of a gaussian beam is the point of minimum diameter. Diffraction causes a gaussian beam to increase in diameter or spread as it propagates from the waist position. This is referred to as the far field divergence of the beam and is related to the waist diameter by

$$\theta = \frac{\lambda}{\pi d_w}$$

where $\lambda$ is the wavelength, $d_w$ waist is the diameter, $\theta$ given in radians. As this relation shows, a larger waist results in less beam divergence. The typical values for waist and divergence of beams produced by He-Ne and Ar+ lasers range from .5 to 1.2 mm diameter with 0.6 to 1.2 mrad full angle divergence. A point to note; it is not possible to perfectly collimate a laser beam. The beam may be enlarged using a beam expander thus greatly

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reducing, but never eliminating, the far field divergence. The waist location of the raw laser beam depends upon the cavity configuration of the laser and design of the output reflector but most often lies very near the output mirror.

Other properties of laser beams are coherence length and polarization. As discussed previously, the coherence of two intersecting beams will affect their degree of interference (i.e. it is necessary for the intersecting beams to be coherent to interfere adequately). Coherence is again a function of laser design and is also related to the length of the cavity. Typically, a He-Ne laser has a coherence length of several tens of centimeters while the Ar+ laser beam may remain coherent for only a centimeter or less. Loss of coherence is minimized by insuring that equal optical path lengths are traveled by all beams which are expected to interfere with one another. The polarization state of beams that are expected to interfere should be linear and parallel to each other. Both He-Ne and Ar+ lasers are available as either unpolarized (random orientation) or linearly polarized. To effect a linear polarization, the laser is designed such that only one polarization state is amplified within the cavity. This is accomplished by adding Brewster angle windows to the ends of the plasma tube which contains the gas. The angle of the windows causes reflection of all but the desired polarization state. Lasers used for LDV should be polarized although it is possible to filter a randomly polarized beam allowing only linearly polarized light to pass, but with a large loss in beam power.
3.0 OPTICAL PRINCIPLES AND PRACTICES

Optical phenomena may be described by geometrical or physical optics theory. Physical optics treats light as a wavelike phenomena. This approach is generally required in dealing with coherent light sources and interferometry. Geometrical optics theory ignores the wave nature of light and treats the light as a collection of rays that travel on geometrical paths determined by simple principles.

3.1 Light Propagation Through Transparent Media

Light travels through a vacuum in straight lines at a constant velocity \( c \approx 3 \times 10^8 \) m/sec irrespective of color or wavelength. In a transparent media, the velocity of light is reduced and becomes wavelength dependent. The vacuum velocity is used to normalize the actual velocity by forming the index of refraction, \( n \),

\[
n = \frac{c}{v}
\]  

Since the speed of light in transparent media is slower than the vacuum speed, the index of refraction of all media is greater than one. The paths along which light propagates are called rays. When a ray intersects an interface between two transparent media, a portion will transmit into the media and the remainder will be reflected as shown in figure 3.1.

The angle between the surface normal and the incident ray is equal to the angle between the surface normal and the reflected ray. The transmitted ray is refracted or turned forming a new angle to the surface normal. The relationship between the incident and transmitted ray angles is called Snell's Law, which states that

\[
\frac{\sin \phi_i}{\sin \phi_t} = \frac{n_2}{n_1}
\]  

These rules for reflection and refraction at an interface can be derived simply using Huygen's principle. Huygen's principle is a geometrical method for finding, from a known shape of a portion of a wavefront at some instant, what shape will occur at a later instant. The principle states that every point on the wavefront may be considered as a source of small secondary wavelets, which spread out spherically at the local velocity of propagation. The wavefront at a new time is found by reconstructing a surface tangent to the secondary wavelets, figure 3.2.

When \( n > n_{\text{crit}} \) (figure 3.3), the transmitted ray refracts to a larger angle from the normal than the trajectory of the incident ray. Hence, for a glass-air interface, the transmitted ray is refracted to 90 degrees for an incident ray angle of 42 degrees. This is the critical angle. For incident ray angles larger than the critical angle, the light is totally reflected (referred to as total internal reflection or TIR) and no light is transmitted.

There are some special cases where the wave-like nature of light is important to the reflection of a light ray from a dielectric media. The wave theory of light allows
Figure 3.1 Reflection and Refraction at a Plane Interface

Figure 3.2 Huygen's Construction for the Propagation of Light
ANGLE OF INCIDENCE LARGER THAN CRITICAL ANGLE OF INCIDENCE

Figure 3.3 Critical Angle
the description of a light ray as having a vector amplitude in the plane normal to the direction of propagation and each vibrational component having some relative phase. If each component is in phase, then the electric field vibrates in the plane containing the propagation vector and the vector sum of the transverse electric field components or the electric field vector (figure 3.4). If the components are out of phase, the electric field vector rotates about the propagation vector, figure 3.5. As discussed earlier, the direction of the electric field vector if referred to as the polarization direction of the light wave.

When the transverse components are equal and exactly 90 degrees out of phase, then the electric field vector rotates about the propagation vector with fixed amplitude; hence, the electric field vector sweeps out a circle in the transverse plane and the light is said to be circularly polarized, figure 3.6.

Consider an incident plane wave linearly polarized so that it’s direction of vibration is normal to the plane of incidence of a dielectric surface, figure 3.7. The wave is refracted at the interface and enters the medium at some transmission angle, \( \phi_t \), whose electric field excites electrons in the media and they, in turn, reradiate optical energy. The reradiated energy comprises the reflected wave whose polarization remains normal to the incidence plane as is the polarization of the refracted wave.

A more interesting case occurs when the polarization of the incident wave is in the plane of incidence (figure 3.8). In this case, the electrons near the surface will oscillate under the influence of the refracted wave. The component of the refracted ray in the direction of the reflected ray can radiate energy into the reflected ray; whereas the component normal to the reflected ray cannot. For the special case when the reflected and refracted rays are normal, no energy can radiate into the reflected ray, and in fact, no reflection occurs. This occurs for a fixed angle of incidence known as Brewster’s angle where \( \theta_r + \theta_t = 90 \) degrees. Hence, from Snell’s Law

\[
 n_i \sin \theta_i = n_t \sin \theta_t \, ,
\]

and since

\[
 \theta_i = 90^\circ - \theta_r = 90^\circ - \theta_i \text{ (recalling } \theta_i = \theta_r) \, .
\]

then

\[
 n_i \sin \theta_1 = n_t \cos \theta_i
\]

or

\[
 \tan \theta_i = n_t / n_i
\]

For transmission from air into glass, the Brewster angle is 56 degrees.

Light can also be absorbed as it propagates through various media. Although good quality optics are designed to minimize this loss, any small percentage that may be absorbed becomes critical when high power lasers and/or highly focussed beams are involved. In such cases, the small amount of absorption results in localized heating of the material.
Figure 3.4  Linear Light

Figure 3.5  Rotation of the Electric Vector in a Right Circular Wave
Figure 3.6 Right Circular Light

Figure 3.7 A Wave Reflecting and Refracting at an Interface
Figure 3.8 Electron Oscillators and Brewster's Law
with accompanying changes in optical and mechanical properties. Optical coatings break
down and stress changes in optical materials lead to unwanted beam distortions. This
phenomena is called thermal blooming and most often results in damaged components.

3.2 Optical Component Quality

The performance of an LDV is closely coupled to the quality of optical components
that constitute the system. Unlike incoherent imaging systems (e.g. cameras), an LDV
requires the careful maintenance of beam wavefronts and coherence throughout the optical
system. A 1mm laser beam may only use a very small area of most standard components
(lenses, mirrors), but a scratch or speck of dust on that area can prove disastrous to overall
performance. Transmissive optics like lenses and windows have the added requirement
that the material of which they are made be free of inclusions (bubbles, impurities) and
internal striations caused by inhomogenieties and stresses which may adversely affect beam
propagation.

Surface quality is important to all components. The interface between two media of
different optical properties must be free of defects which may distort or degrade wavefront
quality. The very process of manufacturing optical components results in the creation of
irregularities on the surface which manifest themselves as micro-scratches and pits (referred
to as digs). Mil-spec scratch and dig numbers are assigned to optical surfaces whenever
surface quality is an issue. Another surface characteristic is 'flatness' which refers to the
devation of a real surface from an idealized reference surface. Flatness is measured by the
number of light wavelengths deviation per unit length from the reference surface. Good
quality optics may be flat to $\lambda/4$ per cm or better. Flatness is achieved through careful
polishing in the final production stages, often by hand, and closely complements the final
scratch and dig figures for the surface.

Aside from the quality of individual optical surfaces, the geometric relationship be-
tween surfaces of the same component is also an important consideration. Parallelism
between the input and output surfaces of a plate beamsplitter or the reflective surfaces of
a rhomboid prism greatly affect beam propagation. In more complex prisms, many surfaces
may be involved and the term 'pyramidal error' refers to the relative alignment of those
surfaces. In general, surfaces which involve reflections as opposed to refractions (especially
at near-normal incidences) have a much greater impact on resulting beam propagation and
should be specified more closely.

Most precision optical components have thin film coatings applied to their surfaces.
Some coatings are designed to protect the surfaces as in the case of transparent overcoats
applied to mirrors to prevent oxidation of the reflective surface. Another broad class of
coatings reduces reflection losses at refractive surfaces and is appropriately named antire-
fection or AR coatings. This type of coating may be wavelength specific and very efficient
($< 0.1\%$ reflection) or broadband with some loss in efficiency ($< 2\%$ reflection). The
antireflection coating is designed to produce reflections from each interface between the
layers of the coating which interfere destructively; hence canceling the reflected wave and
maximizing the transmitted wave. For a single layer coating (figure 3.9), the initial reflection from the air/coating interface, $R_0$, is canceled by the sum of the secondary reflections, $R_1$ through $R_{\infty}$, which are all $\lambda/2$ out of phase with the initial reflection. The $\lambda/2$ phase shift is achieved for $\lambda/4$ thick coatings; and the amplitude of the initial reflection is equal to the sum of the secondary reflections by choosing a coating material with an appropriate index of refraction, namely

$$n_c = (n_{\text{air}} n_{\text{glass}})^{1/2} \approx 1.22$$

A common coating used for increased durability and optical transmission is magnesium fluoride, MgF$_2$, which has a refractive index of 1.38 (i.e., not optimum). However, this coating will reduce the air-glass reflection loss from the uncoated case of 4 percent to 1 percent.

At times, the coating applied to a substrate defines the component. A mirror is simply a reflective thin film applied to a substrate while a line filter is a complex, multilayered coating which only transmits a very narrow wavelength band. Coatings are applied in a variety of ways, but the art of optical coating involves high technology and quite a bit of know-how on the part of the coating engineer. As such, coatings should be treated respectfully as they may be even more delicate than the highly polished surface on which they lie.

The care and handling of optical components is an area too often overlooked by the casual experimenter. Optics left uncovered when not in use accumulate dust which may degrade the performance of an LDV or scratch the surface if improperly removed. Worse still, in high power laser applications, dust particles absorb power and heat-up or burn under the laser beam and damage coatings which quickly leads to the problem of thermal blooming discussed earlier. The oil and acids associated with fingerprints is enough to oxidize stainless steel, no less can happen to optics carelessly mishandled. Proper respect for optical components includes careful treatment and handling. Coatings scratch easily set on hard or rough surfaces - the packing materials in which optics are originally shipped are designed to minimize component degradation and can be used as storage.

As a general rule, optics should never be cleaned; the cleaning process may prove more detrimental to the component then the contamination. Prevention is truly the best medicine - store optics when not in use and cover them whenever possible. When, however, it is determined that a surface is in need of cleaning, care must be exercised to insure that no damage is inflicted to the surface. If dust is the only contaminant, a gentle stream of highly filtered, compressed air from a can designed for that purpose may suffice. The dust may adhere too strongly or other contaminants may be present in which case a solvent cleaning may be necessary. The best procedure involves washing the component in a liquid detergent/water solution followed by rinsing in distilled water. A final wipe using spectroscopic grade acetone and lens tissue should complete the process. The surface is wetted with acetone after which the lens tissue is laid on the surface and slowly dragged across it - no rubbing is involved. The above may be repeated as necessary until careful
Figure 3.9 Single Layer Thin Film Coating
inspection of the component under bright illumination fails to show any contamination
remaining.

3.3 Specific Components

The specific components which comprise most LDV systems will be discussed individ-
ually.

Lenses - The function of a lens is the bending of light rays to a specific purpose; focussing, collimation, or imaging. The operation of a lens can be examined easily using
Snell's law applied to every ray incident on the lens at both the input and output surfaces. The deflection of each ray can be computed so the effect of the lens on an bundle of rays
can be discerned. The curvature of the interface determines the extent of the refraction
and the refractive index of each media also affects the amount of bending. In most cases,
lenses are fabricated from glass and air/glass interfaces are involved.

A lens is formed by two refracting surfaces and the object and image locations are
formed by refractions at each surface. The image formed by the first surface is used as the
object (it may be virtual) for the second surface as shown in figure 3.10a. It is often the
case that the lens thickness is small compared to the object and image distance and most
simple optical designs are based on this 'thin lens' assumption. For a thin lens in air, the
object and image points are defined by

\[ \frac{1}{s} + \frac{1}{s'} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]  

as shown in figure 3.10b.

The focal length, \( f \), of a thin lens is either the object distance of a point object on
the lens axis whose image is at infinity, or the image distance of a point object on the lens
axis at infinity; hence, the equation for the focal length of a thin lens is

\[ \frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]  

Using this definition of the focal length, the common form of the lens equation is

\[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]  

The point from which image and object conjugates are measured is called the principle
plane of the lens and usually lies somewhere between the front and back surfaces. The
magnification of a simple lens is

\[ M = - \frac{s'}{s}. \]
Figure 3.10a  Image Formation by Lens Surfaces

Figure 3.10b  Thin Lens Operation
Conventional lenses have spherical surfaces because such surfaces lend themselves to ease of manufacture. In theory, however, a spherical surface is not ideal for lens design. Non-spherical or aspheric lens with more idealized curvature can be manufactured but at great cost. One result of the use of spherical lenses is a lens error known as spherical aberration in which parallel light input to the lens is caused to focus at different distances dependent on the radial location at which it entered the lens. When the entire lens is used at one time, the result is a blur spot instead of a sharp focus. Also, a simple lens will focus light of different wavelengths at different distances because of the dispersive nature of glass. This is called chromatic aberration and can be compensated by forming a lens with two materials of compensating dispersion. The two elements are usually cemented together to form a single lens called an achromat. In so doing, not only is the chromatic error reduced, but the spherical aberration as well. For this reason, most lenses used in an LDV system are well-corrected achromats. Lenses should always be AR-coated to minimize stray reflections and maximize power throughput.

The practical aspects of using lenses in LDV include recognizing that lenses are used for two purposes in such systems. On the transmitter side, lenses are used to manipulate coherent, gaussian laser beams which may only have diameters of several millimeters yet very high energy densities. On the receiver side lenses are used to collect scattered light and therefore make full use of their apertures. With regard to lens orientation, as a rule of thumb, the side of the lens with the most curvature is oriented towards the longest conjugate of the lens system - if a collimated laser beam is input to a lens, the collimated beam has a conjugate of infinity and should enter the most curved side of the lens. The lens should be aligned normal to the system axis as skewness and tilt contribute to distortion.

Beam Expander - A beam expander is simply a combination of lenses used much like a telescope in reverse. The laser beam enters through the shorter focal length lens and exits through the longer focal length lens. The magnification of the expander is the ratio of the lens focal lengths. A Keplerian expander consists of two positive lenses. The input beam focuses between the two lenses then expands again into the output lens. A Galilean expander uses a negative lens on the input which presents a virtual focus to the output lens (the beam does not focus), but the result is the same. The separation between the two lens is critical to the operation of the expander. To expand a nearly collimated beam (like the raw beam from the laser), the separation between lenses will be exactly the sum of the individual focal lengths, resulting in a nearly collimated and expanded output beam. As this distance is varied, the output beam will either converge or diverge with distance. Laser beams in LDV systems may be expanded to reduce beam divergence over long beam paths or to control the focussed waist diameter of the beam.

Mirrors - Mirrors are used to control the direction of light rays in optical systems. Every mirror used in an LDV system is first surface to prevent the distortions introduced by the mirror substrate. Thin film reflective coatings are arranged into the major categories of metals and dielectrics, which discriminates between the characteristics of the film material. Among the differences between the many types in each category is the reflective efficiencies as a function of wavelength, the durability of the coating, and the cost. Dielectric reflective
coatings can be made very efficient (> 99.5% reflection at one wavelength), are moderately durable (won't rub off during cleaning), are more difficult to produce, and cost about 2-10 times as much as metal coatings. Metal coatings are available in gold, silver, aluminum and a host of other metals. Their reflective efficiencies can run as high as 90%, they are generally much more broadband than dielectrics, but they are more susceptible to damage from scratching and contamination.

Mirrors may be flat or curved, with flat mirrors the more common. As curves they are usually spherical or parabolic and act much like lenses. Curved mirrors are often chosen over lenses when large diameters are required or high energy densities are involved which may damage refractive components. Large scale wind tunnel Schlieren systems are usually built around a pair of curved mirrors. In LDV systems, flat mirrors are used to steer laser beams and to reflect scattered light into detectors. The turning of laser beams is by far the more critical with regard to mirror flatness and surface quality.

Windows - Intrusive material probes have been exchanged for the non-intrusive LDV which requires optical access to the flow. When it is necessary to contain the flow, a window is used to permit the required access. A window is as much an optical component as a lens or mirror and should be selected with as much consideration. Whenever possible, it is desirable to AR coat window surfaces. However, windows by nature are more susceptible to contamination from dust and oil carried by the flow and often require cleaning. It is preferable, in this case, to leave the window uncoated. The cleanliness of windows is very important to the operation of an LDV. As the experiment progresses, the film of oil or vapor which may build up on the flow side of the window causes increasing attenuation of both the incident beams and the scattered light. This film may be invisible to the naked eye but can seriously degrade the performance of the system.

Windows also affect the propagation of light through the optical system. They are, in fact, lenses with surfaces having an infinite radius of curvature and although they do not focus light rays, they displace them nonetheless. On the transmitter side of an LDV, two or more converging beams are incident on the window. Each beam will refract through the window according to Snell's law and emerge into the flow. If the window faces are parallel, the emerging beams will remain parallel to the incident beam direction but will have been displaced by an amount given by,

\[ d = \frac{t \theta^2}{2n_w} \left(1 - \frac{1}{n_w^2}\right) \]  \hspace{1cm} (3.12)

where \( t \) is the window thickness, \( \theta \) the incident angle, and \( n_w \) the window refractive index. Figure 3.11 shows this effect. The focal distance of the receiver system is similarly affected.

Rhomboid Prism - A rhomboid prism is sketched in figure 3.12. This prism is used to displace a laser beam a known, fixed amount while maintaining the parallelism between input and output beam directions. It is, in effect, a set of 45 degree reflective surfaces in a fixed configuration. The reflection occurs within the glass and is due to total internal reflection at the glass/air interface - no coating is applied to the reflecting surfaces. The precision of the rhomboid determines the parallelism of the emergent beam.
Figure 3.11  Beam Displacement Due to a Window
Beamsplitter Prism - The beamsplitter is the heart of the LDV transmitter. To insure an adequate interference pattern at the probe volume, it is necessary to intersect two coherent beams. This is accomplished by splitting one laser beam into two, have them traverse similar optical paths, then intersect at their beam waists. Laser beams can be split in a variety of ways. The stray reflections from most surfaces is a good example of unwanted beamsplitting. For LDV uses, it is desirable to split the beam such that the optical energy is equally partitioned into the resultant beams, coherence is maintained, and path lengths are similar. Keep in mind that the optical path length through glass is longer (by a factor equal to the refractive index) than a similar distance through air. For this reason, plate-type and cube beamsplitters which are cumbersome to work with when path length is important are not the preferred arrangement. Instead, a prism assembly of the type shown in figure 3.13 is used. A special thin film coating is applied to a surface at the junction of the two prisms which produces the energy partition. Both beams traverse the same distance in the glass and emerge coaxial and parallel to the incident beam. This beamsplitter is referred to as 'self-aligning' because the parallelism of the emergent beams is guaranteed by the fabrication process. All coatings have a damage threshold which should be avoided. Be sure that beam powers do not exceed recommended energy densities when using high power (>1 Watt) lasers.

Dispersion Prisms - The wavelength dependence of the refractive index of dielectric materials is used in the design of dispersing prisms. Since the refraction index varies inversely with wavelength, short wavelengths are refracted through larger angles than long wavelengths. Hence, a ray entering a dispersing prism (figure 3.14) will emerge having been deflected by an angle, \( \delta \), known as the angular deviation. The angular deviation depends on three parameters:

1. the prism angle, \( \alpha \),
2. the prism index of refraction, \( n(\lambda) \), which is wavelength dependent, and
3. the angle of incidence.

\[ \delta = \theta_i + \sin^{-1} \left[ (\sin \alpha)(n_2 - \sin^2 \theta_i)^{1/2} - \sin \theta_i \cos \alpha \right] - \alpha \]  

(3.13)

The angular deviation is known to have a minimum when the incidence angle equals the transmission angle. For a 60 degree or equilateral glass prism, the minimum angular deviation, 37 degrees, occurs for an incidence angle of 48 degrees.

Dispersion prisms are used to separate the wavelengths produced by multiline lasers (Ar+ variety). Each wavelength (or color) will emerge from the prism at a different angle whereupon mirrors can be used to 'pick off' and redirect the desired lines.

Wedges - A wedge is a prism with a very narrow apex angle or can be thought of as a window with nonparallel sides. A laser beam will transmit through a wedge but will be deviated from its original path by an angle \( D \), given by

\[ D = (n - 1)A \]  

(3.14)
Figure 3.12  Rhomboid Prism

Figure 3.13  Beamsplitter Prism
where $n$ is the refractive index and $A$ is the apex angle of the wedge. In practice, a set of two wedges is installed back-to-back and each is allowed to rotate independently about its axis. The resulting configuration allows an emergent beam to be directed to any point within a cone of angle $2\theta$ (figure 3.15). This beam steering wedge combination is used in LDV systems to redirect beams which may have been necessarily deviated by other components (Bragg cells in particular).

Laser line filters - A line filter is essentially a color filter with a very narrow wavelength passband centered on the beam wavelength of a particular laser. This is accomplished by designing an interference thin film such that all wavelengths save the one of interest tend to interfere destructively with themselves. This coating is applied to a substrate and usually sealed behind another for protection - moisture seems to damage line filters. Line filters are used on the receiver side of LDV systems to prevent stray light and room light from entering the photodetectors. Although they strongly attenuate light from outside their passband, even the desired wavelength will suffer some attenuation. Typically, a HeNe filter (wavelength = 0.6328 microns) with a full width at half maximum passband of 3 nanometers will only transmit 50% of the HeNe laser light.

Another aspect of line filters to be aware of is that of vignetting. The interference coating is designed such that the incident light must travel a specific distance between layers (this distance is usually a multiple of $\lambda/2$). If the light incident on the filter is not normal due to a tilted filter or a converging or diverging cone of rays, the paths between layers traveled by individual rays will not meet the design criteria of the filter and may not behave as expected. The center of the filter passband actually shifts depending on incident angle and the ‘tightness’ of the filter. This is not usually a problem if the filter is oriented normal to the incident light and the cone of light entering the filter has an f-number greater than about 5. Also, line filters usually have one highly reflective silvered surface and another colored surface. The silvered surface should be oriented toward the incident light.

Dichroic filters - Dichroic filters are designed to reflect all wavelengths above or below a certain value while transmitting the remainder of the spectrum. They are also referred to as dichroic mirrors because of their reflective properties. A dichroic is a multilayer interference coating applied to a transparent substrate. Their function in LDV is color separation; usually on the receiver side. Dichroics are not as efficient as dispersion prisms in their wavelength discrimination as they allow a small amount of crosstalk, but the geometry of the LDV system often precludes the use of dispersion prisms.

Polarization filters - These filters are designed to permit the passage of light of only one polarization direction while absorbing all other polarizations. Since most laser beams for LDV are already polarized, these filters serve little purpose except for analyzing the polarization state of the beam. Some LDV applications, however, make use of orthogonally polarized sets of beams for multicomponent systems. In these cases, a polarization filter(s) is used to discriminate one polarization from others. Since this filter absorbs most of what it doesn’t transmit, it’s very easy to damage even with low power lasers.

Polarization rotators - Also referred to as retardation plates or 1/2-wave plates, a
Figure 3.14  Geometry of a Dispersing Prism

Figure 3.15  Wedge Assembly for Beam Steering
polarization rotator will change the polarization direction of an incident plane-polarized beam. They are constructed from materials which exhibit the phenomena of birefringence in which waves of different polarization travel at different speed through the material. In effect, one component of the wave is retarded such that, at emergence, the recombined wave possesses a new polarization. The polarization can be continuously adjusted by rotating the plate about its axis. Their use in LDV systems is to adjust the polarization of a beam so that it matches the input requirements of a component (beamsplitters and dispersion prisms in particular).

Spatial filters (pinholes) - Spatial filters are small holes placed in otherwise opaque materials which block all light except that which gets through the hole. On the receiver side of an LDV, they are used in conjunction with the receiver lens to limit the extent of the spatial volume from which scattered light is collected. The pinhole is located at the axial back focal point of the receiver lens system. Only light scattered from the front axial focal point (which coincides with the beam intersection) will pass through the pinhole to the detector.

On the transmitter side, pinholes may be used to improve or ‘clean-up’ the beam. Invariably, a laser beam contains diffraction noise, flash-lamp or plasma illumination and transverse mode structure. Much of this can be removed by the procedure of spatial filtering. The beam is focused by a high quality lens through a tiny pinhole (figure 3.16). The intensity distribution of the focused laser beam depends upon the wavelength, lens diameter and focal length. For a uniform intensity laser beam striking the lens a diffraction pattern produced at focus has a central maximum (Airy’s disk) whose diameter is

\[ d_a = 2.44 f_e \lambda / d_e \]  

(3.15)

The aperture having this diameter will pass 84 percent of the collimated light striking the lens, while removing most of the uncollimated light. Actually, most laser beams of interest are gaussian in intensity profile rather than uniform and the above equation is not applicable. For cw lasers, pinhole diameters range from a few microns in diameter upward, depending on \( f \).

Photomultiplier Tubes - Photodetectors for LDV use include solid state silicon photodiodes of the avalanche and PIN types as well as vacuum tube photomultipliers (PMT). The advantages of the PMT over the solid state devices are low noise, high gain, and gain variability while the disadvantages include fragility, low quantum efficiency, and the need for a high voltage source. The PMT is usually the detector of choice because LDV signals derived from wind tunnel studies are often of very low level. A PMT is comprised of a photocathode, a series of intermediate amplification stages called dynodes, and an output anode. Light striking the photocathode is absorbed resulting in the emission of photoelectrons into the dynode structure. At each dynode, one incident electron results in the emission of several identical electrons. This process is repeated through each stage with a typical final gain of as much as \( 10^7 \) at the anode. At the anode, a signal is extracted which represents the electrical analog of the light scatter signal. The signal is then fed into a
preamplifier, designed to match the impedance and frequency characteristics of the signal before being routed to signal processing electronics.

Photomultipliers are sensitive to light even when power has not been applied to them. The tubes' dark current or background noise will increase after exposure to ambient light or elevated temperature and may require several hours to 'quiet down'. They should be stored in the dark when not in use. With power applied, a PMT becomes extremely light sensitive. A direct or stray laser beam incident on the photocathode may irreparably damage the PMT. High scattered light levels may also damage the PMT - be sure to shield the tube whenever a stationary scatterer is placed in the probe volume for alignment purposes.

3.4 Gaussian Beam Optics

The use of lasers requires a knowledge of the propagation characteristics of gaussian beams. In this section, a brief description of how these gaussian beams of light are transformed on their passage through lenses, telescopes (beam expanders), and other lens combinations is given.

Optics for laser applications, in some ways, are more critical in the component design requirement than for other imaging applications. However, the designs are simpler because the laser applications in most cases involve on-axis operations which eliminates the need for asymmetrical aberration corrections. Lasers are essentially monochromatic so chromatic aberrations are not a concern.

Light scattered from dust, damaged coatings, and other surface defects as well as inclusions can produce noise when dealing with coherent imaging systems. This is especially true of LDV receiver systems.

Laser light is generally coherent over a relatively large volume so it does not have the usual problems associated with imaging incoherent sources. That is, each part of the wavefront will act as if it originated from the same point source and so the emergent wave is precisely defined. Beginning with a well-defined wavefront permits more precision in focusing and the control of the beam. This does, however, require designing the optical components such that diffraction and aberration effects are considered.

Although a Gaussian laser beam wavefront may originate at or pass through a point where the wavefronts are perfectly plane with all parts of the wave moving in precisely parallel directions, the wavefronts soon acquire curvature and begin to spread. The spreading parameters describing the radius of curvature $R(Z)$ and the radius of the $1/e^2$ irradiance contour is given by

$$R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$

and

$$w(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}.$$
where $z$ is the distance propagated from the plane where the wavefront was flat, $\lambda$ is the wavelength of light, $w_0$ is the radius of the $1/e^2$ irradiance contour at the plane where the wavefront was flat, $w(z)$ is the radius of the $1/e^2$ contour after the wave has propagated a distance $z$, and $R(z)$ is the wavefront radius of curvature after propagating a distance $z$, $R(z)$ is infinite at $z = 0$, passes through a minimum at some finite $z$, and rises again toward infinity as $z$ is further increased, asymptotically approaching the value of $z$ itself. The plane $z = 0$ is the location of a Gaussian waist which is a place where the wavefront is flat and $w_0$ is called the waist radius.

$R(z)$ approaches $z$ asymptotically for large $z$ and $w(z)$ asymptotically approaches

$$w(z) \approx \frac{\lambda z}{\phi w_0}.$$  \hfill (3.18)

The contour of the $1/e^2$ irradiance asymptotically approaches a cone of angular radius

$$\Theta = \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}.$$  \hfill (3.19)

This value is the far field angular radius (or divergence) of the Gaussian beam. The vertex of the cone lies at the center of the waist, figure 3.17.

To determine the size of the focal spot formed when the laser beam is focused with a diffraction limited (aberration free) lens, the asymptotic formula

$$w(z) = \frac{\lambda z}{\pi w_0}$$

may be used. In this case, the distance $z$ becomes the focal length, $f$, of the lens and $w(z) = w(f) = w$ is the $1/e^2$ contour radius of the beam at the lens. $w_0$ is then the contour radius of the waist formed near the lens focal point. Hence

$$w_0 \approx \frac{\lambda f}{\pi w}$$  \hfill (3.21)

or for a combination of laser and laser line focusing lens, the formula should be modified to become

$$w_0 \approx \frac{1.33 \lambda f}{\pi w}.$$  \hfill (3.22)

The factor $4/3$ enters because of the careful balance of spherical aberration and the diffraction inherent in the design of the lens.

The waist in front of the lens may have originated from a laser, for example, which is located at the exit mirror. If the lens is placed a distance $d_1$ from the beam waist of radius $w_1$ (figure 3.18), the lens produces another waist $w_2$ at distance $d_2$. The far field angles $\Theta_1$ and $\Theta_2$ of the beams are computed from

$$\theta_1 = \frac{\lambda}{w_1} \text{ and } \theta_2 = \frac{\lambda}{\pi w_2}.$$  \hfill (3.23)
Figure 3.16 Spatial Filtering

Figure 3.17 Gaussian Beam Divergence Contour
Figure 3.18  Beam Transformation by a Lens
From these two angles follow the beam radii $w_{1f}$ and $w_{2f}$ in the two focal planes of the lens where the image of the far field appears,

$$w_{1f} = f\Theta_2 = \frac{\lambda f}{\pi w_2}$$
$$w_{2f} = f\Theta_1 = \frac{\lambda f}{\pi w_1}$$

(3.24)

According to the imaging rules, corresponding centers of curvature are images of each other.

The waist $w_2$ can be expressed in terms of the known laser waist, $w_1$, and distance to the lens, $d_1$, as follows:

$$w_2 = \frac{1}{w_1} \left[ \left(1 - \frac{d_1}{f} \right)^2 + \frac{1}{f^2} \left(\frac{\pi w_1}{\lambda} \right)^2 \right]^{-1/2}.$$  

(3.25)

A corresponding relation for the spacing of the waist, $d_2$, between the lens and the beam waist $w$ is given by

$$d_2 = f + \frac{(d_1 - f)f^2}{(d_1 - f)^2 + \left(\frac{\pi w_1}{\lambda} \right)^2}.$$  

(3.26)

These relationships can be used to predict the location of the waist and its diameter for an LDV system. Notice that with parallel beams, the beams will cross at one focal length from the principal planes of the lens. However, the waist $w$ will occur at the focal point of the lens only if the lens is located at one focal length from the waist within the laser ($d_1 = f$). A beam collimator may be used to essentially make $d_1$ appear to be at a much larger distance than $f$. In this case, the waist $w_2$ will also form at the focal point of the lens.

### 3.5 Laser Safety

The laser is a source of extremely intense light having characteristics that are very different from the light emitted from conventional sources. The user must be aware of these characteristics of laser light and the proper safety precautions before attempting to operate the laser. The energy level of the beam is high enough to cause serious damage to the eye with possible loss of vision if the beam were to pass directly into the eye. The laser beam can cause serious flesh burns, ignite flammable materials, and damage sensitive optical equipment. Since the beam is collimated and coherent, the energy in the beam remains high and dangerous even at great distances from the laser source. In addition, the high voltages and high amperages associated with laser operation also pose a safety hazard. The user is, therefore, advised to observe the following safety precautions:

1. Every laser or laser product contains instructions in the proper usage of the product - follow the manufacturers recommendations.
2. Limit access to the laser to those familiar with the equipment. Keep the laser out of the hands of inexperienced and untrained personnel.

3. Do not operate the laser in the presence of inflammable, explosive, or volatile substances such as alcohol, gasoline, or ether.

4. When the laser is on and the output beam is not terminated in an experiment or optics system, the beam should be blocked. Use a laser power meter or some other nonreflective, nonflammable object.

5. Never look directly into either the main laser beam or any of the stray beams. Never sight down a beam into its source.

6. Do not allow reflective objects to be placed in the laser beam. Light scattered from a reflective surface can be as damaging as the original beam. Even objects such as rings, watch bands, and metal pencils can cause a hazard.

7. Post warning signs and limit access to the laser area when the laser is in operation.

8. Set up experiments so the laser beam is not at eye level.

9. Set up a ‘trap’ for transmitted beams. A folded sheet of metal, painted black, works well. Also set up shields to prevent strong reflections from going beyond the area needed for the experiment.

10. Protective eyewear is available for all varieties of visible lasers. Eyewear should be worn during any time that the laser is energized.

11. In those cases when visual access to the beam is necessary (as in system alignment), insure that the laser is turned down to minimum power, the area is clear of unnecessary personnel, and extreme caution is used to prevent exposure.
4.0 DUAL BEAM LDV BASICS

The discussions leading up to this chapter have been a prelude to the workings of the LDV. They were necessary so that the following description of the dual beam LDV could proceed with the reader having some understanding of practicalities involved in implementing such a system.

4.1 Description

In this section, a general description of the dual beam laser Doppler velocimeter is given. A detailed description of the set up and operating procedures will be given in a later section.

Although a number of optical configurations have been devised to obtain velocity measurements using the Doppler effect, the dual beam system is the best type for most applications. Figure 4.1 shows the basic components of a single component LDV system. The laser beam is split into two equal intensity beams and focused to an overlap region which partially defines the sample volume. Small particles ($\lesssim 1\text{pm}$) that will follow the flow and yet provide an adequate signal are required. The particles passing through the sample volume scatter light, a part of which enters the receiver lens aperture. This light is focused onto the small aperture on the photodetector. If the receiver is located off-axis, the intersection of the image of the aperture and the focused laser beam form a very small sample volume. This technique can be used to effectively limit the detected light from only those particles that cross the overlap region of the beams.

Within the beam intersection region, light from the two incident beams interferes to create a fringe pattern. These fringes form parallel planes which lie perpendicular to the plane of the incident beams yet parallel to the beam bisector. The spacing, delta, between successive fringe planes is given by

$$\delta = \frac{\lambda}{2\sin(\theta/2)}$$

(4.1)

where $\lambda$ is the light wavelength and $\theta$ is the beam intersection angle. As particles transit the probe volume, they scatter light in proportion to the light incident upon them which varies with the intensity of the fringe pattern. It is very important to have a large degree of beam overlap to insure that the highest fringe visibility occurs. Most commercial LDV system optics have been specifically designed to meet this requirement through the use of 'self-aligning' beamsplitters or beam steering wedge assemblies.

Photomultiplier tubes are used to convert the detected light to an electronic signal. The familiar Doppler burst signal is shown in figure 4.2. After the signal is high-pass filtered, the time between zero crossings on the positive slope of the signal is measured to determine the frequency. The velocity is then the product of the frequency and fringe spacing. The velocity measured, however, is only the velocity component perpendicular to
the fringe planes. In order to measure two or all three velocity components simultaneously, a more sophisticated LDV system is required and is discussed later.

Although the velocity can be determined from fundamental principles, it is always wise to test the entire system by measuring the light scattered from an object moving at a velocity that is known a priori. A rotating disk with a mark at a known radius is probably the most convenient means to do this. Using this technique, the system can be checked to determine if the velocity is the same over the detectable limits of the beam intersection.

The selection of the type of laser is invariably determined by the power requirements. HeNe lasers are very reliable and convenient but are limited to about 50 milliwatts. Ar-gon ion lasers are most commonly used for LDV applications and have the advantages of providing a broad power range, several wavelengths of light from blue to green that can be separated for two and three component systems, and are relatively reliable. Photomultiplier tubes also have a better efficiency at the shorter wavelengths of the Ar+ laser.

An Ar+ laser can be configured in the single line or multiline mode. If configured in multiline mode, a dispersion prism or other means is needed to separate the wavelengths before passing the beam through the system. A pair of equilateral prisms or other combination of prisms is used.

Beam collimation and expansion may be used to reduce the diameters of the focused beams, figure 4.3. The focal or waist radius, is given by the following expression

\[ w \approx \frac{4 f \lambda}{\pi D} \]  

(4.2)

where \( f \) is the lens focal length and \( D \) is the incident beam diameter. Beam expansion is required to maintain the waist diameter while increasing the focal length. Also, decreasing the waist by a factor of 2 will increase the incident intensity on the particles by a factor of 4 and improve upon the spatial resolution. This is a much more efficient means for improving the signal-to-noise ratio (SNR) than increasing the laser power. Increased laser power can lead to thermal blooming with a net loss in performance.

In some cases, the expansion of the beam before it is split is limited by the size of the beam splitter or the Bragg cell (acousto-optic frequency shifter) aperture. The beam expansion can also be implemented after the beam splitter and Bragg cells, figure 4.4. However, the beam separation is increased proportionately.

It is good practice to collimate the laser beam even if expansion is not used. A laser produces a gaussian beam that has a waist located near the front mirror at a position depending on the design of the laser cavity. When the distance between the laser waist and the point one focal distance in front of the transmitter lens is much less than the Rayleigh distance of the laser beam given as

\[ s_{\text{laser}} = \frac{\pi w^2_{\text{laser}}}{\lambda} \]  

(4.3)

then the focal waists of the beams will coincide with the crossover region. If the waists do not coincide, then the minimum beam diameter will not be located at the crossover
Figure 4.1  Single Component LDV Transmitter

Figure 4.2  Doppler Burst
Figure 4.3  Beam Expansion
and the interference fringes will not have a uniform spacing over their full extent, figure 4.5. A collimator is thus used to adjust the focal waists by producing a new focal waist that can be positioned one focal length in front of the transmitting lens to insure that the waists occur at the crossover. The collimator may be adjusted while observing the focused laser beams. Adjustments are made until the minimum diameter coincides with the beam crossover.

The size of the waist cannot be reduced without limit. Diffraction limits the extent to which a gaussian beam may be focussed, even under ideal conditions. Also, most processors require the signal to possess a minimum number of fringes to be validated. The processing method most often employed in dual beam LDV is the counter processor which is a single event device. Only signals which originate from a single particle in the probe volume will be accurately processed for velocity. Light scattered simultaneously from multiple particles may be processed erroneously if it is even validated. The diameter of the probe volume is adjusted to insure that the probability of multiple particle occurrences is low.

Placement of the receiver with respect to the beam intersection is not critical for dual beam LDV, however, there are advantages to placing the receiver at certain locations to gain the greatest signal amplitude and signal visibility. As we shall see in the section on particle requirements, most of the light interacting with the particle is scattered in a direction concentrated about the forward direction. However, off-axis collection angles usually allow good noise isolation by admitting only light scattered at the sample volume to reach the photodetectors.

Figure 4.6 shows a typical receiver configuration for a single component system. The goal is to image all of the light scattered at the measuring volume onto the photodetector aperture. Two receiver lenses are used both of which should at least be achromats. Achromatic lenses can partially correct for spherical aberrations. Since lenses are designed to bring light from infinity (collimated light) to focus at their focal length, two lenses placed back-to-back provide the best image quality. The alternate system shown can be used to compress the size of the imaging system. A mirror with precise x-y adjustments provides a convenient means to make fine adjustments to the alignment.

The use of high quality receiver optics cannot be overemphasized. If a lens forms an aberrated image, only a fraction of the light entering the lens aperture may reach the detector. In addition, a blurred image will not provide adequate rejection of spurious light. A high quality lens purchased for one to two thousand dollars can increase the signal-to-noise by a factor of two or more. Use of twice the laser power to achieve a similar effect may cost as much as an order of magnitude more than the lens. Thermal blooming and safety considerations also weigh against the use of high power lasers. Usually lenses of f/no. equal to about 5 are used. Although smaller f/no. lens would appear to provide more efficient, light collection, the increase in spherical aberrations usually negates these gains.

In some applications, backscatter light detection may be desirable, figure 4.7. Although the intensity of the light scattered is less by approximately two orders of magnitude as compared to forward scatter, the optical access to the flow field and traversing
Figure 4.4 Alternate Beam Expansion

Figure 4.5 Beam Intersection at other than the Beam Waist
of the system are simplified. Receiving the light on-axis through the same lens as the beam is transmitted is not recommended. The reflections from the lens and scatter from dust, inclusions and other imperfections will compete with the signal to reduce the overall signal-to-noise ratio.

4.2 Two-Component LDV Systems

In most applications, at least two orthogonal velocity components need to be measured simultaneously. The most effective way to do this is to separate the colors from an Argon ion laser with a dispersion prism, figure 4.8, and use the blue ($\lambda = 0.488\mu m$) and the green ($0.5145\mu m$) beams for the two components. A typical optical arrangement is shown in figure 4.9. The beams are directed through coaxial optical paths with mirrors and displacement prisms. All four beams must focus and cross at the same point. This is best accomplished by magnifying the beams at the sample volume using a microscope objective. A system of this configuration is referred to as a two component LDV. There are other, less effective methods by which two velocity components can be measured (e.g. using orthogonally polarized beams of the same wavelength).

The receiver optics require a means to separate the signals by color and direct them to the respective photodetectors. A dichroic beamsplitter or a polarizing beamsplitter may be used. The dichroic beamsplitter should have a sharp cutoff on the transmitted wavelength. Nonetheless, some of the second wavelength will be reflected and transmitted. Laser line filters are used to stop the unwanted light at the detectors. This approach can result in a significant loss of signal. A better approach is to rotate the polarization on one of the beams and use a polarization beamsplitter in the receiver. The separation by polarization tends to be more efficient but laser line filters are still recommended to exclude cross talk and ambient light.

4.3 Seed Particle Requirements

Perhaps the single most inconvenient characteristic of the laser Doppler velocimeter technique is the need for seed particles. The LDV actually measures the velocity of small particles that are assumed to move with the flow. If these particles do not have the appropriate size and concentration distribution, the results may be adversely affected. Because of the need for seeding in gas flows is greater than for liquids and gas flows are more frequently investigated in aerodynamics research, emphasis will be placed on defining the criteria for these applications.

Particle sizes of the order 1$\mu m$ are often quoted as adequate for most gas flows and can respond to turbulence frequencies exceeding one kHz. In actuality, particle size distributions are skewed with a large population of small particles, figure 4.10. The small particles may be below the detectable limit of the instrument but will contribute to the background noise. Although few in number, there will be large particles that are easily detectable but may not adequately track the flow.
Figure 4.6 Typical LDV Receiver

Figure 4.7 Backscatter Geometry
Figure 4.8 Argon Laser Beam Dispersion by Prisms
Figure 4.9  Two Component LDV System
Figure 4.10  Seed Particle Size Distribution
Often, particle number densities are quoted when trying to establish flow seeding requirements. However, in most applications it is the realizable data rate that is of interest. The data rate is related to the particle flux given by

\[ F = A(d)V(d)N(d) \]  

(4.4)

where \( A(d) \) is the LDV sample volume cross section, \( V(d) \) is the velocity, and \( N(d) \) is the particle number density of particles. Clearly, both the number density and velocity affect the data rate. If there is a need to follow spatial scales in the flow, the concentration or number density of the particles (particles/ m³) will have to be increased accordingly.

Although this seldom occurs in gas flows associated with aerodynamic applications, too many particles can introduce phase noise on the signals. That is, more than one particle may be present in the sample volume at one time. These particles will inevitably be out of phase and have different amplitudes. The resultant signal which is the summation of the individual signals will have reduced visibility and an apparent increase in the noise level. At high particle concentrations, the instrument will not operate satisfactorily.

Uniformity in the particle number density is required. Since the velocity probability density functions are dependent upon the particle flux, velocity biasing will occur if the velocity is correlated with the number density. For example, if the slow moving fluid also has a lower particle number density than the faster moving fluid, the velocity distribution will be biased to the higher speeds. Such situations can occur in free shear layer flows where the two streams originate from a different source or are separately seeded. Boundary layer measurements can also be affected if particles are entrained by the wall or are forced from the wall by thermophoresis.

The situation is especially difficult when attempting to seed high speed jet flows. In such cases, a common plenum can be seeded and hopefully, significant particle loss will not occur in the separate streams. Surrounding fluid entrained by the jet must have the same particle number density.

The upper size of the particles acceptable for LDV applications is dependent upon response requirements of the particle to the flowing fluid. Strictly, the response depends on the particle drag and its mass. Large low density particles would be advantageous from both drag and light scattering considerations. Restricting our attention to spherical particles, the Stokesian particle dynamics are described by

\[ \frac{dV_p}{dt} + \gamma V_p = \gamma v(t) \]  

(4.5)

where \( V_p \) is the particle velocity, \( v \) the fluid velocity, and \( \gamma \) is the response parameter. \( \gamma \) is given by

\[ \gamma = \left( \frac{12\nu}{d_p^2} \right) \left( \frac{3\rho}{2\rho_p + \rho} \right) \]  

(4.6)

where \( \nu \) is the fluid kinematic viscosity, \( \rho \) and \( \rho_p \) are the fluid and particle densities, and \( d_p \) is the particle diameter. The differential equation may be solved for the velocity and
position of a particle in a time-varying flow field. The general solution is

\[ V_p(t) = \gamma e^{-\gamma t} \int V(t)e^{\gamma t}dT + Ce^{-\gamma t} \tag{4.7} \]

and

\[ x_p(t) = x_0 + \int_0^t V_p(t)dt \tag{4.8} \]

For example, if a particle is subjected to a step change in gas velocity as follows

\[ t < 0 \quad V = V_0 \]
\[ t \geq 0 \quad V = V_1 \]

the particle velocity is

\[ \frac{V_pV_1}{V_0V_1} = e^{-\gamma t} \tag{4.9} \]

The results for various \( r \)'s are shown in figure 4.11. The distance travelled in time, \( t \), is

\[ x_p = V_1 \left[ t - \frac{1}{\gamma} \left( \frac{V_0V_1}{V_1} \right) (e^{-\gamma t} - 1) \right] \tag{4.10} \]

These results can be used to evaluate the characteristics of seed particles. In high speed flows, the particle velocities can be measured as they pass through a shock. Information on the size and distribution can be acquired with this technique.

The lower size limit on particles that are useful for LDV applications is based on the sensitivity of the system. Sufficient light must be scattered and adequate detector sensitivity must be available to provide a sufficient signal-to-noise ratio. The amount of light scattered depends on the particle diameter and refractive index, angle of detection, polarization of the incident light and its wavelength. For spherical homogeneous particles of arbitrary size immersed in a homogeneous environment, the light scattered is described exactly by the Mie theory.

The theory and representative results are described in a number of text books. At least a basic understanding of the light scattering phenomena are required for LDV applications. The Mie theory is required for calculating the light scattered by particles with diameters on the order of the wavelength of light (0.5μm). Simplified theories for refraction, reflection, and diffraction may be used for larger particles with the advantage that better insight can be had on the details of the phenomena. The scattering parameter, \( \alpha = \pi d/\lambda \). The scattering intensity varies approximately as the diameter squared for particles larger than 1μm. For smaller particles the scattered light varies as \( d^4 \) to \( d^6 \). Typical scattering diagrams are shown in figure 4.12 for particles in the size range of 0.5 to 5μm. Each ring represents an order of magnitude change in the scattering coefficient and each dot represents 5 degrees.
\[ \gamma = \frac{18 \mu_t}{\rho_p d_p^2} \]

**NOTE:**

FLUID: AIR (SP, 80°F)
SPHERICAL PARTICLE
STOKES LAW

Figure 4.11 Variation of Response Parameter with Density and Diameter
Figure 4.12  Polar Scattering Diagrams
Small particles tend to have much less angular dependence whereas larger particles scatter significantly more light in the forward direction. This is true for dielectric particles. Particles having high conductivity scatter more light in the backwards direction (relative to the incident beam) so are very suitable for the backscatter mode of operation. Before selecting the receiver location, it is often useful to obtain at least a general picture of the theoretical scattering diagrams. These are available in the sources given in the bibliography for additional study.

A caveat that should be recognized is that accurate particle size measurements are very difficult. Thus, when the manufacturers of particle generators of any type quote a size distribution produced by the device, treat these data with skepticism. Inquire as to how these data were obtained and question the technique used.

4.4 Frequency Shifting

The use of frequency shifting is most often associated with the need to resolve the 180 degree directional ambiguity (the inability to know which of the two ways a particle traversed the probe). However, there are other conditions requiring frequency shifting that are equally important. A brief description of the consequences of frequency shifting will help in understanding why it is useful. The methods of frequency shifting will be discussed along with the effect on the resolution and complications to the system.

An electro-optical device introduced into one or both beams of the LDV can produce a frequency shift in one beam relative to the other. When beams interfere, the interference will have a time-varying phase shift which will make the fringes move at the difference frequency. That is, a stationary particle in the sample volume will produce a sine wave with a frequency which is at the difference in the shift frequencies of each beam. The fringes will move from the higher frequency beam to the lower with reference to their orientation as they exit the transmitter lens, figure 4.13. A particle moving through the sample volume will produce a signal given as

\[ f_{\text{signal}} = | f_{\text{shift}} \pm f_D | \]  

where the sign is + for particles moving opposite to the fringe motion. The effect on the frequency spectrum is shown in figure 4.14.

It is easy to see how the direction, whether positive or negative can be determined. In highly turbulent and recirculating flows, frequency shifting is essential to the prevention of large measurement errors. For highly turbulent flows, the frequency bandwidth may be too large to enable proper filtering of the signal. The high pass filter must effectively separate higher frequency Doppler component from the pedestal before processing the signal. If there are too few Doppler cycles in the signal this may not be possible. By introducing a frequency shift, the signal frequency is increased resulting in a compression of the relative bandwidth due to the turbulence. There will also be a proportionate increase in the number of cycles in the Doppler burst. Care must be exercised so that not more than one
Figure 4.13 Frequency Shift Orientation

Figure 4.14 Frequency Spectrum Upshift
reading is made on the slower moving particles. This is especially serious in separated flows wherein some particles can be moving very slowly and the high speed counter (≈ 1μsec turn-around) can read the particle many times.

Another biasing error can be avoided by using frequency shifting. Since some of the signal processors (Macrodynne) require a fixed number of cycles in the burst, particles passing the fringes at oblique angles, figure 4.15, may not be measured. With frequency shifting, even particles with trajectories parallel to the fringes can be recorded. The requirement on the shift frequency can be determined from the particle transit time through the sample volume. That is, if \( N \) cycles are required, the frequency shift \( f \) must be

\[ f_s = N \times \frac{u_p}{w} \]  

where \( u_p \) is the speed of the fastest particle and \( w \) is the diameter of the focused laser beams.

Of the frequency shifting techniques available, Bragg cells satisfy the greatest range of applications, provide the most accurate and stable frequencies, and are as convenient as the other methods. Bragg cells are acousto-optic devices consisting of a solid crystal or a liquid cell driven by a piezoelectric transducer. The transducer generates waves in the Bragg cell. If a laser beam is directed through the cell at the Bragg angle, deflected beams deflected from the transmitted beam will appear. The Bragg angle is given by

\[ \sin \phi_b = 1/2(\lambda/\lambda_a) \]  

where \( \lambda \) and \( \lambda_a \) are the optical and acoustic wavelengths respectively. The Doppler frequency shift produced by the Bragg cell is equal to the ultrasonic frequency, \( f \).

Bragg cells must be large enough such that the laser beam passing through the medium at the Bragg angle crosses at least one full wavelength of the ultrasonic wave. For Bragg cells of practical size, the Bragg effect occurs at frequencies greater than 10 MHz. Typical crystal Bragg cells used in LDV applications operate at around 40 MHz.

Either one or two Bragg cells may be used to attain the desired frequency shift. With a single Bragg cell, the optical frequency will be 40 MHz plus or minus the Doppler frequency. This usually requires additional electronic down mixing to obtain the desired frequency for processing. If the flow speeds are low, it may be advantageous to keep the optical frequencies lower. In this case, two Bragg cells are required, one in each beam. The Bragg cells are driven slightly above and below the design frequency of 40 MHz. A shift frequency equal to the difference of the Bragg cell frequencies is produced at the sample volume.

A rotating diffraction grating may be used as an inexpensive means for obtaining relatively low frequency shifts, figure 4.16. Light emerging from a diffraction grating is deflected at angles

\[ \phi_k = \sin^{-1} \left[ \frac{k \lambda}{s} \right] \]  

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where $k$ is the diffracted order and $s$ is the line spacing of the grating. When it is rotated, the usual Doppler shift considerations prevail such that the frequency difference of the diffracted beam from the incident beam is

$$f_s = kN\omega_r$$

(4.15)

where $\omega_r$ is the rotational frequency. The radius of the grating does not affect the shift frequency which simplifies the alignment.

Diffraction gratings are limited to frequency shifts of approximately 5 MHz and lower. If low turbulence levels are to be measured, vibration of the rotating grating may present problems.

In general, nothing comes without losses. The Bragg cells steer the beams out of parallel. Specially cut beamsplitter cubes or wedge prisms are needed to correct the beam direction back to parallel. Higher order diffracted beams must be blocked to prevent loss of signal visibility. To some extent, the beams are distorted to a slightly elliptical shape and care must be taken to ensure that they overlap completely at the crossover.
Figure 4.15 Oblique Particle Trajectory thru Fringe Volume

Figure 4.16 Rotating Grating for Frequency Shift
5.0 SIGNAL PROCESSING

In this section, general operational principles of the available methods for signal conditioning and processing will be discussed. Some of the specific features of the LDV signals will be considered as they relate to the signal processing requirements.

The signals which are characterized as ‘burst’ signals consist of a pedestal component which is a result of the Gaussian intensity distribution of the laser beam and a Doppler component. The Doppler or high frequency component arises from the particle passing the interference fringes. In addition to these frequency components there is the inevitable noise produced by distortions to the laser beams, flare light from optical components and other surfaces in the flow field, particles outside of the measurement volume, and electronic noise. Phase noise may also be present if the particle number density is high and there are frequent occurrences of more than one particle in the sample volume.

Most often, ideal signals are shown as examples of a ‘typical’ Doppler burst, figure 5.1. However, in some difficult measurement environments the signals may deteriorate to where they are almost indistinguishable from the noise. In such circumstances, the temptation is to close down the bandpass filter around the assumed frequency. Unfortunately, the noise passed through a narrow band filter will look like the signal. To avoid such self-deception, some requirements of signal filtering will be discussed.

Although the Doppler frequency does not normally change during individual signals because of the size of the sample volume relative to the turbulent scales, large changes in frequency can occur from one signal to another. The range of frequencies can exceed an order of magnitude in some highly turbulent flows. In this case frequency shifting can suppress the relative frequency range but with some loss in resolution as a penalty.

With the most commonly used signal processors, the pedestal or low frequency component of the signal must be removed by a high pass filter. The high pass filter cutoff must be low enough to avoid excessive attenuation of the Doppler frequency information. However, if the filter is set too low the pedestal will not be fully removed producing a signal as shown in figure 5.2. If this filter distortion large enough, for example, due to a large signal, the frequency at the trailing part of the signal can be interpreted incorrectly by as much as an order of magnitude.

The difficulty in selecting the appropriate filter setting arises from both the need to process a relatively large frequency range and the burst nature of the Doppler signals. Typically, fixed filter bandwidths are used with a bandwidth of eight. If, for example, the fringe spacing is such that there are 12 fringes to the $1/e^2$ point of the beams and the average signal has 12 cycles that are above the threshold, the separation between the pedestal and Doppler frequencies is approximately 1 to 12. Consider the two signals shown in figure 5.3, representing velocities that differ by a factor of 10. It is easy to see that the low frequency component of the signal from the fast particle approaches the high frequency component of the slow one. Closing the filter bandwidth to eliminate noise can result in the attenuation of one end of the frequency range and bias the velocity distribution.
Figure 5.1  Doppler Burst

Figure 5.2  Filtered Signals

signal distortion due to improper filter setting
The need for an adequate frequency bandwidth determines the fringe spacing to beam diameter and the dynamic range. However, frequency shifting can remove this constraint. Counter processors which are most commonly used in aerodynamic applications most often process a fixed number of fringes. If there are too few cycles in the burst signal, it will not be processed, too many and only the first \( N \) cycles will be used reducing the processing accuracy. These constraints on the signal must be considered in the optical system design.

Several methods are available for processing LDV signals but in air flow studies the particle concentrations are relatively low which favors the use of counter processors. Signal processing systems that produce information in the frequency domain are dependent on the transit time of the Doppler burst and are in general sensitive to the amplitude of the signal. Counter processors obtain measurements in the time domain which are free from transit time broadening errors.

The basic principles of counting methods can be seen with the aid of figure 5.4. High pass filtering is first required to remove the pedestal. If this is done properly, the zero crossings will be independent of the signal amplitude and represent the period of the Doppler difference frequency. Typically, the electronic system will average the period over a number of cycles in the burst, preferably all of the cycles that are above the threshold. However, measurements in the time-domain are independent of the burst duration so the number of cycles in the burst does not influence the measurements. With noise on the signals, averaging over the greatest number of cycles available can ameliorate the effects of any erroneous counts.

A signal is detected when the signal voltage exceeds a threshold level which is set just above the baseline noise level. When the threshold is exceeded, three counters are started on the next zero crossing. Two counters measure the Doppler period and one counts the number of cycles. On systems requiring a preset \( N \) cycles, the counters stay on until the \( N \)th cycle count is reached. If the hysteresis level is reached without exceeding the threshold, the counters will reset and get ready for the next signal. One of the counters is set for measuring \( N/2 \). At the end of the burst, a comparison is made such that data is accepted only if

\[
\left| 1 - \frac{2\tau_{N/2}}{\tau_N} \right| \leq \epsilon
\]  

(5.1)

where \( \epsilon \) is a selected error on the periodicity of the signal. This method serves to reject signals with some types of noise errors. It also throws out signals that may have had insufficient cycles and were completed on the succeeding burst.

In some systems (Macrodyne) the signal level must exceed a positive threshold, pass through zero and exceed a negative threshold in that order before the succeeding zero crossing is accepted. This three level detection scheme is very effective in discriminating signals from noise. Since the system can reset within one cycle, starts on noise at the low level initial part of a signal will not result in the loss of the entire signal as with the aforementioned periodicity check. For example, figure 5.5 shows a possible noise condition that will pass the periodicity check. The three level logic would not allow the signal to be
Figure 5.3  Filtered Signals at Different Frequencies

Figure 5.4  Counting Processor Principles
processed since it would not cross the levels in the proper sequence. Logic circuits of this type take only one zero crossing between the two discrimination level crossings and ignore others.

When the three level condition is met the Doppler period counter is initiated and stays on for N cycles. A high speed crystal stabilized clock (100 MHz to 1 GHz) is enabled and turned off at the end of N counts, figure 5.6. The gate can open at any time so the count error can be as much as +/- 1 clock count. Because high speed counters are now used, this error is generally insignificant. As an example, if the Doppler difference frequency is 20 MHz and the clock frequency is 500 MHz, for eight cycles the clock counts will be

\[
N_{\text{clock}} = \frac{(500 \times 10^6/\text{sec})(8 \text{ cycles})}{20 \times 10^6/\text{sec/cycle}}
\]

\[
N_{\text{clock}} = 200
\]

The maximum count error is 1/200 which is much less than the errors that may result due to noise. It should be apparent that the signal quality is the first concern in achieving accurate measurements.
Figure 5.5 Example of Dropped Cycle
Figure 5.6  Processor Operating on a Fixed Cycle Count
6.0 AMES' LDV SETUP AND OPERATION

In this section, some of the specific requirements involved in setting up and operating the LDV will be addressed. Clearly, each step cannot be described in detail. However, some of the less obvious conditions that may affect the quality of the data will be identified.

It is assumed that all of the components of a two-dimensional LDV system are available and the system needs to be set up and aligned. The Argon-ion laser should be turned on and given time to warm up at relatively low power (follow the laser operating instructions and the laser safety rules). Alignment should always be initiated with the laser at minimum power. Because the beam may move when the laser is turned to the operating power as a result of thermal effects in the cavity, final alignment should be made at the operating power level.

For reasons of safety, the laser beam intensity must be reduced during alignment. A convenient attenuator for this purpose consists of a half wave plate to rotate the polarization and a polarizing beamsplitter, figure 6.1, with a trap to dump the reflected beam. The half wave plate is also required for adjusting the polarization incident to the color separator.

Color Separation

A color separator is used to spread the laser lines so that they may be directed through their respective optical paths. The most effective way to accomplish this with perfect color separation is to use dispersion prisms. To obtain maximum light transmission through the prisms, the beam polarization should be parallel to the plane defined by the incident and refracted beams. If the two prism method shown in figure 6.2 is used, the prisms will also turn the beams which, with a little planning, may be used to advantage. At some distance from the prisms, the beams will have sufficient separation so mirrors can be used to direct the individual beams to the next optical components.

Beam Expander

A beam expander/collimator may be used before the color separator or a separate one may be used in each beam depending on the requirements. Remember that the Bragg cell apertures will limit the beam expansion used to that point.

Beam Splitting

Although a number of beamsplitters were identified, the beamsplitter shown in Figure 6.3 has the most desirable features. It produces two parallel beams at a fixed separation (50mm) despite small changes in its orientation with the beam. The beams will be of approximately equal intensity if the plane of polarity of the incident beam is perpendicular to the plane through the exit beams. Recall that equal beam intensities are required to achieve high signal visibility and hence, a good signal-to-noise ratio. An additional half wave plate will be required to rotate the polarization on the orthogonal velocity component.

Frequency Shifting

Frequency Shifting is required in most two-component systems, at least on the component that is orthogonal to the general flow direction. If a single Bragg cell is used,
Figure 6.1 Beam Attenuation

Figure 6.2 Beam Separation
an optical path compensator (glass of similar length as the Bragg cell crystal) should be used. The Bragg cell will deflect the frequency-shifted beam to one half the Bragg angle. Thus, a beam steering prism is needed to bring the beams back to parallel. This may be accomplished by measuring the beam separation at the steering prisms and at a distant point. The beams are adjusted to make them parallel which means the separation must be equal at the near and distant points and the beams must be in the same plane. Otherwise, the beams will not overlap completely at the sample volume.

Beam Transmitter

The four beams may be passed through the above optical components on separate paths and then recombined or they may be coaxial. The later method provides compactness whereas the former is more flexible and in some cases, easier to handle. If the coaxial approach is used, the beam of one wavelength must be directed parallel to the other at a preset separation. A displacement prism is then used to steer the beams onto the axis between the beams of the other wavelength, figure 6.4. When separate beam paths are used, the orthogonal pairs of beams are formed using a narrow rectangular mirror, Figure 6.5. This method offers greater control in steering the four beams onto a common axis.

Depending upon the focal length of the transmitter lens, the beam spacing may not be optimum. In such cases, rhomboid prism beam displacers may be used to adjust the separation. Recall that the beam intersection angle determines the fringe spacing in the sample volume and at least 8 fringes should be available for processing. Thus, a definite combination of beam diameter and intersection angle is required. Beam expansion of the four beam matrix may be required when using a long focal length transmitter lens. A beam expansion of 3, for example, also expands the separation of the beams by 3. This method of beam expansion maintains the ratio of fringe spacing to focused beam diameter. That is,

\[ \frac{w}{\delta} \approx \frac{1.22 \frac{\lambda f}{d_B}}{\frac{\lambda f}{d_B}} = 1.22 \frac{S}{d_B} \]  

(6.1)

where \( f \) is the transmitter focal length, \( s \) and \( d_B \) are the beam separation and diameter at the transmitter lens.

The four beams must all focus to a common point in the flow and be approximately coaxial. Using the steering mirrors for the appropriate beam pair, the focused beams are adjusted to visually overlap. A microscope objective or a simple lens with a short focal length is then used at the focal volume to magnify the beam overlap region. While observing the magnified beams, fine adjustments are made such that all the beams overlap at a common point. This operation must be carried out carefully if a high percentage of coincident velocity measurements are to be made.

Receiver

The receiver system consists of a large pair of lenses to collect the light, a color separator, mirrors, laser line filters, and photomultiplier tubes, figure 6.6. Since lenses are designed to image light from a distant source (collimated light) to a point which is
Figure 6.3 Beamsplitter

Figure 6.4 Coaxial Beam System using Rhomboid Prism
at the focal length of the lens, the best performance will be achieved if the lenses are used in this manner. Thus, the first lens receives light from the sample volume and produces a collimated beam. The second lens is used to focus the collimated light onto the detector apertures. As previously discussed, the quality of the receiver lenses is very important to the acquisition of reliable data. This is especially true when making boundary layer measurements where the ability to reject flare light is important. Unfortunately, the commonly used receiver lenses are not of very high quality. The size and behavior of the focused light (blur circle) will give a good indication of the lens quality.

Achromatic lenses used in the receivers have different radii of curvatures. As a general rule, the side of the lens with the greatest curvature must be on the side of the collimated beam. Another way of remembering this is to recognize that the angle between the incident rays and the lens surface should be minimized.

In order to align the receiver, a convenient means for producing scattered light is required. Usually, a piece of transparent tape on a post or a microscope slide is used. The scatterer is carefully located at the laser beam intersection. The visible beam on the scatterer is then imaged by the receiver optics. The receiver may be located at an off-axis forward or backscatter angle to minimize the sample volume size and back-ground light. Remember that the scattering intensity decreases rapidly with angle from the transmitted beams.

Because both colors are collected, a color separator is needed. If orthogonal polarizations were used as required for the beamsplitters, a polarization beamsplitter can be used to separate the signals. In the forward scatter configuration, the scattered light undergoes only insignificant rotation of the polarization. Laser line filters must be used on each of the photodetectors to prevent cross talk and to block ambient light. Mirrors with x-y adjustments are suggested for both components to steer the scattered light into the photodetector apertures. The apertures on the photodetectors should be just large enough to admit the blur circle of the focused light. In some cases, a slit may provide a better flare light isolation without truncating the extent of the fringe volume.

Detectors

With the optics aligned, the scatterer should be removed before turning on the voltage to the detectors. Photomultiplier tubes provide the best means for converting scattered light into an electrical signal and amplifying it. They have a definite high voltage range that will provide good signal-to-noise. This level usually depends on the type of PMT used, the background noise, and the scattered light levels received.

Preamplifiers are usually located in the photodetector housings to further increase the signal amplitude and drive the signal over relatively long cables. The preamplifier is kept close to the detectors in order to increase the signal level before additional noise enters the signal cables. It is always good practice to amplify the signal as much as possible with the photomultiplier tube before going to the preamplifier which inevitably produces electronic noise.

Particles produced by a nebulizer or other device should be blown through the sample
Figure 6.5  Forming Orthogonal Beam Pairs with Mirrors

Figure 6.6  Two Color Receiver
volume and the signals observed on an oscilloscope. The signals produced should be exceptionally free from noise if reliable measurements are to be made in the actual flow field. Careful observation of the signal characteristics can provide important clues to problems with the system operation. For example, the shape of the signals shown in figure 6.7 indicate if the receiver is imaging the beam crossover region. The signal visibility or amplitude modulation gives some indication of the relative size of the scattering particles to the interference fringe spacing. If the particle number density is too high, coincident signals will occur, Figure 6.7, which decreases the signal-to-noise ratio. The signals also provide information needed for properly setting the filters.

Filtering

The bandpass filters serve to remove the low frequency pedestal and the high frequency noise from the signal. As described in the previous section on processing, the pedestal must be removed completely. If the pedestal is not properly removed, the zero crossings and hence, the period of the signal, will show amplitude dependence and be in error. Proper high pass filtering will produce symmetric burst signals, figure 6.8. Since the frequency may vary over a relatively wide range, a large number of signals should be observed to ensure proper filtering over the entire spectrum. The measuring velocity probability density distribution can also be used to quickly calculate the frequency range. A real-time data display is valuable for this reason.

Low pass filtering is not as critical since its purpose is to remove the higher frequency noise. If the signal amplitude is partially attenuated, no errors will be introduced except that some signals from smaller particles may be lost. There is the danger of filtering the white noise such that the remaining noise may be processed as a signal.

Signal Processing

The processors most commonly used at Ames are produced by Macrodyne. These processors have good resolution (2.5 nanoseconds) and have superior noise rejection circuitry. A brief description of this particular counter processor will be given as an aid in understanding the manual. Some requirements in setting the system up will also be discussed.

The Macrodyne 3000 series processor is a high speed counter with a 125 MHz clock (1GHz is also available). Proper operation of the processor requires some knowledge of how it works, what the controls are and what they do, and the conditions under which the processor may produce erroneous data.

As previously mentioned, when the particle seeding rate is low the LDV signals will be intermittent bursts. Under these conditions, the counter processor is the best choice as a means for obtaining the Doppler difference frequency. If the seeding is high, the signals will be almost continuous with random amplitude and phase fluctuations. The counter will still function but cannot be expected to produce accurate results to better than ± 1/2 cycle.

The counter processor determines the Doppler difference frequency in the time domain. Timing over a selected number of cycles in the burst signal is accomplished with a crystal
Figure 6.7 Unfiltered Signal Shapes
HIGH QUALITY

HIGH PASS SET TOO LOW, PEDESTAL LEAKAGE

EXCESSIVE D.C.

SATURATED

LOW SIGNAL-TO-NOISE RATIO

MULTIPLE PARTICLES

Figure 6.8 Filtered Shapes
controlled 'clock' oscillator. The frequency of the oscillator must be much higher than
the Doppler difference frequency to achieve good resolution. The clock count is initiated
when the high pass filters signal exceeds a set threshold level. A preselected number
of cycles are processed and used to stop the counter. Greater accuracy may be obtained
by processing more cycles. Some processors, (e.g., Aerometrics) count all of the cycles in
the burst and average the result over the number of cycles counted. This capability is
especially important if the seed particles are polydisperse and produce a range of signal
amplitudes. In such cases, there is the possibility of completing the preset number of cycles
on the early part of the signal which has a lower signal-to-noise and slew rate. There is
also the possibility of reading signals from larger particles more than once.

Although the importance of attaining good quality signals cannot be overemphasized,
in many practical cases the signals will be noisy. Therefore, each burst must pass certain
criteria before it is accepted. The so-called periodicity check is used to reject signals with
insufficient cycles, signals produced by two or more particles in the sample volume at a
time, and noise. Two counters are used to make period measurements over a different
number of cycles, usually 4 and 8. The ratio of the two measurements are then compared
to one-half and if they agree to a specified percentage, the signal is validated. The problem
with this technique is that the entire signal may be rejected because of a noise cycle early
in the burst. However, if the method is coupled with the three-level validation circuitry
used in the Macrodyne and some other machines, it can be effective in rejecting spurious
noise.

The three-level validation requires that a signal, figure 6.9, must exceed a a positive
threshold level, pass through zero, and exceed a negative threshold level before the next
zero crossing is counted as a cycle. Such logic has proven to be very effective in rejecting
noise. If the conditions are not met on any cycle, the counter resets and starts again
almost immediately. Thus, noisy cycles on the start of a burst need not cause the loss of
an otherwise good signal.

With these general comments out of the way, the characteristics of the Macrodyne
3000 series processor can be treated explicitly. The manual should be used when setting
up the system. However, the explanations given in the manual are lacking in clarity so the
next few pages will be an attempt to add some details to the descriptions.

The signals presented to the processor must have a sufficient number of cycles (4,
8, 16) that cross the threshold after being high pass filtered. Observation of the signals
using an oscilloscope can be used to determine their relative levels. A storage scope may
be useful for lower frequency signals. If the signals are too low, the best way to amplify
them is to increase the PMT voltage. A preamplifier should also be installed at the PMT.
Since electronic noise will be picked up on the transmission lines, the signal level should
be increased as early as possible.

Assuming that the signals are between ≈ 50 and 500mV, the PMT signal can be
connected to the BNC marked 'Photo Detector'. The amplifier switch on the FED module
should be at X1, figure 6.10. Amplification after passing the signal through long lines
also amplifies the noise. The signal may be monitored using the BNC marked 'Amplified

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Figure 6.9 Three Level Detection Scheme
Input'. Changing the FED Amplifier from X1 to X10 will demonstrate that the signal and noise are amplified together.

Filter Setting

While monitoring the 'Filtered Output' on an oscilloscope, the high pass (HP) and low pass (LP) filters can be set. The filter settings for 3dB point values (where the signal is attenuated to one-half its value) are 0.5, 2, 4, 8, 16, 32, and 64 MHz. The 'OFF' position provides no filtering and 'Auto' uses diodes to clip the burst later in the circuit.

Before setting the filters, the approximate frequency of the signals can be estimated from the oscilloscope traces. The approximate number of cycles in the burst or the duration of the burst (pedestal frequency) should also be estimated. As a rough estimate, the Gaussian pedestal can be regarded as one-half of a sine wave cycle. This frequency must be attenuated by the high pass filter. Because of the velocity fluctuations, this frequency will vary proportionally. Since the high pass filter is set to attenuate all frequencies lower than the set frequency, the setting should be made based on the highest frequency bursts. If frequency shifting is used, the 'Doppler' frequency will be much higher than the pedestal frequency so the setting will not be as critical. When the HP filter is set too high, the Doppler frequency of the slow moving particles will be attenuated. This can bias the velocity to higher frequencies.

The low pass filter serves to attenuate higher frequency noise. Although tempting, it is not wise to set the LP filter too close to the Doppler frequency because the 'white' noise will then be filtered to look like the Doppler frequency. This makes it more difficult for the front end logic to reject noise.

Threshold Selection

Perhaps the most frequently adjusted control is the threshold selector. Unfortunately, no definitive criteria are available to determine how the setting should be made. With good signals, the setting is relatively easy. However, when the seed particles have a broad size range as is often the case, most samples may be accepted from the large particles if the threshold is too high. If set too low, noise may pass as signals. Often, the shape of the velocity probability distribution or turbulence intensity are presupposed and the threshold is set accordingly. This is at best risky.

As a suggestion, the threshold setting should be calibrated to determine the corresponding threshold voltage level. Observe the background noise level and set the threshold such that only occasional noise spikes cross the level. One laser beam may be blocked to see if the noise on the burst is being processed as a signal. A few samples per second passing the rejection logic is acceptable if the data rate is several hundred per second.

The 'Schmitt Trigger' BNC may be used to monitor when the signal is passing the multilevel threshold logic. A Schmitt trigger is used to produce square waves initiated by the filtered signal zero crossings. It produces square waves only when the multilevel logic is validating the cycles.

Overload Setting
The overload sets the level of a signal peak detector. Signals that have amplitudes that are greater than the set level are rejected. This prevents the measurement of velocity from large particles that may not follow the gas phase fluctuations. Also, if the signal gain is too high, the amplifiers may saturate which produces a distortion on the signal. The overload serves to reject these signals.

Comparator Accuracy

This control selects the allowable difference between the short count and the long cycle count (2/4, 4/8, 8/16) for signal validation on the periodicity check. The rejection based on periodicity best serves to discard data obtained by starting on one burst and ending on another or other gross errors. Thus, the comparator accuracy can be set fairly high. Setting 11 turns off the periodicity validation.

Range

The range setting essentially divides the number of clock counts in the Doppler burst to fit the 10 bit (1023) digital output. In order to determine the range, an estimate of the Doppler frequency \(f_D\) can be used with the following

\[
N = \frac{f_c \times N_{\text{cycles}}}{f_D \times 2^{\text{exp}}} \quad (6.2)
\]

where \(f_c = 500\) MHz (the 125 MHz clock is divided into 4 phases). \(N\) cycles is the number of cycles selected for processing (2, 4, 8, or 16), \(f_D\) is the signal frequency (Doppler plus shift) and EXP is the range selection. \(N\) must be less than 1024 or an overflow will occur. For example if the signal frequency is \(\approx 20\) MHz, then

\[
N = \frac{(500 \times 10^6) \times 8 \text{ cycles}}{20 \times 10^6 \times 2^0}
\]


so a range of 0 is okay. It would also work for 16 cycles selected giving 400 counts for better resolution. This value is presented at the digital I/O as a 10 bit mantissa plus exponent, namely

\[
D_1, D_2 \ldots D_{10} \times 2^{\text{exp}} \quad (6.3)
\]

In the above example, the output would be

\[
001010000000 \quad 0000
\]

Mantissa EXP

Since the clock counts are produced at 500 MHz, each clock count is 2.5 nanoseconds. Thus,

\[
t_d = \frac{N \times 2.5}{N_{\text{cycles}}} \quad \text{nanoseconds} \quad (6.4)
\]
is the Doppler period. When measuring 8 cycles, the Doppler period is

\[ t_d = D_q D_B \ldots D_0 \times 2^{+\text{exp}} \times \left( \frac{2.5}{8} \right) \text{ns} \]  

(6.5)

Note that the clock counts were divided by $2^{\text{exp}}$ which means that the time of each count is $2.5\text{ns} \times 2^{\text{exp}}$. Eight cycles were used so the average period for eight cycles is the time divide by $2^3$. The manual apparently has an error in the evaluation.
7.0 SUMMARY

It has been the intention of this primer to provide a starting point for potential users of the dual beam LDV, particularly those systems in use at the NASA Ames Research Center. Although much can be gained by reading about the complexities of the LDV, there is no substitute for experience. The pitfalls that have been pointed out have all been fallen into by others. They are, by no means, a complete list of the problems that can be encountered in LDV applications just as this primer is not a complete handbook on LDV. Instead, it is meant as a collection of observations that have been recorded by researchers who, themselves, have had to learn by trial. Perhaps the most important lesson to be learned in maintaining an advancing technology is to build on the learned experiences of others while, at the same time, adding to that bulk of knowledge. As a closing suggestion, it is strongly recommended that all users of LDV equipment within the Branch maintain a logbook detailing their experience with the equipment and describing the application. These notes would remain with the equipment and not be meant so much as a technical report but as a guidebook to subsequent users.
BIBLIOGRAPHY


# LASER DOPPLER VELOCIMETRY PRIMER

## Abstract
Advanced research in experimental fluid dynamics requires a familiarity with sophisticated measurement techniques. In some cases, the development and application of new techniques is required for difficult measurements. Optical methods and in particular, the laser Doppler velocimeter (LDV) are now recognized as the most reliable means for performing measurements in complex turbulent flows. And such, the experimental fluid dynamicist should be familiar with the principles of operation of the method and the details associated with its application. Thus, the goals of this primer are to efficiently transmit the basic concepts of the LDV method to potential users and to provide references that describe the specific areas in greater detail.

**END DATE**

Jun. 7, 1988