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ANALYSIS OF A CLOSED-KINEMATIC CHAIN ROBOT MANIPULATOR

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SUMMARY

This report presents the research results from the research grant entitled: "Active Control of Robot Manipulators", sponsored by the Goddard Space Flight Center (NASA), under Grant Number NAG-780, obtained between January 1, 1988 and June 30, 1988.

In this report, we concern with a class of robot manipulators built based on the closed-kinematic chain mechanism (CKCM). This type of robot manipulators mainly consists of two platforms, one is stationary and the other moving, which are coupled together through a number of in-parallel actuators. Using spatial geometry and homogeneous transformation, we derive a closed-form solution for the inverse kinematic problem of a six-degree-of-freedom manipulator, built to study robotic assembly in space. Iterative Newton Raphson method is employed to solve the forward kinematic problem. Finally the equations of motion of the above manipulator are obtained by employing the Lagrangian Method. Study of the manipulator dynamics is performed using computer simulation whose results show that the robot actuating forces are strongly dependent on the mass and centroid locations of the robot links.

1. INTRODUCTION

The in-parallel actuated manipulator whose design is based on the concept of *closed-kinematic chain mechanism* (CKCM) has recently attracted considerable research interest [3]-[11]. Because of its closed mechanism, this group of manipulators has several advantages over the conventional open-chain type. Manipulators with CKCM have smaller positioning errors due to the non-cantileverlike configuration and consequently have greater positioning ability, as compared to those with the open-chain type. Furthermore, they provide higher force/torque and greater payload handling capability for the same number of actuators. CKCM concept has been widely applied to design several industrial robots [10]-[11].

Implementation of the CKCM concept was first appeared in the design of the Stewart mechanism [1] consisting of two platforms driven by in-parallel actuators. After its introduction, the Stewart platform has been proposed in numerous robotic applications [2]-[4]. Yang and Lee [5] investigated the inverse kinematic and workspace problems of the Stewart platform. Unlike the case of the open-chain type, manipulators with in-parallel actuators possess closed-form solutions to its inverse kinematic problem as learned by many investigators. However, in most cases the forward kinematic problem must be solved iteratively using numerical methods. A general closed-form solution for this problem is not known at this time. Besides the forward kinematic problem, the dynamics of CKCM manipulators has also been an active research area. Due to its highly complicated mechanism, the dynamics of CKCM manipulators was not investigated as extensively as that of the open-chain type. Newton-Euler approach was employed by Do and Yang [6] to study the inverse dynamics of CKCM manipulators. Sugimoto [7] combined the Newton-Euler approach and the so-called "*Motor Algebra*" to analyze the kinematic and dynamical problems of manipulators driven by in-parallel

actuators. Applying the Lagrangian approach, the dynamics of a two-degree-of-freedom and a three-degree-of-freedom CKCM manipulators was derived by Nguyen and Pooran [8] and by Lee and Shah [9], respectively. For control purposes, the Lagrange-Euler approach is more attractive than the Newton-Euler approach because it provides closed-form dynamical equations in any selected coordinate system.

In this report, we concern with a closed-kinematic chain mechanism that was applied to build a six-degree-of-freedom robot at the *Goddard Space Flight Center* (NASA) to study potential robotic applications in the space station [10]. In order to obtain an effective control scheme for this robot, the dynamics and kinematics analysis of CKCM robot manipulators should be performed. This report is structured as follows: First, we present a closed-form solution to the forward kinematic problem of the above type of CKCM manipulators by applying spatial geometry and the concept of homogeneous transformations. Then using Newton-Raphson Method, an iterative solution is obtained for the forward kinematic problem. After that, the manipulator dynamics is derived by employing the Lagrangian approach. Finally, we present results of the computer simulation, performed to study the manipulator dynamics.

2. KINEMATIC ANALYSIS

Figure 1 presents the structure of a six-degree-of-freedom CKCM manipulator driven by 6 in-parallel actuators. The manipulator mainly consists of a stationary base platform and a moving platform coupled together through the actuators. Two coordinate systems are defined, the fixed Cartesian coordinate system {XYZ} whose origin is at Point B, centroid of the base platform, and the moving coordinate system {xyz} whose origin is at Point C, centroid of the moving platform. The assignment of the above coordinate

systems complies with the right hand rules. A_i and B_i are the attachment points of the ith actuator to the base and moving platforms, respectively. In this mechanism, there is a symmetric distribution of each pair of ball joints on the base and moving platform with respect to the three radii located at 120 degrees apart from each other on the platforms. The coordinates of the attachment points B_i with respect to {XYZ} and A_i with respect to {xyz} are given by

$$\underline{B}_i = [\ R\cos(\Lambda_i)\quad R\sin(\Lambda_i)\quad 0\ ]^T \tag{1}$$

and $$\underline{a}_i = [\ r\cos(\lambda_i)\quad r\sin(\lambda_i)\quad 0\ ]^T \tag{2}$$

where R and r are the radii of the fixed and movable platforms, respectively and $\underline{a}_i$ is the position vector of the ball joints at A_i with respect to {xyz}. Λ_i and λ_i are computed by

$$\Lambda_i = 60\ (i-1)\ \text{deg};\ \lambda_i = 60\ (i-1)\ \text{deg},\ \text{for } i = \text{odd}; \tag{3a}$$

$$\Lambda_i = \Lambda_{i-1} + \theta_B \text{deg};\ \lambda_i = \lambda_{i-1} + \theta_A\ \text{deg},\ \text{for } i = \text{even}, \tag{3b}$$

for $i=1,2,\ldots,6$.

In (3a) and (3b), θ_A and θ_B are the angles between two consecutive ball joints at A_i and B_i, respectively.

2.1 Inverse Kinematics

The inverse kinematic problem is formulated as to determine the required manipulator link lengths corresponding to a desired configuration (desired position and orientation) of the moving platform expressed in {XYZ}. The kinematic equations can be derived by considering the vector diagram for the ith actuator, illustrated in Fig. 2. All vectors are expressed with respect to {XYZ}. Vector $\underline{\ell}_i = [\ell_{ix}\ \ell_{iy}\ \ell_{iz}]^T$ can be expressed by

$$\underline{\ell}_i = \underline{p}_i - \underline{B}_i \tag{4}$$

However

$$\underline{p}_i = \underline{A}_i + \underline{d}. \tag{5}$$

Substituting (5) into (4) yields

$$\underline{\ell}_i = \underline{A}_i - \underline{B}_i + \underline{d} \tag{6}$$

In Eq. (6), vector A_i represents the coordinates of the attachment points A_i with respect to {XYZ}, while in Eq.(2) its coordinates is given by vector a_i with respect to {xyz}. If the orientation of Frame {xyz} with respect to Frame {XYZ} is specified by ZYX Euler angles (α β γ), meaning that the orientation of Frame {xyz} is identical to that of a frame resulted after first rotating about the Z-axis an angle α, then about the Y-axis an angle β, and finally about the X-axis an angle γ, then the corresponding rotation matrix is given by

$$T = \begin{bmatrix} C_\alpha C_\beta & C_\alpha S_\beta S_\gamma - S_\alpha C_\gamma & C_\alpha S_\beta C_\gamma + S_\alpha S_\gamma \\ S_\alpha C_\beta & S_\alpha S_\beta S_\gamma + C_\alpha C_\gamma & S_\alpha S_\beta C_\gamma - C_\alpha S_\gamma \\ -S_\beta & C_\beta S_\gamma & C_\beta C_\gamma \end{bmatrix} \tag{7}$$

where for compactness we have defined $S_\alpha \triangleq \sin(\alpha)$ and $C_\alpha \triangleq \cos(\alpha)$. Now since $\underline{A}_i = T\underline{a}_i$, we obtain from (6)

$$\underline{\ell}_i = T\underline{a}_i + \underline{d} - \underline{B}_i \tag{8}$$

In Equation (8), vectors $\underline{a}_i$ and $\underline{B}_i$ are known fixed vectors, the desired orientation and desired postion (the position of the origin) of the moving platform with respect to {XYZ} are contained in the rotation matrix T and the vector $\underline{d}$, respectively where

$$\underline{d} = [x_c \quad y_c \quad z_c]^T. \tag{9}$$

Therefore, using (8) Vector $\underline{\ell}_i$ can be solved for a desired configuration of the moving platform. After solving for $\underline{\ell}_i$, the required corresponding lenght of the ith actuator can be computed by

$$\ell_i = (\ell_{ix}^2 + \ell_{iy}^2 + \ell_{iz}^2)^{1/2} \tag{10}$$

Defining vector $\underline{\Phi} = [\Phi_1 \ \Phi_2 \ \Phi_3 \ \Phi_4 \ \Phi_5 \ \Phi_6]^T = [x \ y \ z \ \alpha \ \beta \ \gamma]^T$ containing the moving platform Cartesian configuration and vector $\underline{\ell} = [\ell_1 \ \ell_2 \ \ell_3 \ \ell_4 \ \ell_5 \ \ell_6]^T$

containing the six actuator lenghts as joint variables, the Jacobian matrix J of the manipulator which comprises the partial derivative of Φ_i with respect to the joint displacements ℓ_j is defined as

$$J = [\partial\Phi_i/\partial\ell_j]. \tag{11}$$

As we will see in the next section, there exist no closed form expressions of Φ in terms of ℓ because the forward kinematic problem must be solved iteratively. Therefore, the computation of the manipulator Jacobian matrix can be done by first using the inverse kinematic equation given by (10) to compute the inverse Jacobian as

$$J^{-1} = [\partial\ell_i/\partial\Phi_j], \tag{12}$$

and then inverting the result obtained in (12).

In order to prepare the dynamical analysis, we proceed to compute the angular velocity vector $\underline{\omega}$ defined by

$$\underline{\omega} = [\omega_X \quad \omega_Y \quad \omega_Z]^T. \tag{13}$$

where ω_i denotes the angular velocity of the moving platform around the i-axis of Frame {XYZ}, for i = X, Y, Z. Using the Euler Angles relationship and the rotation matrix in (7), it is simple to find the following:

$$\underline{\omega} = \begin{bmatrix} -S_\beta\dot{\alpha} + \dot{\gamma} \\ C_\beta S_\gamma\dot{\alpha} + C_\gamma\dot{\beta} \\ C_\beta C_\gamma\dot{\alpha} - S_\gamma\dot{\beta} \end{bmatrix}. \tag{14}$$

Differentiating $\underline{\omega}$, the angular acceleration is obtained by

$$\dot{\underline{\omega}} = \begin{bmatrix} -S_\beta\ddot{\alpha} - C_\beta\dot{\alpha}\dot{\beta} + \ddot{\gamma} \\ -S_\beta S_\gamma\dot{\alpha}\dot{\beta} + C_\beta C_\gamma\dot{\alpha}\dot{\gamma} + C_\beta S_\gamma\ddot{\alpha} - S_\gamma\dot{\beta}\dot{\gamma} + C_\gamma\ddot{\beta} \\ -S_\beta C_\gamma\dot{\alpha}\dot{\beta} - C_\beta S_\gamma\dot{\alpha}\dot{\gamma} + C_\beta C_\gamma\ddot{\alpha} - C_\gamma\dot{\beta}\dot{\gamma} - S_\gamma\ddot{\beta} \end{bmatrix} \tag{15}$$

2.2 Forward Kinematics

The forward kinematic problem is formulated as to find the actual position and orientation of the moving platform when a set of actuator lengths ℓ_1, ℓ_2, ..., ℓ_6 as joint variables are given. This situation occurs in the case of position feedback in which the length information provided by six joint position sensors is to be converted into the actual configuration of the moving platform. With the information available, we are faced with a problem of solving six simultaneous nonlinear equations for six unknowns, namely x, y, z, α ,β ,γ . At the present time, no symbolic solution for this problem exists. Therefore, a natural way is to seek an iterative approach to solve the above problem. Among currently available numerical methods, Newton-Raphson algorithm is recommended to treat the above problem, as proposed in [11]. This method consists of first defining a function of the unknowns, $f(\Phi)$ such that

$$f(\underline{\Phi}) = 0 \tag{16}$$

where $$\underline{\Phi} = [x \quad y \quad z \quad \alpha \quad \beta \quad \gamma\ ]^T \tag{17}$$

and $$f(\underline{\Phi}) = [f_1(\underline{\Phi}).....f_6(\underline{\Phi})]^T \tag{18}$$

and iteratively applying the formula

$$\underline{\Phi}_{k+1} = \underline{\Phi}_k - J_f^{-1} f(\underline{\Phi}_k) \tag{19}$$

where J_f, the Jacobian matrix of f is given by

$$J_f = [\partial f_i(\underline{\Phi})/\partial \Phi_j] \tag{20}$$

until certain accuracy is met.

To utilize the available information on the actuator lenght ℓ_i, provided from the joint position sensors, we define the function $f_i(\underline{\Phi})$ as

$$f_i(\underline{\Phi}) = \underline{\ell}_i^T \underline{\ell}_i - |\ell_i|_a^2 \tag{21}$$

where $|\ell_i|_a$ is the actual lenght of the ith actuator provided from the joint position sensor mounted on the ith joint. Obviously we can see that (21)

satisfies (16). Substituting (8) into (21) yields

$$f_i(\underline{\Phi})=(T\underline{a}_i+\underline{d}-\underline{B}_i)^T(T\underline{a}_i+\underline{d}-\underline{B}_i)-|\ell_i|_a^2 \qquad (22)$$

For a given set of initial conditions of Φ, first we compute (22) and then (19) using (20). Equation (19) is repeated until some predetermined convergence criterion (desired accuracy) is met.

Two drawbacks of the Newton-Raphson method are discussed now. First, it requires a significant amount of computational time. Second, the evaluation of the inverse Jacobian at each iteration may create the singularity problem. One solution to the first problem is to select an appropriate initial estimate based on a design model estimated at the desired moving platform configuration [12]. This will reduce the number of iterations and also save some computation time for convergence. The singularity problem can be avoided by modifying the robot trajectory planner.

3. DYNAMICAL ANALYSIS

The Lagrangian formulation describes the behavior of a dynamic system in terms of work and energy stored in the system rather than of forces and moments of the individual members involved [13]. Using this approach, the closed-form dynamical equations can be derived systematically in any selected coordinate system. The general form of Lagrangian equations of motion for an N degree-of-freedom manipulator is

$$\tau_j = d/dt(\partial L/\partial \dot{q}_i) - \partial L/\partial q_i \qquad (23)$$

where $$L = K-P \qquad (24)$$

q_i is the generalized coordinates, and τ_j is the generalized force or torque. K denotes the kinetic energy and P the potential energy . The total kinetic energy of the manipulator consists of the kinetic energy of the moving platform (translational and rotational motion with respect to the base coordinate system {XYZ}) and the kinetic energy of the links due to the

rotational motion of the link about the ball joints and the translational motion of the prismatic joints. Thus

$$K = \frac{1}{2} MV_c^2 + \frac{1}{2}\sum_{j=1}^{3} I_j\omega_j^2 + \frac{1}{2}\sum_{i=1}^{6} (m\ell_{ci}^2\dot{\theta}_i^2 + m_1\dot{\ell}_i^2) \qquad (25)$$

where M is the mass of the moving platform, m the total mass of Link i, and m_i the mass of the moving parts of Link i. The angular velocity and the moment of inertia of the moving platform about the j axis are denoted by ω_j and I_j, respectively, for j=x,y, and z. In (25) ℓ_{ci} is the distance between the center of each link and the ball joint at B_i. The velocity vector of Point C is given by

$$V_c = (\dot{x}_c, \dot{y}_c, \dot{z}_c) \qquad (26)$$

Similarly, the total potential energy of the manipulator consisting of the potential energy of the moving platform and the links is given by

$$P = Mgz_p + mg\sum_{i=1}^{6} \ell_{ci}\sin(\theta_i) \qquad (27)$$

where the angle formed between Link i and the base platform surface is represented by θ_i whose sine function is

$$\sin(\theta_i) = \ell_{iz}/\ell_i \ . \qquad (28)$$

Substituting (25) and (27) into (24) and applying (23), we obtain after some mathematical manipulations the following equations of motion for the CKCM robot manipulator:

$$\tau_j = \sum_{i=1}^{6}\left\{m\ell_{ci}^2\left[\ddot{\theta}_i \frac{\partial\dot{\theta}_i}{\partial\dot{\Phi}_j} + \frac{d}{dt}\left(\frac{\partial\dot{\theta}_i}{\partial\dot{\Phi}_j}\right)\dot{\theta}_i\right] + 2m\dot{\theta}_i\left(\frac{\partial\dot{\theta}_i}{\partial\dot{\Phi}_j}\right)\ell_{ci}\dot{\ell}_{ci}\right.$$

$$+ m_1\left[\ddot{\ell}_i\frac{\partial\dot{\ell}_i}{\partial\dot{\Phi}_j} + \frac{d}{dt}\left(\frac{\partial\dot{\ell}_i}{\partial\dot{\Phi}_j}\right)\dot{\ell}_i\right] - m\left[\ell_{ci}\left(\frac{\partial\ell_{ci}}{\partial\Phi_j}\right)\dot{\theta}_i^2 + \ell_{ci}^2\left(\frac{\partial\dot{\theta}_i}{\partial\Phi_j}\right)\dot{\theta}_i\right]$$

$$- m_1 \dot{\ell}_i \frac{\partial \dot{\ell}_i}{\partial \dot{\Phi}_j} + mg \frac{\partial \ell_{ci}}{\partial \Phi_j} \sin\theta_i + mg\ell_{ci} \cos\theta_i \frac{\partial \theta_i}{\partial \Phi_j} \Bigg\}$$

$$+\sum_{k=1}^{3} \Bigg\{ M\ddot{x}_k \frac{\partial \dot{x}_k}{\partial \dot{\Phi}_j} + I_k \Bigg[\dot{\omega}_k \frac{\partial \omega_k}{\partial \dot{\Phi}_j} + \frac{d}{dt} \Big(\frac{\partial \omega}{\partial \dot{\Phi}_j} \Big) \omega_k \Bigg]$$

$$- \Bigg[M\dot{x}_k \frac{\partial \dot{x}_k}{\partial \Phi_j} + I_k \omega_k \frac{\partial \omega_k}{\partial \Phi_j} \Bigg] \Bigg\} + Mg \frac{\partial x_3}{\partial \Phi_j} \qquad \text{for } j=1,2,\ldots,6 \tag{29}$$

where it is noted that (29) is derived for the generalized coordinates Φ_j. In (29) for compactness, the notation k is used to indicate Cartesian coordinates such that x_1, x_2 and x_3 stand for x, y, and z, respectively, and $\dot{x}_1$, $\dot{x}_2$, and $\dot{x}_3$ stand for $\dot{x}, \dot{y}$, and $\dot{z}$, respectively, etc. Similar notation is used for j = 1,2,...6 to indicate the Cartesian configuration $x, y, z, \alpha, \beta, \gamma$ such that for example, τ_1 and τ_6 stand for τ_x and τ_γ, respectively. It is also noted that Equation (29) represents the relationship between the generalized force/torqe τ_j applied to the manipulator and the corresponding generalized coordinates Φ_j.

In order to determine the actuating forces F_j along the links, the virtual work concept is used [13] where the relationship between the joint force F_j and the generalized force τ_j is found as

$$F = J^T \tau \tag{30}$$

where J is the Jacobian matrix of the manipulator whose inverse is given in (12). Eqn. (30) represents the actuating forces in the links of a six-degree-of-freedom manipulator having j actuators. In the above development, the joint friction is assumed to be negligible.

The above generalized equations were used to derive equations for kinematics and dynamics of a 2-degree-of-freedom CKCM robot manipulator.

Results showed that the derived equations for this special case are identical to those derived by applying the Lagrangian approach to the robot [14]. This fact verifies the correctness of the generalized equations.

4. SIMULATION STUDY

In order to gain some insight of the manipulator behavior, the kinematics and dynamics developed in previous sections are now studied using computer simulation. The software packages System Simulation Language (SYSL) developed by E^2 Consulting and Matlab developed by MatWorks, Inc were employed to simulate the robot motion and compute the required actuating forces. Using the inverse kinematics, we first computed the time history of the actuator lengths to track a semi-circle on the x-y plane of the base coordinate system, described by $x^2+y^2=(0.20)^2$ (see Fig. 3). The time history of the actuator lenghts were then applied to the dynamical equations to compute the required actuating forces. Study of the influence of the total mass and centroids of the links was performed and presented below where the system parameters used in this study were m=0.450 kg, m_1=.060 kg, M=4.500 kg, r=0.18 m, R=0.45 m, θ_a=10 and θ_b=110 degrees.

Case 1: *The mass of the links changed from .450 to 0 kg.*

The study results are recorded in Figure 4 where the solid line represents the profile of the actuating forces in Links 1 to 6 for m=0.450 kg and the dotted line for m = 0 kg. As the figure shows, link mass increase results in force increase which is expected in a dynamical system.

Case 2: *The link centroids described by lci=a+bli, where a represents the system parameter and b was changed from 3cm to 0cm.*

The study results for Links 1 to 6 are showed in Figure 5 where the solid

line represents the case when b=3cm (original position) and the dotted line for b=0cm. The results shows that moving the link centroids closer to the base frame reduces the required actuating forces.

Case 3: *The mass of the platform changed from 4.5 kg to 8 kg.*

The results are shown in Figure 6 where the solid line represents the profile of the actuating forces in Links 1 to 6 for M = 4.5 kg and the dotted line for M = 8 kg. It is seen that increasing the mass of the platform will increase the actuating forces.

5. CONCLUSIONS

In this report, the analysis of kinematics and dynamics of a six-degree-of-freedom in-parallel actuated manipulator was presented. This type of manipulator is built based on the CKCM that consists mainly of two platforms coupled via a number of in-parallel actuators. A closed-form solution was obtained for the inverse kinematic problem. Newton-Raphson iterative method was proposed to solve the forward kinematic problem. Using the Lagrangian approach, we derived a set of generalized dynamical equations that can be employed to derive equations of motion for CKCM manipulators having up to six degrees of freedom. Computer simulation was performed to study the effect of variation in link mass and link centroids. Simulation results showed that reduction in actuating forces can be achieved if the link mass is reduced and/or the centroid is moved closer to the base frame. Future research is directed to studying effective control schemes such as adaptive or learning for the trajectory and/or force control of the above CKCM robot manipulator.

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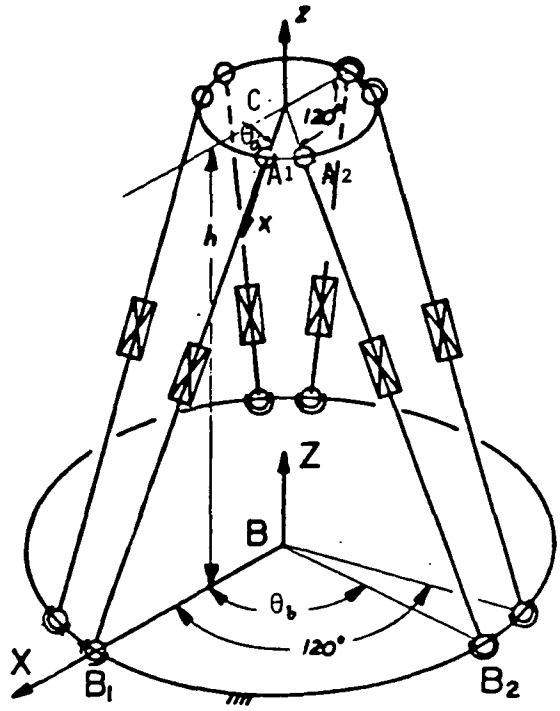

Fig. 1: The Six-degree-of-freedom in-parallel actuated manipulator

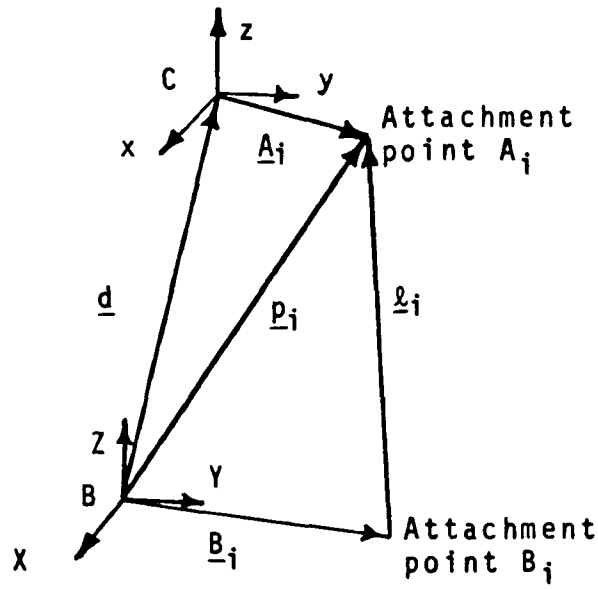

Fig. 2: Vector diagram for ith actuator

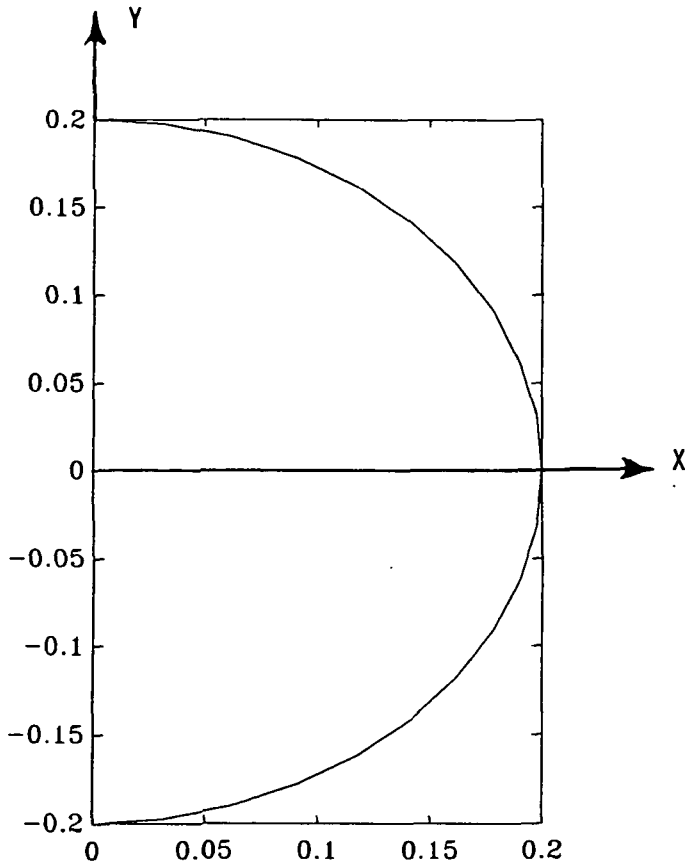

Fig. 3: The desired path

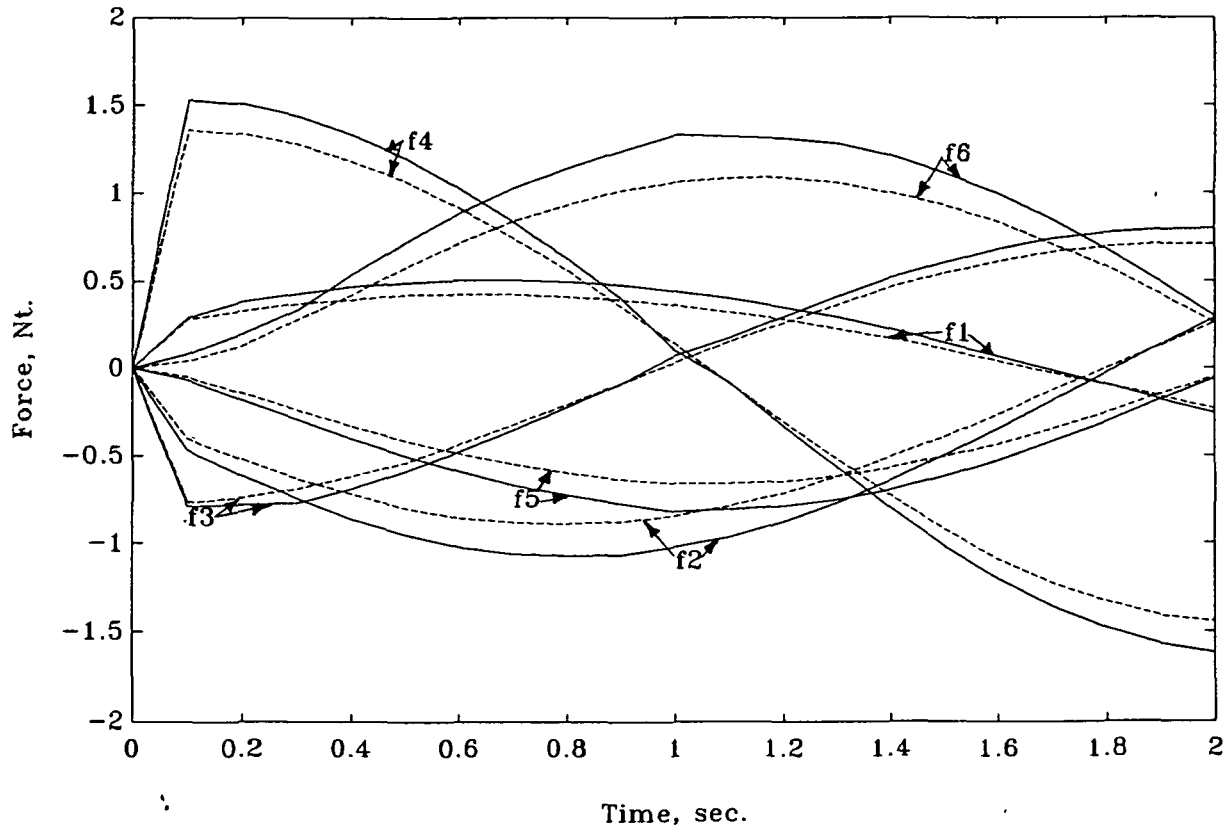

Fig. 4: The Influence of Link Masses on Actuating Forces

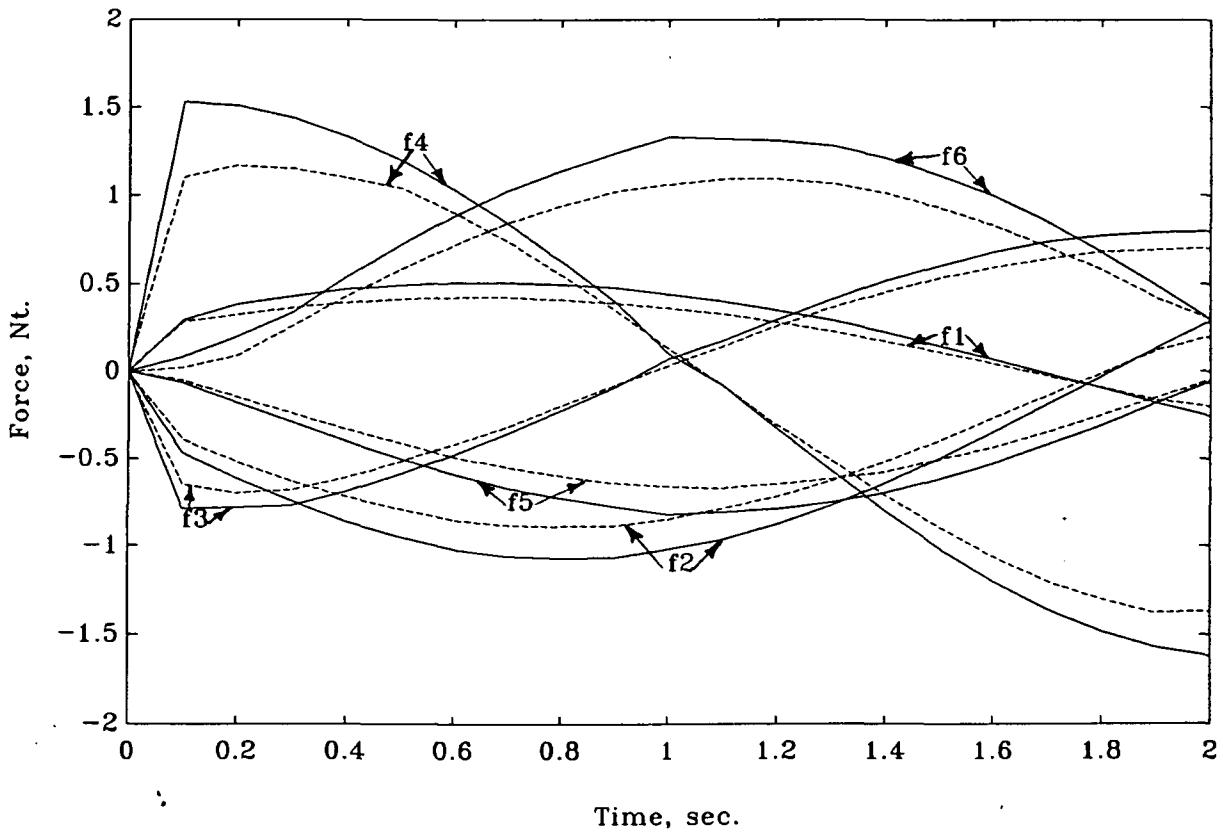

Fig. 5: The Influence of Link Centroids on Actuating Forces

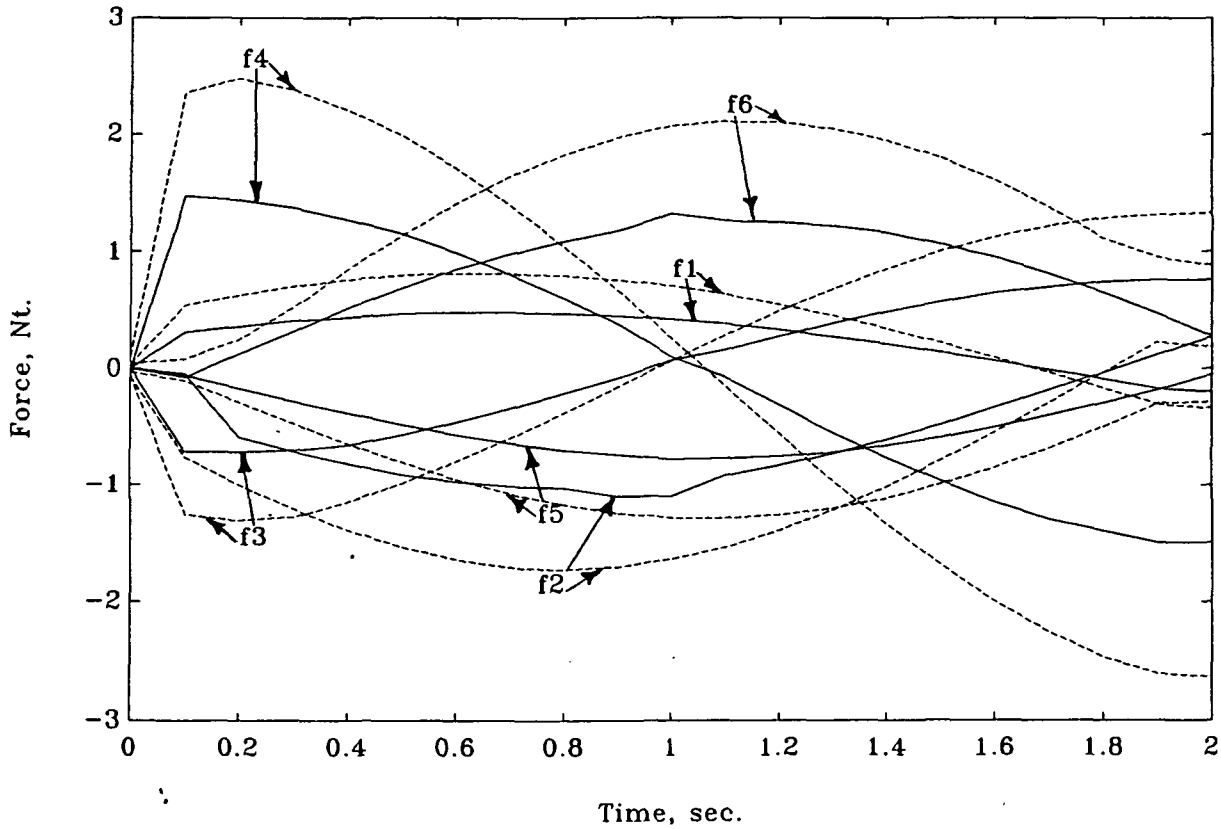

Fig. 6: The Influence of Platform Mass on Actuating Forces