Relativistic Jet Models for the BL Lacertae Object Mrk 421 during Three Epochs of Observation

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Abstract

Coordinated observations of the nearby BL Lacertae object Mrk 421 obtained during May 1980, January 1984, and March 1984 are described. These observations give a time-frozen picture of the continuous spectrum of Mrk 421 at X-ray, ultraviolet, optical, and radio wavelengths. The observed spectra have been fitted to an inhomogeneous relativistic jet model like the one described in Hutter and Mufson (1986). In general, the models reproduce the data well. Many of the observed differences during the three epochs can be attributed to variations in the opening angle of the jet and in the angle that the jet makes to the line of sight. The jet models obtained here are compared with the homogeneous, spherically symmetric, synchrotron self-Compton models for this source computed by Makino et al. (1987). Our models are also compared with the relativistic jet models obtained for other active galactic nuclei.
I. Introduction

In a previous paper (Mufson et al. 1984, Paper I), we presented time-frozen spectra of the BL Lacertae objects Mrk 180 and Mrk 501 which were obtained during a campaign of coordinated, multifrequency observations from the radio to the X-ray. In Paper I we fitted these time-frozen spectra with a variety of simple models, including the synchrotron-self Compton model (SSC) and a relativistic jet model (Hutter and Mufson 1986, Paper II). The conclusion we reached in Paper I was that the relativistic jet model provided the most reasonable description of the data sets.

The nearby (z = .031) BL Lacertae object in Mrk 421 (B2 1101 + 38) is identified by its nonthermal power law continuum in the radio (Colla et al. 1975; Owen et al. 1978), optical (Maza, Martin, and Angel 1978; Mufson et al. 1980), and X-ray (Cooke et al. 1978); the absence of emission lines in its optical spectrum (Ulrich et al. 1975); and the strong variability observed at all wavelengths (Miller 1975; Ricketts, Cooke, and Pounds 1976; Purton, Kojoian, and Dickinson 1977). The optical polarization (Maza, Martin, and Angel 1978) and flat radio spectrum (O'Dell et al. 1978) observed in this source are also common characteristics of BL Lacertae objects. In addition, Mrk 421 has an X-ray spectrum with a variable, high-energy (> 10 keV) component (Mushotzky et al. 1979).

Photometric and spectroscopic observations indicate that there is an extended optical envelope surrounding Mrk 421 which has the properties
of a giant elliptical galaxy (Ulrich et al. 1975; Miller, French, and Hawley 1978; Mufson et al. 1980; Hickson et al. 1982)). The picture of Mrk 421 which emerges is that of a typical BL Lacertae object in which an active galactic nucleus is embedded at the core of a giant elliptical galaxy.

In a recent paper (Makino et al. 1987), coordinated, multifrequency observations of Mrk 421 during January 1984 and March 1984 were presented and analyzed in terms of the SSC model. In this paper, we add a third set of coordinated, multifrequency observations of Mrk 421 from May 1980. We then discuss all three sets of data in terms of the relativistic jet model, with the goal of determining how the jet parameters vary with time. In addition, we compare our results to the SSC model fits of Makino et al. (1987), looking in particular for differences in these two approaches. Finally, we compare the characteristics of our jet models for Mrk 421, Mrk 180, and Mrk 501 with jet models computed by others.

II. Observational Data

The data sets used in the fits are from 3 epochs - May 1980, January 1984, and March 1984. The two data sets for the 1984 observations are reported and discussed in Makino et al. (1987). The 1980 data set reported here was analyzed in the same manner as the data for Mrk 180 and Mrk 501 reported in Paper I. The reader is referred to those papers for details.
a) Analysis of the 1980 Observations

The 1980 data set used in the model fits is given in Table 1. A brief discussion of the observational results is given below.

1) X-Ray Regime

The X-ray data for 1980 were obtained by the IPC and MPC aboard the HEAO-2 satellite (Giacconi et al. 1979). Since the IPC field in the Mrk 421 observations has a strong, bright X-ray source within 1 arc min of the optical position of the active galactic nucleus, we associate the X-ray source with the BL Lacertae object. The absence of other strong X-ray sources in the IPC field argues that the MPC flux we observed also comes from Mrk 421.

In Table 1 we give the details of the HEAO-2 observations. The IPC X-ray data reported here are the reprocessed PHASE 2 data. The power law spectral fits to the IPC data are given in the IPC data output. In Figure 1 we show contours of \( \chi^2 / \text{dof} \) in the \((\alpha, N_H)\) plane for these fits. The errors quoted in Table 1 are our 1 \( \sigma \) estimates from this figure. Although \( (\chi^2 / \text{dof})_{\text{min}} = 3.9 \) suggests that the fit is marginal, the fitting algorithm classifies it as "good", its highest quality indicator. This suggests that systematic errors contribute to \( \chi^2 \). We consequently regard the HEAO-2 power law fit as acceptable. At 4.5 keV, the IPC fit gives a flux density of \( \approx 3 \mu\text{Jy} \) which is greater than the TENMA X-ray flux density observed in March 1984 and January 1984 (Makino et al. 1987). We note that Heiles (1975) measures \( N_H < 1.7 \times 10^{20} \text{ cm}^{-2} \) at 21 cm, in good agreement with these IPC results.
The MPC data have also been fit with a power law spectrum. These results are also given in Table 1. In these fits the column density of hydrogen is unreliable since it is difficult to determine soft X-ray absorption with the MPC. It appears from the MPC data that the spectrum is steepening at higher energies. We regard the MPC spectral fits to be of lower overall quality than the IPC fits.

For the purposes of our fitting algorithm as described in Paper II, we have broken up the TENMA spectral observations into two power laws - a low energy (1.5-4.5 keV) component and a high energy (>4.5 keV) component. For the March 1984 observations, the X-ray spectrum appears to be steepening and the power laws were taken to be: log S (Jy) = -5.79 - 1.7[log ν(Hz) - 17.68] at low energies and log S (Jy) = -6.15 - 4.9[log ν(Hz) - 17.99] at high energies. The errors in the slopes are approximately ±0.3 and ±0.5 for the low energy and high energy power laws, respectively; the errors in the normalizations are approximately ±0.02 and ±0.15 for the low energy and high energy power laws, respectively. For January 1984, the observations were more difficult to analyze. The low energy power law was taken to be of the form log S (Jy) = -5.47 - 1.6[log ν(Hz) - 17.68] at low energies; the errors are ±0.02 and ±0.2 in the normalization and spectral index, respectively. The high energy data, however, are much more difficult to characterize by a single power law. We therefore bracketed the high energy data with two power laws: a "flat" power law of the form log S (Jy) = -6.11 - .14[log ν(Hz) - 17.99] and a "steep" power law of the form log S (Jy) = -6.11 - 2[log ν(Hz) - 17.99].
In Figures 2, 3, and 4, the X-ray data for the 3 epochs are plotted. In Figure 3, the low energy and high energy X-ray power laws are represented by double solid lines. The "flat" and "steep" high energy X-ray power laws bracket the high energy X-ray emission.

ii) Ultraviolet Regime

The ultraviolet observations were obtained using the IUE satellite (Boggess et al. 1978). Single spectral observations of Mrk 421 were made in the low-dispersion mode (effective resolution approximately 10 Å) with both the SWP (1150-1950 Å bandpass) and LWR (1900-3200 Å) cameras. The image numbers of the observations are given in Table 1. Spectral images were assembled from geometrically and photometrically rectified data samples in the line-by-line files of the IUE guest observer tapes. The long exposure, faint images of Mrk 421 were corrected for radiation events and fixed pattern noise as discussed in Paper I. The power law fit to the IUE data is given in Table 1. The fit listed in Table 1 is a composite of the SWP and LWR data. The spectral index in May 1980 appears to be flatter than it was in 1984. The $\chi^2$/dof for this fit, however, is not as good as it is for the two 1984 IUE data sets. The integrated flux (1200 Å - 3000 Å) for May 1980 is $5.8 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, which is similar to the value for January 1984 and greater than that for March 1984 (Makino et al. 1987).

iii) Optical Regime

The ground based optical photometry consists of broadband BVRI
measurements made using the 1.5-m telescope of the University of Arizona Mt. Lemmon Observatory. The data were acquired using dc techniques. Extinction and transformation coefficients were determined from observations of primary UBVRI standards (Johnson et al. 1966). The V band flux density of $1.7 \times 10^{-2}$ Jy is significantly greater than any of the V band measurements made in January or March 1984 (Makino et al. 1987).

iv) Radio Regime

The radio data were obtained with the University of Michigan computer controlled 26-m radio dish. The flux densities given are means of the observations made that month weighted by their photometric errors. The errors quoted are 1σ. The data do show evidence for variability at the 15-25% level. However, the shape of the spectrum always remains quite flat. Consequently, in our model fits we used the flux at the 8.0 GHz and flux densities at 4.8 GHz and 14.5 GHz extrapolated from the average spectral index observed between 1980-1982, $\alpha \approx 0$. The 14.5 GHz flux density we used is consistent with the 14.5 GHz observed in 1984 (Makino et al. 1987).

v) Composite Spectra

In Figures 2, 3, and 4 we have plotted the data for the 3 epochs. We note that although the flux in the different bandpasses has varied between May 1980 and January/March 1984, only the changes at optical and X-ray wavelengths show any evidence for correlated behavior.
Examination of figures 2, 3, and 4 show that both the optical and X-ray have declined over the three epochs of observation.

III. Relativistic Jet Models for Mrk 421

We have used a relativistic jet model, similar to that of Blandford and Konigl (1979) and Konigl (1981), to model the simultaneous radio, optical, ultraviolet, and X-ray observations of Mrk 421. Relativistic jet models like these can explain many properties of AGNs (Paper II). There is a great deal of observational evidence suggesting that jet-like features are common in AGNs (Konigl 1986).

Details of the model and the fitting procedure can be found in Paper II.

a) The Model

In our model the relativistic motion takes the form of conical jet of opening half-angle $\phi$ which is viewed at an angle $\theta$ to its axis. It is assumed that the jet is free with no external pressure to confine the expansion in the direction perpendicular to the axis. For a free jet whose material is expanding with bulk velocity $\beta_j = v_j/c$, the opening half-angle is given by $\phi \approx 1/\gamma_j$, where $\gamma_j = (1-\beta_j^2)^{-1/2}$ is the Lorentz factor of the jet plasma.

In the comoving frame of the expanding jet gas, the emission is assumed to be locally described by the homogeneous synchrotron self-Compton mechanism. It is further assumed that the population of
relativistic electrons is continuously reacclerated so that the
distribution of electrons with respect to energy, \( n_e(\gamma_e) d\gamma_e = K_{\gamma_e}^{-\left(2\alpha e + 1\right)} d\gamma_e \), is independent of time. Continuous reacceleration is
required since the lifetimes of the synchrotron electrons radiating at
high energies are so short compared with the lifetime of the object.
The well-tangled magnetic field strength \( B \) and the particle number
density \( K \) along the jet are of the form

\[
B = B_1 (r/1 \text{ pc})^{-m},
\]

and

\[
K = K_1 (r/1 \text{ pc})^{-n}.
\]

where \( r \) is the radial coordinate measured from the jet apex. As viewed
by an observer on earth, the radiation is beamed in the forward
direction by a factor \( D_j = \gamma_j^{-1} (1 - \beta_j \cos \theta)^{-1} \), where \( \gamma_j \) and \( \beta_j \) measure the
bulk velocity of the jet gas. By beaming its radiation in the forward
direction, these models avoid the problem of huge quasar luminosities
from extremely small volumes. As discussed in Konigl (1981) and Paper
II, each volume element in the jet contributes a local component of
nonthermal emission to the composite jet spectrum. The local jet
spectrum can be characterized by several distinct emission regions.
Below a frequency \( \nu_{\text{sm}} \), the source is self-absorbed and its flux density
is proportional to \( \nu^{-2.5} \). Below \( \nu_{\text{sm}} \), very little flux is contributed by
any element of the jet to the total BL Lacertae continuum. The
optically thin spectrum above \( \nu_{\text{sm}} \) has a spectral index \( \alpha_e \) up to a
frequency $v_{sb}$, where the spectral index abruptly increases to $\alpha_e + 0.5$ due to synchrotron radiation losses. The highest energy electrons within the jet have a radius-independent Lorentz factor of $\gamma_{eu}$; above the frequency at which these electrons radiate, the spectrum drops off exponentially. Thus $\gamma_{eu}$ gives the frequency of the upper limit to the synchrotron spectrum at a particular position. We assume that the time evolution of $\gamma_{eu}$ is slow compared with the reacceleration time; under these circumstances the local synchrotron spectrum remains stationary. The lower limit to the local synchrotron emission at a particular position is given by $v_{min}$.

The superposition of many different regions of synchrotron emission in the jet produces a composite nonthermal spectrum which is flat below $v \leq v_{SM}$, but which has a constantly varying spectral index for $v > v_{SM}$. In the observable radio and optical-ultraviolet wavelength regions, the spectrum can be approximated by two power laws:

$$S_{obs}(v) = \begin{cases} S(v_{SM})(v/v_{SM})^{-\alpha_{s1}} & v \leq v_{SM} \\ S(v_{SM})(v/v_{SM})^{-\alpha_{s2}} & v_{0} \leq v \leq v_{uv} \end{cases}$$

(3a)

(3b)

where $\alpha_{s1}$ is the spectral index which applies in the radio region and $\alpha_{s2}$ is the spectral index in the ultraviolet-optical region. The frequency at which the composite synchrotron spectrum begins to steepen from $\alpha_{s1}$ is given by $v_{SM}$; the flux density at this frequency is $S(v_{SM})$.

The jet itself can be characterized by an inner radius $r_M$ and an outer
radius \( r_u \). As discussed in Paper II, the inner boundary, \( r_M \), is
determined from the frequency \( v_{SM} \) at which the spectral index begins to
steepen from \( \alpha_{s1} \), the spectral index in the radio, to \( \alpha_{s2} \), the spectral
index in the optical-ultraviolet. Similarly, \( r_u \) can be determined from
the intensity of synchrotron emission at its low frequency cutoff
\( v_{\text{min } s} \); at this radius the emission is dominated by electrons radiating
at \( v_{\text{min } s} \).

Finally, the ratio of the strength of the optical nonthermal emission
from the BL Lacertae object to the strength of the stellar optical
emission from the giant elliptical galaxy is characterized by \( R_V \), the
ratio of these strengths in the V band. For sources with large values
of \( R_V \), the continuum is dominated by quasar emission; for sources with
small values of \( R_V \), the continuum has a continuum dominated by galaxy
emission.

b) Results of the Model Calculations

The best-fitting models of the spectra of Mrk 421 during the three
epochs of observation are shown in Figures 2, 3, and 4 superposed onto
the data from the coordinated observations. The models were computed by
varying the 9 parameters \( \theta, \phi, v_{SM}, S(v_{SM}), \alpha_e, v_{\text{min } s}, \alpha_{s1}, \alpha_{s2}, \) and
\( R_V \). The parameter \( \alpha_{s1} \) was held fixed at the observed value of the radio
spectral index. In our fits we began by setting \( \alpha_{s2} \) equal to the
observed uv spectral index since the IUE spectra are presumably pure
synchrotron emission uncontaminated by stellar emission from the galaxy.
However, since the fits are relatively sensitive to the exact value of
we allowed this parameter to vary within $3\sigma$ of its observed value. A $\chi^2$-statistic was then minimized to find the best fitting models. This $\chi^2$-statistic was based upon the observed values of the radio flux at 4.8, 8.0, and 14.5 GHz; the U, B, V, and R optical fluxes; and the normalization constants ($S_0$) and spectral indices ($\alpha$) at UV, soft X-ray, and hard X-ray wavelengths. The number of degrees of freedom (dof) in these fits is thus $13-9 = 4$. Since we have no U measurement for May 1980, dof = 3 for this model. In these models the galaxy colors were fixed at the K-corrected colors of a normal giant elliptical galaxy (Hutter and Mufson 1981), although Hickson et al. (1982) suggest that the host galaxy of Mrk 421 may be redder than normal.

In Table 2 the best-fit model parameters are given. In this table we have given two fits for the January 1984 data. These two fits are computed using the "flat" and "steep" power law representations of the TENMA high energy X-ray spectrum, as discussed above. The $\chi^2$/dof for the January/March 1984 data sets show that these fits are acceptable. The $\chi^2$/dof for May 1980 suggests that the model fit is only marginally adequate.

These results can be most easily understood from the graphical displays of the models. In Figure 2, the superposition of the computed jet model onto the observations suggests that the model is adequate given the relatively low overall quality of the data. The poorer $\chi^2$/dof, compared with the models for the other data sets, is easily understood from an inspection of the figure. In particular, the inconsistency of the IPC and MPC data present constraints on the fitting
procedure which are impossible to overcome. This figure shows, however,
that the spectrum is adequately represented by synchrotron emission from
the radio to the X-ray. In Figure 3, both computed models are shown
superposed onto the data. This figure demonstrates that both models are
similar in the radio, optical, and ultraviolet bandpasses and reproduce
the data quite well. In the X-ray, the emission from the models has
been broken into a synchrotron component (dash-dot curve), which is
similar for both models, and two self-Compton components (dashed
curves), which represent fits to the "flat" and "steep" high energy
power laws. It is clear that a self-Compton component is necessary to
explain the high energy emission in these models. We recognize that the
observed high energy tail could simply be an additional component of
synchrotron emission. Nevertheless it is encouraging that the variable,
hard spectral component (Mushotzky et al. 1979) can be explained by a
single jet structure whose characteristics change with time. Neither of
these models adequately account for the infrared data. But the fit
procedure we used does not include infrared data in the $\chi^2$ statistic.
The model for March 1984 in Figure 4 shows how a suitable modification
of the fit parameters could account for the infrared emission. If the
composite spectrum were more steeply curving in the infrared and $\nu_{SM}$
were at a lower frequency, then the infrared emission would simply be
galaxy emission. In Figure 4, the superposition of the model onto the
data shows that the model reproduces the data well in all bandpasses as
is clear from the value of $\chi^2$/dof.
IV. Discussion

a) Physical Parameters of Model Jets

In Table 3, the physical parameters derived from the fits are given. The first two parameters characterize the degree of relativistic motion and Doppler beaming present in the models. The jet in Mrk 421 is clearly relativistic at all times. Due to small compressions/expansions in the jet opening angle $(\phi)$, however, the Doppler beaming factor varies. The Doppler beaming factor is an important characteristic of jet models because it explains the brightness of quasars without resorting to huge luminosities. Since the emission from a jet is restricted to small solid angles, the jet emits only a small fraction of the luminosity of a source radiating over $4\pi$. Consequently, the variations in $D$ and the wobble in the jet with respect to our line of sight $(\theta)$ easily account for the variability we observe in the source flux from Mrk 421.

The four parameters $m$, $B_\perp$, $n$, and $K_\perp$ describe the magnetic field and the particle density as a function of radius in the jet (eq[1] and eq[2]). Table 3 shows that the run of magnetic field and relativistic particle density is similar for all fits. As discussed in Paper II, an isotropic $(B_\perp \approx B_\parallel)$ magnetic field should scale as $B \propto r^{-1}$. Table 3 shows that the values of $m$ are consistent with this scaling law. In addition, for a freely expanding conical jet, conservation of particle number in a free jet requires that $n = 2$. As can be seen in Table 3, conservation of particle number apparently holds in Mrk 421 during all observations. Finally, Konigl (1981) demonstrates that the special case $m = 1$ and $n = 2$ corresponds to equipartition between magnetic field and
particle energy. Table 3 shows that the jet in Mrk 421 is consistent with equipartition at all epochs.

Table 3 further shows that the inner jet radius, \( r_M \), is extremely small, close to the Schwarzschild radius for a \( 10^9 M_\odot \) black hole. The implication is that the jet penetrates close to the most probable AGN engine. In this table we have not given a value for \( r_u \), the outer radius of the jet. For Mrk 421, the value of \( r_u \) cannot be consistently determined since the low frequency synchrotron flux is dominated by emission from an extended, steep-spectrum halo component rather than emission from a jet (Weiler and Johnston 1980). The parameter \( r(v_{sb} = v_{su}) \) indicates the radius at which radiation losses become important. It is the point at which the local synchrotron spectral index \( \alpha_e \) changes to \( \alpha_e + 0.5 \). The frequency at which \( v_{sb} = v_{su} \) indicates the frequency radiation losses become important. For all the models this occurs in the ultraviolet. The final parameter given is \( v_{su}(r_M) \) which is the frequency of the termination of the synchrotron spectrum. Any emission above this frequency arises from Compton emission or another component of synchrotron emission. For January 1984 and March 1984, this occurs in the X-ray and would have been in principle observable.

As discussed, one advantage of an inhomogeneous relativistic jet model is that it can naturally account for the variability observed in Mrk 421 during the 3 epochs of observation. In detail, the comparison of Figures 2, 3, and 4 shows that there was little variation in the radio flux during the 3 epochs, but a decline in the optical and soft X-ray
flux from May 1980 through January 1984 and March 1984. Our jet models reproduce this variability by changing the slope of the synchrotron spectrum and varying the overall level of the Compton emission. In the models, the weakest high frequency synchrotron spectrum occurs in January 1984. This occurs because the jet at this epoch has its lowest flow speed $\gamma_j$ and highest density. Consequently, synchrotron radiation losses steepen the local spectral index to $\alpha \approx \alpha_e + 0.5$ over a significant fraction of the jet's length [small value of $r(v_{sb} = v_{su})$]. In addition, the upper cutoff in the electron energy distribution ($\gamma_{eu}$) is lowered at low flow speeds. Both effects combine to lower the total synchrotron emission from the jet at high frequencies. Interestingly, although the low flow speed leads to a lower synchrotron flux at X-ray energies, the increased particle density results in a higher level of Compton emission at this epoch (Compton emission $\alpha \frac{n_e^2 D^{-2}}{\gamma_j}$; Paper II).

The added contribution of the Compton spectral component actually results in a slightly higher overall level of X-ray flux in January 1984 over that two months later. Since the flux of the Compton emission is $\alpha D_{\gamma_j}^{-2}$, the large variations seen in the X-ray flux (particularly in the hard X-ray component) are not surprising when viewed in the context of the jet models.

In the radio, the lack of significant variability is to be expected since the emission is from extended regions of the jet plasma. For the January and March 1984 models, where the radio spectra particularly flat, the outer radius of the emitting region of the jet ($r_u$) is very large (Table 3). In these models a flat radio spectrum ($\alpha_{s1} \approx 0$)
results from the combined emission from many optically thin regions of spectral index $\alpha_e \approx 0.5$, all of which differ in their low frequency turnover $\nu_{\text{sm}}$. At frequencies below 1 GHz, the spectrum again begins to rise due to the extended halo of emission (Weiler and Johnston 1980).

Variations in the optical region are the result of fluctuations in the flux of synchrotron emission beamed toward the observer. That is, changes in the viewing angle and opening angle of the jet add a greater or lesser component of nonthermal emission to the constant component of galaxy emission. These in turn lead to variations in $R_\nu$ (Table 2) or variations in the color and intensity of the optical emission. The very blue colors of Mrk 421 (Hutter and Mufson 1981) are the consequences of large values of $R_\nu$.

b) Comparison with SSC Models

In Makino et al. (1987) homogeneous and spherical SSC models were fit to the data from January 1987 and March 1987. In many ways these SSC models are quite complementary to the relativistic jet models constructed here. As in jet models, SSC models demand a population of relativistic electrons with a power law energy distribution which emit a local power law spectrum of synchrotron radiation. In both the jet models and the SSC models for Mrk 421, these electrons must be continuously reaccelerated in situ to prevent synchrotron losses from removing the highest energy electrons from the assumed time-independent distribution of relativistic particles. In both models, the low frequency radio flux arises from an extended component; the optical flux
is due to a combination of beamed quasar emission and galaxy emission; and the high energy X-ray tail seen in January 1984 is the result of self-Compton emission.

There are differences in the two approaches, however. In particular, the relativistic jet models described here fit a set of model parameters to the data; the SSC models only offer a range of model parameters based upon specific choices for the source size, the frequency of the synchrotron self-absorption turnover, and the flux density at the turnover. Often the model parameters in SSC models are relatively uncertain since they depend on the input parameters to very high powers. Further, relativistic jet models explain flux variations in a more natural way. Slight wobbles in the jet viewing angle and compressions/expansions in the jet opening angle would be expected in an inhomogeneous jet. Variations in source size, the explanation preferred in SSC models, seems somewhat more unphysical. But there is a price to pay for using relativistic jet models. Jet models have a great number of parameters (9 in our case) which must be fit to data which are often too poorly known to pin down the parameters adequately.

In detail it is difficult to compare many of the parameters of the SSC models given by Makino et al. (1987) and the jet models described here. In jet models many of the source parameters vary with position in the jet and in SSC models many of the source parameters are uniform throughout the radiating volume. However, a few parameters can be compared directly. The Doppler factor, $D_j$, which characterizes the degree of relativistic bulk motion in the source, is much greater in the
jet models than the value for the SSC models. The cooling times for the electrons emitting the bulk of the emission in the optical-uv is in the range of hours to days for both models.

c) Comparison with Jet Models of Mrk 180 and Mrk 501

In Paper I, relativistic jet models were fitted to time-frozen spectra of the nearby BL Lacertae objects Mrk 180 and Mrk 501. In Paper II these two jet models, along with a preliminary model of Mrk 421, were discussed and compared. The jet models for Mrk 421 discussed here, however, were constructed in a slightly different manner than those for the other two sources. In this paper, 9 jet parameters were varied in the fitting procedure, whereas in Paper I only 5 parameters \([v_{\text{SM}}, S(v_{\text{SM}}), v_{\text{min}}, s', \gamma_{\text{e}}, R_{\text{V}}]\) were varied in the fit. Nevertheless, many of the individual comparisons between the sources discussed in Paper II are still valid here.

Both Mrk 180 and Mrk 421 have values of \(D_{\text{J}}\), the Doppler beaming factor, which indicate that relativistic bulk motion is present in their jets. Mrk 421 is clearly the most relativistic; its jet is most closely aligned to our line of sight and its opening angle is smallest. This accounts for its large value of \(R_{\text{V}}\) and its very blue color (Hutter and Mufson 1981). The jet in Mrk 501, however, is pointed sufficiently far from our line of sight that there is no need to invoke relativistic beaming for this source; the colors of this source are most like a normal giant elliptical galaxy (Paper I). The structure of the magnetic fields in Mrk 421 and Mrk 501 are consistent with a locally isotropic
field \((B_\perp \approx B_\parallel)\) over the length of the jet. In fact this assumption was implicitly made in the synchrotron and Compton radiation formulae we used in our models. Mrk 180, however, does not appear to follow this scaling law. As discussed, the run of relativistic particle number with radius is different in Mrk 421 than it is in Mrk 180 and 501. In Mrk 421, the most relativistic of the jets, the particle number is apparently conserved \((n \approx 2)\); in Mrk 180 and Mrk 501, however, particle injection is apparently occurring \((n < 2)\). This is difficult to understand physically and is most likely a statement about the imprecision with which we compute these parameters.

As discussed in Paper II, a comparison of the energy density in the relativistic electrons and the energy density in the magnetic field at several radii shows that the magnetic energy density dominates the flow at small jet radii, while at \(r \gtrsim 1 \text{ pc}\) the relativistic particle energy dominates the flow. This behavior is consistent with models for the formation of relativistic jets (Blandford and Payne 1982; Rees et al. 1982; Phinney 1983) where the rotational energy of a black hole is extracted and transferred to the relativistic particles by the magnetic field. Near the black hole engine, where the transfer is taking place, the magnetic field would carry the most significant fraction of the energy. Farther out, the energy has been transferred to the particles and they would carry the bulk of the energy. Since this type of hydromagnetic jet acceleration is thought to occur when the accretion rate onto the central black hole is low (Blandford, Begelman, and Rees 1984), it is most likely to be operating in low power AGN's like Mrk
It is interesting to note that in the SSC models of Makino et al. (1987) the magnetic energy density dominates the relativistic particle energy.

d) Further Comparison of Relativistic Jet Models

Urry and Mushotzky (1982) and Worrall et al. (1984a,b,c; 1986) have constructed relativistic jet models for many sources. In all these models, the authors conclude that relativistic jets provide a satisfactory fit to the data. Values of magnetic field strength, particle densities, and Doppler factors cover ranges similar to those reported in Papers I and II.

V. Conclusions

Coordinated radio, optical, ultraviolet, and X-ray observations of the BL Lac object Mrk 421 have been presented. These observations give a time-frozen picture of the continuous spectrum of Mrk 421 at each of three epochs. The observed spectra have been fitted with an inhomogeneous relativistic jet model which generally reproduces the data well. Many of the observed differences between the spectra at the different epochs can be attributed to slight variations in the jet direction and speed which can account for the integrated intensity variations through changes in the shaped of the composite synchrotron spectrum of the jet and (in the X-ray) through the level of the self-Compton emission. This is the same conclusion previously found from the application of this model to the BL Lacertae objects Mrk 180.
and Mrk 501. Further, the differing degrees of variability in the individual bandpasses also arise naturally in this model. Models similar to the one presented here have been shown by other authors to give similarly consistent descriptions of the emitting regions in other blazars. The model's principal disadvantage lies in the large number of parameters which must be specified on the basis of often limited data.

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Table 1
1980 Observations of Mrk 421

a) X-Ray Observations:

<table>
<thead>
<tr>
<th>UT Date</th>
<th>Count Rate (cts/s)</th>
<th>Range (keV)</th>
<th>Spectral Index (a)</th>
<th>log N_H (cm^-2)</th>
<th>log S_o (Jy) @ 2 keV</th>
<th>x^2/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPC:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>1.06±.05</td>
<td>0.2-4.0</td>
<td>-1.2±.2</td>
<td>(9.5±1.8) x10^{19}</td>
<td>(7.6±.6) x10^{-6}</td>
<td>3.91</td>
</tr>
<tr>
<td>12-17 May</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| MPC: | 2.45±.09 | 1.3-8.2 | -2.0±.5 | -------- | (8±3)x10^{-6} | 0.3 |

b) Ultraviolet Observations:

<table>
<thead>
<tr>
<th>UT Date</th>
<th>Camera (a)</th>
<th>Exposure Time (sec)</th>
<th>Spectral Time Index (a)</th>
<th>log S_o (mJy) @ log v = 15.2</th>
<th>x^2/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>SWP (9025)</td>
<td>12,600</td>
<td>-0.63±.2</td>
<td>-2.4±.02</td>
<td>1.5</td>
</tr>
<tr>
<td>17 May</td>
<td>LWR (7779)</td>
<td>5,400</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c) Optical Observations:

<table>
<thead>
<tr>
<th>UT Date</th>
<th>Aperture (arcsec)</th>
<th>B (mag)</th>
<th>V (mag)</th>
<th>R (mag)</th>
<th>I (mag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980 May 12-18</td>
<td>16.8</td>
<td>13.96±0.2</td>
<td>13.44±0.02</td>
<td>12.82±0.02</td>
<td>12.21±0.02</td>
</tr>
</tbody>
</table>

---

d) Radio Observations:

<table>
<thead>
<tr>
<th>UT Date</th>
<th>$S_{4.8}$(Jy)</th>
<th>$S_{8.0}$(Jy)</th>
<th>$S_{14.5}$(Jy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980 May</td>
<td>---</td>
<td>0.62±0.03</td>
<td>---</td>
</tr>
<tr>
<td>1980 Nov</td>
<td>0.74±0.06</td>
<td>0.66±0.03</td>
<td>0.58±0.05</td>
</tr>
<tr>
<td>1981 Nov</td>
<td>0.73±0.05</td>
<td>0.76±0.03</td>
<td>0.83±0.04</td>
</tr>
<tr>
<td>1981 Dec</td>
<td>0.76±0.03</td>
<td>0.81±0.02</td>
<td>0.76±0.02</td>
</tr>
<tr>
<td>1982 Mar</td>
<td>0.83±0.04</td>
<td>0.79±0.05</td>
<td>0.78±0.02</td>
</tr>
<tr>
<td>&lt;S&gt;</td>
<td>0.77±0.05</td>
<td>0.74±0.03</td>
<td>0.76±0.03</td>
</tr>
</tbody>
</table>

---

1 HEAO-2 processing algorithm specifies fit as "good".

2 All observations with Mt. Lemmon 1.5 m telescope.
Table 2

Relativistic Jet Model Fits to Mrk 421 for 3 Epochs

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (rad)</td>
<td>.04</td>
<td>.15</td>
<td>.09</td>
<td>.06</td>
</tr>
<tr>
<td>$\phi$ (rad)</td>
<td>.03</td>
<td>.12</td>
<td>.08</td>
<td>.06</td>
</tr>
<tr>
<td>$v_{sM}$ (Hz)</td>
<td>$4.7\times10^{12}$</td>
<td>$1.4\times10^{13}$</td>
<td>$2.1\times10^{13}$</td>
<td>$3.5\times10^{12}$</td>
</tr>
<tr>
<td>$S(v_{sM})$ (Jy)</td>
<td>.45</td>
<td>.19</td>
<td>.16</td>
<td>.31</td>
</tr>
<tr>
<td>$\alpha_e$</td>
<td>.44</td>
<td>.40</td>
<td>.48</td>
<td>.47</td>
</tr>
<tr>
<td>$v_{min}$ (Hz)</td>
<td>$1.3\times10^9$</td>
<td>$5.8\times10^5$</td>
<td>$8.8\times10^4$</td>
<td>$1.1\times10^4$</td>
</tr>
<tr>
<td>$\gamma_{eu}$</td>
<td>$1.7\times10^5$</td>
<td>$4.2\times10^4$</td>
<td>$3.5\times10^4$</td>
<td>$5.7\times10^4$</td>
</tr>
<tr>
<td>$\alpha_{s1}^2$</td>
<td>0.3</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>$\alpha_{s2}$</td>
<td>.93</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>$R_V$</td>
<td>2.4</td>
<td>4.2</td>
<td>4.1</td>
<td>1.6</td>
</tr>
<tr>
<td>$\chi^2/dof$</td>
<td>7/3</td>
<td>6.5/4</td>
<td>6.1/4</td>
<td>3.5/4</td>
</tr>
</tbody>
</table>

2 held fixed in these fits at the observed radio spectral index.
## Table 3

Derived Physical Parameters of Jets\(^1\)

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_j)</td>
<td>38</td>
<td>8</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>(D_j)</td>
<td>22</td>
<td>7</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>(m)</td>
<td>.85</td>
<td>.79</td>
<td>.79</td>
<td>.96</td>
</tr>
<tr>
<td>(B_1) (G)</td>
<td>.22</td>
<td>.06</td>
<td>.04</td>
<td>.06</td>
</tr>
<tr>
<td>(n)</td>
<td>1.9</td>
<td>2.1</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>(K_1) (cm(^{-3}))</td>
<td>.16</td>
<td>.33</td>
<td>.47</td>
<td>.26</td>
</tr>
<tr>
<td>(r_M) (pc)</td>
<td>9.7x10(^{-4})</td>
<td>7.4x10(^{-5})</td>
<td>7.2x10(^{-5})</td>
<td>1.0x10(^{-3})</td>
</tr>
<tr>
<td>(r(\nu_{\text{sb}} = \nu_{\text{su}})) (pc)</td>
<td>13</td>
<td>2.1</td>
<td>2.3</td>
<td>2.4</td>
</tr>
<tr>
<td>(\nu_{\text{sb}} = \nu_{\text{su}}) (Hz)</td>
<td>(6.2x10^{15})</td>
<td>(1.0x10^{15})</td>
<td>(1.5x10^{15})</td>
<td>(5.4x10^{15})</td>
</tr>
<tr>
<td>(\nu_{\text{su}}(r_M)) (Hz)</td>
<td>(2.1x10^{20})</td>
<td>(3.5x10^{18})</td>
<td>(5.4x10^{18})</td>
<td>(9.5x10^{18})</td>
</tr>
</tbody>
</table>

References


Figure Captions

Figure 1: Contours of $\chi^2$/dof in the (spectral index, hydrogen column density)-plane for power law fits to the IPC X-ray data. The $\chi^2$/dof data were generated by the HEAO-2 fitting algorithm.

Figure 2: The multifrequency spectrum of Mrk 421 from the observations of May 1980. The data plotted are given in Table 1. The radio and optical data are shown as diamonds; the power law fits to the IUE, IPC, and MPC data are shown as solid lines with error bars indicating 1 σ errors in the flux density at the ends of the bandpass. Superposed onto the data as a dotted line is the best fitting relativistic jet model. The component of emission due to self-Compton emission is shown as a dashed line. The fit parameters used to describe the jet are given in Table 2.

Figure 3: The multifrequency spectrum of Mrk 421 from the observations of January 1984. The data plotted are taken from Makino et al. (1987). The radio and optical data used in the fits are shown as diamonds. Infrared data and additional radio data not used in the fits are shown as X's. The power law fit to the IUE data is shown as a solid line; 1 σ errors at the end of the bandpass are indicated. The TENMA observations have been broken into a single low energy and two high energy
("flat" and "steep") power laws, as described in the text. The low energy X-ray power law and the "flat" and "steep" high energy X-ray spectra are represented by a double solid line. The approximate 1 $\sigma$ errors in the low energy component are indicated. Superposed onto the data is the best fitting relativistic jet models for both the "flat" and the "steep" spectrum. At the high energy end, the model flux density has been broken into synchrotron (dash-dots) and self-Compton (dashes) components. The fit parameters used to describe both the "flat" and "steep" jets are given in Table 2.

Figure 4: The multifrequency spectrum of Mrk 421 from the observations of March 1984. The data plotted are taken from Makino et al. (1987). The radio and optical data used in the fits are shown as diamonds. Infrared data and additional radio data not used in the fits are shown as X's. The power law fits to the IUE and TENMA data are shown as solid lines; 1 $\sigma$ errors at the end of the bandpass are indicated. Superposed onto the data as a dotted line is the best fitting relativistic jet model. The component of emission due to self-Compton emission is shown as a dashed line. The fit parameters used to describe the jet are given in Table 2.