Analytical Approximation of a Distorted Reflector Surface Defined by a Discrete Set of Points

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1. Introduction

Reflector antennas in a space environment are subjected to continuous variation in temperature distribution, and are thus distorted from its true geometrical shape (typically parabolic, hyperbolic, elliptical, etc.). The distorted reflector surface has in general a very complicated shape and hence cannot be represented with an analytical expression. The analysis of a distorted reflector antenna defined by a set of discrete surface points requires the use of numerical techniques. Many numerical techniques for analyzing the performance of reflector antennas defined by a set of points have been extensively reported in the open literature (refs. 1-4). The most popular of these techniques represent the reflector surface either globally or locally by using polynomial splines (refs 5,6). The polynomial splines and other techniques require the surface points to be orderly labeled. This is an undesired characteristic because it can result in a nonunique approximation for the desired reflector surface. The order of the spline polynomial necessary to best fit the reflector surface points is in general unknown a priori, and therefore it involves a trial and error procedure for checking the accuracy of the interpolation.

The distorted reflector surface points can be best approximated by two analytical components, an undistorted surface component and a surface error component. The undistorted surface component is a best fit paraboloid polynomial for the given set of points and the surface error component is the deviation of the actual surface points from the best fit paraboloid. This residual error component is then described with a sinusoidal Fourier series expansion. This approximation technique is insensitive to the labeling of the reflector surface points and can describe the surface errors very accurately. Similar to a time signal the spatial spectrum of the surface error component is unique to the reflector under consideration. Therefore spatial spectra can be utilized as a performance index for comparing distortion profiles in reflector antennas.
2. Description of the Problem

The best analytical representation of the distorted reflector antenna surface that uniquely identifies the surface errors can be obtained as follows: the reflector surface points are separated into two components, a best fit paraboloid component and a sinusoidal Fourier series expansion of the residual (ref. 7). Figure 1 illustrates a conceptual layout of the problem under consideration.

In analyzing large reflector antenna performance it is necessary to accurately characterize the reflector surface points. Any deviation from its ideal geometry causes the antenna performance to degrade. The surface error component provides an independent performance index against which distorted reflector antennas can be compared. The surface error component by definition is a sinusoidal Fourier series expansion of the difference between the actual reflector surface points and the best fit paraboloid geometry. Information such as root mean square value (RMS), peak surface error and two-dimensional distortion profile can be calculated from the surface error component. Fourier coefficients in the series expansion represent the spatial spectrum that uniquely identifies the distorted reflector under study. The best fit paraboloid surface represents the reflector antenna surface in an average sense. When the surface error is zero the best fit paraboloid surface reduces to the ideal or the design surface geometry.

3. Numerical Results and Discussion

Figure 2 represents a block diagram of a computer simulation of the above stated problem. The reflector surface points are usually obtained from a holographic, photogrammetric or any other surface detection technique. The computer algorithm was tested with known distortion profiles and TRASY-SYNA-nda-NASTRAN (Ref.1) simulated thermal distortions superimposed into a reflector antenna geometry.

The distortion profiles considered for the simulation are described in Table 1 and the reflector geometry illustrated in figure 3. Case A in Table 1 describes a small (λ/20 or less) distortion profile and case B illustrates a large (in the order of several wavelengths) distortion profile. The frequency used for the simulation was 10 GHz. A set of equally spaced data points (100,100) were needed to generate the distorted reflector surface. The estimated surface errors and best fit paraboloid surface geometry are described in Table 2.

Figures 4(a) and (b) show the estimated surface profile corresponding to case A and case B and their respective far field radiation patterns. The results presented in Table 2 are in good agreement with the
input distortion by Fourier coefficients in Table 2. It was found that the Fourier matrix had higher order coefficients with non zero values but they were one order of magnitude less than the lowest amplitude of the Fourier coefficient presented in Table 2.

The technique was compared with results obtained by using a spline polynomial fit for approximating the distorted reflector surface points (ref. 1). In this case the distorted reflector surface points were obtained by simulating thermal deformation with a TRASYS-SYND-NASTRAN computer programs. In brief, TRASYS and SINDA are used to characterize the in-orbit thermal environment; NASTRAN calculates the thermally induced mechanical distortions. Figure 5 shows the temperature distribution on the reflector antenna surface for the case under consideration. The reflector geometry input to the thermal programs is presented in fig. 6. The frequency considered was 28.75 GHz and (100,100) surface points were used for the analysis. The far field radiation pattern corresponding to the thermal simulation case is presented in figure 7. The continuous line pattern corresponds to the polynomial spline algorithm and dotted line pattern corresponds to the best fit paraboloid and Fourier series expansion approximation. There are no major differences between the beam direction and sidelobe levels, indicating a good agreement between the two techniques. The best fit paraboloid and Fourier series algorithm was very slow: 3 hr. of c.p.u. time in an IBM 370 computer. The spline polynomial algorithm takes about 1/2 hr. on a CRAY XMP computer. The long computation can be justified as a trade-off to obtain vital information about the distorted surface characteristics. These are the amplitude spectra of the surface error, the distorted surface profile, RMS value and the largest deviation on the reflector antenna surface. These are not directly available from any of the other existing techniques.

4. Concluding Remarks

One advantage of the developed technique is that it can be easily implemented to any existing reflector antenna secondary pattern computational method. It can easily be extended to nonparabolic reflectors surfaces (spherical, planar, hyperbolic, elliptical, etc.) by modifying the least square polynomial approximation. In applications involving the fabrication and design of precision reflector antennas the technique can be used as a computer aided tool. Information such as the average focal length, a root mean square of the surface error, surface error profile and the amplitude spectra of the reflector antenna under consideration are easily accessible from the algorithm. One draw back of the algorithm is that it is slow. This can be improved by using a faster computer such as a CRAY and optimizing the computer codes.
REFERENCES


## TABLE 1. - DISTORTION PROFILE PARAMETERS

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<th>Case</th>
<th>$d_{11}$</th>
<th>$d_{12}$</th>
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<td>3.000</td>
<td>-1.000</td>
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<td>-0.3000</td>
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## TABLE 2 - ESTIMATED DISTORTION PROFILE PARAMETERS

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</table>

Estimated focal length:
- Case A: 3.290105 ft
- Case B: 3.313091 ft
Q. 1-Distorted reflector surface points separated into an undistorted surface component and a surface error component.
Fig. 2 - Computer implementation for obtaining an analytical representation for a reflector surface defined by a set of points.
Fig. 3 - Offset parabolic reflector geometry
Fig. 4 - E-plane radiation pattern and their estimated distortion profile.
Fig. 4 - Concl.
Fig. 5 - Temperature distribution on the reflector surface
Fig. 6 - Offset parabolic reflector geometry
Fig. 7 - E-plane radiation pattern
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Reflector antennas on earth orbiting spacecrafts generally cannot by described analytically. The reflector surface is subjected to a large temperature fluctuation and gradients, and is thus warped from its true geometrical shape. Aside from distortion by thermal stresses, reflector surfaces are often purposely shaped to minimize phase aberrations and scanning losses. To analyze distorted reflector antennas defined by discrete surface points, a numerical technique must be applied to compute an interpolatory surface passing through a grid of discrete points. In this paper, the distorted reflector surface points are approximated by two analytical components; an undistorted surface component and a surface error component. The undistorted surface component is a best fit paraboloid polynomial for the given set of points and the surface error component is a Fourier series expansion of the deviation of the actual surface points, from the best fit paraboloid. By applying the numerical technique to approximate the surface normals of the distorted reflector surface, the induced surface current can be obtained using physical optics technique. These surface currents are integrated to find the far field radiation pattern.