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Dynamic Characteristics of a Vibrating Beam With Periodic Variation in Bending Stiffness

John S. Townsend

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DEFINITION OF SYMBOLS

Symbol	Definition
A	Peak stiffness variation, A = (EImax – EImin) / 2
Anm	Factor defined by equation (A-2)
dx	Differential element length
d/dx , (′)	Denotes differentiation with respect to position
EIa	Geometric average stiffness of beam design, EIa = (EImax + EImin) /2
EI(x)	Bending stiffness function
k	Dummy variable referring to mode number
L	Span length
Lt	Period of the stiffness function
m	Dummy variable referring to mode number
$\overline{m}$	Mass per unit length
n	Vibration mode number
Un(x)	Denotes nth mode eigenfunction, bending
x	Distance along span measured relative to support
$\tilde{x}$	Dimensionless horizontal coordinate, $\tilde{x} = x/L$
δ_{mn}	Kronecker delta function, equation (10)
ϵ	Stiffnesss perturbation parameters, $\epsilon = A/EIa$
λ_n	Dimensionless eigenvalue, $\lambda^2_n = (\overline{m}L^4/EIa)\ \omega^2_n$
λ_{n0}	Zeroth-order eigenvalue solution
λ_{n1}	First-order eigenvalue solution
λ_{n2}	Second-order eigenvalue solution
π	Pi
ρ	Stiffness parameter, $\rho = 1,2,3$, i.e., the number of half periods $\rho = 2\ L/Lt$
ϕ_n	Dimensionless Eigenfunction, $\phi_n = Un/L$

ϕ_{n0}	Zeroth-order eigenfunction solution
ϕ_{n1}	First-order eigenfunction solution
ϕ_{n2}	Second-order eigenfunction solution
ω_n	Denotes nth mode frequency, bending
∂	Partial derivative notation
$\int$	Integral sign
Σ	Summation sign

DYNAMIC CHARACTERISTICS OF A VIBRATING BEAM WITH PERIODIC VARIATION IN BENDING STIFFNESS

INTRODUCTION

Vibrating beam theory has been considered extensively in the literature for any number of variable property states, ranging from structures with changing cross-sectional geometry to those of a composite nature. Solutions are obtained either in closed-form for a few simple cases, or they are pursued using numerical techniques. In the present study, a perturbation expansion technique applicable to continuous systems is used to develop a closed-form solution to the problem of a vibrating beam with bending stiffness periodic in the spatial coordinate. Results are compared to a finite element solution and verified experimentally using forced vibration of a test span. To the knowledge of the author, this specific beam problem (static or dynamic) has not been addressed in the literature.

Application of periodic stiffness is recognized in the field of vortex-induced motion of transmission power lines [1]. In recent years a conductor, known as twisted-paired, has been developed that uses a variable diameter design to provide a changing conductor profile into the wind. Twisted-paired conductors are constructed by twisting together two identical standard round conductors with 360 deg twists occurring at set intervals along the span. The periodic nature of the twist causes a periodic variation in bending stiffness. Variable profile diameter results in non-uniform shedding of vortices, and hence excitation frequencies, along the span. Multiple vortex frequencies act to minimize wind energy transfer and detune vibration response. In conductor systems, influence of bending stiffness effects becomes extremely important in the vicinity of supports.

The purpose of this report is to characterize the dynamic bending behavior of beams with periodic stiffness variation. Also, the models developed will provide insight into the behavior of similar type systems with changing property states.

EQUATIONS OF MOTION AND BASIC ASSUMPTIONS

Consider the problem of the transverse vibrations of a straight beam with periodic variation in bending stiffness along its length. The beam is assumed to be simply supported and long compared to its cross-sectional dimensions, and dynamic shear distortions and rotary inertia are negligible. We will also make the usual simplifying assumptions that Hooke's Law holds and plane sections remain plane. Figure 1 shows a free body sketch of a differential element of the vibrating beam.

We will proceed from the well-known differential equation of motion for the normal mode response of an undamped beam

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2U_n(x)}{dx^2}\right] - \bar{m}\,\omega^2{}_n\,U_n(x) = 0 \tag{1}$$

where EI(x) is the bending stiffness function, $\overline{m}$ is the mass per unit length, $U_n(x)$ is the nth normal mode displacement, and ω_n is the nth normal mode frequency. The bending stiffness function is given by

$$EI(x) = EIa + A\cos\frac{2\pi}{L_T}x \tag{2}$$

where EIa is the geometric average stiffness of the beam design, EIa = [EImax + EImin]/2; and A is the peak stiffness variation, A = [EImax – EImin]/2. The period of the stiffness function is Lt. Figure 2 plots the function. Notice, at x = 0, the maximum flexural stiffness occurs, and the periodicity of the function is an even multiple of the span length. This particular stiffness function is characteristic of twisted-paired systems [1].

An equation of motion that models an undamped, vibrating beam with periodic bending stiffness is determined by combining equations (1) and (2). The result is a fourth-order differential equation with variable coefficients. A closed-form approximate solution to this boundary value problem is obtained by using a variation of the Rayleigh-Schrodinger expansion [2,3]. The solution in closed-form is extremely useful, since it clearly displays the influence of system parameters on response. Nayfeh [4] presents an application of a similar perturbation formulation for a simple linear second-order eigenvalue problem.

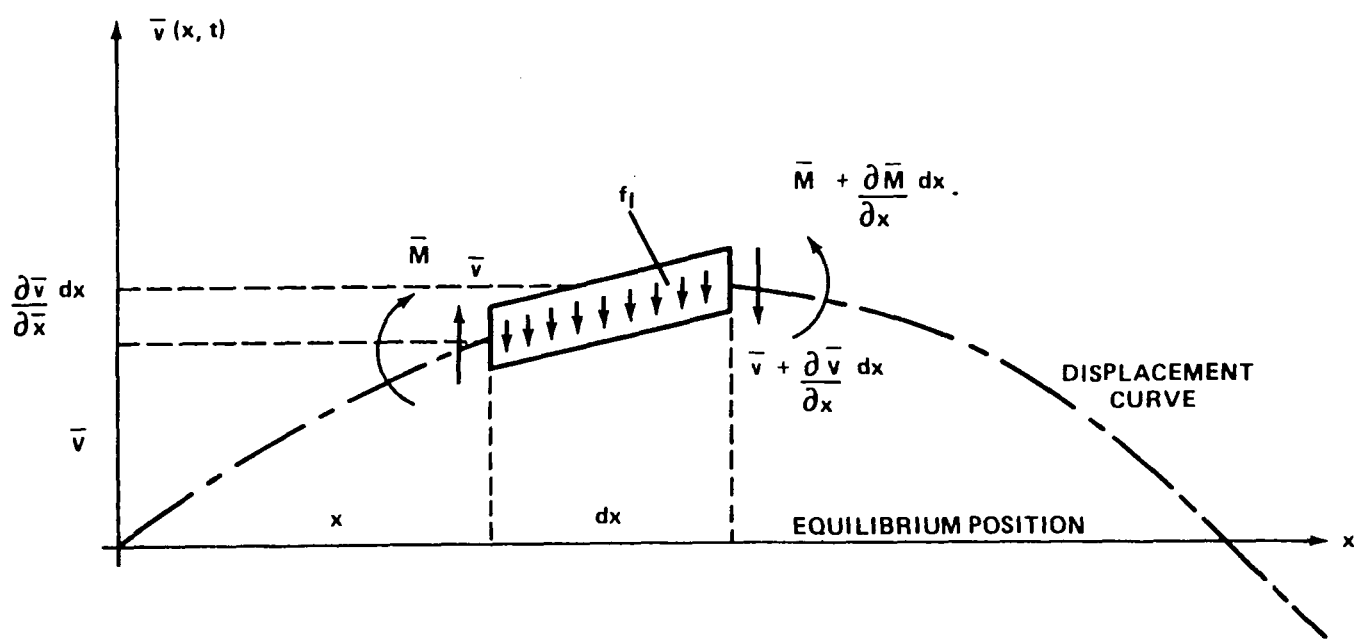

Figure 1. Differential element of a vibrating beam.

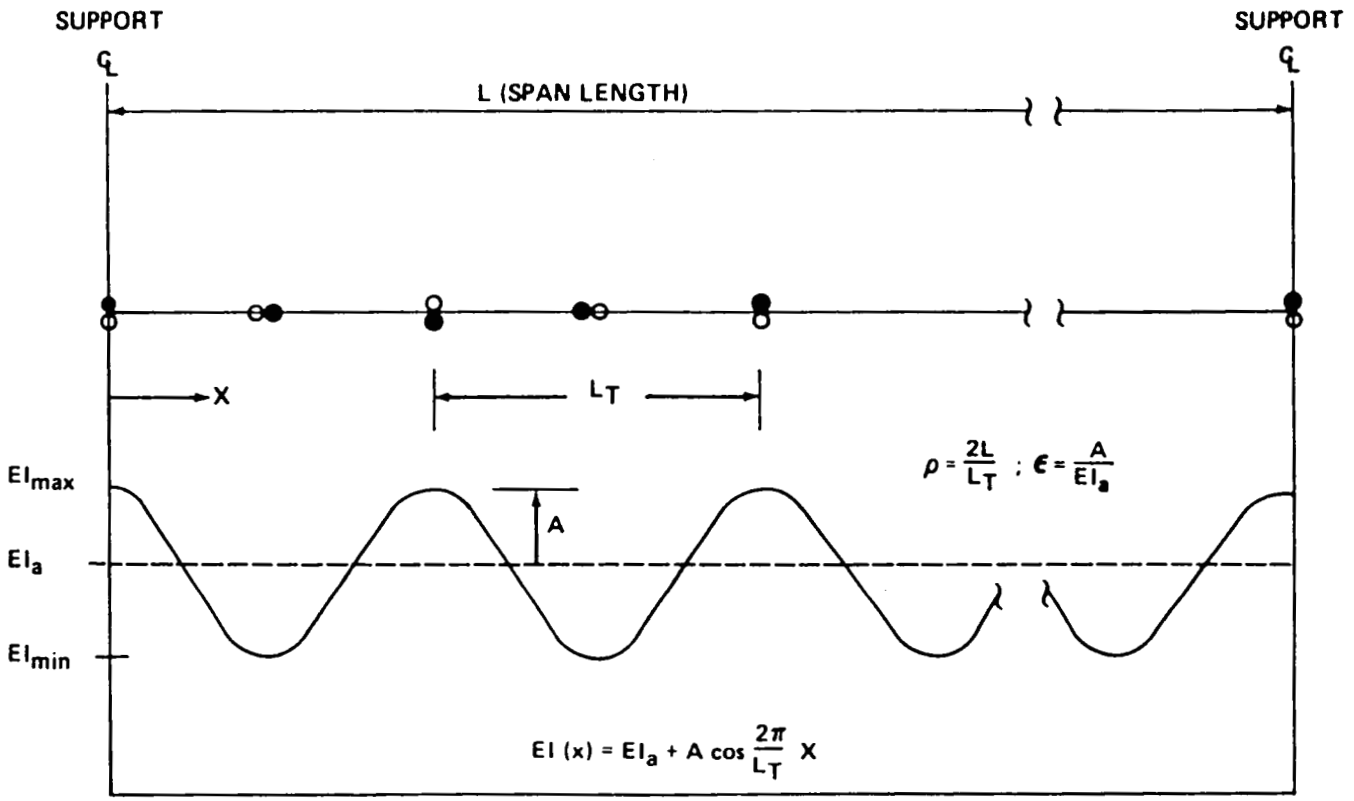

Figure 2. Periodic bending stiffness function.

A dimensionless form of the governing motion equation is

$$\frac{d^2}{d\tilde{x}^2}\left\{\left[1+\epsilon\cos\rho\pi\tilde{x}\right]\phi_n''\right\}-\lambda^2{}_n\phi_n=0 \tag{3}$$

(See nomenclature section for a definition of terms.)

and the corresponding boundary conditions for the case of simple supports are

$$\phi_n(0)=\phi_n(1)=0 \tag{4}$$

and

$$\phi_n''(0)=\phi_n''(1)=0 \tag{5}$$

The quantity $[1 + \epsilon \cos \rho\pi x]$ is the dimensionless bending stiffness. For convenience, the tildes are dropped in the remaining analysis. The coefficient ϵ is a measure of the magnitude of the stiffness variation, $\epsilon = A/EIa$, and the parameter ρ is equal to the number of half periods of the stiffness function in a given span, $\rho = 2L/Lt$.

PERTURBATION EXPANSION SOLUTION

The solution (ϕ,λ) of equation (3) is a function of the independent variable x and the parameters ϵ and ρ. If the parameter ϵ is equal to zero, the equation reduces to the case of a vibrating beam with uniform flexural stiffness whose eigenfunctions and eigenvalues are given, respectively, by

$$\phi_n = \sqrt{2}\sin n\pi x \tag{6}$$

$$\lambda^2{}_n = n^4\pi^4 \tag{7}$$

The $\sqrt{2}$ coefficient is arbitrary. It is picked so that the eigenfunctions are normalized according to the integral function,

$$\int_0^1 \phi^2{}_n\,dx = 1 \tag{8}$$

The above eigenfunctions are orthonormal; i.e.,

$$\int_0^1 \phi_n(x)\,\phi_m(x)\,dx = \delta_{mn} \tag{9}$$

where, δ_{mn}, the Kronecker delta function is specified as

$$\delta_{mn} = \begin{cases} 0 & m \neq n \\ 1 & m = n \end{cases} \tag{10}$$

When ϵ does not equal zero, equations (6) and (7) are no longer valid and corrections must be added to them. An approximate solution is obtained by expanding both the eigenfunction and the square of the eigenvalue in the form of a power series in ϵ; i.e.,

$$\phi_n = \phi_{n0} + \epsilon\,\phi_{n1} + \epsilon^2\,\phi_{n2} + \ldots \tag{11}$$

$$\lambda^2{}_n = \lambda_{n0} + \epsilon\,\lambda_{n1} + \epsilon^2\,\lambda_{n2} + \ldots \tag{12}$$

where ϕ_{n0} and λ_{n0} are the eigenfunction and the square of the eigenvalue when ϵ equals zero; equations (6) and (7). An asymptotic expansion is generally valid only if ϵ is small. By definition, the parameter ϵ for beams with periodic stiffness may not be small, but it is always less than one. Corrections of the higher order terms are therefore negligible, and the series converges eventually to the correct solution.

Substituting equations (11) and (12) into equation (3) and equating like powers of ϵ through order ϵ^2, we get the following system of differential equations:

$$0(\epsilon^0): \phi_{n0}{}^{iv} - \lambda_{n0}\,\phi_{n0} = 0 \tag{13}$$

$$0(\epsilon^1): \phi_{n1}{}^{iv} - \lambda_{n0}\,\phi_{n1} = \lambda_{n1}\,\phi_{n0} - (d^2/dx^2)\,[\phi_{n0}''\cos\rho\pi x] \tag{14}$$

$$0(\epsilon^2): \phi_{n2}{}^{iv} - \lambda_{n0}\,\phi_{n2} = \lambda_{n2}\,\phi_{n0} + \lambda_{n1}\,\phi_{n1} - (d^2/dx^2)\,[\phi_{n1}''\cos\rho\pi x] \tag{15}$$

The problem of finding an approximate solution to equation (3) is now simplified to one of obtaining sequential solutions to equations (13), (14), and (15). To illustrate the procedure, the first-order correction is formulated in Appendix A. Equations (A-6) and (A-8) define the correction terms of the eigenvalues and eigenfunctions.

Using equation (A-8) and recalling that $\phi_n = \phi_{n0} + \epsilon\,\phi_{n1}$ where $\rho = 1,2,3,\ldots$, the general eigenfunction solution ϕ_n is expressed in terms of dimensionless parameters for the case when $n \neq \rho/2$ as

$$\phi_n = \sqrt{2}\sin n\pi x - \frac{\epsilon\, n^2}{\sqrt{2}}\left[\frac{(n+\rho)^2}{(n+\rho)^4 - n^4}\right]\sin(n+\rho)\pi x - \frac{\epsilon\, n^2}{\sqrt{2}}\left[\frac{(n-\rho)^2}{(n-\rho)^4 - n^4}\right]\sin(n-\rho)\pi x \tag{16}$$

If $n = \rho/2$, the last term in the series is secular and hence vanishes.

Using equations (7), (12), and (A-6), the eigenvalue solution λ_n is given in terms of the first-order correction. The result is expressed in terms of the dimensionless variables as

$$\lambda_n = n^2\pi^2\,[1 - (\epsilon/2)]^{1/2} \tag{17}$$

Equation (17) is valid only for the vibration mode $n = \rho/2$. For all other vibration states the first-order correction term is zero. The second-order perturbation solution of the eigenvalue is determined using the same techniques previously developed for finding the first-order terms. Details of the formulation are outlined in Reference 1. The general eigenvalue solution λ_n of the second-order expansion is given as

$$\lambda_n = n^2\pi^2\left[1 - (\epsilon/2) - (\epsilon^2/4)\left[\frac{(n+\rho)^4}{(n+\rho)^4 - n^4}\right] - (\epsilon^2/4)\left[\frac{(n-\rho)^4}{(n-\rho)^4 - n^4}\right]\right]^{1/2} \tag{18}$$

The first-order correction term in equation (18) is equal to zero for the vibration modes where $n \neq \rho/2$. For the case defined by $n = \rho/2$, the last term in the second-order correction is specified to vanish (i.e., this term is secular from ϕ_{n1} solution). Notice that the eigenvalue λ_n simplifies to the case of a beam with uniform stiffness when the perturbation parameter ϵ is equal to zero.

The general behavior of a beam with periodic bending stiffness variation is given by equations (16) and (18). As the vibration state approaches the anomaly occurring at $n = \rho/2$, the eigenfunction solution deviates from a simple sine wave displacement curve to a mode shape comprising other harmonics. For the case when $n = \rho/2$, the eigenfunction returns to the sine wave shape. The eigenvalue solution responds in a similar nature. At $n = \rho/2$ a jump in the eigenvalue occurs, since for this mode the harmonics of the stiffness function are secular. Results of the closed-form perturbation analysis have been checked using finite element results and results of experimentation.

EIGENFUNCTIONS AND EIGENVALUES

Effects of the perturbation terms on the eigenfunction solution are exemplified in Figure 3. The magnitudes of the bracketed terms in equation (16) are plotted as a function of the ρ/n ratio. Two distinct ranges are apparent; $\rho/n < 1$ and near $\rho/n = 2$. In these ranges, the perturbation effects are the strongest. At $\rho/n = 2$, a vibration state is defined where the lengths of the vibration loops match the period of the stiffness function. Based on the stiffness definition given, this mode defines maximum stiffness at the nodes of the vibration loops and minimum stiffness at the antinodes. At $\rho/n = 1$, both the maximum and minimum stiffnesses occur at the node positions in alternating sequence along the span.

Figures 4 and 5 give eigenfunction comparisons of vibration modes in the general modal solution range. The vibration displacement amplitudes are normalized and plotted versus the normalized horizontal span coordinate (x/L). Recall, boundary conditions are simple support, and beam orientation at the supports is for maximum stiffness. Figure 4 plots eigenfunction solutions near the anomaly $\rho/n = 2$, where $\rho = 64$ and $\epsilon = 0.4$. The effect of periodic stiffness is to modulate the displacements of those vibration modes approaching the anomaly at $n = \rho/2$, or for this case mode 32. Similar displacement curves as patterned for modes 31 and 33 are characteristic of all vibration modes near the anomaly. Close examination of the eigenfunctions reveals that the node (or antinode) locations are adjusting themselves along the span, and the longer vibrating loops result in lower midloop displacement amplitudes. Apparently, the beam attempts to minimize the elastic strain energy stored within the dynamic span by adjusting the lengths of the vibrating loops until the same average bending stiffness exists across each individual loop. Equalizing the loop stiffnesses may require the loops to have different lengths depending on the vibration mode, and a longer loop has greater mass. An equal partitioning of potential energy and thus kinetic energy between each of the loops results in lower vibration amplitudes for the longer vibrating loops. Loop stiffness calculations verify this reasoning.

At the anomaly, modulation in the mode shape disappears, since for this case the individual loop stiffnesses are equal (i.e., the lengths of the vibration loops match the period of the stiffness function). The same basic reasoning holds true for the case where $\rho/n = 1$ (Fig. 5). Here also, the mode shape is sinusoidal – no modulation; and average loop stiffnesses are equivalent with maximum and minimum values defining the nodes of each loop. Additional cases identified in Figure 5 are for small and large values of ρ/n. As ρ/n approaches zero, perturbations in the mode shape increase. At the opposite end of the scale, where the ρ/n parameter goes to infinity, the displacement shape is sinusoidal. Figure 6 characterizes the eigenfunction solution as a function of the perturbation parameter, ϵ. As the magnitude of Epsilon (i.e., the stiffness variation) increases, the modulation effects become more pronounced.

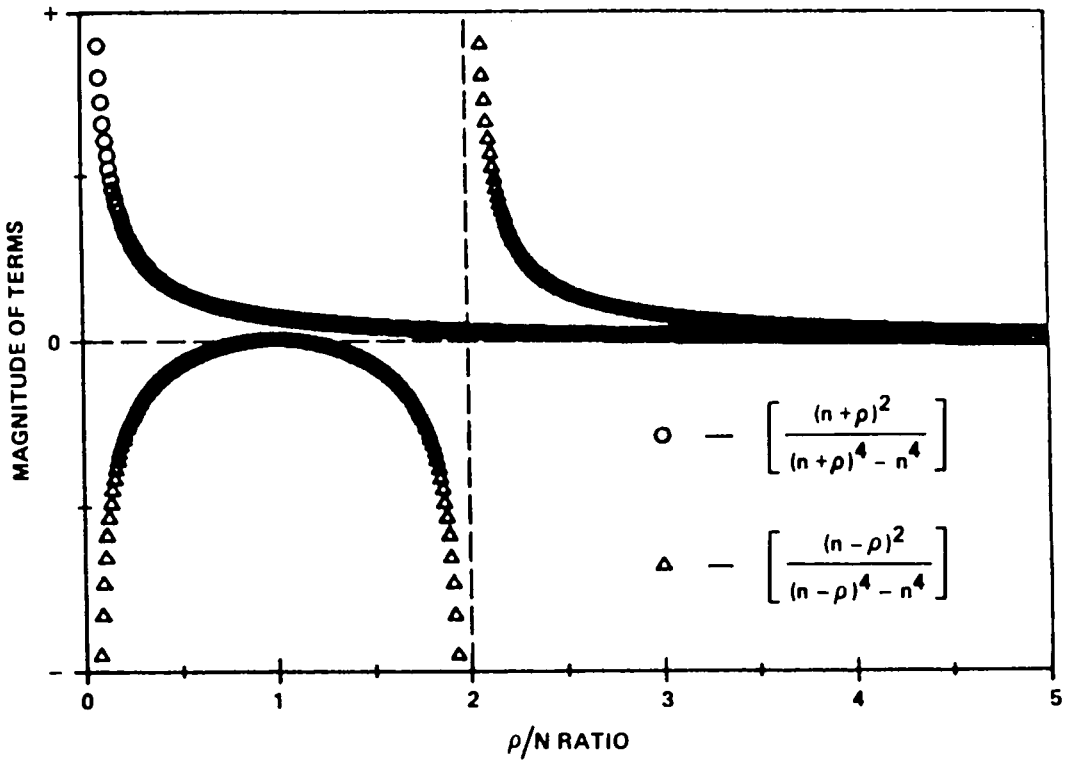

Figure 3. Effects of perturbation terms on eigenfunction.

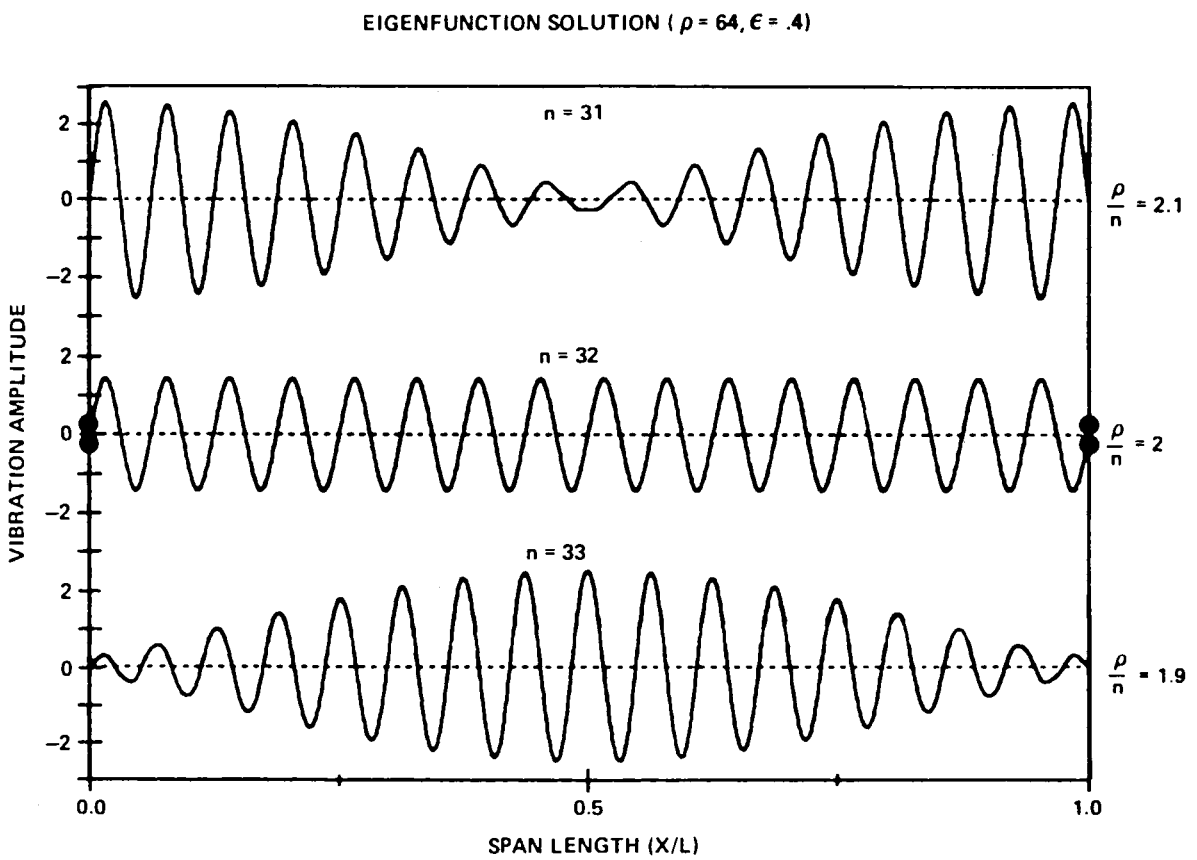

Figure 4. Eigenfunction solutions near the anomaly, ρ/n = 2.

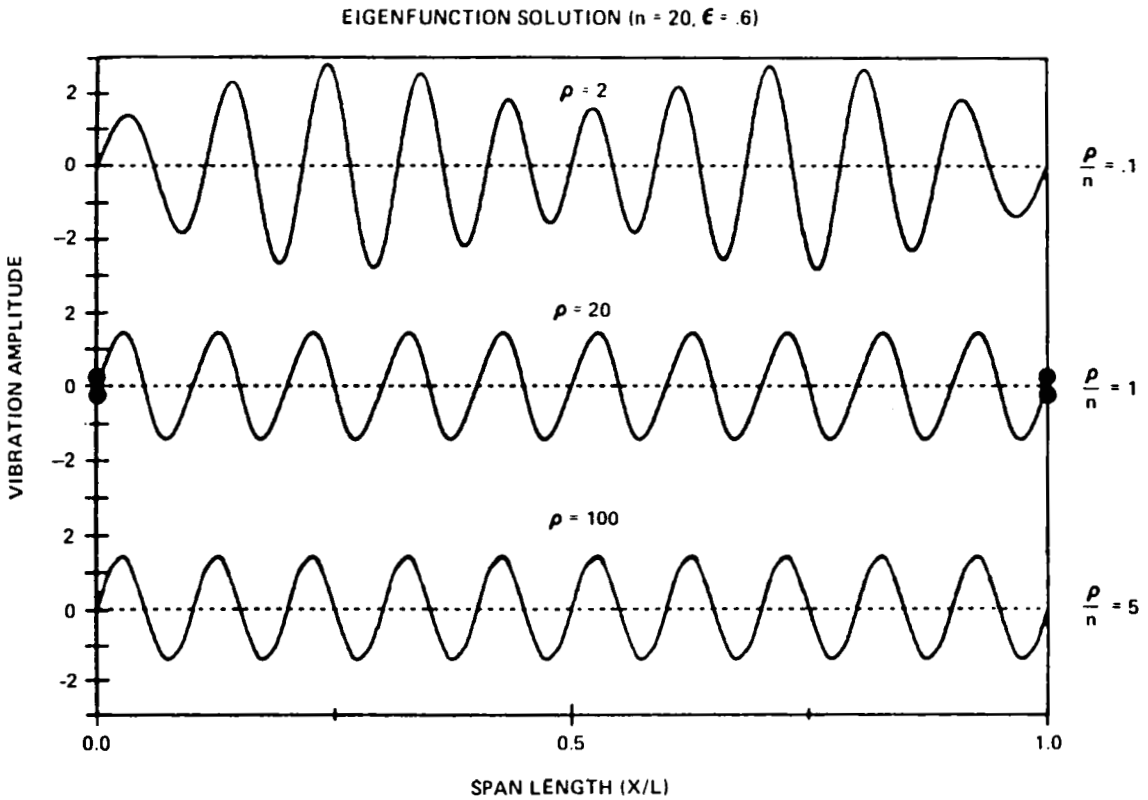

Figure 5. Eigenfunction solutions in characteristic ranges of vibration.

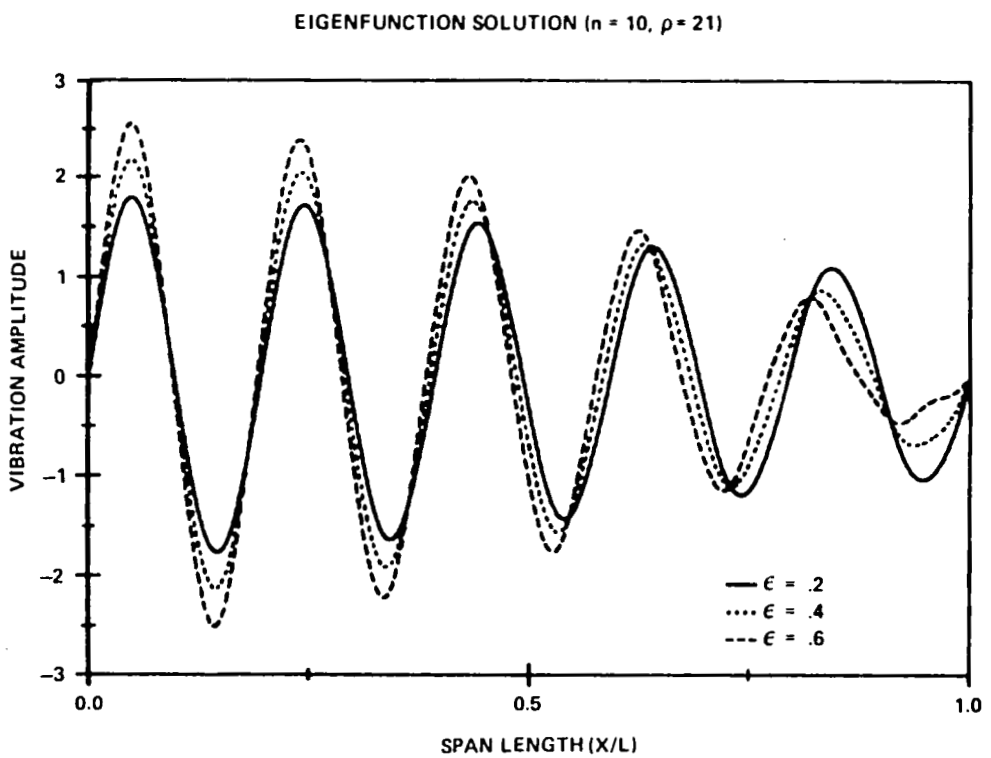

Figure 6. Eigensolution as a function of epsilon.

Eigenvalue solutions of the second order perturbation expansion for different cases of ρ are plotted in Figure 7 versus the respective vibration mode. At the anomaly, ρ/n = 2, a jump in the eigenvalue occurs. The intensity of the jump increases with ρ. Physically, the jump identifies a sharp change in frequency (or stiffness) between characteristic modes of vibration. Mathematically, the jump is equivalent to removing the secular nature of the stiffness function from the eigenfunction solution.

The closed-form perturbation expansion solution has been verified through comparisons with a finite element solution [1]. Although the findings are not formally documented herein, agreement between the analytical results is excellent. Some discrepancy does occur in the vicinity of the stiffness anomaly. This is apparently due to a sudden change in bending stiffness. Nevertheless, the qualitative picture remains the same. Two characteristic effects of periodic bending stiffness on dynamic response are determined: (1) periodic stiffness forces an anomaly in the system which results in a jump in the natural frequency, and (2) periodic stiffness acts to modulate the modal displacements in distinctive ranges of ρ/n. A qualitative explanation of the modulation and its effects on beam response is given in terms of energy principles.

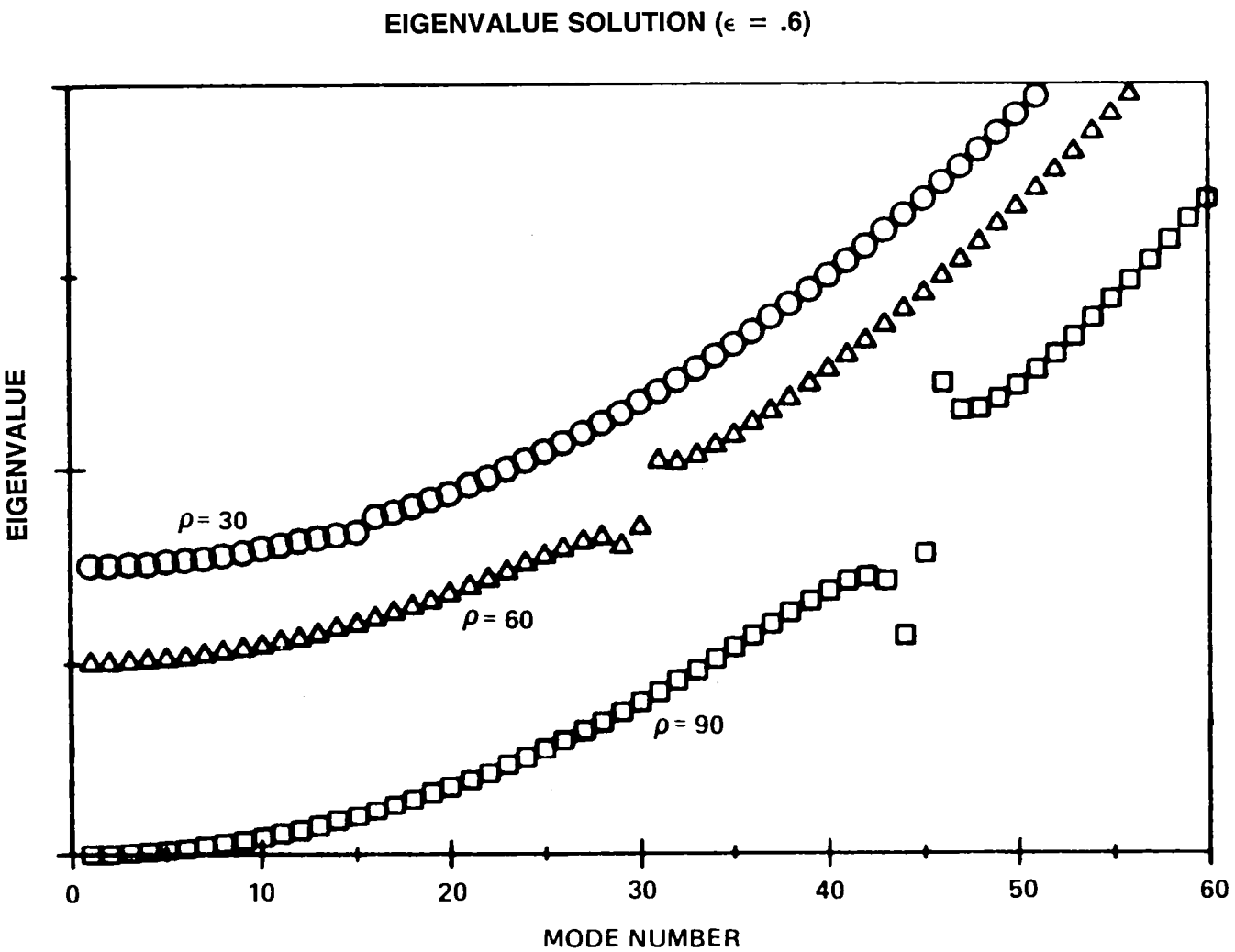

Figure 7. Eigenvalue solution for different cases of ρ.

EXPERIMENTAL VERIFICATION

A series of tests were designed to investigate the dynamic response of beam type systems which have a periodic variation in bending stiffness. A stiff-string structure, known as a twisted-paired conductor, was the test candidate in the program. Table 1 summarizes the experimental test parameters and Figure 8 shows a photograph of the test span. Periodic variation in diameter profile of the twisted-paired conductor is compared to the uniform diameter of a standard conductor design. In stiff-string systems, elastic strain energy is stored in tension and bending. If tension is constant along the span, then tension has minimal effect toward equalizing the variable flexural stiffness of the vibrating loops [1]. In other words, tension effects do not mask the effects of stiffness variation.

Experimental data are compared to the finite element results for free vibration since the fixed boundary conditions are applicable. This type of boundary support keeps the end losses to a minimum. Internal damping of the conductor was also reduced by applying a high tension line force. The procedure of minimizing conductor system damping is necessary; higher harmonics are difficult to excite if mechanical damping is significant. The testing program used forced vibration response to study free vibration. If the span is tuned properly to a single natural frequency, contributions from all other harmonics are minimal. The vibration exciter unit was positioned near the span center to eliminate even harmonics from the general response. The added mass of the moving shaker element and span attachment fixture resulted in a shortening of the drive loop, and thus a lowering of its vibration amplitude. No attempt was made to decouple shaker mass from the conductor span. A V-scope attached to the center of each vibrating loop was used to measure the midloop amplitude displacements. The device is inexpensive and its accuracy is remarkable at 0.01 in.

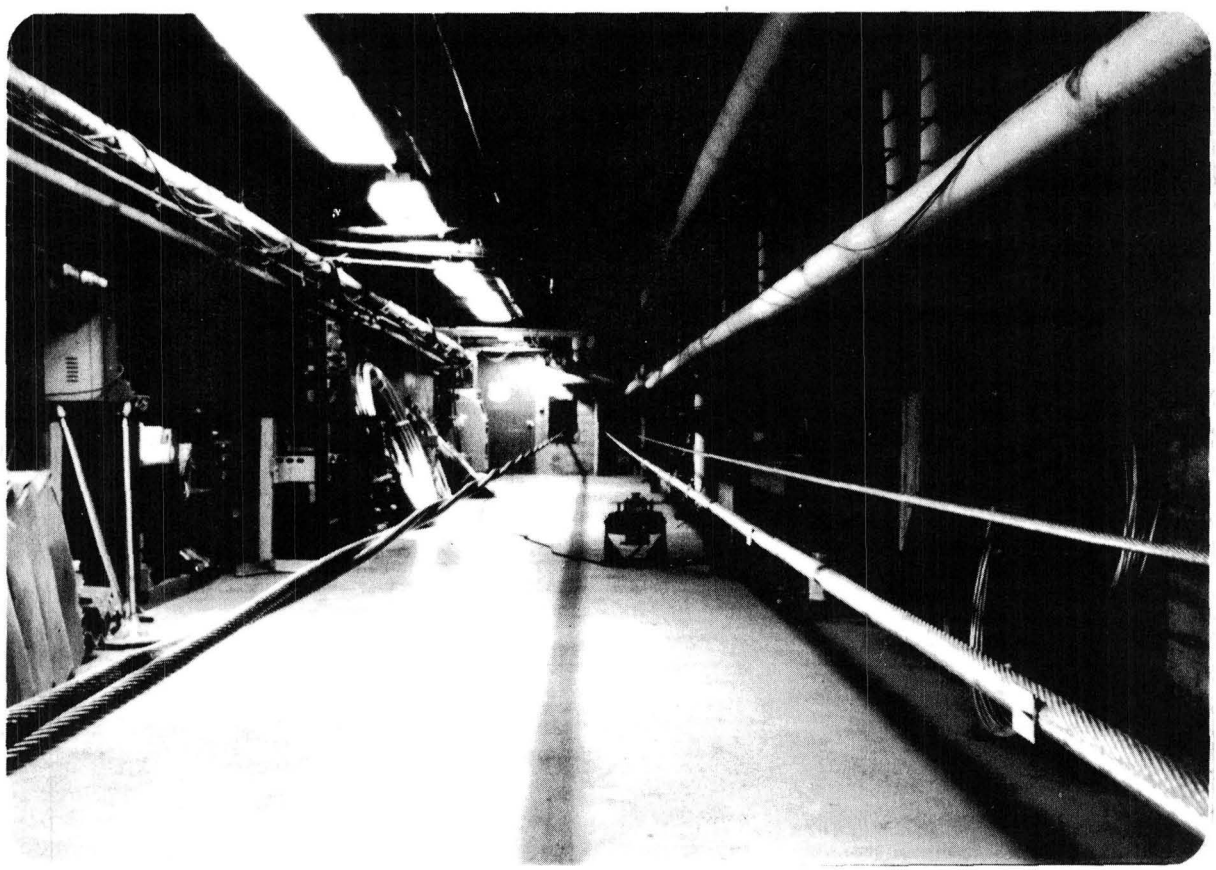

Figure 8. Photograph of test span, twisted-paired conductor.

TABLE 1. SUMMARY OF TEST PARAMETERS

Span Tension, T	9919 lb
Span Length, L	144 ft
Period of Stiffness, L_T	4.5 ft
Stiffness Parameter	64
Bending Stiffness, lb-in.^{2}	
EI min.	386785
EI max.	2185202
EIa	1285993.5
Stiffness Parameter	0.7
Weight, lbs/ft	1.164
Fixed Boundary	—
Support Orientation Stiffness	EI min.
Drive Unit	MB C-11c Shaker
Weight of Moving Element of Shaker	2.20 lb
Weight of Span Attachment Fixture	0.77 lb

Typical test results are presented in Figures 9 and 10. Finite element displacement amplitudes are normalized using a method previously outlined [equation (8)]. Experimental amplitudes are normalized to one of the measured values – chosen in arbitrary fashion. Span length is used to nondimensionalize the horizontal coordinate. Since mode shape is symmetric about the span center, data results are shown only for half the span. Figure 9 gives the comparison for mode 27, an eigenfunction near the anomaly in the system occurring at $n = 32$. Although the comparison is not exact, the modulation in the eigenfunction response is proved physically to exist. The same general results are reported for all other modes near the anomaly. Figure 10 compares the experimental and analytical data of mode 19, an eigenfunction well removed from the stiffness anomaly. Agreement between the analytical and test data is excellent and the mode shape is sinusoidal. Some discrepancy does occur in the node positions near the drive location where the measured loop lengths are shorter. Shacker attachment changes the stiffness and mass of the drive loop. Shifting of the nodes tends to compensate for these effects.

CONCLUSIONS

The dynamic response of beams with periodic stiffness contrasts significantly with the vibration behavior of standard beams. Linear vibration theory was used to develop a stiffness model and characterize response. Using a perturbation expansion, a closed-form solution of free vibration was formulated for the case of simple supports and periodic stiffness variation. The technique worked exceptionally well when the stiffness parameter was slowly varying. Applications of variable tension, mass, and area are natural extensions of the theory. The main conclusions are summarized below.

1. Periodic bending stiffness forces an anomaly in the system which corresponds to the vibration state where the loop length matches the period of the stiffness function. Physically, the anomaly denotes the vibration mode for which loop stiffness changes most rapidly. The result is a jump in natural frequency. The perturbation solution loses some accuracy for those vibration modes near the anomaly; however, the qualitative characteristics of the response remain the same.

2. The stiffness parameter acts to modulate the modal displacements in two distinct ranges of vibration: $\rho/n < 1$ and near $\rho/n = 2$. Experimental evidence is presented which supports these findings.

3. Modulation in modal displacements is explained in terms of energy principles. The beam attempts to minimize the elastic strain energy stored within a dynamic span by adjusting the lengths of the vibrating loops until the same average bending stiffness exists across each individual loop. Equalizing the loop stiffnesses may require the loops to have different lengths depending on the vibration mode, and a longer loop has greater mass. An equal partitioning of potential energy and thus kinetic energy between each of the loops results in lower vibration amplitudes for the longer vibrating loops.

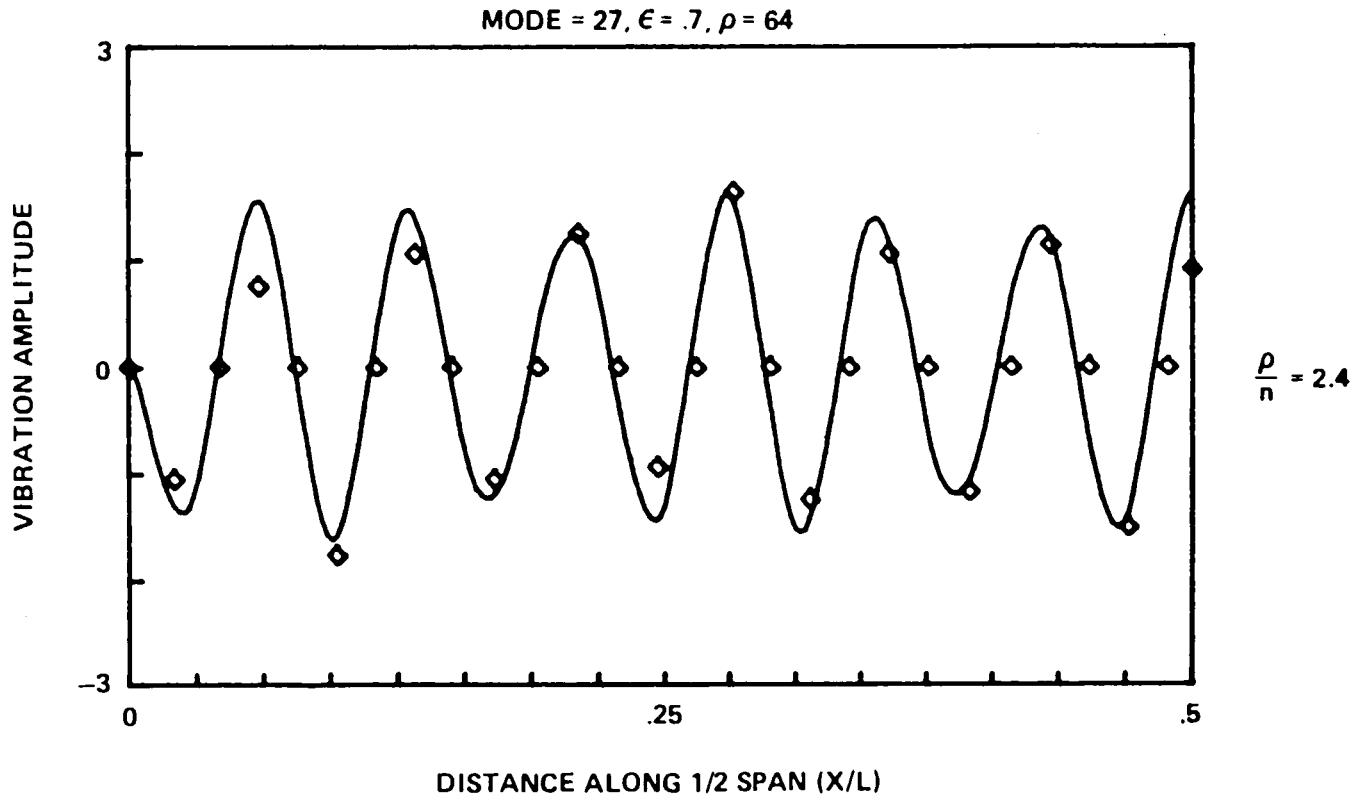

Figure 9. Comparison of eigensolution with experimental test valves, mode 27.

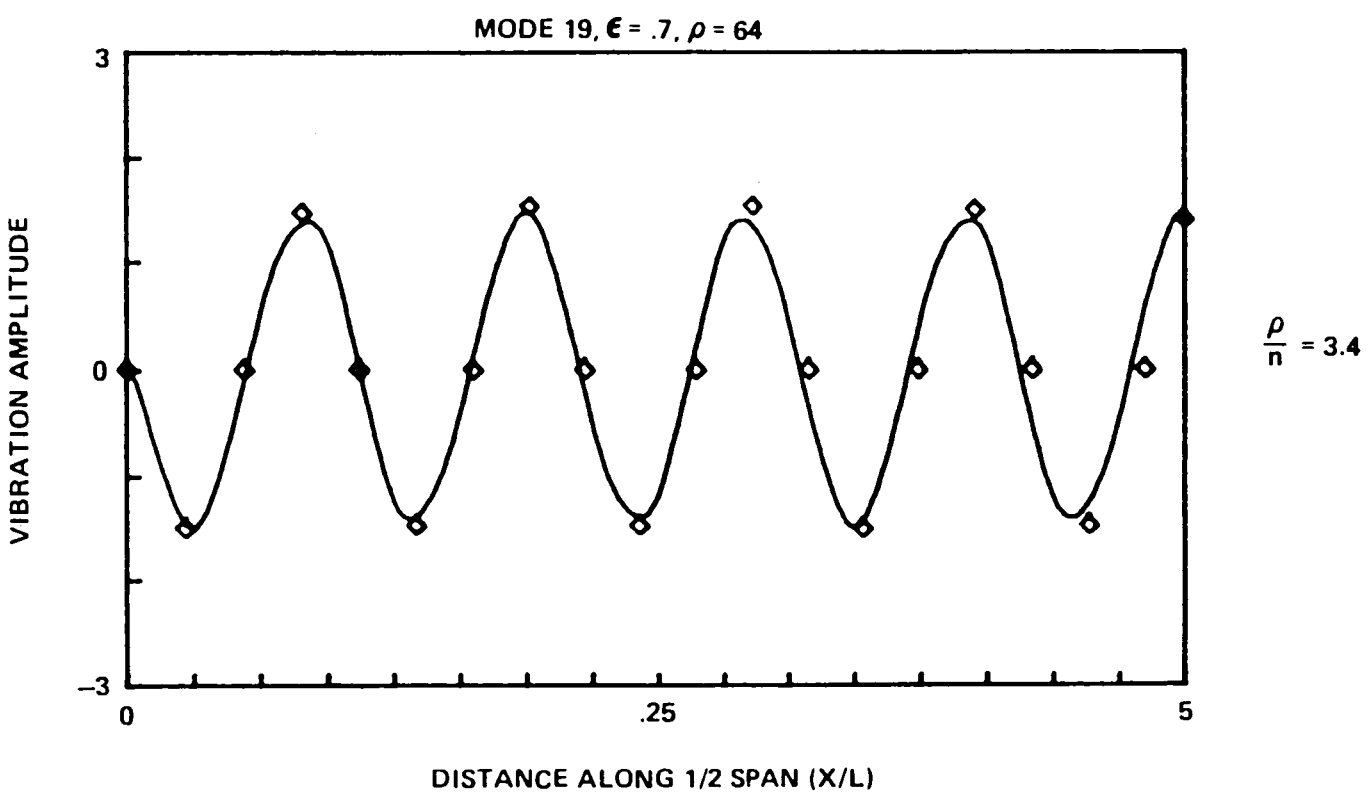

Figure 10. Comparison of eigensolution with experimental test valves, mode 19.

APPENDIX A

FIRST-ORDER CORRECTION TO THE PERTURBATION SOLUTION

The first step in the first-order correction to the perturbation solution is to substitute the zeroth-order solution, equations (6) and (7), into equation (14). After taking the appropriate derivatives and using trigometric identities, the result simplifies to

$$\phi_{n1}{}^{iv} - n^4 \pi^4 \phi_{n1} = \sqrt{2}\{\lambda_{n1} \sin n\pi x - (n^2\pi^4/2)(n+\rho)^2 \sin[(n+\rho)\pi x] - (n^2\pi^4/2)(n-\rho)^2 \sin[(n-\rho)\pi x]\} \quad \text{(A-1)}$$

Next, assume that the solution ϕ_{n1} can be expressed as a linear combination of the zeroth-order eigenfunctions ϕ_{n0};

$$\phi_{n1} = \sqrt{2} \sum_{m=1}^{\infty} A_{nm} \sin m\pi x \quad \text{(A-2)}$$

This solution satisfies the boundary conditions on ϕ_{n1}. Taking derivatives and substituting into equation (A-1) yields

$$\sum_{m=1}^{\infty} \pi^4(m^4 - n^4) A_{nm} \sin m\pi x = \lambda_{n1} \sin n\pi x - (n^2\pi^4/2)(n+\rho)^2 \sin(n+\rho)\pi x$$
$$- (n^2\pi^4/2)(n-\rho)^2 \sin(n-\rho)\pi x \quad \text{(A-3)}$$

Multiplying equation (A-3) by $\sin k\pi x$ and integrating from 0 to 1 using the orthonormal property, equation (9), we obtain

$$\pi^4[k^4-n^4] A_{nk} = \lambda_{n1} \delta_{nk} - n^2\pi^4 (n+\rho)^2 \int \sin(n+\rho)\pi x \sin k\pi x dx$$
$$- n^2 \pi^4 (n-\rho)^2 \int \sin(n-\rho)\pi x \sin k\pi x dx \quad \text{(A-4)}$$

If $k = n$, the left-hand side of equation (A-4) vanishes, hence

$$\lambda_{n1} = n^2 \pi^4 (n+\rho)^2 \int_0^1 \sin(n+\rho)\pi x \sin n\pi x dx$$
$$+ n^2\pi^4 (n-\rho)^2 \int_0^1 \sin(n-\rho)\pi x \sin n\pi x \, dx \quad \text{(A-5)}$$

The above integral expressions evaluate only when $n = \rho/2$, that is when the vibration mode number corresponds to the span length-stiffness function ratio ($n = L/Lt$). This is the anomaly that makes the periodic stiffness problem so interesting.

Equation (A-5) then calculates the eigenvalues of the first-order expansion as

$$\lambda_{n1} = -n^4\pi^4/2 \qquad \text{when} \quad n = \rho/2 \tag{A-6}$$

Note, $\lambda_{n1} = 0$ for all other values of n. The above condition removes the secular terms from the solution when $k = n$. If $k \neq n$, then equation (A-4) simplifies to

$$A_{nk} = [-n^2(n+\rho)^2/(k^4-n^4)] \int_0^1 \sin(n+\rho)\pi x \sin k\pi x dx$$
$$+ [-n^2(n-\rho)^2/(k^4-n^4)] \int_0^1 \sin(n-\rho)\pi x \sin k\pi x \, dx \tag{A-7}$$

Because of the stipulations on the parameter ρ, Ank in equation (A-7) calculates non-zero values only for the two cases: (1) $k = n + \rho$ and (2) $k = n - \rho$, where $n \neq \rho/2$. The general solution of the eigenfunction ϕ_{n1}, given by equation (A-2), is finally expressed as

$$\phi_{n1} = \frac{-n^2(n+\rho)^2}{\sqrt{2}\,[(n+\rho)^4-n^4]} \sin(n+\rho)\pi x$$

$$- \frac{n^2(n-\rho)^2}{\sqrt{2}\,[(n-\rho)^4-n^4]} \sin(n-\rho)\pi x + \sqrt{2}A_{nn} \sin n\pi x \tag{A-8}$$

For the vibration mode corresponding to $n = \rho/2$, the second term in equation (A-8) vanishes, since for this mode it is secular in nature. Keeping this in mind we can say that the ϕ_{n1} solution is valid for all vibration states where $\rho = 1,2,3,\ldots$. The coefficient A_{nn} is determined by the normalizing function

$$\int_0^1 2\,\phi_{n0}\,\phi_{n1}\,dx = 0 \tag{A-9}$$

For this case, A_{nn} is calculated to equal zero.

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16. ABSTRACT

A detailed dynamic analysis is performed of a vibrating beam with bending stiffness periodic in the spatial coordinate. Using a perturbation expansion technique the effects of system parameters on beam response are explored. It is found that periodic stiffness acts to modulate the modal displacements from the characteristic shape of a simple sine wave. The results are verified by a finite element solution and through experimental testing.

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