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Prepared by

H. A. Hassan
Project Coordinator
Department of Mechanical and Aerospace Engineering
North Carolina State University
Raleigh, North Carolina 27695-7910
The major accomplishments during this period are summarized in the enclosed abstract which was submitted for the Aerospace Sciences meeting. Other approaches to turbulence modeling are being pursued.
A One Equation Turbulence Model for Transonic Airfoil Flows

R. A. Mitcheltree*
North Carolina State University, Raleigh, North Carolina

M. D. Salas†
NASA Langley Research Center, Hampton, Virginia

H. A. Hassan‡
North Carolina State University, Raleigh, North Carolina

Abstract

A one equation turbulence model based on the turbulent kinetic energy equation is presented. The model is motivated by the success of the Johnson-King model and incorporates a number of features uncovered by Simpson’s experiments on separated flows. Based on the results obtained, the model duplicates the successes of algebraic models in attached flow regions and outperforms the two equation models in detached flow regions.

Introduction

This work is the first phase of an ongoing research effort designed to develop turbulence models for separated turbulent flows over transonic airfoils. It

*Research Assistant, Mechanical and Aerospace Engineering, Student Member AIAA.
†Head, Theoretical Aerodynamics Branch, Transonic Aerodynamics Division, Associate Fellow AIAA.
‡Professor, Mechanical and Aerospace Engineering, Associate Fellow AIAA.
is generally accepted that algebraic turbulence models are not suited to
describe non-equilibrium turbulent flows. This was clearly brought out by
the extensive work of Simpson and his associates (see reference 1 for a
summary). The next logical step is to use a higher order closure model.
Unfortunately, the work of Patel, Rodi and Scheurer [2] showed that most of
the $k-\epsilon$ models and related two equation models are unable to predict some
simple flows. Moreover, the work of Coakley [3] showed that predictions
of two equation models are not much better than algebraic models in the
presence of separation.

The recent model of Johnson and King [4], which is based on a simplified
version of the turbulent kinetic energy equation, preformed well in regions
where non-equilibrium turbulent effects are important. However, it did
not always preform well in regions where equilibrium turbulent flow exists.
This negative result does not diminish the significance of their contribution.
Rather, it suggests that more physics exists in the turbulent kinetic energy
equation that is yet to be explored and exploited.

Because of the above, an effort was undertaken to develop a one equa-
tion turbulence model based on the turbulent kinetic energy equation for
both attached and detached flows. Simpson made two relevant conclu-
sions. First, scaling within and outside separated flow regions is not the
same. Second, the eddy viscosity assumption is incorrect in the backflow
region of a separating boundary layer. Thus, the model developed retains
the accepted form of the turbulent kinetic energy equation but adjusts the
turbulent shear stress and the scaling depending on the local flow conditions.

Most of the previous work using one equation models employed the
same length scale for turbulent stresses and turbulent dissipation. This led
to numerical difficulties in the near wall region with the result that most
researchers lost interest in such a model. The difficulties can be avoided
by using two different length scales in the near wall region, one for the
turbulent stresses and one for turbulent dissipation. Specification of the
length scales requires close attention to scaling near the wall.

The Navier-Stokes code employed is based on that used by Swanson
and Turkel [5]. The turbulent kinetic energy equation is coupled to the
conservation equations and a four-stage Runge-Kutta time stepping scheme
is employed. The Reynolds averaged Navier-Stokes equations that use the
Johnson-King model or higher order closure models cannot be started from
a uniform flow condition. The usual procedure is to use an initial solution obtained from a flow solver that incorporates an algebraic turbulence model. This inefficient starting procedure is not necessary in the present work.

**Approach**

The equations governing turbulent flow past airfoils are the Reynolds averaged conservation equations of mass, momentum and energy and the turbulent kinetic energy equation. When Favre's mass averaging and indicial notation are used the conservation equations are:

\[ \rho_{,t} + (\rho \tilde{u}_j)_{,j} = 0. \]  \hspace{1cm} (1)

\[ (\rho \tilde{u}_i)_{,t} + \left[ \rho \tilde{u}_j \tilde{u}_i + \delta_{ij} \tilde{P} - \left( \tilde{\tau}_{ij} - \rho u_i^n u_j^n \right) \right]_{,j} = 0. \]  \hspace{1cm} (2)

\[ (\rho \tilde{E})_{,t} + \left[ \tilde{u}_j (\rho \tilde{E} + \tilde{P}) + \tilde{q}_j + \rho u_j^n h^n - \tilde{u}_i \left( \tilde{\tau}_{ij} - \rho u_i^n u_j^n \right) \right]_{,j} = 0 \]  \hspace{1cm} (3)

where

\[ \tilde{\tau}_{ij} = \mu \left[ \tilde{u}_{i,j} + \tilde{u}_{j,i} - \frac{2}{3} \delta_{ij} \tilde{u}_{m,m} \right] \]  \hspace{1cm} (4)

\[ \tilde{q}_j = -\lambda \tilde{T}_{ij} \]  \hspace{1cm} (5)

\[ \tilde{P} = \tilde{\rho} (\gamma - 1) \left[ \tilde{E} - \frac{1}{2} \tilde{u}_i \tilde{u}_i \right] \]  \hspace{1cm} (6)

where \( \rho \) is the density, \( \tilde{u}_i \) is the mean velocity in the direction of \( x_i \), \( \tilde{E} \) and \( h \) are the total energy and enthalpy per unit mass, \( \tilde{P} \) is the mean pressure. \( \mu, \lambda \) and \( \tilde{T} \) are respectively, the molecular viscosity and conductivity and temperature. In attached flow regions, the turbulent stress and heat flux are determined from the eddy viscosity (Bousinessq) approximation.

\[ \overline{\rho u_i^n u_j^n} = -\mu_t \left[ \left( \frac{\partial \tilde{u}_j}{\partial x_i} + \frac{\partial \tilde{u}_i}{\partial x_j} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \tilde{u}_m}{\partial x_m} \right] + \frac{2}{3} \delta_{ij} \tilde{\rho} k \]  \hspace{1cm} (7)

\[ \overline{\rho u_i^n h^n} = \frac{\mu_t}{Pr_t} \left( \frac{\partial \tilde{h}}{\partial x_i} \right) \]  \hspace{1cm} (8)
Pr is the turbulent Prandtl number. \( \mu_t \) is the eddy viscosity and \( \tilde{k} \) is the mean turbulent kinetic energy per unit mass.

\[
\dot{\rho}\tilde{k} = \frac{1}{2} \rho u''_i u''_k
\]  

(9)

For detached flow the above relations are replaced by correlations obtained from Simpson's experimental data.

The modeled turbulent kinetic energy equation is;

\[
(\dot{\rho}\tilde{k}) = \left[ \dot{\rho}\tilde{u}_j \tilde{k} - \nu (\dot{\rho}\tilde{k}) \right]_{ij} - c_k \frac{\tilde{k}}{\rho} \rho u''_i u''_j (\dot{\rho}\tilde{k})_{ii} = -\rho u''_i u''_j \tilde{u}_{ij} - \rho \epsilon
\]  

(10)

where \( \epsilon \) is the turbulent kinetic energy dissipation rate.

\[
\epsilon = \frac{k^{\frac{3}{2}}}{\ell_c}
\]  

(11)

c_k is a constant (0.1). \( \ell_c \) is the dissipation length scale and \( \nu \) is the molecular kinematic viscosity. In a one equation turbulence model the eddy kinematic viscosity is given by

\[
\nu_t = C_\mu \sqrt{\tilde{k} \ell_\mu}
\]  

(12)

where \( C_\mu \) is a constant and \( \ell_\mu \) is the turbulent length scale.

Simpson emphasized the importance of normal turbulent stresses in separating flows regions. Because of this, the governing equations retain all normal stresses.

**The Length Scales**

To complete the formulation of the model, the length scales must be defined. The expressions are summarized here and a complete derivation will be given in the paper. All variables are non-dimensionalized with the chord being the characteristic length scale and \( (P_\infty/\rho_\infty) \) the characteristic velocity. Letting

\[
L_c = C_{t} y \left[ 1 - \exp \left( \frac{-R_s \sqrt{k} \nu}{A_s} \right) \right]
\]

\[
L_\mu = C_{t} y \left[ 1 - \exp \left( \frac{-R_s \sqrt{k} \nu}{A_p} \right) \right]
\]

\[
F(y) = y \left[ 1 - \exp \left( \frac{-R_s \sqrt{k} \nu}{A_p} \right) \right]
\]
then

\[ \ell_c = \min(L_c, y_{\text{max}}) \]
\[ \ell_\mu = \min(L_\mu, y_{\text{max}}) \]

where \( y_{\text{max}} \) is the value of \( y \) at which \( F(y) \) is maximum. \( \omega \) is the vorticity and \( R_e \) is the Reynolds number. The constants in the definition are

\[ C_l = \kappa C_\mu^{-1/4} \]
\[ C_\mu = 0.09 \]
\[ \kappa = 0.41 \]
\[ A_\varepsilon = 2C_l \]
\[ A_\mu = 76. \]
\[ A_F = 47.47 \]

Results and Discussion

The results presented here are for a flat plate at zero angle of attack and for an NACA 0012 airfoil at a variety of Mach and Reynolds numbers and angles of attack. Fig. 1 shows flat plate results for a free stream Mach number, \( M_\infty \), of 0.5, \( R_e = 10^{6} \) and an angle of attack, \( \alpha \), of zero. As is seen in the figure, good agreement is indicated for the skin friction, Fig. 1a, velocity distribution in the near wall region, Fig. 1b, and the one seventh power law, Fig. 1c.

The next set of calculations are for an NACA 0012 airfoil. All calculations employed a 160 X 79 C-grid. The normal spacing of the first point off the wall was \( 2 \times 10^{-5} \). The first case considered is that for \( M_\infty = 0.5, R_e = 2.89 \times 10^{6} \) and \( \alpha = 0 \). Fig. 2(a) compares the pressure distribution with the measurements of Ref. 6. Fig. 2(b) compares the skin friction coefficient with a boundary layer calculation given in Ref. 7. As is seen in the figure, good agreement is indicated. The difference in the skin friction coefficients at the leading and trailing edges are a result of much finer resolution in the boundary layer calculation. Another comparison with the
experiment of Ref. 6 is given for $M_\infty = 0.756$, $\alpha = 0$, and $R_e = 4.01 \times 10^6$. Again excellent agreement with experiment is obtained.

All of the above cases are for $\alpha = 0$. Fig. 4 compares predictions of this theory with the experiment of Ref. 8 for $\alpha = 1.49^\circ$ and $R_e = 9.6 \times 10^6$. As is seen in the figure, both upper and lower surface pressure distributions are well predicted by the present one equation model.

Simpson noted that in the backflow region of a separating boundary layer, the eddy viscosity assumption is incorrect. Because of this, Simpson's measurements are used to develop correlations for the turbulent stresses in the backflow region. Unfortunately, there is scatter in the data. Because of this, the results presented in Fig. 5 ($M_\infty = 0.799$, $R_e = 9.6 \times 10^6$, $\alpha = 2.26^\circ$) should be considered preliminary.

Details of the separated models together with comparisons for the RAE 2822 and the Cast 10 airfoils will be presented in the paper. Based on the results obtained to date, it is concluded that one equation models, properly formulated, are capable of out performing two equation models at reduced computational cost.

References


Acknowledgments

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Figure 1a. Turbulent flat plate flow ($M = 0.5, Re = 10^6$): comparison of skin friction distributions.

Correlation, $M_\infty \approx 0$.

$C_f = 0.0592 Re_x^{-0.2}$

○ One Eq. Model
Figure 1b. Turbulent flat plate flow (M = 0.5, Re = 10^6):
comparison of velocity profiles.
Figure 1c. Turbulent flat plate flow (M = 0.5, Re = 10^6): comparison of near wall velocity profiles.
Figure 2a. Turbulent flow over a NACA 0012 airfoil (M = 0.5, Re = 2.89x10^6): pressure distributions.
Figure 2b. Turbulent flow over an NACA 0012 airfoil (M = 0.5, \text{ Re } = 2.89 \times 10^6): skin friction distributions.
Figure 3. Turbulent flow over an NACA 0012 airfoil (M = 0.756, \( \alpha = 0^\circ \), Re = 4.01x10^6): pressure distributions.
Figure 4. Turbulent flow over an NACA 0012 airfoil ($M = 0.7$, $\alpha = 1.49^\circ$, $Re = 9 \times 10^6$): pressure distributions.
Figure 5a. Turbulent flow over an NACA 0012 airfoil (\( M = 0.799, \alpha = 2.26^\circ, Re = 9 \times 10^6 \)): pressure distributions
Figure 5b. Turbulent flow over an NACA 0012 airfoil (M = 0.799, \( \alpha = 2.26^\circ \), Re = 9.1x10^6): pressure distributions