

N88 - 23233

PARALLEL COMPUTER METHODS FOR EIGENVALUE EXTRACTION

Fred Akl*
Department of Civil Engineering
Ohio University
Athens, Ohio

ABSTRACT

This paper presents a new numerical algorithm for the solution of large-order eigenproblems typically encountered in linear elastic finite element systems. The architecture of parallel processing is used in the algorithm to achieve increased speed and efficiency of calculations. The algorithm is based on the frontal technique for the solution of linear simultaneous equations and the modified subspace eigenanalysis method for the solution of the eigenproblem. Assembly, elimination, and back-substitution of degrees-of-freedom are performed concurrently by using a number of fronts. All fronts converge to and diverge from a predefined global front during elimination and back-substitution, respectively. In the meantime, reduction of the stiffness and mass matrices required by the modified subspace method can be completed during the convergence/divergence cycle, and an estimate of the required eigenpairs can be obtained. Successive cycles of convergence and divergence are repeated until the desired accuracy of calculations is achieved. The advantages of this new algorithm in parallel computer architecture are discussed.

*NASA Resident Research Associate. Work funded by NASA grant NAG3-762 (monitor: L.J. Kiraly) and performed on-site at the Lewis Research Center for the Structural Dynamics Branch.

GENERALIZED EIGENPROBLEM

- New parallel algorithm for the solution of large-scale eigenproblems in finite element applications
- Assumptions
 - (1) Linear elastic finite element models
 - (2) n lower order eigenpairs are required, i.e., $\omega_1^2 \leq \omega_2^2 \leq \dots \omega_n^2$
 - (3) $[K]$ is positive-definite
 - (4) $[M]$ is semipositive definite

$$[K] [\Phi] = [M] [\Phi] [\Omega]$$

N – DEGREES OF FREEDOM

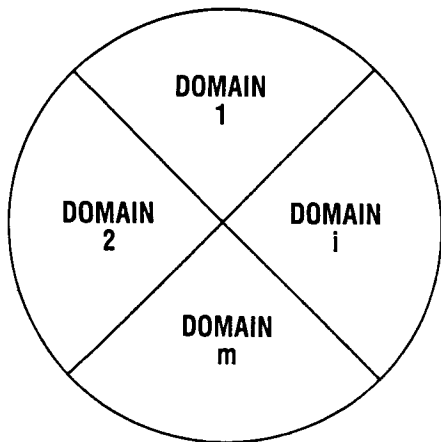
REQUIRED n EIGENPAIRS, $n \leq N$

$[K]$ POSITIVE-DEFINITE

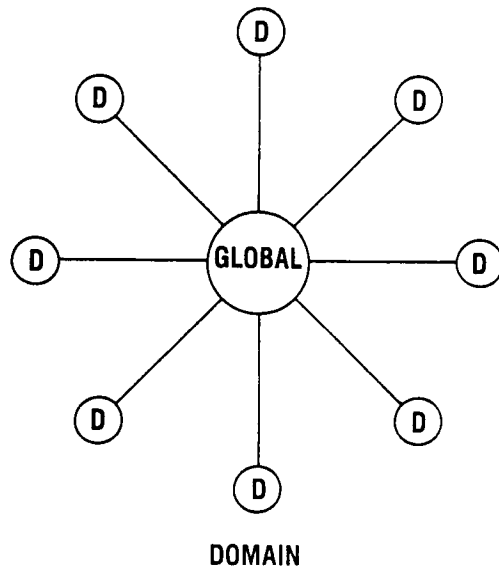
PARALLEL ARCHITECTURE

- Consider a parallel computer with $(m+1)$ processors (tasks).
- Designate the first processor as a global processor (task).
- Designate the remaining m -processors as domain processors (tasks).
- A finite element model can be divided into a number of domains equal to m .
- A star architecture (or tree) is the first to be investigated.

**FINITE ELEMENT MODEL
SUBDIVIDED INTO m DOMAINS**



STAR ARCHITECTURE



CD-88-31669

MAJOR COMPUTATIONAL TASKS

Three major steps of large computational requirements

- (1) Creation of element stiffness and mass matrices
- (2) Extraction of a set of eigenpairs
- (3) Solution of a set of simultaneous linear equations

The merits of selecting the modified subspace method for step 2 and the frontal solution for step 3 are discussed in the next viewgraphs.

$$[K] [\Phi] = [M] [\Phi] [\Omega]$$

- (1) CREATION OF K^e AND M^e
- (2) EIGENSOLUTION (MODIFIED SUBSPACE)
- (3) EQUATION SOLVER (FRONTAL SOLUTION)

MODIFIED SUBSPACE METHOD

The Modified Subspace Method iterates simultaneously for a subset of eigenpairs $[\phi, \omega^2]$ of the generalized eigenproblem:

- (1) Let $[V]_1$ be n starting eigenvectors.
- (2) Operate on each $[V]_\ell$ as follows:

$$[V]_{\ell+1}^* = [K]^{-1} [M][V]_\ell = [K]^{-1} [B]_\ell, \ell = 1, 2, 3, \dots$$

- (3) Modify $[V]_{\ell+1}^*$ to increase convergence rate by one third on average

$$[V]_{\ell+1}^* \leftarrow [V]_{\ell+1}^* - \beta_\ell [V]_\ell$$

where $\beta_\ell = 0$ for $\ell = 1$ and $\ell > 11$

$$\beta_\ell = 0.5 (1 + r_{\ell-1}) / \omega_n^2 \quad 1 < \ell \leq 11$$

$r_{\ell-1}$ are the interval points of the 11th order Labatoo rule $[-1, 1]$

- (4) Project K and M onto the required subspace.
- (5) Solve the auxiliary eigenproblem to obtain $[Q]_{\ell+1}$ and $[\Omega]_{\ell+1}$.
- (6) An improved set of eigenvectors $[V]_{\ell+1}$ can be obtained.
- (7) Test for convergence on ω_n^2 . Repeat steps 2 to 6 until desired accuracy is achieved.

$$[K]_{\ell+1}^* = \sum_e [V]_{\ell+1}^{*T} [K][V]_{\ell+1}^*$$

$$[M]_{\ell+1}^* = \sum_e [V]_{\ell+1}^{*T} [M][V]_{\ell+1}^*$$

AUXILIARY EIGENPROBLEM

$$[K]_{\ell+1}^* [Q]_{\ell+1} = [M]_{\ell+1}^* [Q]_{\ell+1} [\Omega]_{\ell+1}$$

IMPROVED EIGENVECTORS

$$[V]_{\ell+1}^e = [V]_{\ell+1}^{*e} [Q]_{\ell+1}$$

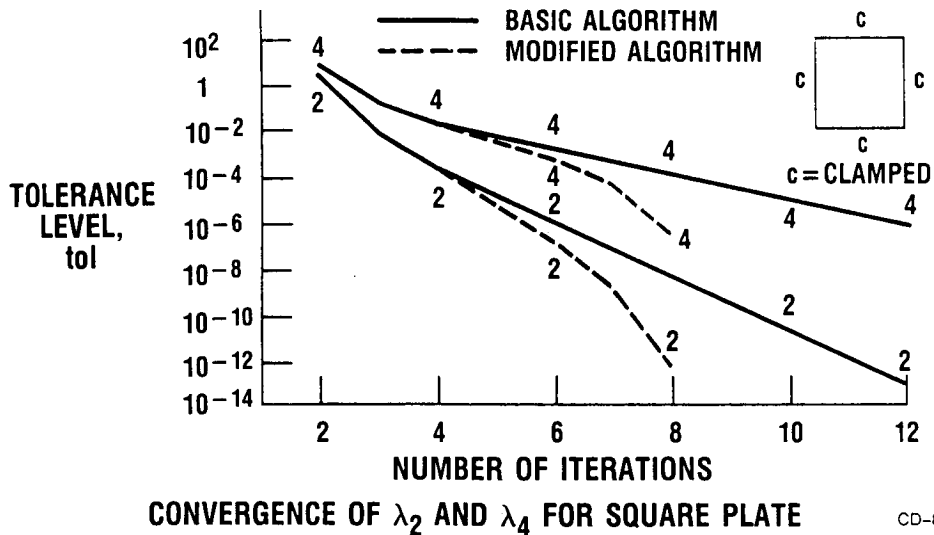
RATE OF CONVERGENCE

Rate of convergence of the modified subspace is 33 percent faster on average compared to the classical subspace method.

The figure shows typical behavior.

Most computations are performed on an element-by-element basis.

BASIC SUBSPACE	$\approx \frac{\omega_i^2}{\omega_{n+1}^2}$
MODIFIED SUBSPACE	$\approx \frac{\omega_i^2}{\omega_{n+1}^2} \frac{1 - \beta_\rho \omega_{n+1}^2}{1 - \beta_\rho \omega_i^2}$



FRONTAL SOLUTION

- (1) Gauss elimination technique
- (2) Underlying philosophy based on processing of elements one by one
- (3) Simultaneous assembly and elimination of variables
- (4) The optimum frontal width at most equal to the optimum bandwidth
- (5) Numbering of nodes - no impact on optimality; numbering of elements - important to minimize the frontal width
- (6) More efficient for solid elements and elements with midside nodes
- (7) Requires a prefront to determine last appearance of each node
- (8) Lends itself to parallel solutions

WITHIN EACH DOMAIN

$$k_{ij} \leftarrow k_{ij} - \sum \left[\frac{k_{is} k_{sj}}{k_{ss}} \right]$$

$$b_{iq} \leftarrow b_{iq} - \sum \left[\frac{k_{is} b_{sq}}{k_{ss}} \right]$$

DOMAIN PROCESSING

Multitasking on the Cray supercomputer provides tools for implementing the frontal solution concurrently within a number of domains. Assembly and elimination for the i -th domain is assigned to a task (processor). Simultaneous assembly and elimination of variables within the domains is performed in parallel tasks until the domain fronts reach their respective global fronts. However, it is instructive to analyze the set of simultaneous equations for the i -th domain assuming that the domain stiffness matrix and right-hand sides are fully assembled before Gauss elimination is performed.

For domain i

$$[K][V]_{\ell+1}^* = [B]_{\ell} \quad \text{at iteration } \ell$$

Elimination gives

$$U_d V_d^* + K_d V_F^* = B_d$$

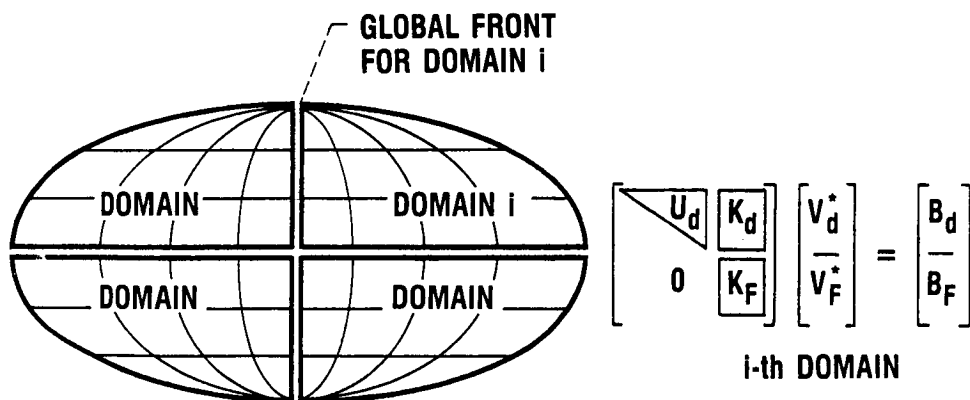
$$K_F V_F^* = B_F$$

where U_d upper Δ matrix for domain i

V_d^* variables within domain i

V_F^* variables along global front of domain i

B_d and B_F are right-hand sides for domain variables and global fronts, respectively.



$$U_d V_d^* + K_d V_F^* = B_d$$

$$K_F V_F^* = B_F$$

GLOBAL FRONTS

$$K_{FF} = \sum^m K_F$$

$$B_{FF} = \sum^m B_F$$

where m = total numbers of domains

$$K_{FF} V_{FF} = B_{FF}$$

Solution for V_{FF} is then obtained by using the frontal solution on the global fronts. Since V_{FF} is a superset of all V_F^* , back-substitution for V_d^* follows concurrently within the domains.

$$K_{FF} = \sum^m K_F$$

$$B_{FF} = \sum^m B_F$$

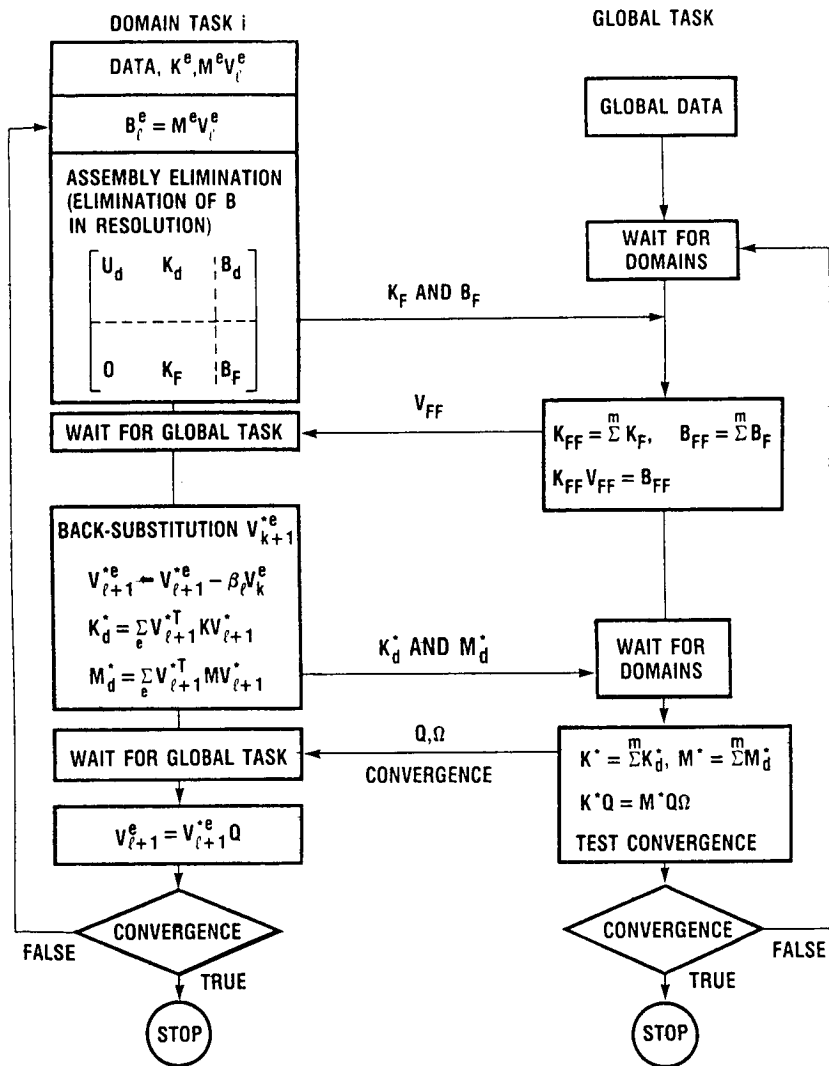
WHERE m = TOTAL NUMBER OF DOMAINS

$$K_{FF} V_{FF} = B_{FF}$$

IMPLEMENTATION

Successful implementation of the new parallel algorithm depends on

- (1) Maximizing the efficiency of communication links between the global task and the domains
- (2) Minimizing sequential computational steps
- (3) Multithreaded I/O



CONCLUSIONS

Parallel solution method for eigenvalue extraction for linear elastic finite element models has been successfully implemented on the Cray supercomputer by using the multitasking environment. Preliminary results are encouraging and extensive testing of the new algorithm is currently progressing. The new algorithm enhances the speed-up of similar sequential solution methods. Both the frontal method for the solution of the set of simultaneous equations arising in finite element problems and the modified subspace method for the solution of a subset of eigenpairs offer a new algorithm which has been efficiently parallelized. The parallel tasks are associated with recognizable finite element domains rather than dissected blocks of abstract equations. Moreover, the complexity of data management and data flow normally associated with parallel solution methods is avoided in this new algorithm.

REFERENCES

- Adams, L., and Ortega, J., 1982, "A Multi-Color SOR Method for Parallel Computation," Proceedings of the 1982 International Conference on Parallel Processing, IEEE, New York, pp. 53-56.
- Akl, F.A. and Hackett, R.M., 1986, "A Multi-Frontal Algorithm for Parallel Processing of Large Eigenproblems," AIAA 27th Structures, Structural Dynamics, and Materials Conference, Part 2, AIAA, New York, pp. 395-399.
- Baudet, G., 1978, "Asynchronous Iterative Methods for Multiprocessors," Journal of the Association for Computing Machinery, Vol. 25, pp. 226-244.
- Bostic, S.W. and Fulton, R.E., 1986, "Implementation of the Lanczos Method for Structural Vibration Analysis on a Parallel Computer," AIAA 27th Structures, Structural Dynamics, and Materials Conference, Part 2, AIAA, New York, pp. 400-410.
- Bostic, S.W. and Fulton, R.E., 1985, "A Concurrent Processing Implementation for Structural Vibration Analysis," AIAA 26th Structures, Structural Dynamics, and Materials Conference, Part 2, AIAA, New York, pp. 566-572.
- Crockett, T.W. and Knott, J.D., 1985, "System Software for the Finite Element Machine," NASA CR-3870.
- Darbhamulla, S.P., Razzaq, Z., and Storaasli, O.O., 1986, "Concurrent Processing in Nonlinear Structural Stability," AIAA 27th Structures, Structural Dynamics, and Materials Conference, Part 1, AIAA, New York, pp. 545-550.
- Gannon, D., 1980, "A Note on Pipelining a Mesh Connected Multiprocessor for Finite Element Problems by Nested Dissection," Proceedings of the 1980 International Conference on Parallel Processing, IEEE, New York, pp. 197-204.
- Rheinboldt, W.C. (Panel Chairman), 1985, "Report of the Panel on Future Directions in Computational Mathematics, Algorithms, and Scientific Software," Society of Industrial and Applied Mathematics, New York.
- Salama, M., Utku, S. and Melosh R., 1983, "Parallel Solution of Finite Element Equations," Proceedings of the 8th Conference on Electronic Computation, American Society of Civil Engineers, Houston, Texas, pp. 526-539.
- Schendel, U., 1984, Introduction to Numerical Methods for Parallel Computers, Trans. by B.W. Conolly, Ellis Horwood Limited, New York.
- Silbar, M.L., 1985, "The Pursuit of Parallelism," MOSAIC, National Science Foundation, Vol. 16, No. 3, pp. 8-17.
- Storaasli, O.O., and Bergan, P., 1986, "A Nonlinear Substructuring Method for Concurrent Processing Computers," AIAA 27th Structures, Structural Dynamics, and Materials Conference, Part 2, AIAA, New York, pp. 13-20.
- Storaasli, O.O., Peebles, S.W., Crockett, T.W., Knott, J.D. and Adams, L., 1982, "The Finite Element Machine: An Experiment in Parallel Processing," NASA-TM 84514.