

## Angular distribution of turbulence in wave space

By G. Coleman<sup>1</sup>, J.H. Ferziger<sup>1</sup>, J.P. Bertoglio<sup>2</sup>

### 1. Introduction

As experience with the one-point closure models for turbulence in current use has not been completely satisfactory, people have begun to search for other ways to predict turbulent flows. One alternative that has been suggested is large eddy simulation (LES) which, together with its more exact relative, direct numerical simulation (DNS), has had considerable success in the prediction of turbulent flows. These methods are beginning to serve as partial substitutes for turbulence experiments.

It is perhaps natural that people should regard these new methods as panaceas. More careful consideration will lead one to be more cautious. DNS and LES have been applied only to the simplest low Reynolds number turbulent flows. The prospects for a large increase in the range of applicability of DNS in the near future are very small. For LES, the prospects are somewhat brighter.

The range of flows that has been treated with LES to date is only a little broader than that treated by DNS. The Reynolds numbers are somewhat higher but the geometries are almost as restricted. Three items pace the growth of LES applications. The first is computational resources: speed, memory (both fast and archival), and number of processors available. The second is numerical methods; there is, and perhaps always will be, a need for faster algorithms applicable to a wide range of geometries. Finally, there are the subgrid models required by LES; this is the focus of the present work.

In simulations done to date, the Reynolds numbers were such that most of the turbulence energy resided in the resolved scales. Under these circumstances, the results are relatively insensitive to the quality of the model used for the subgrid scale (SGS) component of the turbulence. As one pushes LES to higher Reynolds numbers or more complex flows, the model quality becomes a more important issue. It is safe to say that, if the models in current use are applied to these more difficult flows, the results will be of reduced quality. Thus the development of improved SGS models must be of highest priority if LES is to become an engineering tool.

SGS models in current use are, for the most part, based on the same ideas as one-point closure models. To obtain significant improvements, new ideas will probably be needed. It is here that turbulence theories may have a role to play.

1 Stanford University

2 Ecole Centrale de Lyon

## 2. Turbulence theories

There is a wide range of turbulence theories. The modern ones deal with the distribution of the turbulence in Fourier or wavenumber space. Use of Fourier transforms implies that their applicability is limited to homogeneous turbulence; however, their importance lies in the fact that they contain information about the length scales of turbulence, something notably lacking in one-point closure models. Extensions to inhomogeneous flows may be possible, but it is unlikely that these theories will ever be applied directly to the complex engineering flows. Nonetheless, they may be of use in the development of SGS models. In particular, one may be able to regard the turbulence as locally homogeneous and apply the theory to the prediction of the SGS turbulence. The objective is to obtain the best of both worlds: the ability of LES to simulate inhomogeneous flows and that of theory to provide length scale information.

In selecting a candidate turbulence theory on which to base an SGS turbulence model, one should be guided by the following principles. The theory should be successful in predicting homogeneous flows. The computation time should not be too large. Finally, it should be capable of simplifications that will render it practical for use as an SGS model.

There is no space to review turbulence theories here. Let it suffice to say that, of the theories that we considered, the Eddy Damped Quasi-Normal Markovian (EDQNM) model appears to have the brightest prospects. It meets the criteria set forth in the preceding paragraph to a higher degree than its competitors. The EDQNM model is based on simplifications of the moment equations in Fourier space. The quasi-normal assumption replaces the fourth order moments by their values for a Gaussian distribution. Eddy damping is introduced to restore some of the important interactions removed by the quasi-normal hypothesis. Finally, the Markovian assumption removes history effects that complicate the analysis. The result is a system of non-linear integral equations for the second moments in Fourier space. These are also the Fourier transforms of the two-point correlation functions; hence this is a two-point closure method.

Solving the equations of EDQNM is not trivial. In the absence of further simplifications, it is necessary to solve a coupled system of non-linear integral equations in three dimensional wave space. This has been done for homogeneous flows with excellent results. However, when using EDQNM as an SGS model for an inhomogeneous flow, it is necessary to solve these equations at each point at every time step. Although EDQNM has been applied as an SGS model for homogeneous isotropic turbulence, it is clearly impractical for more complex flows without additional simplifications. Such simplifications have been used. For isotropic turbulence, one can integrate over angles analytically and reduce the equation to one involving a single independent variable. In other flows, the symmetries can be used to provide other, less dramatic, simplifications. In the work reported here, we investigated possible simplifications in the homogeneous flow of most direct relevance to engineering applications: homogeneous sheared turbulence.

### 3. Angular Distributions

One way to simplify EDQNM is to assume that the distributions of the second moments in wave space can be represented as a sum of a small number of simple functions. The equations could then be reduced to a set of non-linear algebraic equations for the parameters. This would greatly reduce the cost of EDQNM and could render it practical for use as an SGS model for inhomogeneous flows.

It is well-known that, in the inertial subrange, the spectral distribution of the energy obeys a power law. The viscous range can be represented by using a cutoff, the details of which should not be important at high Reynolds numbers. Since the full simulation data we will use as the basis of the current work is at Reynolds numbers lower than those at which the model is to be applied, and the spectral distributions are nearly always smooth, it was felt that there is little point in investigating the distributions in wavenumber. We therefore concentrated on the angular distribution in wave space; caution is required because the results obtained may not apply at higher Reynolds numbers.

The data on which our analysis is based represent isotropic turbulence which has been sheared at a rate  $S = d\bar{u}/dy$  until  $St = 12$ ; the initial turbulence Reynolds number based on microscale was approximately 50. The data, originally generated by Mike Rogers, were supplied to us in the form of the Fourier-transformed velocity field by Moon J. Lee.

The data were converted from Cartesian to spherical coordinates in wave space. The  $k_y$  direction was chosen as the pole of the spherical system while the  $k_z$  direction was chosen as the origin for the azimuthal angle.

The angular distribution of the converted data was examined. At each wavenumber, contours of each of the significant second moments ( $E_{11}$ ,  $E_{22}$ ,  $E_{33}$ , and  $E_{12}$ ) and the total energy were plotted as functions of the two spherical angles; only results at the largest wavenumber for which an entire shell was available will be presented here. The distribution was found to be smooth enough that it can probably be represented as a sum of a small number of functions. The energy is concentrated near the poles, indicating the presence of small scales in the  $k_y$  direction caused by shear-thinning of the eddies.

In order to further determine what is needed to fit the angular distribution, we plotted the energy components on lines on which one of the angles is held fixed. Figure 1 shows the results as a function of azimuthal angle for fixed polar angle while figure 2 shows the energy as a function of polar angle for fixed azimuthal angle. The distribution in polar angle can be fit with the first two terms of a Fourier series while the distributions in azimuthal angle appear to require three terms. Thus, approximately six terms should suffice to fit the angular distribution of each component. If the distribution in wavenumber can be assumed, a total of eighteen parameters should be the upper limit of what is needed to represent the subgrid turbulence. With further experience, we may be able to reduce the number somewhat. We estimate that using an eighteen parameter algebraic SGS model would approximately double the cost of LES, a not unreasonable price if the Reynolds numbers can be increased sufficiently.

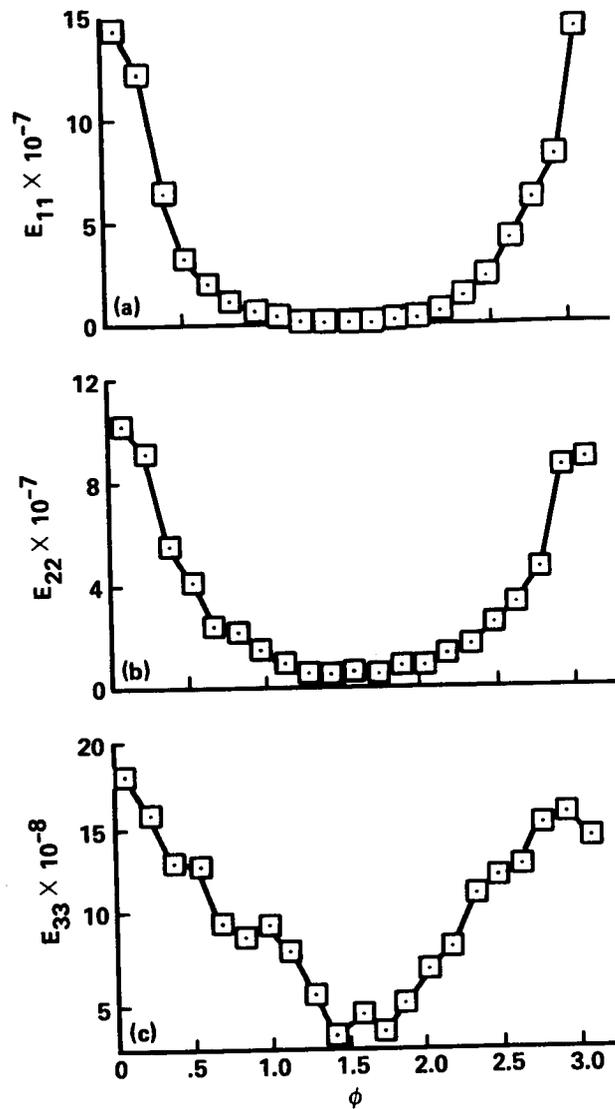


FIGURE 1. Distribution of Reynolds stresses over azimuthal angle  $\phi$  at constant polar angle  $\theta = 1.1794$ .

#### 4. Future Work

We intend to continue the work described above. We will attempt to fit the distribution of the second moments with a few functions as described above and determine how many parameters are needed more precisely. At some later date, we will try to perform an EDQNM calculation of homogeneous sheared turbulence using the parameter set suggested by these fits. The results will be compared to the original data used in this work and with the results of an EDQNM calculation carried out in the usual way.

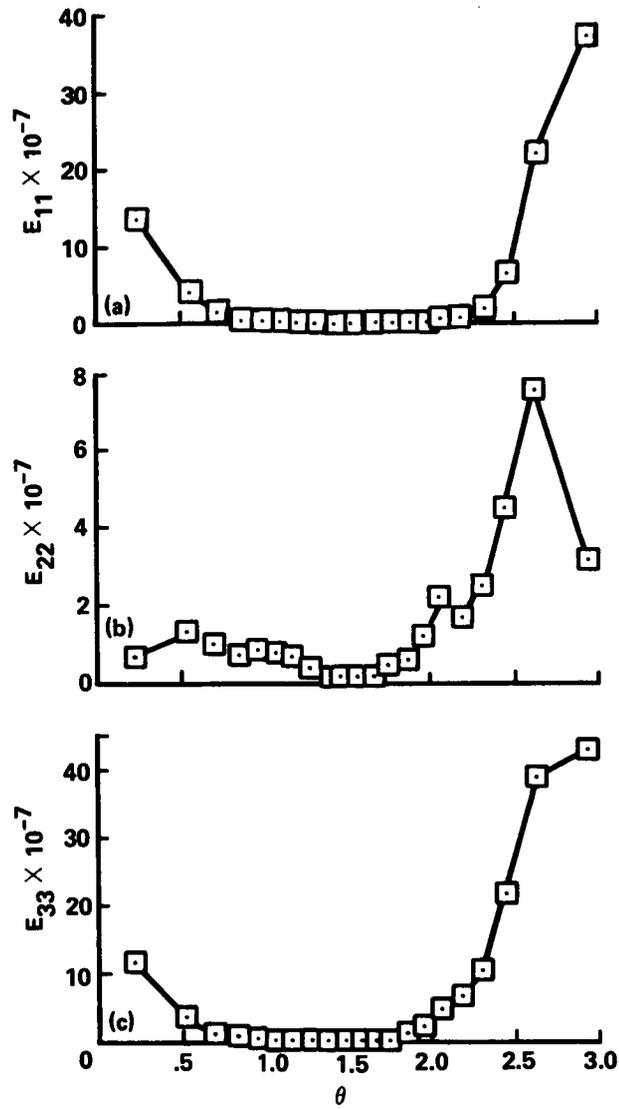


FIGURE 2. Distribution of Reynolds stresses over polar angle  $\theta$  at constant azimuthal angle  $\phi = 1.5708$ .

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