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BASIC RESEARCH FOR THE GEODYNAMICS PROGRAM
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Goddard Space Flight Center
Greenbelt, Maryland 20771

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The Ohio State University
Research Foundation
1314 Kinnear Road
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BASIC RESEARCH FOR THE GEODYNAMICS PROGRAM

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Prepared for
NASA/Goddard Space Flight Center
Greenbelt, Maryland 20771

The Ohio State University
Research Foundation
Columbus, Ohio 43212

July, 1988
PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The Science Advisor is Dr. David E. Smith, Code 921, Geodynamics Branch, and the Technical Officer is Dr. Robert J. Coates, Code 601, Crustal Dynamics Project, Space and Earth Sciences Directorate, both at Goddard Space Flight Center, Greenbelt, Maryland 20771.

Although this report covers activities from July 1, 1987, through June 30, 1988, most of the described work was performed before the end of 1987, at which time continuation of the project was in some question. Funds were depleted, the graduate students graduated, and it appeared that a final report would complete the project requirements. This is why the usual semiannual report for July-December, 1987, was not submitted. New funds are now expected as of July 1, 1988, and the project will resume in the beginning of the academic year, October 1, 1988.
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**Appendix 3** Orbit Determination for the Global Positioning System Satellites, First Quarterly Status Report, January, 1988

1. ACTIVITIES

1.1 Earth Rotation Parameter Determination from Different Space Geodetic Systems

Final report completed and published (Dept. of Geodetic Science and Surveying Rep. 375). Summary presented at the IUGG General Assembly in Vancouver. The paper is included here as Appendix 1 and is to be published in the Bulletin Géodésique in 1988 or 1989.

1.2 Utilization of Range-Difference Observations in Geodesy

Final report completed and published (Dept. of Geodetic Science and Surveying Rep. 384). Summary presented at the IUGG General Assembly in Vancouver. The paper is included here as Appendix 2 and is to be published in the Bulletin Géodésique in 1988 or 1989.

1.3 An Algorithm for Crustal Deformation Analysis

Final report completed and published (Dept. of Geodetic Science and Surveying Rep. 382).

1.4 Orbit Determination for the Global Positioning System of Satellites

This work item is jointly sponsored by NGS. The progress report is in Appendix 3.

1.5 Reference Frames for Geodynamics

An earlier work on the subject has been updated and presented at the International Summer School of Theoretical Geodesy "Theory of Satellite Geodesy and Gravity Field Determination," May 23 - June 3, 1988, Assisi, Italy, to be published in the Proceedings. See Appendix 4.
2. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time
Clyde C. Goad, Co-Investigator, part time
George C. Dedes, Research Associate, part time
Brent A. Archinal, Graduate Research Associate, part time

3. TRAVEL

George C. Dedes, Clyde C. Goad, Ivan I. Mueller
Vancouver, Canada Aug. 9-22, 1987
To attend XIX General Assembly of the IUGG. Papers presented. No project support.

George C. Dedes
Greenbelt, Maryland Oct. 21-23, 1987
To attend 11th Crustal Dynamics Project Meeting, Goddard Space Flight Center, and to present two papers.

Ivan I. Mueller
San Francisco Dec. 7-11, 1987
To attend Annual Fall Meeting of the American Geophysical Union. No project support.

Ivan I. Mueller
Austin, Texas Jan. 11-12, 1988
To attend Geodynamics Laser Ranging Systems Workshop and to chair a session. No project support.

George C. Dedes
Pasadena, Calif. Mar. 22-24, 1988
To attend 12th Crustal Dynamics Project Meeting and 3rd Annual GPS Workshop at Jet Propulsion Laboratory. No project support.

Ivan I. Mueller
Assisi, Italy May 23 - June 3, 1988
To attend International Summer School of Theoretical Geodesy "Theory of Satellite Geodesy and Gravity Field Determination" and to present lectures. No project support.
4. DEPT. OF GEODETIC SCIENCE AND SURVEYING REPORTS
PUBLISHED TO DATE UNDER PROJECT SUPPORT

No.

262  The Observability of the Celestial Pole and Its Nutations  
     by Alfred Leick, June, 1978

263  Earth Orientation from Lunar Laser Range-Differencing  
     by Alfred Leick, June, 1978

284  Estimability and Simple Dynamical Analyses of Range (Range-Rate and Range-Difference)  
     Observations to Artificial Satellites  
     by Boudewijn H.W. van Gelder, December, 1978

289  Investigations on the Hierarchy of Reference Frames in Geodesy and Geodynamics  
     by Erik W. Grafarend, Ivan I. Mueller, Haim B. Papo, Burghard Richter, August, 1979

290  Error Analysis for a Spaceborne Laser Ranging System  
     by Erricos C. Pavlis, September, 1979

298  A VLBI Variance-Covariance Analysis Interactive Computer Program  
     by Yehuda Bock, May, 1980

299  Geodetic Positioning Using a Global Positioning System of Satellites  
     by Patrick J. Fell, June, 1980

302  Reference Coordinate Systems for Earth Dynamics: A Preview  
     by Ivan I. Mueller, August, 1980

320  Prediction of Earth Rotation and Polar Motion  
     by Sheng-Yuan Zhu, September, 1981

329  Reference Frame Requirements and the MERIT Campaign  
     by Ivan I. Mueller, Sheng-Yuan Zhu and Yehuda Bock, June, 1982

337  The Use of Baseline Measurements and Geophysical Models for the Estimation of Crustal  
     Deformations and the Terrestrial Reference System  
     by Yehuda Bock, December, 1982

338  On the Geodetic Applications of Simultaneous Range-Differencing to Lageos  
     by Erricos C. Pavlis, December, 1982

340  A Comparison of Geodetic Doppler Satellite Receivers  
     by Brent A. Archinal, November, 1982 (partial support)

348  On the Time Delay Weight Matrix in VLBI Geodetic Parameter Estimation  
     by Yehuda Bock, July, 1983

351  Model Choice and Adjustment Techniques in the Presence of Prior Information  
     by Burkhard Schaffrin, September, 1983

370  Positioning with NAVSTAR, the Global Positioning System  
     by Ziqing Wei, October, 1986
Determination of Earth Rotation by the Combination of Data from Different Space Geodetic Systems
by Brent A. Archinal, February, 1987

An Algorithm for Crustal Deformation Analysis
by Huseyin Baki Iz, September, 1987

Baseline Estimation from Simultaneous Satellite Laser Tracking
by George C. Dedes, October, 1987

Publications and Presentations Since Mid-1985


Mueller, Ivan I., “From 100 m to 100 mm in (About) 25 Years,” keynote address, Proc. 4th International Geodetic Symp. on Satellite Positioning, April 28–May 2, 1986, Austin, Texas.

Hilla, Stephen A., “Processing Cycle Slips in Nondifferenced Phase Data from the Macrome
1000 Receiver,” Proc. 4th International Geodetic Symp. on Satellite Positioning,
April 28–May 2, 1986, Austin, Texas.

Wilkins, G.A. and I.I. Mueller, “Rotation of the Earth and the Terrestrial Reference Sys-
EoS, Transactions of the American Geophysical Union, 67, No. 31, Au-

Mueller, Ivan I. and George A. Wilkins, “Earth Rotation and the Terrestrial Reference F-
Applications of Space Techniques for Geodesy and Geodynamics (COS-
Symp. 2, July 30, 1986), D. Reidel Publ. Also, 7th International Symp. on Recent C-
Movements of the Earth, Tallinn, USSR, Sept. 8-13, 1986. Also at Pulkovo Observ-

Archinal, Brent A. “Combination of Data from Different Space Geodetic Systems for De-
Determination of Earth Rotation Parameters,” 11th Crustal Dynamics Project Prin-
Investigators Meeting, October 2–24, 1986, Greenbelt, Maryland. Also, Proc. of IAU-
Symp. No. 128, Earth Rotation and Reference Frames for Geodesy and Geodynamics,

Project Principal Investigators Meeting, October 20-24, 1986, Greenbelt, Maryland.

Mueller, Ivan I., “Surveying with the Global Positioning System,” Argentine Geophys-
Geodetic Association, 14th Scientific Meeting on Geophysics and Geodesy, October 27-
1986, Mendoza, Argentina.

Mueller, Ivan I., “Terrestrial and Celestial Reference Frames: Concepts and Realiz-
Argentine Geophysical and Geodetic Association, 14th Scientific Meeting on Geophys-

Mueller, Ivan I., “International Earth Rotation Service,” Intercosmos Symp. on Use of Ar-
Satellite Observations for Geodesy and Geophysics, May 18-23, 1987, Szentendre, Hun-

Dedes, George C. and Ivan I. Mueller, “Differential Laser Observations for Bas-
Determinations,” 3rd International Conf. on Wegener-Medlas Project, May 25-27,
Bologna, Italy.

Archinal, Brent A. and Ivan I. Mueller, “Further Considerations on Combining Earth Re-
Observations from Different Space Geodetic Systems,” pres. at Symp. 4, Variations in
Rotation, XIX General Assembly of the IUGG, August 18-19, 1987, Vancouver, Ca-
Also, accepted for publication in Bulletin Géodésique, 1988.

Dedes, George C. and Ivan I. Mueller, “Baseline Estimation with Semidynamic and Geo-
Satellite Methods,” pres. at XIX General Assembly of the IUGG, August 18-19,

Dedes, George C., “Baseline Estimation from Simultaneous Satellite Laser Range Diff-
res. at 11th Crustal Dynamics Project Principal Investigators Meeting, October 21-23,
Goddard Space Flight Center, Greenbelt, Maryland.

Dedes, George C., “GPS Orbit Determination in the Personal Computer Environment,” p-
11th Crustal Dynamics Project Principal Investigators Meeting, October 21-23,
Goddard Space Flight Center, Greenbelt, Maryland.


Further Considerations on Combining Earth Rotation Observations From Different Space Geodetic Systems

Abstract

Additional results are presented concerning a study that considers improvements over present Earth Rotation Parameter (ERP) determination methods by directly combining observations from various space geodetic systems in one adjustment. Earlier results are extended, showing that in addition to slight improvements in accuracy, substantial (a factor of three or more) improvements in precision and significant reductions in correlations between various parameters can be obtained (by combining Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR) to Lageos, and Very Long Baseline Interferometry (VLBI) data in one adjustment) as compared to results from individual systems. Smaller improvements are also seen over the weighted means of the individual system results. Although data transmission would not be significantly reduced, negligible additional computer time would be required if (standardised) normal equations were available from individual solutions. Suggestions for future work and implications for the new International Earth Rotation Service (IERS) are also presented.

1 Introduction

In the past, Earth Rotation Parameters (ERP) have been determined using data from only one observational system at a time, or by the combination of parameters previously obtained in such determinations. The question arises as to whether combining observations from several systems in one adjustment would provide better ERP results than combining the ERP time series determined by the individual systems or than the ERP determined from any single system. One would expect there to be some improvement, but the question is one of how much improvement.

To look at this problem, it was decided to perform a simulation study, using realistic networks of Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR) to Lageos, and Very Long Baseline Interferometry (VLBI) stations. A simulation approach was taken so that "correct" ERP values would be available as a standard of reference, and to allow looking at very high observational data rates. Only these three observational systems were considered, since it is clear that most other methods provide ERP results of at least several times lower accuracy. In addition, it was decided to look at several short ERP recovery periods, as these periods are currently of the highest interest. The overall length of the simulated data period was kept to 15 days, in order to minimise the computer resources used and ignore long period model effects.

The models used to simulate and recover the ERP have been kept fairly simple, reflecting the overall geometry of the situation only, and ignoring (modeled or unmodeled) systematic errors or
system weighting differences. Since various methods of ERP determination and not the observational systems themselves are being compared, and the interest is only in the increase in accuracy relative to the "correct" values, this assumption seems reasonable. For example, if an ERP series obtained from one system were degraded by systematic errors in the observations, it is assumed that all combination ERP series (using the same data) being compared to it would be degraded by the same amount. The series specifically available for comparison are those from: a) each of the three observational systems, b) the weighted means of the results from a), c) the combination of the normal equations generated in a), and d) a grand solution with the data of all systems (a fully iterated normal equation solution). If the observational systems themselves were to be compared, complete modeling of systematic errors would be needed, and investigations done into the relative weighting of the systems.

Some results of this study were previously presented in [Archinal, 1988]. Here, we again present a review of the simulation assumptions and solution methods, and results on the accuracy of the recovered ERP with respect to the simulated ERP series. Additionally, new results on the precision and correlation of the recovered series are presented, along with comparisons of the amounts of data and computation time. Finally, we conclude with comments on other advantages of these methods, suggestions for future work, and implications seen for the IERS. A more complete description of this study is given in [Archinal, 1987].

2 Simulation Assumptions

In any simulation experiment, the results are entirely dependent on the set of assumptions made. These are discussed here in regard to the modeling of the geometry, station networks, and the simulated ERP values used to create the simulated data.

2.1 Geometric Models

For the LLR observations, a satellite in a Keplerian orbit about the Earth with the same elements as the Moon is assumed. For SLR, a satellite is assumed with the same Keplerian orbit as Lageos, but affected by the central mass and J_2 of the Earth (the latter so that the node of Lageos's orbit realistically regresses). Both of these orbits are solved for with 6 parameters weighted at the 1 meter level. For VLBI, a real IRIS radio source catalog was assumed. The positions of those sources were essentially fixed, with weights of 50 μas in right ascension and 50 μas in declination, and with the right ascension of one source completely fixed. Fixed values for the Earth's angular velocity, precession, and nutation were assumed, except for the variations in the angular velocity supplied by the simulated ERP (see 2.3 below). Stations are assumed to be observing continuously (when the targets are above a 15° elevation angle) in order to compare ERP determination at the highest possible levels of accuracy of the individual systems.

2.2 Station Networks

The stations chosen are stations which were realistically expected to operate at high data rates as of the 1986-1987 period, and are listed in Table 1. The instruments available at or near each location are also shown in that table. Of all the stations only two are not in operation at the present time, i.e., the Simeiz and Richmond LLR/SLR instruments (although a transportable SLR system has now operated at Richmond). Random noise has been added to all of the observations, with standard deviations for the lasers as shown (agreeing with [Schutz, et. al., 1985; Coates, 1985]), and for the VLBI delays as 0.1 ns. Normal point observations are assumed every 10 minutes for LLR and every 2 minutes for SLR when possible. For the VLBI observations an actual IRIS schedule [W. Carter, 1984, personal communication] was shifted in time as needed. No correlations between any observations were assumed.
2.3 Simulation of ERP

To create the simulated data, ERP were themselves simulated by superimposing sine curves with amplitudes and periods derived from variations seen in real ERP data [Robertson and Carter, 1985]. Adding real trends (from 5 day IRIS data) to these values, a 6 hour step function was generated for all 3 ERP components over the 15 day period. These values were used to generate the simulated observations, and as a standard of reference for 6 hour ERP recovery. For longer periods, the step functions were averaged over time to obtain reference values.

3 Simulation and Solution Methods

The data are simulated using the geometric models, station and target definitions, simulated ERP, and observational accuracies just described. The primary software used for simulation of the data was the program GEODYN [Putney, 1977] (provided via NASA/GSFC).

Individual system solutions were also performed with the same software, using a Bayesian least squares technique. Normal equation combination solutions involving all of the systems’ data were performed by adding the normal equations generated in the individual solutions and solving the combined set of equations (via the SOLVE program, also provided by GSFC). These solutions were iterated to convergence in order to obtain what is called here a "grand solution." The comparison of normal equation combination solutions with grand solutions may thus show whether the computational work of iteration is indeed necessary. Finally, weighted means of the individual systems' ERP series were also taken (in locally written software), using the recovered ERP series standard deviations to determine the weights.

4 "Accuracy" of the Recovered ERP

It is important to emphasise here that by "accuracy" we refer to how close the recovered ERP values are to their "correct" simulated values. The RMS difference for each series with respect to the correct values have been computed and portrayed relative to the best method in Table 2. The results are shown symbolically for each period, with an "s" designating the best method(s), a "+" methods with a factor of 1 to 2 difference, a "−" a factor of 2 to 3, and a blank greater difference. It is easily seen that: a) the normal equation combination or grand solutions always give the best or nearly the best results, b) the weighted mean (or perhaps the VLBI) solution alone is nearly as good, c) that VLBI generally gives better Y than X polar motion (due to the strong geometry of the IRIS network for determining Y), and d) that LLR gives the best long period UT1-UTC, but poor polar motion values.

5 Precision and Correlations of the ERP

Besides the "accuracy" of these various methods, precision and correlation estimates may also be compared. While the accuracy estimates may give a better indication of which method is best, they are inherently dependent on the actual data (its noise, etc.). Estimates of the precision on the other hand are not dependent on the specific observations, but only on the geometry of a particular situation. The correlations (obtained from the same variance-covariance matrix as the precision estimates) also provide information on how well the ERP and other parameters may be recovered.

5.1 Precision of the ERP Results

The overall ERP precision is considered first, with the numerical results presented in Table 3. For each method and in each ERP recovery period, the average and maximum standard deviation is
given. To more directly compare the methods for each recovery period, multiples of the best method for that period are shown in parentheses.

For all of the recovery periods, the grand solution always gives the smallest average and maximum standard deviation. The weighted mean gives the next smallest values for short (6 and 12 hour) ERP periods, while the normal equation combination solution also gives smaller such standard deviations than the individual system results alone. This is as expected since all the combination solutions contain more observations and the "square root of n" rule should approximately apply. Except for the VLBI average standard deviation for 6 hour ERP, all of the individual systems give standard deviations about 3 or more times worse than that of the grand solution. The relative values for LLR and VLBI are comparable, except for 6 hour ERP when LLR is about 4 to 6 times worse. Apparently due to the correlations of UT1—UTC with orbital parameters, SLR always gives the worst standard deviations (except for the LLR maximum standard deviation for 6 hour ERP).

Looking at the actual values of the standard deviations shows an initial increase in the 6 hour values in going to the 12 hour values, and then a gradual decrease in going up to the values for 5 day ERP recovery (with SLR being an exception, due to the correlation of UT1—UTC with orbit errors — see below). This is explained by looking at the simulated ERP. For 6 hour recovery, the ERP can be recovered with the same fluctuations with which it was simulated. For longer periods, the recovered values are actually averages of changes which still occur in the data every 6 hours. In effect, we have introduced a model error by not always recovering the ERP over the same periods at which it exists in the data (6 hours). As the recovered ERP period increases greatly from 6 hours, the 6 hour fluctuations average out more, giving smaller standard deviations again (but never as small as at 6 hours). This strongly emphasizes the importance of using ERP recovery periods consistent with the periods of change in the actual ERP. Otherwise a modeling error (for the ERP recovery) is being committed.

Table 4 summarizes these results further, but also includes a summary of the precision of the individual ERP components of X polar motion, Y polar motion, and UT1—UTC. The summary is given similarly to the accuracy summary in Table 2. For each period, the most accurate method is designated with an "*", methods with standard deviations up to twice as high with a "+", and two to three times as high with a "—". For each method, the average standard deviation for the individual components are summarised (under "X", "Y", and "U") along with the overall average and maximum standard deviation (under "A" and "M"). The individual ERP component average standard deviation summaries indicate that the weighted mean solution usually gives the lowest polar motion standard deviations, while the grand solution always gives the lowest UT1—UTC standard deviations. The weighted mean standard deviations probably appear so optimistic because no correlations are considered for that solution. The normal equation combination ERP and the SLR polar motion standard deviations are all usually within a factor of 2 of the lowest values. VLBI and the weighted mean solutions provide values within a factor to 3. All LLR ERP and the SLR UT1—UTC standard deviations are quite large in comparison, due to the large biases which can (and do) exist for parameters determined in those solutions. As discussed for Table 3, it becomes quite clear that for the overall ERP standard deviations, the grand solution always provides the smallest values. The weighted mean and normal equation combination solutions provide values normally only 1 to 2 times as high. The individual systems only sporadically were capable of values even as little as 3 times as high.

5.2 Parameter Correlation Results

Turning now from the precision estimates obtained for the parameters, we consider the correlations among them and between the ERP and other parameters. Table 5 gives such a summary for those correlations which are significant. This table shows the maximum or range of (the absolute values of) all correlations greater than 0.2. We have divided the correlations first according to solution method and then parameter type. Correlations with the lunar and Lageos orbit parameters
Correlations with and among radio source positions were all less than 0.2. Due to software limitations, these orbit and source position parameter correlations were not available in the combination solutions. Correlations were ignored in the weighted mean solutions.

Immediately obvious is the greater number of significant correlations for the individual systems than for the combination solutions. The only significant correlations in the combination solutions were among the polar motion and UT1—UTC parameters during the same period. These were nearly the same for both the normal equation combination and grand solutions, with values of 0.2 to 0.6. Even correlation among polar motion and UT1—UTC at different times was negligible. VLBI gave similar results, except with generally higher correlations (0.5 to 0.8), and with 5 day ERP recovery, correlations of polar motion with UT1—UTC at other times, of up to 0.3. The lunar and Lageos orbit parameters showed wide ranging correlation among themselves, ranging from 0 to 1. Unlike any other method, SLR showed negligible correlation among polar motion parameters. However, orbit parameter correlations with polar motion were noticeable for 6 and 12 hour ERP (0.3 to 0.6) and with UT1—UTC for 5 day ERP (0.2 to 0.3). The correlations of UT1—UTC with UT1—UTC of other periods, and with X and Y orbit components was always quite high, from 0.975 to 1 in all cases. This clearly demonstrates the poor separability of UT1—UTC parameters from orbit (XY plane) orientation parameters, but shows that the correlation decreases slightly from 1 as the ERP period becomes shorter. The LLR solutions have a wide range of significant correlations, but no extremely high ones except among the orbit parameters, and between polar motion and UT1—UTC if 6 hour ERP recovery is done. The correlations of UT1—UTC with UT1—UTC of other periods, and with Z axis orbit parameters increases with ERP period, from near 0.5 or 0.6 to 0.9. The correlations among polar motion parameters are similar or slightly less than the VLBI values.

6 Comparison of Data Amounts, Computation Time

Since the amount of data being transmitted and the computational speed may be important factors in the operation of (at least a "rapid") earth rotation service, some results from the simulations concerning this are presented here.

6.1 Comparison of Amounts of Input Data

Table 6 summarises the amounts of the three possible types of "data" generated in the simulation: observations, normal equations, and ERP series. The "observations" include here normal point laser ranges and VLBI delays and delay rates. Under the assumptions (including the highest possible data rates) of this study, the SLR network is capable of generating the largest amount of data, with VLBI generating only just over half as much, and LLR about a tenth as much. So it is likely that from several hundred to several thousand records (i.e., nearly card images) would be generated by each system, even with much less observing. Since some of this would be sent daily, instead of being accumulated over the 15 days of data here, this amount could feasibly be sent via an electronic mail system, although at e.g., 1200 baud, this could be expensive and perhaps very time consuming.

We also see that the size of the normal equations depends little on the observational system but is almost entirely dependent on the number of ERP (and other parameters) being solved for. For long ERP recovery periods (2 or 5 days) the size is fairly small, but as the ERP period shortens to 6 hours or (or as additional parameters are added such as would occur in practice) the size will increase greatly, easily exceeding that of the original data, which is already at its highest possible levels. Unless the number of parameters is kept small, the transmission of the data itself would probably be just as economical. It is also obvious that the amount of data in an ERP series itself is always fairly trivial compared to the amount of data or normal equations used to generate them. As is commonly done now in practice, the transmission of this data by electronic mail would be a very low cost procedure.
6.2 Comparison of Amounts of Computer Time

Although relatively computer and software dependent, we now look at the computer time required for our simulation solutions which is summarised in Table 7. Some conclusions are:

1. The actual weighted mean solutions, or combination and solution of normal equations, require small amounts of computer time. It is the creation of the normal equations and the individual system solutions which require large amounts of time.

2. A great savings of computer time would result if normal equations were saved from any individual system solutions already being done.

3. The normal equation combination solution (being a "single iteration" solution) requires much less time than the fully iterated individual solutions and than their weighted mean combination solution. Of course this is at the expense of a not completely converged solution.

4. The ERP recovery period affects the total computer time very little.

5. Ignoring data preprocessing, it is clear that SLR and then the LLR individual solutions or normal equation set ups are very computer intensive, with the VLBI solutions less so. With real data solutions, all of these computations would take even longer due to the additional modeling which would be done, especially for the LLR and SLR computations which would have much more extensive orbital models.

6. Considering the programs, computer, and simple models that are in use, the computer time for the individual solutions and/or normal equation set ups is quite large. Small computers would not efficiently be able to do the individual solutions and/or normal equation set ups, but only the normal equations combination or some type of weighted mean solution of ERP series obtained from elsewhere. And, if the normal equations of all these systems are set up at one time, even a large mainframe might be pressed to accomplish such a task, unless program efficiency was increased or a vector or array processor was in use. In practice, doing solutions every few days instead of with 15 days of data may reduce this problem somewhat, and the use of vector or array processing might substantially eliminate it.

7 Advantages of Normal Equation Combination and Grand Solutions

After studying at length the idea of combination solutions, particularly by the combination of normal equations, several advantages of this method or the use of grand solutions over other methods of ERP determination have become obvious:

1. Combining normal equations even allows us to combine equations that could not be solved on their own, i.e., singular sets of equations can sometimes be added and a solvable set obtained. If applied carefully, i.e., if the user checks that the final system is really non-singular, this might be a useful feature. For example, when one or more of the systems has a small amount of data, be it a single satellite pass at one station, single station LLR data, or one baseline VLBI data, the data can still be combined together so that if enough is available overall a solution can be obtained. This technique is extremely powerful in that it may allow the handling of periods of sparse data from any or all systems, possibly even when no solution can be made from each of the systems involved alone.

2. By combining data from different systems, we end up obtaining better values for parameters that may normally be highly correlated with other parameters. For example, it has been
shown here that SLR normally can not give UT1—UTC without biases (at least with 1 to
5 day recovery periods) due to the inseparability (high correlation) of UT1—UTC with the
orientation of Lageos's orbit. However, if we do a combination solution, the UT1—UTC value
is forced to its correct solution by the LLR and VLBI data and the Lageos orbit parameters
are also then improved as well. Strengthening the orbit could in turn strengthen other model
parameters (if they are included) such as gravity field coefficients, station coordinates, etc.
For "quick look" solutions, it is not likely that many such parameters would be solved for, but
for "final" long arc solutions, the additional accuracy obtainable for many parameters might
be very important. For the ERP themselves, the strengths of each individual ERP method
(e. g., LLR for UT1—UTC, SLR for polar motion and LOD, VLBI for Y polar motion and
UT1—UTC) would all be "automatically" combined.

3. Using normal equation solutions allows the combined normals to be formed first and then
different weights to be used on parameters (or constraints if a constraint model is being used)
if necessary. The addition of new observations or deletion of old ones is also quite easy without
recreating all the normals. This property would be useful for handling e. g., observations
which become available at the last minute, or observations found to be bad for one reason
or another. The solution of the normal equations will still be needed every time, as well as
the usual determination of variance-covariance matrix, but in practice, this is not always done
entirely or even needed. However, as we have shown, the solution of the equations can be done
very efficiently in comparison to setting them up.

Finally, we present two points which are not only advantages of the proposed methods, but might
be considered as the very reasons for adopting them:

4. In order to combine the normal equations, the models and approximate values used must be
carefully matched. This makes it necessary to make the models for each observational system
consistent with each other, and assures that recovered parameters are indeed truly compatible
with each other (e. g., all in one unified reference system at one scale, with the same constants
in use, etc.). Also if the individual systems' normals are then solved, the results can be
compared knowing the same models, constants, etc. are in use. Such comparisons are only
currently possible if each systems' software uses the same set of standards (e. g., the MERIT
Standards [Melbourne, et al, 1983]).

5 One of the most important advantages of doing normal equation combination or grand solu-
tions, perhaps even more so than the high accuracy shown in this study for their ERP solutions,
is their ability to easily unify reference frames when solving for station positions. Provided that
sufficient colocated stations exist, the normal equation combination or grand solutions auto-
matically provide a single Terrestrial Reference System (TRS) and a single Celestial Reference
System (CRS) for all systems which have data included. This means that by default, the bi-
ases between the currently existing TRS's and CRS's of each system are eliminated (assuming
that most station coordinates are solved for), thus establishing what could then be the new
Conventional TRS and Conventional CRS.

8 Further Work

A few possibilities are suggested for future work. This is especially true if it is felt that the slight
improvements in the ERP results provided by the normal equation combination or grand solutions
or their other advantages listed above justify further research. Specific suggestions are:

1. The simulation experiments could be repeated with other observing schedules to see how the
ERP are recovered by the various methods during periods of sparse data.
2. It has been assumed that slight changes in weighting for the orbit, radio source positions, station positions, etc. parameters, would have no effect on the ERP results. This might be investigated further.

3. To study the effects of (modeled and unmodeled) "systematic error," the simulations should also be repeated using complete models. Adding biases to the observations (e.g., for tropospheric refraction) and seeing how the ERP are recovered when such unmodeled biases exist would also be a worthwhile study.

4. Simulations similar to the ones done here could be done to see how well the data combination solution can recover reference frame biases.

5. It is obvious that the experiments in this study could be carried out with real data, although this is not too strongly recommended initially since the true ERP would then no longer be available as a standard of comparison. In addition, software would still need to be developed capable of handling all types of real data to be used.

6. Finally, in the late stages of this research it has come to our attention that it may be possible to "iterate" the normal equation combination solution without reforming the normal equations completely (as was done here for the grand solutions). This could be done by correcting the constant vector of the normal equations for the changes in the parameters from their initial approximate values. The normal coefficient matrix would remain unchanged, and thus not rigorously correct, but assuming a nearly linear solution, it could still be used with the converted constant vector to compute new parameter values, a smaller a posteriori variance of unit weight, and other adjustment results [Estes, 1983, pp. 2-18 to 2-18.1]. This method might provide substantial savings in computer time over the grand solution method.

9 Implications for the IERS

The results of this study do have some implications in regard to IERS, and some comments on these and recommendations to the IERS are given here.

Since there are some advantages in obtaining ERP via the methods of normal equation combination and grand solutions over current methods, it is felt that at least some further research (as outlined in the last section) should be done. The IERS itself might conduct and/or encourage such research, depending on its final operational configuration.

In addition, in anticipation of the possible use of these methods (experimentally or operationally) with real data, it is suggested that software developers be encouraged to provide for complete documentation of models used, and options and formats for normal equation and variance-covariance matrix output in addition to ERP parameter output. In any case, better documentation and the adoption and use of standards have many other well known benefits in addition to being helpful for the combination solutions discussed here.

10 Summary

We have seen here (and in [Archinal, 1988]) that although the normal equation combination and grand solutions do provide higher accuracy ERP results (relative to the "correct" ERP values) than the other methods, the accuracies are not much higher than provided by weighted mean solutions or some of the individual system solutions.

This paper has also shown that the combination methods usually provide at least a factor of 3 better overall precision and (except for SLR polar motion) a factor of two better precision over the results of the individual systems. The normal equation and grand solution methods also again
usually provide higher precision than the weighted mean, with results at times a factor of two or more better for UT1–UTC.

It must be admitted that this level of improvement alone might not justify the extra work in obtaining such solutions. However, the much lower correlation between some parameters, and the other advantages of these methods make them quite attractive for the purposes of estimating ERP and other parameters, and/or in establishing reference systems.

Savings in data transmission would probably not result from using any normal equation combination method, but assuming that the individual system solutions were being done anyway, and compatible sets of normal equations were made available, negligible computer time would be required to obtain solutions from such sets.

References:

B. A. Archinal (1987) : *Determination of Earth Rotation by the Combination of Data from Different Space Geodetic Systems*, Ph.D. dissertation (The Ohio State University, Columbus, Ohio 43210). Available as Department of Geodetic Science and Surveying Report No. 375 (1958 Neil Avenue, Columbus, Ohio 43210) February.


Table 1: Station Positions and Assumed Accuracies

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude °  '  &quot;</th>
<th>Longitude °  '  &quot;</th>
<th>Laser Accuracy</th>
<th>System Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grasse, France</td>
<td>43 45 6 55</td>
<td>5.0</td>
<td>L—S</td>
<td></td>
</tr>
<tr>
<td>Wettsell, F.R.G.</td>
<td>49 09 12 53</td>
<td>7.1</td>
<td>S—V</td>
<td></td>
</tr>
<tr>
<td>Graz, Austria</td>
<td>47 07 15 30</td>
<td>3.8</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Matera, Italy</td>
<td>40 42 16 37</td>
<td>13.9</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Simeiz, U.S.S.R.</td>
<td>44 32 34 01</td>
<td>10.0</td>
<td>L—S</td>
<td></td>
</tr>
<tr>
<td>Yargadee, Australia</td>
<td>-29 03 115 21</td>
<td>2.3</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Simosato, Japan</td>
<td>33 34 135 56</td>
<td>9.7</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Orroral, Australia</td>
<td>-35 38 148 57</td>
<td>5.0</td>
<td>L—S</td>
<td></td>
</tr>
<tr>
<td>Maui, HI, U.S.A.</td>
<td>20 43 203 44</td>
<td>4.2</td>
<td>L—S</td>
<td></td>
</tr>
<tr>
<td>Huahine, French Polynesia</td>
<td>-16 44 208 58</td>
<td>9.7</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Quincy, CA, U.S.A.</td>
<td>39 59 239 03</td>
<td>2.8</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Pt. Davis, TX, U.S.A.</td>
<td>30 41 255 59</td>
<td>8.4</td>
<td>L—S—V</td>
<td></td>
</tr>
<tr>
<td>Richmond, FL, U.S.A.</td>
<td>25 40 279 37</td>
<td>10.0</td>
<td>L—S—V</td>
<td></td>
</tr>
<tr>
<td>Greenbelt, MD, U.S.A.</td>
<td>39 01 283 10</td>
<td>3.4</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Arequipa, Peru</td>
<td>-16 28 288 30</td>
<td>14.5</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Westford, MA, U.S.A.</td>
<td>42 37 288 30</td>
<td>-</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>Herstmonceux, U.K.</td>
<td>50 52 359 39</td>
<td>4.7</td>
<td>S</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Laser accuracy is in cm. VLBI delay accuracy is 0.1 ns. System Type: L—LLR, S—SLR, V—VLBI.
Table 2: Relative RMS Differences for All Methods and ERP Recovery Periods

<table>
<thead>
<tr>
<th>Recovery Period</th>
<th>LLR</th>
<th>SLR</th>
<th>VLBI</th>
<th>weighted mean</th>
<th>normal eq.</th>
<th>grand sol.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XYU</td>
<td>XYU</td>
<td>XYU</td>
<td>XYU</td>
<td>XYU</td>
<td>XYU</td>
</tr>
<tr>
<td>6 hours</td>
<td>++</td>
<td>-</td>
<td>-</td>
<td>++</td>
<td>-</td>
<td>***</td>
</tr>
<tr>
<td>12 hours</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>++</td>
<td>*</td>
<td>+</td>
</tr>
<tr>
<td>1 day</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>*</td>
</tr>
<tr>
<td>2 days</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>*</td>
</tr>
<tr>
<td>5 days</td>
<td>*</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>*</td>
</tr>
<tr>
<td>all periods</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes:
X - X polar motion RMS difference from correct value
Y - Y polar motion RMS difference from correct value
U - UT1-UTC RMS difference from correct value
"normal eq." is the normal equation combination solution.
"all periods" indicates the minimum RMS difference multiple for all recovery periods.
* best method(s) (smallest RMS difference)
+ RMS difference multiple is between 1 and 2.
- RMS difference multiple is between 2 and 3.
(blank) RMS difference multiple is greater than 3.
Table 3: Comparison of Average and Maximum Standard Deviations

<table>
<thead>
<tr>
<th>ERP periods</th>
<th>Method</th>
<th>Ave. S.D.</th>
<th>Max. S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No./length</td>
<td></td>
<td>mas</td>
<td>mas</td>
</tr>
<tr>
<td>61 6 hour</td>
<td>LLR</td>
<td>1.6 (9.4)</td>
<td>4.9 (16.7)</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>2.4 (13.7)</td>
<td>4.1 (14.0)</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>0.4 (2.2)</td>
<td>0.8 (2.9)</td>
</tr>
<tr>
<td></td>
<td>normal eq.</td>
<td>0.3 (2.0)</td>
<td>0.6 (2.1)</td>
</tr>
<tr>
<td></td>
<td>grand sol.</td>
<td>0.2 (1.0)</td>
<td>0.3 (1.0)</td>
</tr>
<tr>
<td></td>
<td>wt. mean</td>
<td>0.2 (1.3)</td>
<td>0.4 (1.4)</td>
</tr>
<tr>
<td>31 12 hour</td>
<td>LLR</td>
<td>2.1 (3.1)</td>
<td>5.5 (4.5)</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>10.3 (15.2)</td>
<td>18.0 (14.5)</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>2.1 (3.1)</td>
<td>4.3 (3.5)</td>
</tr>
<tr>
<td></td>
<td>normal eq.</td>
<td>0.8 (1.2)</td>
<td>1.5 (1.2)</td>
</tr>
<tr>
<td></td>
<td>grand sol.</td>
<td>0.7 (1.0)</td>
<td>1.2 (1.0)</td>
</tr>
<tr>
<td></td>
<td>wt. mean</td>
<td>0.8 (1.1)</td>
<td>1.5 (1.2)</td>
</tr>
<tr>
<td>16 1 day</td>
<td>LLR</td>
<td>1.9 (2.9)</td>
<td>3.0 (2.7)</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>18.1 (27.5)</td>
<td>31.5 (28.9)</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>2.3 (3.5)</td>
<td>4.3 (3.9)</td>
</tr>
<tr>
<td></td>
<td>normal eq.</td>
<td>0.7 (1.1)</td>
<td>1.2 (1.1)</td>
</tr>
<tr>
<td></td>
<td>grand sol.</td>
<td>0.6 (1.0)</td>
<td>1.1 (1.0)</td>
</tr>
<tr>
<td></td>
<td>wt. mean</td>
<td>0.8 (1.2)</td>
<td>1.5 (1.4)</td>
</tr>
<tr>
<td>8 2 day</td>
<td>LLR</td>
<td>1.5 (3.3)</td>
<td>2.0 (2.8)</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>23.6 (51.2)</td>
<td>40.9 (56.3)</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>1.6 (3.5)</td>
<td>2.9 (4.0)</td>
</tr>
<tr>
<td></td>
<td>normal eq.</td>
<td>0.5 (1.1)</td>
<td>0.8 (1.1)</td>
</tr>
<tr>
<td></td>
<td>grand sol.</td>
<td>0.5 (1.0)</td>
<td>0.7 (1.0)</td>
</tr>
<tr>
<td></td>
<td>wt. mean</td>
<td>0.6 (1.3)</td>
<td>1.1 (1.5)</td>
</tr>
<tr>
<td>4 5 day</td>
<td>LLR</td>
<td>1.4 (3.8)</td>
<td>2.0 (4.1)</td>
</tr>
<tr>
<td></td>
<td>SLR</td>
<td>31.2 (84.1)</td>
<td>54.0 (109.)</td>
</tr>
<tr>
<td></td>
<td>VLBI</td>
<td>1.3 (3.4)</td>
<td>1.9 (3.8)</td>
</tr>
<tr>
<td></td>
<td>normal eq.</td>
<td>0.4 (1.1)</td>
<td>0.6 (1.1)</td>
</tr>
<tr>
<td></td>
<td>grand sol.</td>
<td>0.3 (1.0)</td>
<td>0.5 (1.0)</td>
</tr>
<tr>
<td></td>
<td>wt. mean</td>
<td>0.5 (1.3)</td>
<td>0.8 (1.6)</td>
</tr>
</tbody>
</table>

Notes:
1. Values in parenthesis show multiples of lowest value in column for that period.
2. Standard deviations for UT1—UTC were converted from square ms to square mas before computing all values.
3. Summary for solutions with 15 days of simulated data (created with changes in ERP every 6 hours).
Table 4: Relative Std. Dev. for All Methods and ERP Recovery Periods

<table>
<thead>
<tr>
<th>Recovery Period</th>
<th>LLR</th>
<th>SLR</th>
<th>VLBI</th>
<th>weighted mean</th>
<th>normal eq.</th>
<th>grand sol.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>XXYUAM</td>
<td>XXYUAM</td>
<td>XXYUAM</td>
<td>XXYUAM</td>
<td>XXYUAM</td>
<td>XXYUAM</td>
</tr>
<tr>
<td>6 hours</td>
<td>++</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>12 hours</td>
<td>++</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>1 day</td>
<td>--</td>
<td>++</td>
<td>-</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>2 days</td>
<td>-</td>
<td>++</td>
<td>-</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>5 days</td>
<td>++</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
<tr>
<td>all periods</td>
<td>++</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>++</td>
<td>+</td>
</tr>
</tbody>
</table>

Notes:
X - X polar motion average standard deviation
Y - Y polar motion average standard deviation
U - UT1-UTC average standard deviation
A - average ERP standard deviation
M - maximum ERP standard deviation
"normal eq." is the normal equation combination solution.
"all periods" indicates the minimum standard deviation multiple for all recovery periods.
- best method(s) (smallest standard deviation)
+ standard deviation multiple is between 1 and 2.
- standard deviation multiple is between 2 and 3.
(Blank) standard deviation multiple is greater than 3.
Table 5: Summary of Range of Correlations Between Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>6 hours</th>
<th>12 hours</th>
<th>1 day</th>
<th>2 days</th>
<th>5 days</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LLR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X vs. Y</td>
<td>.76</td>
<td>.1-7</td>
<td>.6</td>
<td>.6</td>
<td>.6</td>
</tr>
<tr>
<td>XY vs. UT1</td>
<td>.97</td>
<td>.2-5</td>
<td>.4</td>
<td>.4</td>
<td>-</td>
</tr>
<tr>
<td>XY vs. other UT1</td>
<td>.3</td>
<td>-</td>
<td>.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>UT1 vs. other UT1</td>
<td>.5</td>
<td>.5</td>
<td>.7</td>
<td>.82</td>
<td>.82-.91</td>
</tr>
<tr>
<td>XY vs. SVXY</td>
<td>.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>XY vs. SVZ</td>
<td>.67</td>
<td>.48</td>
<td>-</td>
<td>-</td>
<td>.3</td>
</tr>
<tr>
<td>UT1 vs. SVXY</td>
<td>.73</td>
<td>.3-8</td>
<td>.85</td>
<td>.8-9</td>
<td>.88-.96</td>
</tr>
<tr>
<td>UT1 vs. SVZ</td>
<td>.95</td>
<td>.1-.999</td>
<td>.998</td>
<td>.998</td>
<td>.998</td>
</tr>
<tr>
<td><strong>SLR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UT1 vs. other UT1</td>
<td>.988-.996</td>
<td>.998-.999</td>
<td>.999-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>XY vs. SVXY</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>XY vs. SVZ</td>
<td>.57</td>
<td>.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>UT1 vs. SVXY</td>
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<td>.995-.999</td>
<td>.999-1</td>
<td>.999-1</td>
<td>1</td>
</tr>
<tr>
<td>UT1 vs. SVZ</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.2-.3</td>
</tr>
<tr>
<td>SV</td>
<td>.999</td>
<td>.1-.9</td>
<td>.1-1</td>
<td>.1-1</td>
<td>.2-1</td>
</tr>
<tr>
<td><strong>VLBI</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X vs. Y</td>
<td>.5-.81</td>
<td>.5-8</td>
<td>.7-8</td>
<td>.73-.75</td>
<td>.73-.75</td>
</tr>
<tr>
<td>XY vs. UT1</td>
<td>.5-.78</td>
<td>-</td>
<td>-</td>
<td>.5-6</td>
<td>.52-.56</td>
</tr>
<tr>
<td>XY vs. other UT1</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>.3</td>
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<td><strong>normal equation combination</strong></td>
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</tr>
<tr>
<td>X vs. Y</td>
<td>.4</td>
<td>.6</td>
<td>.4-6</td>
<td>.3-5</td>
<td>.4-5</td>
</tr>
<tr>
<td>XY vs. UT1</td>
<td>.6</td>
<td>.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td><strong>grand solution</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X vs. Y</td>
<td>.2-5</td>
<td>.34-.55</td>
<td>.35-.42</td>
<td>.39-.44</td>
<td>.39-.44</td>
</tr>
<tr>
<td>XY vs. UT1</td>
<td>.2-6</td>
<td>.4-.5</td>
<td>.36-.50</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes:
1. Maximum or range of absolute value of correlations shown.
2. Correlations below 0.2 not shown (not listed, or "-" given).
3. Abbreviations: "X", "Y" - polar motion, "UT1" - UT1-UTC, "SV" - Cartesian state vector for Moon (LLR) or Lageos (SLR). ("SVXY" implies X-Y plane SV parameters. "SVZ" implies Z axis SV parameters.)
4. Correlations between ERP and state vectors/radio source positions not available in combination solutions due to software limitations.
### Table 6: Comparison of Amounts of Input Data For 15 Day Solutions

<table>
<thead>
<tr>
<th>System</th>
<th>Data (obs., bytes)</th>
<th>Normals (bytes)</th>
<th>ERP (parms.)</th>
<th>ERP (bytes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 hours</td>
<td>2030, 143 kb</td>
<td>155724</td>
<td>183</td>
<td>2196</td>
</tr>
<tr>
<td>12 hours</td>
<td>same</td>
<td>46284</td>
<td>93</td>
<td>1116</td>
</tr>
<tr>
<td>1 day</td>
<td>same</td>
<td>15864</td>
<td>48</td>
<td>576</td>
</tr>
<tr>
<td>2 days</td>
<td>same</td>
<td>6264</td>
<td>24</td>
<td>288</td>
</tr>
<tr>
<td>5 days</td>
<td>same</td>
<td>3192</td>
<td>12</td>
<td>144</td>
</tr>
<tr>
<td>SLR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 hours</td>
<td>24489, 1720 kb</td>
<td>155724</td>
<td>183</td>
<td>2196</td>
</tr>
<tr>
<td>12 hours</td>
<td>same</td>
<td>46284</td>
<td>93</td>
<td>1116</td>
</tr>
<tr>
<td>1 day</td>
<td>same</td>
<td>15864</td>
<td>48</td>
<td>576</td>
</tr>
<tr>
<td>2 days</td>
<td>same</td>
<td>6264</td>
<td>24</td>
<td>288</td>
</tr>
<tr>
<td>5 days</td>
<td>same</td>
<td>3192</td>
<td>12</td>
<td>144</td>
</tr>
<tr>
<td>VLBI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 hours</td>
<td>14086, 990 kb</td>
<td>202764</td>
<td>183</td>
<td>2196</td>
</tr>
<tr>
<td>12 hours</td>
<td>same</td>
<td>73164</td>
<td>93</td>
<td>1116</td>
</tr>
<tr>
<td>1 day</td>
<td>same</td>
<td>32664</td>
<td>48</td>
<td>576</td>
</tr>
<tr>
<td>2 days</td>
<td>same</td>
<td>17688</td>
<td>24</td>
<td>288</td>
</tr>
<tr>
<td>5 days</td>
<td>same</td>
<td>11928</td>
<td>12</td>
<td>144</td>
</tr>
</tbody>
</table>

Notes:
1. "bytes" for the data is computed as the number of observations times 72 bytes/observation (as in the GEODYN binary format).
2. "bytes" for the normal equations is the number of bytes used to store the normals in GEODYN E-matrix format.
3. "parms." for ERP is the number of ERP recovery periods times 3 (for X and Y polar motion, and UT1-UTC).
4. "bytes" for ERP is determined from the number of parameters times 3 (for time, parameter values, and standard deviation) times 4 bytes.
5. Additional information, such as station reports, problem reports, calibration data, model information, etc. is not considered. Delay rates are included in the VLBI observations.
Table 7: Comparison of Computer Time For 15 Day Solutions

<table>
<thead>
<tr>
<th>System</th>
<th>ERP Recovery Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6 hours</td>
</tr>
<tr>
<td><strong>LLR</strong></td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td>284</td>
</tr>
<tr>
<td>normals only</td>
<td>192</td>
</tr>
<tr>
<td><strong>SLR</strong></td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td>1210</td>
</tr>
<tr>
<td>normals only</td>
<td>887</td>
</tr>
<tr>
<td><strong>VLBI</strong></td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td>266</td>
</tr>
<tr>
<td>normals only</td>
<td>76</td>
</tr>
<tr>
<td>weighted mean</td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td>7</td>
</tr>
<tr>
<td>with solution</td>
<td>1767</td>
</tr>
<tr>
<td>normal equation combination</td>
<td>15</td>
</tr>
<tr>
<td>with normals</td>
<td>1170</td>
</tr>
<tr>
<td>grand solution</td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td>1969</td>
</tr>
</tbody>
</table>

Notes:
1. All times are CPU seconds on an IBM 3081D.
2. GEODYN 8210.7 used for LLR, SLR, VLBI solutions and setup of normals, SOLVE 8212.0 used for the data combination solution, and local software for wt. mean solution.
3. The LLR, SLR, and VLBI solutions were done with 3 (outer) iterations. The normal equation combination solution is a 1 iteration solution, while the grand solution has, in effect 2 (outer) iterations.
4. The VLBI values were doubled to account for delay rate observation processing.
5. The “with solution” and “with normals” include the times for the LLR, SLR, and VLBI solutions and normal equation setups respectively.
BASELINE ESTIMATION WITH SEMIDYNAMIC AND GEOMETRIC SATELLITE METHODS

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ABSTRACT

Accurate differential positioning via dynamic satellite methods is a complicated process. In an attempt to simplify this process a semidynamic method has been investigated in a real data environment. In this method quasi-simultaneous observations from pairs of stations are transformed to Simultaneous Range Differences (SRD's). With this transformation it is anticipated to reduce the effects of orbital and observational residual biases and, therefore, to obtain baselines the accuracy of which are less sensitive to the overall orbital accuracy and yet compatible to that of the observations. Using laser range observations to Lageos collected during the MERIT Main Campaign, baselines have been estimated via both the SRD and the geometric methods. Baselines estimated via the geometric method are independent of orbital errors and any inconsistencies affecting the implementation of the Terrestrial Reference Frame, and therefore they have been used in the present study as standards of comparison. From this comparison it was concluded that for baselines of regional extent, the SRD method is very efficient and at least as accurate as the more complex dynamic methods.

1. INTRODUCTION

In the geometric method (Veis, 1960; Mueller, 1964), the observed satellite positions are treated as auxiliary independent points in space, and they are only used to relate the observations geometrically via the resulting space networks. This process necessitates use of simultaneous observations without any reference to the fact that the satellite moves along a path (orbit) defined by its physical environment. Consequently, baselines estimated via the geometric method are free of errors affecting the orbit and the implementation
of the Terrestrial Reference Frame (TRF). The accuracy of these baselines depends on the geometry implied by the spatial distribution of the available observations, on their accuracy, and on whether the motions of the observing stations have been modeled properly for the time span of the observations.

In the dynamic and semidynamic methods, the observed satellite positions are constrained to lie on a space curve (Schwarz, 1969) which should resemble the satellite orbit under question to the required degree of accuracy.

Accurate baseline estimation via dynamic methods requires highly sophisticated orbital modeling and the proper implementation of the TRF, which requires either simultaneous determination of the Earth Rotation Parameters (ERP's) or utilization of a consistent set of ERP's obtained in a separate step (Pavlis and Mueller, 1983).

Accurate baseline estimation through semidynamic methods is not as complicated because the required orbital model is simpler and from the dynamic parameters only the position and orientation of the arcs involved are usually determined to "best" fit the available observations. Adjustment for the ERP's is not necessary since proper implementation of the TRF is guaranteed by the use of simultaneous observations. The orbital model can be further simplified if a proper combination (differencing) of the observations cancels or at least reduces the errors caused by the model simplifications. Because of this, the laser observations to Lageos have been transformed to simultaneous range differences (Fig. 1). The potential of using SRD observables for baseline estimation had been studied earlier in the simulated environment by Pavlis (1982).

The MERIT Main Campaign (Sept. 1983 - Oct. 1984) resulted in extensive simultaneous laser tracking of Lageos (Wilkins and Mueller, 1986), making it possible to study the performance of the SRD method on a continental scale. Fig. 2 shows the laser stations involved in the present study.

2. EDITING LASER OBSERVATIONS AND THE GENERATION OF SIMULTANEOUS RANGES

Since Lageos is a passive satellite, it is not possible for coobserving stations to observe exactly simultaneously, even if the same part of the orbit has been coobserved. Implementation, however, of either the geometric or the SRD methods requires strict simultaneity and, thus, interpolation of the observed laser ranges. A successful interpolation also includes the detection
AD = overlapping period for stations 1 and 2

$r_{2,m}^o$ = $m$th range observation at station 2

$r_{1,j}^I$ = $j$th interpolated range at station 1

$r_{2,m}^o - r_{1,j}^I = j$th SRD observable

Fig. 1 Simultaneous range differencing (SRD).
Fig. 2 Location of the American stations used.
and rejection of erroneous observations and the selection of the proper interpolation method.

After appropriately correcting the observed ranges for speed of light (299792458 m/s), refraction (Marini and Murray, 1973), center of mass (0.24 m), and earth tide displacement (Melbourne et al., 1983), the detection and rejection of erroneous observations was accomplished with the one-dimensional data snooping procedure originated by Baarda (1968). This procedure was implemented by interpolating the observed laser ranges on a pass-by-pass basis (Dedes, 1987). Chebychev polynomials were used as base functions. The effectiveness of this procedure in editing laser range observations is shown in Fig. 3 as an example for one representative pass recorded by the station Quincy, California (7109). These residuals indicate the presence of outliers of about 150 meters in the observed ranges (left plot). These outliers have been effectively eliminated with the data snooping procedure (right plot). Although this procedure is very effective, it is relatively slow and therefore expensive (ibid.)

The proper selection of the interpolation method was based on an analysis of available interpolation methods, i.e., global and piecewise interpolations (ibid.) This analysis indicated that the effects of the data gaps in the piecewise interpolation are not uniformly distributed over the interval of approximation and the noise level of the interpolated ranges is twice as large as that of the observed ones. With the global interpolation, on the other hand, the effects of the gaps are uniformly distributed over the interval of approximation and they can be kept well below the noise level of the observations. Because of this, global interpolation was chosen to generate simultaneous ranges (SR's) and simultaneous range differences. As of the base functions, the Chebychev polynomials offered an excellent choice because they result not only in a well-conditioned normal equation matrix but also in an even distribution of the residuals (Carnahan, 1969).

The SR observations for the geometric method were obtained by first identifying passes continuously coobserved (i.e., data gaps smaller than 60 seconds) by four or more stations. For each of these passes the station with the least observations was identified. At its observing epochs simultaneous observations for all of the remaining stations were generated. The maximum gap of 60 seconds was chosen because it does not degrade the interpolation and because when gaps are longer than 60 seconds they tend to be several
minutes long (ibid.) At least four coobserving stations are needed to avoid "critical configurations," for which the solution is singular (Blaha, 1971; Tsimis, 1973).

As of the generation of the SRD observables, the observing stations were divided into pairs of stations having quasi-simultaneous observations. For each of these pairs the station with the least observations was identified, and at its observing epochs observations for the alternate station were generated. The SRD observables were then obtained by subtracting the actually observed ranges from the corresponding interpolated ranges of the alternate station (Fig. 1).

3. STEADY STATE RESPONSE BASELINE ESTIMATION

In the present study, it is assumed that steady state response has been reached when the accuracy of the estimated baselines cannot be further improved by incorporating additional observations. It is also assumed that the steady state response has been reached when the incorporation of additional observations does not change the lengths of the baselines at the centimeter level. Incorporation of additional observations leads to a steady state response because it "improves" the weight coefficient matrix of the adjusted parameter vector (Blaha, 1971). Observations of higher accuracy (Dedes, 1987) and/or constraints on estimable parameters (Van Gelder, 1978) also improve the weight coefficient matrix of the parameter vector, thereby leading again to steady state response. The parameter vector in the SRD solutions contains corrections to the earth-fixed coordinates of the observing stations and to the initial state vectors of all the satellite arcs. In the geometric solutions the adjusted parameter vector contains corrections to the coordinates of the observing stations and to all the observed satellite positions. The coordinates of the stations are used to estimate the baseline lengths and their statistics. As the weight coefficient matrix improves, the unscaled variances of the coordinates of the stations also improve. This improvement leads to reduced unscaled variances for the baselines and, therefore, to steady state. This, however, is true only if the nonlinearity of the models allows such reductions to take place.

Thus incorporation of additional observations, observations of better accuracy, and constraints on estimable parameters constitute the three major factors leading to a steady state response. In the present study this
response for both the SRD and the geometric solutions has been reached by balancing the contribution of the first and third factors because one has no control over the second factor once the observations are made. Furthermore, the goal has been to restrict the contribution of the third factor as much as possible because steady state response reached on the basis of constraining estimable parameters may be affected severely by the errors affecting those parameters (ibid.)

3.1 Steady State Response SRD Baseline Estimation

The underlying characteristic inherently present in any factor leading to a steady state response is the ability to strengthen the geometric characteristics of the SRD solutions. With a strong geometry steady state response can be reached from relatively few observations. Strong geometry in the SRD solutions is present when passes are parallel to the baseline (Pavlis, 1979, 1982). This geometry deteriorates when the observed passes cross the baseline at right angles (Pavlis, 1982) and when the lengths of the coobserved passes become shorter while the length of the baseline increases (Dedes, 1987).

With the above ideas in mind, several single baselines have been estimated using SRD observables. In these solutions the coordinates of one baseline end were held fixed. The coordinates of the other end along with the initial state vectors of all of the satellite arcs were allowed to adjust by assigning through their weight coefficient matrices standard deviations reflecting accuracy estimates for the approximate values of the corresponding parameters (ibid.)

The baseline lengths varied from 8 m to 3700 km. The results at the steady state response are shown in Table 1. Table 2 is an indication of the number of passes needed in this study to achieve steady state response for the different baseline lengths.
Table 1  Baseline Steady State Response of the SRD Solutions

<table>
<thead>
<tr>
<th>Baseline</th>
<th>Length (m)¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>7109-7110</td>
<td>883602.25</td>
</tr>
<tr>
<td>7109-7886</td>
<td>7.74</td>
</tr>
<tr>
<td>7110-7122</td>
<td>1437139.30</td>
</tr>
<tr>
<td>7110-7220</td>
<td>15.24</td>
</tr>
<tr>
<td>7110-7265</td>
<td>274069.48</td>
</tr>
<tr>
<td>7110-7886</td>
<td>883606.34</td>
</tr>
<tr>
<td>7110-7086</td>
<td>1198291.01</td>
</tr>
<tr>
<td>7109-7105</td>
<td>3703351.71</td>
</tr>
</tbody>
</table>

¹ Baseline endpoints coincide with instrument's origin.

Table 2  Steady State Response of the SRD Method

<table>
<thead>
<tr>
<th>No. of Passes Needed</th>
<th>Approx. Occupation Time in This Dataset</th>
<th>Steady State Up to ___ km</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 - 15</td>
<td>1 week</td>
<td>1000</td>
</tr>
<tr>
<td>20 - 25</td>
<td>3 months</td>
<td>1500</td>
</tr>
<tr>
<td>25 - 30</td>
<td>3 months</td>
<td>2500</td>
</tr>
<tr>
<td>50 - 55</td>
<td>8 months</td>
<td>3500</td>
</tr>
</tbody>
</table>

3.2 Steady State Response Geometric Baseline Estimation

Since in the geometric method the observed satellite positions are treated as auxiliary independent points in space, any minimum constraint geometric solution depends entirely on the amount and distribution of observations. The geometric strength of the solutions presented later in this section is based on the examination of Table 3. The bottom part of the second and third columns contains the total number of observations (N), the degrees of freedom (DF), and the a posteriori variance of unit weight (σ₀²) obtained by the minimum constraint solutions on the basis of the data shown in the corresponding columns. The bottom part of the third column contains the same information for two solutions, one minimum constraint and one overconstrained, both of which were obtained on the basis of the data listed in that column. The results of these adjustments are shown in Table 4.
Table 3  Statistics of the Geometric Adjustments

<table>
<thead>
<tr>
<th>Station ID</th>
<th>Sep 83 - May 84</th>
<th>Sep 83 - Aug 84</th>
<th>Sep 83 - Oct 84</th>
</tr>
</thead>
<tbody>
<tr>
<td>7105</td>
<td>7143</td>
<td>18990</td>
<td>19214</td>
</tr>
<tr>
<td>7109</td>
<td>10198</td>
<td>22784</td>
<td>23936</td>
</tr>
<tr>
<td>7112</td>
<td>2884</td>
<td>3467</td>
<td>3467</td>
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<tr>
<td>7122</td>
<td>10996</td>
<td>11284</td>
<td>12212</td>
</tr>
<tr>
<td>7220</td>
<td>1969</td>
<td>1969</td>
<td>1969</td>
</tr>
<tr>
<td>7110</td>
<td>11549</td>
<td>24200</td>
<td>25352</td>
</tr>
<tr>
<td>7062</td>
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<td>841</td>
</tr>
<tr>
<td>7086</td>
<td>1412</td>
<td>4400</td>
<td>4624</td>
</tr>
<tr>
<td>7907</td>
<td>176</td>
<td>245</td>
<td>245</td>
</tr>
<tr>
<td>7082</td>
<td>299</td>
<td>299</td>
<td>299</td>
</tr>
<tr>
<td>7210</td>
<td>712</td>
<td>756</td>
<td>1684</td>
</tr>
<tr>
<td>7265</td>
<td>4395</td>
<td>4395</td>
<td>4395</td>
</tr>
<tr>
<td>7886</td>
<td>—</td>
<td>11859</td>
<td>11859</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>DF</th>
<th>$\sigma_0^2$</th>
<th>$\sigma_0^2$</th>
<th>$\sigma_0^2$</th>
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</thead>
<tbody>
<tr>
<td>52574</td>
<td>14519</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
</tr>
<tr>
<td>105489</td>
<td>29478</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>110097</td>
<td>30630</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
</tr>
</tbody>
</table>

(1) Minimum constraint solution, coordinates fixed: X, Y, Z for 7109; X, Y for 7122; Z for 7105.
(2) Overconstraint solution, coordinates fixed: X, Y, Z for 7109; X, Y, Z for 7122; Z for 7105.
(3) Coordinates fixed to those of (CSR)85L01

N = total number of observations
DF = degrees of freedom
$\sigma_0^2$ = a posteriori variance of unit weight

The geometric strength in each of the minimum constraint solutions is primarily drawn from the stations with the most observations. In assessing this strength it is assumed that stations 7886 and 7220 coincide with stations 7109 and 7110 since these stations are away from each other by 8 m and 15 m respectively. With this in mind it is easily seen from Table 3 that 86 percent of the available observations have been recorded by stations 7105, 7109, 7110 and 7122. These solutions, therefore, tend to be sensitive to how close these stations are from their best fitting plane since with stations in a plane or close to forming a plane six are needed for a nonsingular network (Blaha, 1971). Furthermore, 91 percent of the observations were recorded by stations 7105, 7109, 7122, 7220, 7110, 7062, 7265 and 7886. These stations are
concentrated around two intersected lines defined by stations 7109 with 7122 and 7122 with 7105. Since two intersecting lines belong in the family of second-order curves, it is reasonable to expect that type (B) near singularity (Blaha, 1971) tends to weaken the strength of these solutions. Such singularity occurs when stations lying in a plane and making off-plane observations are not themselves off-curve stations (ibid.) Furthermore, since a relatively large network is employed, the simultaneously observed satellite positions tend to be concentrated in the area above the middle part of the network, and therefore they tend to be closer to a plane. This in turn would lead to type (C) near singularity (ibid.) since off-plane targets are needed to avoid this type of singularity. Thus, due to these near singularities, the geometry is not strong enough to allow steady state response via the minimum constraint solutions. This is confirmed by observing in Table 4 that all the baselines but 7110-7220 and 7109-7886 change their lengths by several centimeters with the incorporation of additional observations (compare solutions A1, B1, C1). The existence of weak geometry is further confirmed by using data set C and by changing the minimum constraints from type 1 to 2 (Dedes, 1987).

The weak geometry will imply, via the minimum constraint solutions, a network with the tendency to shrink towards its center, more specifically towards the area where most of the observations are concentrated. This implies that the scale of the adjusted network is not properly defined. A better definition of the scale in these solutions has been attempted by constraining, in addition to minimum constraints, the third coordinate of station 7122 (Table 4, solution C3). In doing so one implicitly constrains the length of baseline 7109-7122 (which in the geometric method should be an estimable parameter (Mueller et al., 1975)).

Application of this additional constraint brings the unscaled standard deviations of the baselines listed in Table 4 to the millimeter level, thereby indicating that steady state response has been reached. This is confirmed by examining how the steady state response of baselines 7109-7886 and 7110-7220 are related to their unscaled standard deviations.
Table 4  Baseline Steady State Response of the Geometric Solution

<table>
<thead>
<tr>
<th>Baseline</th>
<th>No. of Observ.</th>
<th>Length (m)</th>
<th>Data Solution Set</th>
<th>Solution Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>7109-7110</td>
<td>9,186</td>
<td>883601.637 ± 0.02</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>21,772</td>
<td>.608 ± 0.02</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>22,924</td>
<td>.661 ± 0.02</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td>883601.661 ± 0.02</td>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td>883602.245 ± 0.009</td>
<td>C</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7109-7265</td>
<td>3,363</td>
<td>627043.412 ± 0.02</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>.452 ± 0.01</td>
<td>B</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.535 ± 0.01</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.535 ± 0.01</td>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.988 ± 0.005</td>
<td>C</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7109-7886</td>
<td>11,859</td>
<td>7.746 ± 0.002</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>.746</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.746</td>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>.746</td>
<td>C</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7109-7122</td>
<td>8,644</td>
<td>2280712.335 ± 0.07</td>
<td>A</td>
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</tr>
<tr>
<td>8,932</td>
<td>2.700 ± 0.05</td>
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<td>1</td>
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<tr>
<td>9,860</td>
<td>3.188 ± 0.05</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td>3.188 ± 0.05</td>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td>4.949 ± 0.0005</td>
<td>C</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7110-7122</td>
<td>10,060</td>
<td>1437137.428 ± 0.05</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>10,348</td>
<td>.780 ± 0.04</td>
<td>B</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>11,276</td>
<td>8.187 ± 0.03</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>&quot;</td>
<td>8.187 ± 0.03</td>
<td>C</td>
<td>2</td>
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<td>&quot;</td>
<td>9.288 ± 0.009</td>
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<tr>
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<tr>
<td></td>
<td>.221 ± 0.005</td>
<td>B</td>
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<tr>
<td></td>
<td>.218 ± 0.005</td>
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<td>1</td>
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<tr>
<td></td>
<td>.218 ± 0.005</td>
<td>C</td>
<td>2</td>
<td></td>
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<tr>
<td></td>
<td>.208 ± 0.005</td>
<td>C</td>
<td>3</td>
<td></td>
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<tr>
<td>7110-7265</td>
<td>3,866</td>
<td>274069.453 ± 0.01</td>
<td>A</td>
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<tr>
<td></td>
<td>.383 ± 0.008</td>
<td>B</td>
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<tr>
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<td>.355 ± 0.008</td>
<td>C</td>
<td>1</td>
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<td>.355 ± 0.008</td>
<td>C</td>
<td>2</td>
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<tr>
<td></td>
<td>.474 ± 0.007</td>
<td>C</td>
<td>3</td>
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<tr>
<td>7110-7886</td>
<td>11,859</td>
<td>883605.698 ± 0.02</td>
<td>B</td>
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</tr>
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<td></td>
<td>.751 ± 0.02</td>
<td>C</td>
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<td></td>
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<tr>
<td></td>
<td>.751 ± 0.02</td>
<td>C</td>
<td>2</td>
<td></td>
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<tr>
<td></td>
<td>6.335 ± 0.009</td>
<td>C</td>
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<tr>
<td>7122-7265</td>
<td>4,184</td>
<td>1663980.848 ± 0.05</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1.161 ± 0.04</td>
<td>B</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.555 ± 0.04</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.555</td>
<td>C</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.823 ± 0.005</td>
<td>C</td>
<td>3</td>
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### Table 4 (cont'd)

<table>
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<tr>
<th>Baseline</th>
<th>No. of Observ.</th>
<th>Length (m)</th>
<th>Data Set</th>
<th>Solution Type</th>
</tr>
</thead>
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<tr>
<td>7122-7886</td>
<td>0</td>
<td>2280718.021 ± 0.05</td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.509 ± 0.05</td>
<td>C</td>
<td>1</td>
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<tr>
<td></td>
<td>0</td>
<td>18.509 ± 0.05</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>20.269 ± 0.002</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>7220-7265</td>
<td>0</td>
<td>274066.158 ± 0.010</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.090 ± 0.009</td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.064 ± 0.008</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.064 ± 0.008</td>
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</tr>
<tr>
<td></td>
<td>0</td>
<td>0.189 ± 0.007</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>7265-7886</td>
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<td>627048.351 ± 0.01</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.434 ± 0.01</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.434 ± 0.01</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.887 ± 0.006</td>
<td>C</td>
<td>3</td>
</tr>
</tbody>
</table>

1 Data Sets: A Sep 83 - May 84  
B Sep 83 - Aug 84  
C Sep 83 - Oct 84

2 Solution Type:  
1 Minimum constraint solution, Cartesian coordinates fixed: X,Y,Z for 7109; X,Y for 7122; Z for 7105  
2 Minimum constraint solution, Cartesian coordinates fixed: X,Y,Z for 7109; Y,Z for 7122; Z for 7105  
3 Overconstraint solution, Cartesian coordinates fixed: X,Y,Z for 7109 and 7122; Z for 7105

### 4. Baseline Comparisons

Assuming that steady state response for the baselines shown in Table 4 has been reached, and considering that these baselines are independent of any orbital errors and any inconsistencies affecting the implementation of the Terrestrial Reference Frame, they have been used as standards of comparison to assess the accuracy of the baselines estimated via the SRD and the range dynamic methods. The baselines estimated via the range dynamic methods are those reported by the Central Institute of Physics of the Earth (ZIPE) (Montag et al., 1985) and by the Center of Space Research, University of Texas (Tapley et al., 1985). These baselines were estimated on the basis of observations collected during the MERIT Main Campaign.
Table 5  Baseline Differences (m)

<table>
<thead>
<tr>
<th>Baseline</th>
<th>ZIPE</th>
<th>(CSR)85L01</th>
<th>ZIPE</th>
<th>(CSR)85L01</th>
<th>SRD</th>
</tr>
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<tr>
<td></td>
<td>-SRD</td>
<td>-SRD</td>
<td>GEOM</td>
<td>-GEOM</td>
<td>GEOM</td>
</tr>
<tr>
<td>7109-7110</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>7109-7265</td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>7109-7886</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>7110-7122</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>7110-7220</td>
<td></td>
<td>-0.04</td>
<td></td>
<td>-0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>7110-7265</td>
<td>-0.04</td>
<td>-0.07</td>
<td>-0.03</td>
<td>-0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>7110-7265</td>
<td></td>
<td></td>
<td>-0.02</td>
<td>-0.06</td>
<td></td>
</tr>
<tr>
<td>7122-7265</td>
<td></td>
<td></td>
<td>0.01</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>7220-7265</td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>7265-7886</td>
<td></td>
<td></td>
<td>0.01</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>7110-7886</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7110-7086</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7109-7105</td>
<td>0.06</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of the comparison are shown in Table 5. This table lists the differences of only those baselines for which steady state response was possible either through the SRD or the geometric solutions.

The baseline differences between SRD, ZIPE and CSR solutions listed in the second and third columns of this table are negative for north-south baselines and positive for the east-west baselines. This suggests that the ZIPE and CSR baselines are affected by orbital errors not only because the SRD baselines are largely insensitive to those errors but also because they are in closer agreement with the baselines estimated via the geometric method (Table 5, columns 4-6). The largest differences of the SRD and the geometric baseline estimates from those of the (CSR)85L01 solution are associated with station 7265 (Mohave, Calif.) This station experienced many problems during the MERIT Main Campaign, and therefore it is very likely that the editing of this station in the (CSR)85L01 solution was performed unsuccessfully.

Although the geometric solution C3 (Table 4) was overconstrained to the (CSR)85L01 solution, the baselines estimated via the SRD method are on the average in closer agreement with those of the geometric method than those of both the ZIPE and CSR baselines. Since these comparisons are based on
baselines of up to 1500 km, one can claim safely that for baselines up to that length the SRD method is at least as accurate as the standard dynamic methods and this on the basis of a simple orbital model (next section) and a simple orbit adjustment.

5. RESPONSE OF THE SRD METHOD TO THE SIMPLIFICATION OF THE ORBITAL MODEL

The aim of the present study was not to estimate Lageos's orbit with the highest degree of accuracy but rather to employ orbital models as simple as possible and yet estimate baselines with an accuracy compatible to that of the observations.

Since the temporal variations of the baseline endpoints have been accounted for to the required degree of accuracy and since inconsistencies in the implementation of the Terrestrial Reference Frame do not affect the SRD observables, the errors affecting the SRD baselines are mainly those of the orbit accumulated over the integration periods and not cancelled by the SRD observable. Therefore, the questions to be addressed are as follows:

- Is the sophistication of the orbital model employed in this study sufficient to result in baselines the accuracy of which are compatible to that of the observations?
- If the answer is yes, how much can the sophistication of the orbital model be reduced without affecting the accuracy of the baselines? If the answer is no, how much should the sophistication of the orbital model be enhanced?

To set up the guidelines as to what simplifications, if any, can be applied to the orbital model without affecting the accuracy of the baselines at the centimeter level, several tests were performed, the results of which are shown in Table 6. This table contains the baseline differences obtained as the orbital model was simplified from one containing the PGS1680 12x12 gravity field (Christodoulidis et al., 1985) together with the direct point mass (PM) effects of the Sun and Moon, the tidal (TD) effects due to the Sun and Moon (Diamante et al., 1972), the solar radiation (SR) pressure effects, and the along-track (AT) acceleration effects (Smith et al., 1985; Afonso et al., 1985;
Rubincam et al., 1985), to that containing only a 2x2 gravity field and the direct PM effects of Sun and Moon. The first column of Table 6 lists the orbital models employed to estimate the baselines which were subsequently differenced from those estimated on the basis of the orbital model shown in the title of this table (i.e., 12x12 gravity field+(1)). The resulting differences are shown for only three baselines estimated on the basis of integration periods of up to seven days (7110-7265), up to one hour (7109-7110), and up to three days (7110-7122) respectively.

The perturbations in Lageos's orbit caused by the ocean tides were not included in the orbital model, and that is why they are not shown in Table 6. These perturbations have been ignored because they can reach only as much as 20 percent of the perturbations caused by the tides of the solid earth (Musen, 1973). Inspection of Table 6 (row 3) reveals that the elimination of ocean tidal effects from the orbital model will hardly affect the baselines at the centimeter level.

A careful study of Table 6 reveals that baselines of up to 1500 km estimated via the SRD method will not be affected at the centimeter level if the orbital model includes the following:

- Short arc solutions: 4x4 gravity field and the direct PM effects of the Sun and Moon
- Long arc solutions with arcs up to three days: 8x8 gravity field, the direct PM effects of the Sun and Moon, the TD effects, and the SR pressure effects
- Long arc solutions with arcs up to seven days: 10x10 gravity field, the PM effects of the Sun and Moon, the TD effects, and the SR pressure effects.

Therefore, the sophistication of the orbital model employed in the present study results in baselines being accurate at the centimeter level. This level of accuracy is compatible to that of the laser observations collected during the MERIT Main Campaign.
Table 6  Baseline Differences (in meters) With Respect to Those Using an Orbital Model Including a 12x12 Gravity Field + (1)

<table>
<thead>
<tr>
<th>Force Model Gravity Field + ( )</th>
<th>7110-7265</th>
<th>7109-7110</th>
<th>7110-7122</th>
</tr>
</thead>
<tbody>
<tr>
<td>12x12 + (2)</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>12x12 + (3)</td>
<td>0.02</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>12x12 + (4)</td>
<td>-0.08</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>12x12</td>
<td>5.14</td>
<td>-0.02</td>
<td>-3.15</td>
</tr>
<tr>
<td>10x10 + (2)</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>10x10 + (4)</td>
<td>--</td>
<td>0.00</td>
<td>--</td>
</tr>
<tr>
<td>8x8  + (2)</td>
<td>0.03</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>8x8  + (4)</td>
<td>--</td>
<td>0.00</td>
<td>--</td>
</tr>
<tr>
<td>6x6  + (2)</td>
<td>1.02</td>
<td>0.00</td>
<td>2.02</td>
</tr>
<tr>
<td>6x6  + (4)</td>
<td>--</td>
<td>0.01</td>
<td>--</td>
</tr>
<tr>
<td>4x4  + (2)</td>
<td>0.90</td>
<td>0.00</td>
<td>3.37</td>
</tr>
<tr>
<td>4x4  + (4)</td>
<td>--</td>
<td>-0.01</td>
<td>--</td>
</tr>
<tr>
<td>3x3  + (4)</td>
<td>--</td>
<td>-0.02</td>
<td>--</td>
</tr>
<tr>
<td>2x2  + (4)</td>
<td>--</td>
<td>-0.07</td>
<td>--</td>
</tr>
</tbody>
</table>

(1) (PM) + (TD) + (SR) + (AT)  PM = point mass effects of sun & moon
(2) (PM) + (TD) + (SR)          TD = tidal effects due to sun & moon
(3) (PM) + (TD)                SR = solar radiation pressure effects
(4) (PM)                       AT = along-track acceleration effects

6. CONCLUSIONS AND RECOMMENDATIONS

For regional (as opposed to global) baseline estimations the SRD method is very effective and at least as accurate as the more complex dynamic mode methods, and this on the basis of a simple orbital model and a simple orbit adjustment. This in turn makes it possible, if enough observations are available, to estimate baselines up to 1000 km with centimeter-level accuracy on the basis of only a 4x4 gravity field and the direct PM effects of the Sun and Moon. However, employing a 10x10 gravity field together with the PM effects of the Sun and Moon, the TD effects and the SR radiation pressure effects, baselines of up to 1500 km can be estimated at the centimeter level.

Since the SRD method is insensitive to the inconsistencies affecting the implementation of TRF, simultaneous determination of the ERP's is not necessary, thereby making the use of the SRD method even simpler. Therefore, the SRD method offers an accurate alternative for projects designed to study regional crustal movements (Wegener/Medlas).
Effective implementation of the SRD method requires effort by at least two observing stations to achieve simultaneous tracking. The baselines being estimated should be chosen, if such a choice is feasible, to be closely parallel to the two main groundtracks of the Lageos satellite.

The response of the SRD method should also be studied using normal points and a network setup. Full potential of the SRD method should be utilized at regional crustal motion projects such as Wegener/Medlas, where the proximity of the stations allows for the types of observations needed.

Finally, selected geometric solutions are possible and they can be used effectively as standards of comparison.

Acknowledgments. This work has been carried out under NASA Grant NSG 5265, OSURF Project 711055. Extensive computer support was provided by the Instruction and Research Computer Center of The Ohio State University.

References


Department of Geodetic Science and Surveying

ORBIT DETERMINATION FOR THE GLOBAL POSITIONING SYSTEM SATELLITES.

First Quarterly Status Report
OSURF Project No. 711055

The Ohio State University
Columbus, Ohio 43210

January, 1988
1. Introduction

1.1 Project Goals.

The objective of this investigation is to develop a user-friendly software capable of using satellite pseudo range information to obtain survey quality (1ppm) satellite ephemerides. This software should allow independent modeling of user and satellite clock drift histories either as smoothly varying functions (i.e., low order polynomials) or as epoch-to-epoch independent clock variations. The measurement processor of the software should allow processing of pseudo ranges in interferometric mode.

1.2 Summary of Progress Up to Date.

The original version of the orbit determination software, obtained from NOAA and running on HP-9000, was modified to adhere to the more stringent requirements of the FORTVS compiler available on the OSU main frame. The performed modifications included changes such as reordering of variables within many common blocks, elimination of problematic entry points from several subroutines and double precision representation of all numerical values to avoid loss of significance. Subsequently, this software was transferred from the OSU main frame to an IBM personal computer.

Before proceeding with the enhancement of user interface and epoch-to-epoch clock drift representation, it was considered appropriate to systematically proceed, whenever necessary, with the documentation of the subroutines and the description of the variables listed in the common blocks of those subroutines. This in turn, led not only to the successful expansion of the station and satellite clock models to include epoch-to-epoch clock drift representation but also to a greater flexibility in handling the orbit determination software. As a result the user interface has been enhanced to allow much easier choices regarding station and satellite clock models.

2. Enhancement of User Interface and Epoch-to-Epoch Drift Representation.

The user interface of the orbit determination software is continuously upgraded with the aim of maximizing user choices and at the same time minimizing the a priori information needed in making these choices. To this extent the prototype coding of the software has been enhanced so as to analyze the available data set and determine the ID of the observing stations and the observed satellites as well as the starting epoch and the length of the observing campaign. This information is then displayed and the user is free to choose the stations and/or satellites to be incorporated in the current solution. The user is also allowed to choose the models of the ground and satellite clocks either as low order polynomials or as epoch-to-epoch independent estimates. Furthermore, all of the dynamic and the clock parameters can assume, according to user's choices, any apriori chosen values.

The mathematical model of the pseudo range is a function of the true range, the receiver and satellite clock corrections

\[ R_j = \rho_j + (\Delta t_6_j - \Delta t_5_j) \]  (1)
where

\[ \rho_j = \{(\bar{S}_j - \bar{x}_i)^T(\bar{S}_j - \bar{x}_i)\}^{1/2} \]  
(2)

and

\[ \bar{x}_i = (SNP)^T \bar{G}_i \]  
(3)

The quantities \( R_j \) and \( \rho_j \) are the pseudo and true ranges from receiver \( i \) to the observed satellite position \( j \) respectively. The quantities \( \Delta t_{6j} \) and \( \Delta t_{8j} \) are the corrections applied to the receiver and satellite clocks. These corrections can be treated either as independent epoch-to-epoch estimates or as low-order polynomials. The vectors \( S_j \) and \( \bar{x}_i \) designate the position vectors of the satellite and the ground receiver \( i \) at epoch \( j \) in the J2000.0 mean-of-date system. The matrices \( S, N, \) and \( P \) denote the earth rotation, the nutation and the precession matrices respectively. The vector \( G_i \) denotes the earth-fixed position vector of receiver \( i \).

The normals resulting from eq. (1) are decomposed using Givens transformations (plane rotations) without square roots (Gentleman, 1973) and subsequently are solved with backward substitutions.

The initial testing of the epoch-to-epoch implementation was carried out using pseudo ranges recorded by stations 1, 2 and 3 to satellites 6, 8, 9, and 11. These stations being equipped with TI4100 pseudo rangers were located in Texas, Florida and Massachusetts respectively. In the initial tests, the results of which are shown in Tables 1 and 2, the Cartesian coordinates of stations 1, 2, and 3 were held fixed while the initial state vectors along with the solar radiation pressure parameters of all of the satellite arcs involved were "free" adjusted.

The initial testing of the epoch-to-epoch implementation was carried out using pseudo ranges recorded by stations 1, 2 and 3 to satellites 6, 8, 9, and 11. These stations being equipped with TI4100 pseudo rangers were located in Texas, Florida and Massachusetts respectively. In the initial tests, the results of which are shown in Tables 1 and 2, the Cartesian coordinates of stations 1, 2, and 3 were held fixed while the initial state vectors along with the solar radiation pressure parameters of all of the satellite arcs involved were "free" adjusted.

Table 1

<table>
<thead>
<tr>
<th>Clocks</th>
<th>Offsets(1)(nanoseconds)</th>
<th>Drifts(1)(nanoseconds/second)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OSU(2)-NGS(2)</td>
<td>OSU(3)-NGS(2)</td>
</tr>
<tr>
<td>STA. 1</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>&quot; 2</td>
<td>0.000</td>
<td>-10.836</td>
</tr>
<tr>
<td>&quot; 3</td>
<td>0.000</td>
<td>27.338</td>
</tr>
<tr>
<td>SAT. 6</td>
<td>0.000</td>
<td>epoch-to-epoch</td>
</tr>
<tr>
<td>&quot; 8</td>
<td>0.000</td>
<td>&quot;</td>
</tr>
<tr>
<td>&quot; 9</td>
<td>0.000</td>
<td>&quot;</td>
</tr>
<tr>
<td>&quot; 11</td>
<td>0.000</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

(1) Offset and drift of station 1 are held fixed to zero values.

(2) Clocks of stations 2, 3 and satellites 6, 8, 9, and 11 modeled with second order polynomials (straight lines).

(3) Clocks of stations 2 and 3 modeled as second order polynomials; Clocks of satellites 6, 8, 9 and 11 modeled as epoch-to-epoch independent variations.

Table 1 verifies that the OSU modified version of the software gives the same results for all clock offsets and drifts as that of NGS when the same clock
correction models are applied. However, when all of the satellite clocks are determined as epoch-to-epoch independent variations, the station offsets and drifts change considerably (Table 1). The effects on the initial state vectors for all of the satellite arcs involved are shown in Table 2.

Table 2. Comparison of OSU and NGS Results/Initial State Vectors.

<table>
<thead>
<tr>
<th>SAT.</th>
<th>Initial Position(1)(meters)</th>
<th>OSU(2)−NGS(2)</th>
<th>OSU(3)−NGS(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Initial Velocity(1)(meters/second)</th>
<th>OSU(2)−NGS(2)</th>
<th>OSU(3)−NGS(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>11</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(1) J2000.0 mean of date system.

(2) Offset and drift of station (1) fixed to zero values; Clocks of stations 2, 3 and satellites 6, 8, 9, and 11 modeled with second order polynomials.

(3) Offset and drift of station (1) fixed to zero values; Clocks of stations 2 and 3 modeled as second order polynomials; Clocks of satellites 6, 8, 9 and 11 modeled as epoch-to-epoch independent variations.

Table 2 also verifies that the same results are obtained for the initial state vectors when the clock correction models are the same for both NGS and OSU version of the software. When the satellite clocks are determined on an epoch-to-epoch basis, the adjusted initial state vectors may change by several hundred meters. Although these changes are large enough to warn us about the importance of implementing epoch-to-epoch models, they are not conclusive because the aposteriori standard deviations of the initial position and velocity range from 14 to 110 meters and from 0.001 to 0.01 meters/second respectively. Conclusive answers will be reached in the near future, when the response of the orbits to the epoch-to-epoch estimation will be investigated on the basis of longer integration periods. The integration periods of the test solutions were approximately six hours.

The epoch-to-epoch estimates (e.g., dots) together with the polynomials (e.g., continuous lines) "best" fitting those estimates at the 1% significance level, are plotted in figure 1. The RMS of the polynomial fits range from 5.6 to 7.8 nanoseconds corresponding to 1.68 and 2.34 meters respectively. It is evident from these plots that the behavior of the satellite clocks is far from being linear. This explains the large differences obtained for the adjusted initial state vectors.
EPOCH ESTIMATES (NANoseconds)

SATELLITE 6
STARTING EPOCH 05111912355.7
ORDER 5 RMS 5.6

SATELLITE 8
STARTING EPOCH 051119112355.7
ORDER 7 RMS 7.0

FIG. 1 Epoch-to-Epoch Estimates/Fitting Polynomials
EPOCH ESTIMATES (NANOSECONDS) (x10^1)

Fig. 1 (cont'd)

EPOCH ESTIMATES (NANOSECONDS) (x10^1)

SAT # 9
STARTING EPOCH 65111913925.7
ORDER = 5 RHS = 7.6

SAT # 11
STARTING EPOCH 65111913955.7
ORDER = 6 RHS = 6.8
Presently the coding is being expanded to automatically feed, if the user chooses this option, the statistically determined polynomial order back into the adjustment algorithm and then to reiterate the solution.


The output of the orbit determination software will be modified to accommodate the expansion of the prototype coding regarding the epoch-to-epoch estimates and the enhancements of the user interface. The coding will also be enhanced to allow feedback modeling of the satellite clocks with polynomials having orders inferred from the epoch-to-epoch estimates.

Next, response of the satellite orbits will be investigated by using longer integration periods and by modeling the satellite clocks either as independent epoch-to-epoch variations or as polynomials with orders resulting from the statistical analysis of the epoch-to-epoch estimates. It is important, however, for the success of this investigation also to study the sophistication of the orbital model employed by the orbital integration package, which up to this point has been treated almost as a black box.

Using satellite clock correction models inferred from epoch-to-epoch estimates and the Doppler effects on the satellite transmitted signals, the pseudo ranges will be transformed to Simultaneous Range Differences (SRD's) (i.e., interferometric mode). Since SRD observables are practically free of any satellite clock errors, the satellite orbits estimated on the basis of those observables are decoupled from the satellite clock errors and, therefore, potentially very accurate. Next, holding these orbits fixed and using pseudo ranges, accurate histories of the satellite clocks could be determined either as independent epoch-to-epoch variations or as polynomials with orders resulting from the analysis of the epoch-to-epoch estimates. This could be especially very useful if the clocks exhibit "step-like" behavior due to resets by the master control facility. It is for these reasons that the measurement processor will be enhanced to allow processing of Simultaneous Range Differences.

The ability to estimate the satellite clock offsets as independent epoch-to-epoch estimates allows one to study the clock drift histories before constraining them to follow an a priori chosen behavior. An apriori chosen behavior may, in fact, be very unrealistic because the exact behavior of the satellite clocks is, to large extent, unknown. Epoch-to-epoch estimates could be analyzed to study the stability characteristics of the satellite or ground clocks (McCaskill, et al., 1985). If, furthermore, these estimates constitute realizations of actual clock processes, this will offer an independent verification for the decoupling of dynamic and clock parameters. Epoch-to-epoch estimation could also model very successfully low cost ground receiver oscillators.

Time permitting, the orbit determination software will be modified to accept Earth Rotation Parameters (ERP's) in the format disseminated by BIH and to predict from the broadcast ephemeris a priori initial state vectors for all of the satellite arcs involved. The latter modification would allow automatic calculation of the a priori initial state vectors instead of having to input them manually.
4. References


REFERENCE COORDINATE SYSTEMS: AN UPDATE

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ABSTRACT. A common requirement for all geodetic investigations is a well-defined coordinate system attached to the earth in some prescribed way, as well as a well-defined inertial coordinate system in which the motions of the terrestrial system can be monitored. This paper deals with the problems encountered when establishing such coordinate systems and the transformations between them. In addition, problems related to the modeling of the deformable earth are discussed.

NOTE. This paper is an updated version of the earlier work "Reference Coordinate Systems for Earth Dynamics: A Preview," by the author published in the Proceedings of IAU Colloquium 56 on Reference Coordinate Systems for Earth Dynamics, Sept. 8-12, 1980, Warsaw, Poland, E.M. Gaposchkin and B. Kolaczeck, eds., D. Reidel, 1981. The updates are clearly indicated throughout the text by this type style.
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Appendix 4: Concepts and Methods of the Central Bureau of the International Earth Rotation Service .......................................................... 50
1. INTRODUCTION

Geodynamics has become the subject of intensive international research during the last decade, involving plate tectonics, both on the intra-plate and inter-plate scale, i.e., the study of crustal movements, and the study of earth rotation and of other dynamic phenomena such as the tides. Interrelated are efforts improving our knowledge of the gravity and magnetic fields of the earth. A common requirement for all these investigations is the necessity of a well-defined coordinate system (or systems) to which all relevant observations can be referred and in which theories or models for the dynamic behavior of the earth can be formulated. In view of the unprecedented progress in the ability of geodetic observational systems to measure crustal movements and the rotation of the earth, as well as in the theory and model development, there is a great need for the definition, practical realization, and international acceptance of suitable coordinate system(s) to facilitate such work. Manifestation of this interest has been the numerous specialized symposia organized during the past decade or so, such as those held in Stresa [Markowitz and Guinot, 1968], Morioka [Melchior and Yumi, 1972; Yumi, 1971], Torun [Kojaczek and Weiffenbach, 1974], Columbus [Mueller, 1975b and 1978], Kiev [Fedorov, Smith and Bender, 1980] and San Fernando [McCarthy and Pilkington, 1979]. There seems to be general agreement that only two basic coordinate systems are needed: a Conventional Inertial System (CIS), which in some "prescribed way" is attached to extragalactic celestial radio sources, to serve as a reference for the motion of a Conventional Terrestrial System (CTS), which moves and rotates in some average sense with the earth and is also attached in some "prescribed way" to a number of dedicated observatories operating on the earth's surface. In the latter, the geometry and dynamic behavior of the earth would be described in the relative sense, while in the former the movements of our planetary system (including the earth) and our galaxy could be monitored in the absolute sense. There also seems to be a need for certain interim systems to facilitate theoretical calculations in geodesy, astronomy, and geophysics as well as to aid the possible traditional decomposition of the transformations between the frames of the two basic systems. This scheme is shown in the figure below. The Earth Model block represents the current best knowledge of the geometry and dynamic behavior of the earth, partially deduced from the measurements made at the Dedicated Observatories. This model is continuously improving as more data of increasing accuracy becomes available, and it includes both the local (L) and global (G) phenomena which have theoretical foundations based on physical reality and are mathematically describable. In the final and ideal situation, which may be achieved only after several iterations over an extended period of time, the global part of the model should be identical to the connection between the CIS and CTS frames. Departures (v) from the model (L') observed at the observatories (j) or at other stations (i) are of course most important since they represent new information based on which the model can be improved, after observational random and systematic errors have been taken into proper consideration. The model could eventually include the solid earth as well as the oceans and the atmosphere.
As we will see later, there already seems to be understanding on how the two basic reference systems should be established; certain operational details need to be worked out and an international agreement is necessary. There are, however, a number of more or less open questions which will have to be discussed further. These include the type of interim systems needed and their connections to both CIS and CTS, the type(s) of observatories, their number and distribution, whether all instruments need to be permanently located there or only installed at suitable regular intervals to repeat the measurements; how far the model development should go so as not to become impractical and unmanageable; and how independent observations should be referenced to the CTS, i.e., what kind of services need to be established and by whom.

In order to clarify some of the conceptual aspects of various reference systems and frames, we propose to use specific terms suggested in (Kovalevsky and Mueller, 1981) that have been used somewhat inconsistently in the past.

The purpose of a reference frame is to provide the means to materialize a reference system so that it can be used for the quantitative description of positions and motions on the earth (terrestrial frames), or of celestial bodies, including the earth, in space (celestial frames). In both cases the definition is based on a general statement giving the rationale for an ideal case, i.e., for an ideal reference system. For example, one would have the concept of an ideal terrestrial system, through the statement that with respect to such a system the crust should have only deformations (i.e., no rotations or translations); cf. the Tisserand axes. The ideal concept for a celestial system is that of an inertial system so defined that in it the differential equations of motion may be written without
including any rotational term. In both cases the term “ideal” indicates the conceptual definition only, and no means are proposed to actually construct the system.

The actual construction implies the choice of a physical structure whose motions in the ideal reference system can be described by physical theories. This implies that the environment that acts upon the structure is modeled by a chosen set of parameters. Such a choice is not unique: there are many ways to model the motions or the deformations of the earth; there are also many celestial bodies that may be the basis of a dynamical definition of an inertial system (moon, planets, or artificial satellites). Even if the choice is based on sound scientific principles, there remains some degree of imperfection or arbitrariness. This is one of the reasons why it is suggested to use the term “conventional” to characterize this choice. The other reason is related to the means, usually conventional, by which the reference frames are defined in practice.

At this stage, there are still two steps that are necessary to achieve the final materialization of the reference system so that one can refer coordinates of objects to them. First, one has to define in detail the model that is used in the relationship between the configuration of the basic structure and its coordinates. At this point, the coordinates are fully defined, but not necessarily accessible. Such a model is called a conventional reference system. The term “system” thus includes the description of the physical environment as well as the theories used in the definition of the coordinates. For example, the FK4 (conventional) reference system is defined by the ecliptic as given by Newcomb’s theory of the sun, the values of precession and obliquity, also given by Newcomb, and the Woolard theory of nutation. Once a reference system is chosen, it is still necessary to make it available to the users. The system usually is materialized for this purpose by a number of points, objects or coordinates to be used for referencing any other point, object or coordinate. Thus, in addition to the conventional choice of a system, it is necessary to construct a set of conventionally chosen (or arrived at) parameters (e.g., star positions or pole coordinates). The set of such parameters, materializing the system, define a conventional reference frame. For example, the FK4 catalogue of over 1500 star coordinates defines the FK4 frame, materializing the FK4 system.

Another way of defining the CTS for the deformable earth is through the time varying positions of a number of terrestrial observatories whose coordinates are periodically reobserved by some international service. The frame of this CTS could then be derived from the changing coordinates through transformations containing rotational (and possibly translational) parameters. These transformation parameters computed and published by the service would then define the frame of the system. The service, as part of the system definition, thus would have to make the assumption that the progressive changes of the reference coordinates of the observatories do not represent rotations (and translations) in a statistically significant sense.

It is also necessary to point out that celestial reference systems may be defined kinematically (through the geocentric or heliocentric motion of artificial satellites, moon, planets). Stellar systems, such as the FK4, are hybrid. Furthermore, approximations must be introduced in the model, so that it is not true to say that these systems are realizations of an ideal inertial system. This is why it is appropriate to use the term conventional inertial system (CIS) as a common term for all such celestial systems. The corresponding frames would be defined by either the adopted positions of a set of radio sources (kinematic frame) or the adopted geocentric or heliocentric ephemerides (dynamic frames), all serving for the materialization of the CIS with greater or lesser success (accuracy).

2. CONVENTIONAL INERTIAL SYSTEMS (CIS) OF REFERENCE

2.1 Basic Considerations

The first law of Newton is as follows: "Every body persists in its state of rest or uniform motion in a straight line unless it is com-
peled to change that state by forces impressed on it" [Newton, 1686].
It should be obvious that the above law of inertia cannot hold in any
arbitrary reference frame so that only certain specific reference frames
are acceptable. In classical mechanics, reference frames in which the
above law is valid are called inertial frames. Such "privileged" frames
move through space with a constant translational velocity but without
rotational motion. Another privileged frame in classical mechanics is
the quasi-inertial, which also moves without rotational motion, but its
origin may have acceleration. Such a frame would be, for example, a
non-rotating geocentric Cartesian coordinate system whose origin due to
the earth orbit around the sun would move with a non-constant velocity
vector. Inertial reference frames thus are either at rest or are in a
state of uniform rectilinear motion with respect to absolute space, a
concept also mentioned by Newton and visualized as being observationally
defined by the stars of invariable positions, a dogma in his time.

The refinement of classical mechanics through the theory of rela-
tivity requires changes in the above concepts. The theory of special
relativity allows for privileged systems, such as the inertial frame but
in the space-time continuum instead of the absolute space [Moritz, 1967].
Transformation between inertial frames in the theory of special relativ-
ity are through the so-called Lorentz transformations, which leave all
physical equations, including Newton's laws of motion, and the speed of
light invariant. The special theory of relativity holds only in the
absence of a gravitational field.

In the theory of general relativity, Einstein defined the inertial
frames as "freely falling coordinate systems" in accordance with the lo-
cal gravitational field which arises from all matter of the universe.
Thus the inertial frames lose their privileged status. Concerning the
existence of inertial frames in the extended portions of the space-time
continuum, Einstein [1956] states that
"there are finite regions, where, with respect to a suitably
chosen space of reference, material particles move freely
without acceleration, and in which the laws of special rela-
tivity hold with remarkable accuracy."
In other words, one can state [Weinberg, 1972] that
"At every space-time point in an arbitrary gravitational field,
it is possible to choose a locally inertial coordinate system
such that, within sufficiently small region of the point in
question, the laws of nature take the same form as in unaccel-
erated Cartesian Coordinate system in the absence of gravitation."
(i.e., as in the theory of special relativity). Our sphere of interest,
the area of the solar system, where the center of mass of the earth-moon
system is "falling" in an elliptic orbit around the sun, in a relatively
weak gravitational field, seems to qualify as such a "small region."
Thus we may assume that inertial or quasi-inertial frames of reference
exist, and any violation of principles when using classical mechanics
can be taken into account with small corrections appropriately applied
to the observations and by an appropriate "coordinate" time reference.
The effects of special relativity for a system moving with the earth
around the sun are in the order of $10^{-8}$, while those of general relativ-
ity are $10^{-9}$ [Moritz, 1979]. Since $10^{-8}$ on the earth's surface corre-
ponds to about 6 cm, corrections at least for special relativity effects
are needed when striving for such accuracies. Other than this, the prob-
lem, in the conceptual sense, need not be considered further.
Since the definition of the CIS may be based on dynamical properties of the solar system as well as on the kinematics of extragalactic sources, we are led to distinguish between two kinds of quasi-inertial systems (Fig. 2) (Kovalevsky and Mueller, 1981).

(a) *Conventional kinematical systems*, based on the assumption that the proper motions of some celestial bodies have known statistical properties. In the case of extragalactic sources, it is postulated that remote galaxies have no rotational component in their motions.

(b) *Conventional dynamical systems*, based on the theory of the motion of some bodies in the solar system (including artificial earth satellites) constructed in such a way that there remains no rotational term in the equations of motion.

If in the framework of Newtonian mechanics, both definitions are equivalent, this is not true in the theory of general relativity. A dynamical system of coordinates is a local reference that is locally tangent to the general space-time manifold. In contrast, the kinematical system defined by the apparent directions of remote objects is a coordinate system that is subject to relativistic effects such as the geodesic precession. Even if this is being suitably corrected for, there remains a basic difference between the concepts, and this is another good reason to use the terminology "quasi-inertial" to characterize both kinematical and dynamical systems.

It is now well agreed that the best future CIS will be based on the position of extragalactic radio sources. But even in such a system is due to play a major role among conventional quasi-inertial systems, there may be great advantages, in some cases, to sue a dynamical system. This is the case, for instance, when artificial satellites are used to monitor the earth rotation. This is why a certain hierarchy among these systems has been proposed in which the CIS, based on extragalactic radio sources is designated as a primary system, a role which used to be played by the FK4 System. Other systems, and in particular all the conventional dynamical systems, will have to be connected to the primary system in order to give consistent results (see later).

As mentioned, the actual availability of the systems is obtained through their realization in the form of reference frames. This materialization can be done in two different ways so that one can distinguish between two kinds of reference frames (Kovalevsky and Mueller, 1981):

(a) *Stellar reference frames*. The fiducial points are presently stars or extragalactic radio sources. In case of the latter, it is necessary to provide connection to stellar catalogues, so that the celestial system can be made available to optical instrumentation.

(b) *Ephemeris reference frames*. In such frames, one or several moving objects are used as the materialization of the system (e.g., the GPS). The theory supporting the corresponding reference system provides the apparent ephemeris of the objects as a function of time and the observed successive positions are the fiducial points needed to refer the observations to the system.

It is to be noted that there is not a bi-univocal correspondence between both types of frames and the two sorts of quasi-inertial systems. For instance, the FK4 or FK5 stellar systems are dynamical (due to the method of determination of the equinox), while their frame is stellar.
Fig. 2 Conventional terrestrial and quasi-inertial systems of reference with some possible connections.
2.2 Inertial Systems in Practice

2.21 Extragalactic Radio Source System. This system is attached to radio sources which generally either are quasi-stellar objects (quasars) or galactic nuclei. Very long baseline interferometers rotating with the earth determine the declinations of these sources with respect to the instantaneous rotation axis of the earth, as well as their right ascension differences with respect to a selected source (3C273, NRAO 140, Persei (Algol), etc.). In addition, the observations also determine changes in the earth rotation vector with respect to a selected initial state, the baseline itself, and certain instrumental (clock) corrections. The frame of the Radio Source-CIS can be defined by the adopted true or mean coordinates of appropriately selected sources referred to some standard epoch. The mean coordinates naturally will depend on the model of the transformation from the true frame of date to the adopted mean standard. If, however, the reduction procedure is correct (see more on this later), there are no known reasons for non-radial relative motions of the sources, i.e., for the rotation of the frame. Thus, such a frame could be considered inertial or at least quasi-inertial. The equatorial system of coordinates may be retained for convenience, but the frame could be attached to the sources in any other arbitrary way should this be necessary.

As far as the accuracy of the Radio Source-CIS is concerned, the question has meaning only in the sense of the formal precisions of the source positions in the catalogue. At the Torun meeting, this number was 0".1 [Moran, 1974]; now it is at most 0".01 [Purcell et al., 1980]. It is hoped that within a few years the precision should reach 0".001 (5 x 10^-9). The problem on this level is that the densification of such a catalogue will be very difficult, since only a relatively few well-defined point-like radio sources have been observed. Others have structures such that identification of the center of the radiation with such accuracy may not be possible. This situation may change when the astrometric satellites (see below) are launched.

VLBI instrumentation has undergone considerable development since the initial efforts in the early 1960's. Table 1 describes the primary recording systems (Ma, 1988).
Table 1  VLBI Recording Systems

<table>
<thead>
<tr>
<th>System</th>
<th>In Use</th>
<th>Basic Design</th>
<th>Sample Rate</th>
<th>Tape Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark I</td>
<td>1967-78</td>
<td>Digital recording IBM computer tape</td>
<td>0.72</td>
<td>3</td>
</tr>
<tr>
<td>Mark II</td>
<td>1971-</td>
<td>Digital recording on various TV recorders</td>
<td>4</td>
<td>64-246</td>
</tr>
<tr>
<td>Mark III</td>
<td>1977-</td>
<td>Digital recording</td>
<td>112</td>
<td>13</td>
</tr>
<tr>
<td>Mark IIIA</td>
<td>1984-</td>
<td>Instrumentation recorder</td>
<td>112</td>
<td>164</td>
</tr>
</tbody>
</table>

Two connected element interferometer (CEI) instruments are now regularly used for astrometric measurements. The National Radio Astronomy Observatory interferometer in Green Bank, Wet Virginia, has a 35-km baseline and operates continuously as part of a program to monitor UT1. The Very Large array (VLA) near Socorro, New Mexico, while primarily a mapping instrument is also used for differential and absolute astrometry. It consists of 27 25-m antennas laid out in a Y pattern with the longest arm 21 km.

VLBI networks, since they are composed of independent elements, vary with time and availability. Table 2 shows the stations which have contributed significantly to the current astrometric data base.

Table 2  VLBI Antennas Used for Astrometry (Ma, 1988)

<table>
<thead>
<tr>
<th>Location</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gilmore Creek, Alaska, USA</td>
<td>26 m</td>
</tr>
<tr>
<td>Goldstone Deep Space Station, California, USA</td>
<td>64</td>
</tr>
<tr>
<td>Hartebeesthoek Radio Observatory, So. Africa</td>
<td>26</td>
</tr>
<tr>
<td>Hat Creek Radio Observatory, California, USA</td>
<td>26</td>
</tr>
<tr>
<td>Harvard Radio Astronomy Station, Texas, USA</td>
<td>26</td>
</tr>
<tr>
<td>Haystack Observatory, Massachusetts, USA</td>
<td>37</td>
</tr>
<tr>
<td>Kashima Space Research Center, Japan</td>
<td>26</td>
</tr>
<tr>
<td>Kokee Tracking Station, Hawaii, USA</td>
<td>9</td>
</tr>
<tr>
<td>Kwajalein Atoll, Marshal Islands</td>
<td>26</td>
</tr>
<tr>
<td>Madrid Deep Space Station, Spain</td>
<td>64</td>
</tr>
<tr>
<td>Mojave Base Station, California, USA</td>
<td>12</td>
</tr>
<tr>
<td>National Radio Astronomy Observatory, West Virginia, USA</td>
<td>43</td>
</tr>
<tr>
<td>Onsala Space Observatory, Sweden</td>
<td>20</td>
</tr>
<tr>
<td>Owens Valley Radio Observatory, California, USA</td>
<td>40</td>
</tr>
<tr>
<td>Richmond, Florida, USA</td>
<td>18</td>
</tr>
<tr>
<td>Tidbinbilla Deep Space Station, Australia</td>
<td>64</td>
</tr>
<tr>
<td>Westford, Massachusetts, USA</td>
<td>18</td>
</tr>
<tr>
<td>Wettzell, Fed. Repub. Germany</td>
<td>20</td>
</tr>
</tbody>
</table>

There are at present several catalogs of extragalactic radio sources in the J2000.0 system. They vary considerably in number of sources, distribution of sources, and precision. See Table 3 for a summary (Ma, 1988).
Table 3 J2000.0 Catalogues of Extragalactic Compact Sources

<table>
<thead>
<tr>
<th>Organization</th>
<th>Instrument</th>
<th>Baseline Length (km)</th>
<th>No. of Sources</th>
<th>Uncertainties mas</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRAO</td>
<td>CEI</td>
<td>35</td>
<td>36</td>
<td>20-40</td>
<td>Wade &amp; Johnston, 1977</td>
</tr>
<tr>
<td>NRAO</td>
<td>CEI</td>
<td>35</td>
<td>16</td>
<td>10</td>
<td>Kaplan et al., 1982</td>
</tr>
<tr>
<td>JPL</td>
<td>Mark II</td>
<td>8000-11000</td>
<td>836</td>
<td>300</td>
<td>Morabito et al., 1982-86</td>
</tr>
<tr>
<td>NSF</td>
<td>VLA</td>
<td>&lt;27</td>
<td>700</td>
<td>20-100</td>
<td>Perley, 1982</td>
</tr>
<tr>
<td>JPL</td>
<td>Mark II</td>
<td>8000-11000</td>
<td>117</td>
<td>1-5</td>
<td>Fanselow et al., 1984</td>
</tr>
<tr>
<td>NASA</td>
<td>Mark III</td>
<td>800-6000</td>
<td>85</td>
<td>0.3-13</td>
<td>Ma et al., 1986</td>
</tr>
<tr>
<td>NGS</td>
<td>Mark III</td>
<td>800-6000</td>
<td>26</td>
<td>0.5</td>
<td>Robertson et al., 1986</td>
</tr>
</tbody>
</table>

Ma (1983) intercompared the catalogues of JPL and NASA, based on 45 overlapping sources and found an RMS difference of about 0.005 in both right ascension and declination. A recent study by Arias et al. (1987) intercompared JPL, NASA and NGS 1984-1986 catalogues based on 19-128 overlapping sources and found the directions of the axes of their respective reference frames consistent within 0.003. This is considered a remarkable agreement on account of the diversity of observing strategies and data analysis.

The premier instrument for future radio astrometry will be the Very Long Baseline Array, currently under construction. It will consist of ten 25-m antennas spaced from Hawaii to Puerto Rico, each equipped with ten receivers from .33 GHz to 43 GHz.

Until the VLBA becomes fully operational in the mid-1990's, there are several ongoing programs which will continue to expand and refine the extragalactic catalogue. The NASA Crustal Dynamics Project has a VLBI survey program to expand its catalogue of unresolved sources to take advantage of improvements in sensitivity. The US Naval Observatory is starting an astrometric program using North American VLBI stations to densify the grid of optical/radio sources in the Northern Hemisphere. The JPL survey work will be further refined to support planetary spacecraft navigation using differential VLBI.

2.22 Stellar System. This system will be attached to stars in the FK5 catalogue, i.e., the adopted right ascensions and declinations of the FK5 stars will define the equator and the equinox and thus the frame of the Stellar-CIS. The FK5, to be effective in 1984, will be the fifth fundamental catalogue in a series which began with the FC in 1879 [Fricke and Gliese, 1978]. In the fundamental catalogues the equator is determined from zenith distance (or distance difference) observations of the stars themselves, but the equinox determination also necessitates measurements of the sun or other members of the planetary system. It was always tacitly assumed that coordinate systems attached to the fundamental catalogues were quasi-inertial. However, as more and more observations became available for proper motions and on the various members of the planetary systems, certain small rotations were discovered, which require changes in the positions of the fundamental equator and equinox, in the proper motions and in the precessional constant (all intricately interwoven) when one fundamental catalogue replaces the other. This slow and painstaking process should lead to a quasi-inertial system eventually. We hope that the FK5 will be such a system.

When the FK4 was compiled, a small definitive correction to the declination of FK3 was applied, but there seemed to be no need to change the position of the equinox or the precessional constant [Fricke, 1974]. The FK5 will be a considerably different and improved catalogue. The main changes with respect to the FK4, regarding the issue of the coordi-
nate systems, are as follows [Fricke, 1979a]: (1) New value of general precession in longitude adopted by the IAU in 1976 will be used (more on this later). (2) The centennial proper motions in right ascension will be increased by $0.0086$/century (this number is provisional) to eliminate the motion of the FK4 equinox with respect to the dynamical equinox (the FK4 right ascensions are decreasing with time due to an error in the FK4 proper motions, see below). (3) Rotation of the FK4 equinox at 1950 by the amount of $0.040$ (also a provisional value) so that the FK5 and the dynamic equinoxes will be identical (the FK4 right ascensions at 1950 are too small). (4) Elimination of inhomogeneities of the FK4 system by means of absolute and quasi-absolute observations. (5) Determination of individual correction to positions and proper motions of FK4 stars. (6) Addition of new fundamental stars to extend the visual magnitude from 7.5 to about 9.2. More than 1500 new stars are to be added.

It should be mentioned that the above improvements are possible because of the availability and/or reanalysis of observations of the sun (1900-1970), of lunar occultations (1820-1977), of Mars (1941-1971), of minor planets (1850-1977), and the JPL DE-108 Ephemeris based on optical or radar observations of the sun, planets and some space probes (Mariner 9, Viking). All in all the number of these observations exceeds 350,000. In addition, more than 150 catalogues of star observations have become available since the completion of the FK4 [Fricke, 1979b].

One should also take note here of the FK5sup catalogue, which will contain the FK5 coordinates of a few extragalactic radio sources with radio and optical positions and thus provide the connections between the Stellar-CIS and the Radio Source-CIS, though with somewhat limited accuracy (~0.1'). Improvement of this particular problem is expected from the Space Telescope [Van Altena, 1978] which could increase the number of radio stars, observable by VLBI, in the FK5 to about 50. Such missions (e.g., Hipparcos) could also contribute to the determination of the fundamental equator and equinox with increased accuracies, by observations of the minor planets. This, of course, would mean improved ties with the planetary-CIS (discussed below) which nowadays is based on the observations mentioned in connection with the establishment of the FK5 equator and equinox. The astrometric satellite Hipparcos is described to be able to measure relative positions of some 100,000 stars to a precision of $0.0015$ and annual proper motions to $0.002$ over a lifetime of 2.5 years [Barbieri and Bernacca, 1979]. A second mission ten years later could improve this figure by a factor of 5. This compares well indeed with the precision of ground based observations of $0.04$ at best, requiring something like 50 years to obtain proper motions of comparable precision ($0.002$).

As far as the accuracy of the FK5-CIS is concerned, the question again is meaningful only in the sense of how precise the star positions in the FK5 will be. It is hoped that in the worst regions this will not be worse than $0.02$ in position and $0.0015$ in the annual proper motion. There should be better regions, of course.

The compilation of the FK5 represents a major effort at the Astronomiches Rechen Institut. The comparison of 100 new catalogues with the FK4 permitted the improvement of the individual proper motions of stars by a factor of 2. This part of the work is independent of the reference
system. The regional errors were essentially deduced from 90 absolute or quasi-absolute catalogues (25 new ones in α and 15 in δ) including astrolabe and time catalogues (Schwan, 1986, 1987). The mean precision achieved in the FK5 is 0.02 in position and 0.8 mas per year in proper motion (Kovalevsky, 1988).

An important extension of the FK5 is the International Reference Star (IRS) catalogue which is almost completed and will include about one star per square degree. It will include the AGK3R stars in the northern hemisphere, the SRS (Southern Reference Stars) catalogue in the Southern Hemisphere (Zverev et al., 1986) and some additional stars to insure the homogeneity of the distribution on the celestial sphere (Smith, 1986). A special effort was made to obtain a homogeneous system of proper motions (Corbin, 1978).

Further extensions should be based on the IRS itself or on future larger and more homogeneous catalogues like the HIPPARCOS catalogue mentioned above (Froeschle and Kovalevsky, 1982).

2.23 Dynamical Systems. The dynamics expressed in the equations of motion define a number of non-rotating planes which could be the basis of reference frames. Considering the observable planes that could be the basis of such a Dynamic-CIS, there are the planetary (including the earth-moon barycenter) orbital planes, the equator, the lunar orbital plane, and the orbital planes of certain high flying, thus only slightly perturbed, artificial earth satellites (e.g., Lageos or GPS). Since all of these planes have relative rotations, it is possible to derive a mean plane for a given epoch from an observable apparent plane, or a non-observable invariant plane could be adopted [Duncombe et al., 1974]. At this point, the definition of the origin of the system becomes important also, because relativistic effects necessitate the distinction between proper and coordinate times. In the radio-source or stellar quasi-inertial systems, the question of origin can be settled through appropriate corrections for aberration and parallax, etc., but here it is also necessary that a uniform and unambiguous time scale referenced to a non-rotating frame of specified origin be established (coordinate time). The practical implications of a global coordinate time scale is not treated here, but the problem should not be ignored (cf. [Ashby and Allan, 1978]). In more practical (observational) terms one can distinguish between Planetary, Lunar and (artificial) Satellite CIS's, each frame defined, in theory, by two of the above-mentioned planes, and in practice, by the available ephemerides.
In the case of the planetary systems, the defining planes are the equator and the ecliptic, their intersection being the line of the equinoxes. In practical terms the frame of the Planetary-CIS is defined by the ephemerides of the centers of masses of the planets, including the barycenter of the earth-moon system. The ephemerides, such as the JPL DE-108 mentioned earlier, are based on observations of the sun, the planets, possibly space probes. Since most modern ephemerides are computed through the numerical integration of the orbital equations of motion, the degree of satisfaction that can be obtained depends only on the completeness of the modeling, including the astronomical constants, the determination of the starting conditions and, of course, on the type, accuracy and distribution of the observed data. In this sense each planetary ephemeris defines its own reference frame. These should agree with each other within the observational accuracies. Connection between the Planetary-CIS's and the Stellar-CIS's is through the determination of the equinox and the equator, as explained earlier.

In the case of the lunar system, the main references are the orbital plane of the moon and the equator of the earth. In practice the Lunar-CIS frame is again defined by the lunar ephemeris, which nowadays is most accurately determined from lunar laser observations made from the surface of the earth to reflectors deposited on the lunar surface. For this reason, the adequacy of the definition also depends on how well the lunar rotation (librations) can be computed. Since the most frequently used lunar ephemerides are generally calculated through numerical integration, the above dependence on modeling (especially on the effect of tidal dissipation in the earth), and on initial conditions, apply here also. The identity of the coordinate frame, such defined, may be compared to the other frames to certain accuracies. Lunar occultation of stars, or the earlier Markowitz moon-camera photography, provide a connection to the Stellar-CIS; differential VLBI observations between radio sources deposited on the moon and the extragalactic ones would tie to the Radio Source-CIS. The connection to the Planetary-CIS is through solar eclipse observations, and also through the planetary ephemeris used when calculating the lunar ephemeris. There are also some other looser connections stemming from the orientation of the earth when its non-spherical gravitational effects on the lunar motions are taken into consideration. Present observations reveal a residual rotation (or accelerations) in the order of a few seconds of arc per century squared. This seems to be the present stability (i.e., the accuracy) of this quasi-inertial frame. It is unlikely that without stronger connections to a frame of better stability, this rotation can be eliminated. As it is, the accuracy of this CIS should compare favorably with that defined by the FK5 but only over a period of, say, a decade [Kovalevsky, 1979].
Data types to which modern planetary and lunar ephemerides are adjusted are listed in Table 4. The post-fit rms residuals indicate the accuracy of the data. The values listed without brackets are the units of the original observations; those within brackets give the comparable values for comparison purposes (Williams and Standish, 1988).

### Table 4 Data in Modern Lunar and Planetary Ephemerides (Williams and Standish, 1988)

<table>
<thead>
<tr>
<th>Type of Observation</th>
<th>Time Span</th>
<th>Post-Fit Rms (km)</th>
<th>Residuals((\prime)) No. of Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar Ranging</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>1966-</td>
<td>1.5</td>
<td>[0'002]</td>
</tr>
<tr>
<td>Venus</td>
<td>1965-</td>
<td>1.5</td>
<td>[0'002]</td>
</tr>
<tr>
<td>Mars</td>
<td>1967-</td>
<td>2.2</td>
<td>[0'003]</td>
</tr>
<tr>
<td>Mars Closure</td>
<td>1969-82</td>
<td>0.15</td>
<td>[0'0002]</td>
</tr>
<tr>
<td>Spacecraft Ranging</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ma9 Orbiter (Mars)</td>
<td>1972-73</td>
<td>0.040</td>
<td>[0'0002]</td>
</tr>
<tr>
<td>Viking Lander (Mars)</td>
<td>1976-80</td>
<td>0.007</td>
<td>[0'000003]</td>
</tr>
<tr>
<td></td>
<td>1980-82</td>
<td>0.012</td>
<td>[0'000006]</td>
</tr>
<tr>
<td>Spacecraft Tracking (Range, Doppler)</td>
<td>1973-80</td>
<td>[200, 400]</td>
<td>[0'05]</td>
</tr>
<tr>
<td>Pion&amp;Voy (Jup,Sat)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lunar Laser Ranging</td>
<td>1969-70</td>
<td>0.00100</td>
<td>[0'0005]</td>
</tr>
<tr>
<td></td>
<td>1970-75</td>
<td>0.00030</td>
<td>[0'00016]</td>
</tr>
<tr>
<td></td>
<td>1976-85</td>
<td>0.00015</td>
<td>[0'00008]</td>
</tr>
<tr>
<td></td>
<td>1985-</td>
<td>0.00006</td>
<td>[0'00003]</td>
</tr>
<tr>
<td>Radio Astrometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jupiter, ..., Neptune</td>
<td>1983-</td>
<td>[100, ..., 600]</td>
<td>0'03</td>
</tr>
<tr>
<td>Ring Occultation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uranus</td>
<td>1978-</td>
<td>[1500]</td>
<td>0'1</td>
</tr>
<tr>
<td>Optical Transits (Manual)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sun, Mercury, Venus</td>
<td>1911-</td>
<td>[700]</td>
<td>1'0</td>
</tr>
<tr>
<td>Mars, ..., Neptune</td>
<td>1911-</td>
<td>[150, ..., 10000]</td>
<td>0'5</td>
</tr>
<tr>
<td>Optical Transits (Photoelectric)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mars, ..., Neptune</td>
<td>1982-</td>
<td>[100, ..., 4000]</td>
<td>0'3</td>
</tr>
<tr>
<td>Astrolabe</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mars, ..., Uranus</td>
<td>1961-</td>
<td>[100, ..., 4000]</td>
<td>0'3</td>
</tr>
<tr>
<td>Astrometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pluto</td>
<td>1914-</td>
<td>[15000]</td>
<td>0'5</td>
</tr>
</tbody>
</table>

Earlier ephemerides of the moon and planets, based upon optical observations, have inherited errors directly from the catalogues upon which they have been based. These errors amount to a number of tenths of an arcsecond in angular position and a number of tenths of an arcsecond per century in angular motion; i.e., errors comparable to those that are known to exist in the FK4 fundamental reference system. Modern ephemerides based upon ranging observations show at
least an order of magnitude improvement over their optically based predecessors. Williams and Standish (1988) selected the most important data types and calculated how sensitive these data are to changes in certain ephemeris elements. The sensitivities, in turn, indicate how well each of these elements may be determined through the data fitting, keeping in mind that the statistics of the actual determinations are improved due to the large number of observations but also that there are correlations among the various parameters.

The lunar laser ranging data is sensitive to a change in the lunar mean anomaly and its rate at levels of $0'.0006$ and $0'.02/cy$ respectively. The data is also sensitive to the rate of the lunar longitude with respect to inertial space at a level of $0'.04/cy$. This rate error is dominated by the uncertainties in the precessional rates of the lunar perigee; the precessional rates themselves are due to the perturbations which depend on the orbital elements and gravitational harmonics of the earth and moon. At times away from the data span, the uncertainty $(1'.0/cy^2)$ in the tidally induced acceleration in longitude becomes predominant.

For the planets, the most important data are the ranges to the Viking landers on Mars. Williams and Standish show that these ranges have a remarkable sensitivity to a number of differential angles: the difference in heliocentric longitudes between earth and Mars at a level of $0'.00001$, each longitude with respect to the perihelion of Mars at a level of $0'.00004$ and each longitude with respect to the perihelion of the earth at a level of $0'.0002$. Further, the corresponding level for the inclination of Mars' orbit upon the ecliptic is about $0'.0002$.

Radar ranging to Mercury and Venus determines the longitudes of these planets with respect to the longitude of the earth (and therefore to Mars). These sensitivities are on the order of $0'.005$ and $0'.003$ respectively, since the data are accurate to the level of $1.5$ km. The sensitivities to the inclinations upon the ecliptic are two orders of magnitude worse than that for Mars.

Solar perturbations upon the lunar orbit provide sensitivity to both the differential longitude between the heliocentric earth and the geocentric moon and to the inclination of the lunar orbit to the ecliptic; $0'.001$ and $0'.007$ respectively.

Since the lunar ranges are taken from the spinning earth, sensitivities to the earth's orientation, coupled with the terrestrial coordinates of the observing station, allow determinations of (1) the mutual inclinations of the equator, the ecliptic and the lunar orbital plane ($0'.002$); (2) the longitude of the earth and moon with respect to the dynamical equinox ($0'.005$); and (3) a tie between the ephemeris frame and the terrestrial reference system ($0'.001$ in longitude, comparable to $0.001$ seconds in UTO).

Finally, the fact that the lunar retroreflectors and the Viking landers are situated on the surfaces of the bodies, the ranges are sensitive to the physical orientations of the bodies themselves. The lunar librations affect the LLR data; the spin rate, obliquity and equinox of Mars influence the Viking ranges.

The analytical sensitivity analyses in (Williams and Standish, 1988) have been substantiated by numerical examples though the correspondence is not exact because of differences in numbers of observations, correlations, additional data and other perturbing forces. However, even when all of these factors are considered, it is seen that the dynamical reference system may be determined better than $0'.01$ in position with respect to the dynamical equinox. Further, the mean motions of earth and Mars with respect to inertial space may be determined as well as $0'.003/cy$ during the times of the highly accurate ranging data; the uncertainty for Mars will grow to about $0'.015/cy$ over the course of many decades away from the present data.
In the case of satellite systems, the problem is compounded by additional modeling problems related to the force field in which the satellite moves and by the fact that nowadays there are no direct connections to other frames of reference. Modern satellite tracking techniques (laser, Doppler, etc.) all basically observe ranges or range differences and contain no directional information. The main reference planes, the orbital plane of the satellite and the equator, intersect along the line of nodes, the initial orientation of which therefore must be defined more or less arbitrarily. In the "old days" of satellite geodesy, when satellites were observed photographically in the background of stars, this direction could be determined with respect to the FK4, though not much better than a few tenths of a second of arc. The accumulation of errors in describing the motion of the node with respect to a selected zero point, even for the most suitable high flying and small heavy spherical satellites (Lageos), prevents a Satellite-CIS from being accurate over a long period of time, say beyond several months. In any case, in observational terms such a frame would be defined by the satellite ephemeris made available to the users by organizations which provide for the continuous tracking of the satellite in question. A current example would be the Precise Ephemeris of the U.S. Navy Navigational Satellite (Transit) System. As far as the connections to other systems are concerned, the only accurate possibility seems to be indirectly through the tracking stations. If two observational systems occupy the same station, one observing the satellite, the other, say, the radio sources, either simultaneously or after a short time interval (during which the movement of the station can be modeled), the connection between the satellite and radio source frames can be established. In fact, the now classical disparity between the JPL and SAO frames came to light just through such an arrangement, when the SAO longitudes determined from satellite camera tracking (thus in the FK4 frame) differed by those determined by JPL space probe tracking (in the planetary frame) by an amount (about 0.7 in the early 1970's) consistent with the FK4 equinox motion with respect to the dynamical equinox, mentioned earlier. Only through such continuously maintained connections can the lifetime of a Satellite-CIS be extended, thus its accuracy increased.

2.3 Conclusions

From the above discussion, the following conclusions can be drawn:

1. The most accurate, long-term CIS will be the one attached to extragalactic radio sources. It is accessible through VLBI observations. Other systems can be accurately connected to it by station collocation or the Space Telescope.

2. The CIS attached to the FK5 is somewhat less accurate. Direct access to it is through optical star observations, which by nature are generally less accurate than VLBI observations. Its main value is in defining the fundamental mean system of coordinates and thereby providing a direction (the FK5 equinox) for the time (UT1) definition, and for the possible orientation of the Radio Source-CIS. The latter
function, however, stems from more of a traditional requirement and not from theoretical needs.

3. Of the Dynamical-CIS's, the accuracy of the planetary system should be equivalent to the FK5. The lunar and satellite systems by themselves are suitable for medium-term to short-term work only. Their stability can be extended by connections to the Radio Source-CIS through accurate and continuous observations at collocated stations. Ties between the radio source and the planetary systems may also be available through the proposed Very Large Array (VLA) observations of minor planets. Solar eclipse observations provide a connection between the lunar and planetary systems.

4. If a dynamical system is based on the motion of planets, the ecliptic plays a privileged role and, naturally, the ecliptic is used in the definition of coordinate. Since equatorial coordinates are preferred to ecliptic ones for obvious instrumental reasons, the ecliptic (through its intersection with the equator, the vernal equinox) becomes the natural origin of right ascensions. When the dynamical system is geocentric, the natural reference plane is the Laplace plane whose position depends upon the relative magnitude of the perturbations. For the moon, the solar effects are dominant and, practically, the Laplace plane is the ecliptic and, again, the equinox is the natural origin of equatorial coordinates. In the case of artificial satellites the perturbations due to the earth flattening are predominant so that the Laplace plane is the equator. The equator is, therefore, the natural fundamental plane, but the origin may be arbitrary.

Similarly, the choice of the equinox in the stellar systems is justified by the fact that they are partially dynamical systems based upon planetary theories. However, in the construction of the corresponding stellar frame, the difficulty of maintaining the theoretical origin is so serious that one is led to distinguish between the dynamical equinox which defines the origin of the system and the catalogue equinox which is the origin of the frame. In practice, the actual origins of the stellar reference frames are purely conventional and are not the dynamical equinox.

The situation will become even more conspicuous for frames derived from conventional kinematic systems. Even if, for the sake of continuity, the origin and the fundamental plane of such a system should be close to the equinox and the equator, they should be conventional points defined only by the realization of the corresponding frame. Otherwise, it would be necessary to introduce a complex dynamical model to define the origin at the expense of introducing inaccuracies in the system and an uncertainty in its realization by the frame. In practice, the solution might be analogous to the present situation for the terrestrial longitude system. One would establish an international organization that would provide the coordinates of radio sources in the conventional kinematic frame, taking into account eventual changes in the number and position of the reference sources, due, for instance, to the disappearance or motion of quasars or better measurements, in such a way that the changes should not introduce a rotation (or translation) of the system in the average statistical sense. It is an almost unavoidable conclusion that for geodetic and geodynamic applications the most useful CIS is just such a system (Kovalevsky and Mueller, 1981; Guinot, 1986).

3. CONVENTIONAL TERRESTRIAL SYSTEMS (CTS) OF REFERENCE

As mentioned in the Introduction, the CTS is in some "prescribed way" attached to observatories located on the surface of the earth. The connection between the CTS and CIS frames by tradition (to be preserved) is through the rotations [Mueller, 1969]

\[
\begin{bmatrix}
\text{CTS}
\end{bmatrix} = \text{SNP} \begin{bmatrix}
\text{CIS}
\end{bmatrix}
\]

where P is the matrix of rotation for precession, N for nutation (to be discussed in Section 4), and S for earth rotation (including polar mo-
Polar motion thus is defined as the angular separation of the third (Z) axis of the CTS and the axis of the earth for which the nutation (N) is computed (e.g., instantaneous rotation axis, Celestial Ephemeris Pole, Tisserand mean axis of the mantle (see Section 4)).

Geodynamic requirements for CTS may be discussed in terms of global or regional problems. The former are required for monitoring the earth's rotation, while the latter are mainly associated with crustal motion studies in which one is predominantly interested in strain or strain rate, quantities which are directly related to stress and rheology. Thus for these studies, global reference systems are not particularly important although it is desirable to relate regional studies to a global frame.

For the rotation studies one is interested in the variations of the earth's rotation rate and in the motions of the rotation axis both with respect to space (CIS) and to the crust (CTS). The problem therefore is threefold:
1. To establish a geometric description of the crust, either through the coordinates of a number of points fixed to the crust, or through polyhedron(s) connecting these points whose side lengths and angles are directly estimable from observations using the new space techniques (laser ranging or VLBI). The latter is preferred because of its geometric clarity.
2. To establish the time-dependent behavior of the polyhedron due to, for example, crustal motion, surface loading or tides.
3. To relate the polyhedron to both the CIS and the CTS. For the global tectonic problems only the first two points are relevant although these may also be resolved through point (3).

In the absence of deformation, the definition of the CTS is arbitrary. Its only requirement is that it rotates with the rigid earth, but common sense suggests that the third axis should be close to the mean position of the rotation axis and the first axis be near the origin of longitudes.

In the presence of deformations, particularly long periodic or secular ones, the definition is more problematic, because of the inability to separate rotational (and translational) crustal motions of the crust from those of the CTS.

One geophysical requirement of the reference system is that other geophysical measurements can be related to it. One example is the gravity field. The reference frame generally used when giving values of the spherical-harmonic coefficients is tied to the mean axe of figure of the earth. This frame should be simply related with sufficient accuracy to the CTS as well as to the CIS in which, for example, satellite orbits are calculated. Another example is height measurements with respect to the geoid.

The vertical motions may require some special attention, because absolute motions with respect to the center of mass have an immediate geophysical interest are are realizable. Again, if the center of mass has significant motions with respect to the crust, such a motion will be absorbed in the future CTS, if defined as suggested above. At present there is not compelling evidence that the center of mass is displaced significantly, at least at the decade time scale.

Apart from the geometric considerations, the configuration of observatories should be such that (1) there are stations on most of the major tectonic plates in sufficient number to provide the necessary statistical strength, (2) the stations lie on relatively stable parts of the plate so as to reduce the possibility that tectonic shifts in some stations will not overly influence, at least initially, the parameters defining the CTS frame.

Finally, one should realize that the problem of the geometric origin of the CTS is linked to that of a geocentric ephemeris frame. The center of mass of the earth is directly accessible to dynamical methods and is the natural origin of a geocentric satellite-based dynamical system. But, a such, it is model dependent. And, unless the terrestrial reference frame is also constructed from the same satellites (as is the case in various earth models such as GEM, SAO, GRIM), there may be inconsistencies between the assumed origin of a kinematically obtained terrestrial system and the
center of mass. A time-dependent error in the position of the center of mass, considered as the origin of a terrestrial frame, may introduce spurious apparent shifts in the position of stations that may then be interpreted as erroneous plate motions. To avoid this problem, the parameters defining the CTS frame should include translational terms as well.

3.1 The 1980 Situation

The internationally accepted Woolard series of nutation (the IAU 1979 series became effective only with the 1984 ephemerides)
is computed for the instantaneous rotation axis of the rigid earth, and the Z axis of the CTS is the Conventional International Origin (CIO), defined by the adopted astronomic latitudes of the five International Latitude Service (ILS) stations, located approximately on the 39°08' parallel. These are assumed to be motionless relative to each other, and without variations in their respective verticals (plumb lines) relative to the earth. Thus, conceptually, polar motion should be determined from latitude observations only at these ILS stations. This has been done for over 80 years, and the results are the best available long-term polar motions, properly, but not very accurately, determined. The first axis of the CTS is defined by the assigned astronomic longitudes of time observatories (around 50) participating in the work of the Bureau International de l'Heure (BIH).

Due to the fact that in most geodetic and astronomical applications accurate shorter-term variations of polar motion are needed, which are not available with sufficient accuracy from the ILS observations, polar motion is also determined from latitude and/or time observations at a larger number of observatories participating in the work of the International Polar Motion Service (IPMS), as well as of the BIH. In the resulting calculations the earlier definition of the CIO cannot be maintained. The common denominator being the Woolard series of nutation, observationally the Z axis of the CTS is defined by the coordinates of the pole as published by the IPMS or by the BIH. Thus it is legitimate to speak of IPMS and BIH poles of the CTS (in addition to the CIO). The situation recently has become even more complicated because Doppler and laser satellite tracking, VLBI observations, and lunar laser ranging also can determine variations in the earth rotation vector (including polar motion), some of which are incorporated in the BIH computations. Further confusion arises due to the fact that the BIH has two systems: the BIH 1968 and the BIH 1979, the latter due to the incorporation of certain annual and semiannual variations of polar motion determined from the comparisons of astronomical (optical) results with those from Doppler and lunar laser observations [Feissel, 1980].

Though naturally every effort is made to keep the IPMS and BIH poles of the CTS as close as possible to the CIO, the situation cannot be considered satisfactory from the point of view of the geodynamic accuracy requirement of a few parts in $10^6$. The current accuracy of the pole position is estimated to be 0:01, and that of the UT1, 1 ms ($\pm 5 \times 10^{-9}$) for five-day averages [Guinot, 1978]. These figures, of course, do not include biases from the definition problems mentioned.
From 1984 onward, the IAU 1980 (Wahr, 1981) series of nutation for the nonrigid earth give the space position of the Celestial Ephemeris Pole (CEP) (see later). The CTP officially remained the same as before. Thus conceptually, polar motion was to be determined from latitude observations only at these ILS stations. As described elsewhere in this book, this had been done for over 80 years, and the results are the best available long-term polar motion, properly, but not very accurately, determined. The first axis of the CTS, the Greenwich Mean Astronomical Meridian, was defined by the assigned astronomic longitudes of time observatories participating in the work of the Bureau International de l'Heure (BIH).

3.2 The CTS (1988)

There seems to be general agreement that the new CTS frame conceptually be defined similarly to the CIO-BIH system [Bender and Goad, 1979; Guinot, 1979; Kovalevsky, 1979; Mueller, 1975a], i.e., it should be attached to observatories located on the surface of the earth. The main difference in concept is that these can no longer be assumed motionless with respect to each other. Also they must be equipped with advanced geodetic instrumentation like VLBI or lasers, which are no longer referenced to the local plumblines. Thus the new transformation formula may have the form

$$[\text{OBS}]_{j} = [\text{L}]_{j} + [\text{CTS}]_{j} + \mathbf{v}_{j} = [\text{L}]_{j} + \text{SNP} [\text{CIS}]_{j} + \mathbf{v}_{j}$$

where $[\text{L}]_{j}$ is the vector of the "j" observatory's movement on the deformable earth with respect to the CTS, computed from suitable models (see the figure and Section 4); NP, the nutation and precession matrices computed with the new 1976 IAU constants and the 1979 IAU series of nutation (provided the latter is not going to be changed; see Section 4); and $S$, the rotation matrix between the CTS and the true frame for which the nutation is computed. Variations in $S$ can be determined by a future international service (like the BIH) by comparing repeatedly observed observatory coordinates ($[\text{OBS}]_{j}$), corrected for the modelable deformations ($-U$), and by minimizing the residuals ($\mathbf{v}_{j}$) in the least squares sense.

The $[\text{OBS}]_{j}$ is related to the observatory coordinates ($X_{j}^O$), determined in the terrestrial frame inherent in the observational technique "o", through the well-known transformations involving three translation components ($\delta$), three (usually very small) rotations ($\beta$) and a differential scale factor ($c$):

$$[\text{OBS}]_{j} = X_{j}^O + \delta + R_1(\beta_1^O) R_2(\beta_2^O) R_3(\beta_3^O) X_{j}^O + cX_{j}^O$$

Naturally in the case of techniques which observe directions only (e.g., astrometry), the terms containing translation and scale will be omitted. Equations (1) and (2) together with (3) (and possibly others) may form the observation equations to be used when realizing the new type of CTS. The latter equations derived in (Zhu and Mueller, 1983) relates an ERP series determined by the technique "o", within its own frame of reference, with the parameters of rotation above:
\[
x_p - \beta_2^o + \alpha_1^o \sin \theta + \alpha_2^o \cos \theta = x^o + v_x_p
\]
\[
y_p - \beta_1^o - \alpha_1^o \cos \theta + \alpha_2^o \sin \theta = y^o + v_y_p
\]
\[
\omega_d \text{UT1} + \beta_3^o - \alpha_3^o = \omega_d \text{UT1}^o + v_{\text{UT1}}
\]

(3)

where \(x_p, y_p\) and UT1 are the observed ERP's, \(\omega_d\) the conversion factor, and \(\theta\) the sidereal time. The small rotations \(\alpha^o\) are between the CIS of the service and that associated with the technique \(o\).

The unknowns in the above system of equations to be solved for, in a least squares solution minimizing the square sum of the residuals \(v\), are \([\text{CTS}]_j\) and \(L_j\) for the observatories; \(\beta^o, \beta^o\) and \(c^o\) for the terrestrial frames of the techniques; \(\alpha^o\) for their inertial frames; and finally, the ERP \((x_p, y_p\) and UT1) for the service. If, however, in eq. (3) the ERP's \((x_p^o, y_p^o, \text{UT1}^o)\) are mean values averaged over intervals longer than a day, \(\alpha_1^o\) and \(\alpha_2^o\) cannot be determined, because the \(\sin \theta\) and \(\cos \theta\) terms average to zero in one sidereal day.

As mentioned, the parameters pertaining to the observatories \([\text{CTS}]_j\) and \(L_j\) define the CTS. The others give the relationship of the CTS to the technique "\(o\)" terrestrial frame \((\beta^o, \beta^o, c^o)\); to the CIS \((x_p, y_p, \text{UT1})\); and the latter's relationship to the technique "\(o\)" inertial frame \((\alpha^o)\).

The rotations in eq. (2) can either be determined from the Cartesian coordinates (e.g., Moritz, 1979)) or, for possibly better sensitivity, since the rotation is least sensitive to variations in height, only from those of the horizontal coordinates (geodetic latitude and longitude) (e.g., (Bender and Goad, 1979)). It is, however, unlikely that the rotations will continue to be determined (as presently) from astronomical coordinates, i.e., from the direction of the vertical, for the reason of inadequate observational accuracy. Note that when using this method, the deformations (and the residuals) by definition cannot have common rotational (or translational) components.

As far as the origin of the CTS is concerned, it could be centered at the center of mass of the earth, and its motion with respect to the stations can be monitored either through observations to satellites or the moon, or, probably more sensitively, from continuous global gravity observations at properly selected observatories (Mather et al., 1977). For the former method, the condition

\[
\sum_D w_D \delta_D^o = 0
\]

could be imposed on the above adjustment. The summation would be extended to all the above dynamic techniques \(D\) with given relative weights \(w_D\). A similar condition could also be imposed on the scale extended to techniques defining the best scales (probably VLBI).

The above method of determining ERP or some variation thereof needs to be initialized in a way to provide continuity. This could be done through the IPMS or BIH poles, and the BIH zero meridian, at the selected initial epoch (or averaged over a well-defined time interval, say 1 to 1.2 years), uncertainties in their definition mentioned elsewhere in the book being mercifully ignored.

It is probably not useless to point out that if such a system is established, the most important information for the users will be the ERP and the transformation parameters, but for the scientist...
new knowledge about the behavior of the earth will come from the analysis of the residuals after the adjustment.

The IAU and IUGG recently made practical recommendations on the establishment of such a (or very similar) Conventional Terrestrial System, including the necessary plans for supporting observatories and services by establishing the International Earth Rotation Service, effective 1 January 1988 (Wilkins and Mueller, 1986). The goal of the service is the determination of the total transformation between the CTS and CIS. The service will publish not only ERP determined from the repeated comparisons (the past situation), but also the models and parameters discussed above, i.e., the parameters defining the whole system. (See Section 5.)

3.3 Reference Frame Ties

3.31 Ties Between the CIS Frames

Measurements are inherently more accurate in their "natural" frame and hence should always be reported as such. However, to benefit from the complementarity of the various techniques, knowledge of the frame interconnections (both the rotation and the time-variable offset) is essential; these are summarized in Fig. 3 (Dickey, 1988) and in Fig. 2.

Recent activity in this area is indicated by the number of boxes and lines in Fig. 3, entitled Connections 1986 (the accuracy cutoff here is 0.05); a similar figure in an earlier paper (Williams et al., 1983) had fewer boxes and connecting lines. For example, ten lines instead of fifteen connected the targets with the techniques, and radio stars were listed as prospects for the future. The lunar planetary system, integrated in a joint ephemeris, is by its nature unified by the dynamics (Williams and Standish, 1988). The radio frame is tied to the ephemeris frame in several ways; one is via differential VLBI measurements of planet-orbiting spacecraft and angularly nearly quasars (Newhall et al., 1986). Another is the determination of a pulsar's position in the ephemeris frame (via timing measurements) and the radio frame (via radio interferometry, see Backer et al., 1985). Very Large Array (VLA) observations of the outer planets (Jupiter, Saturn, Uranus and Neptune) or their satellite provide an additional tie between these two frames (Muhleman et al., 1985).

As for an optical-radio frame tie, a preliminary link has been established between the FK5 optical frame and the JPL radio reference frame via the differential VLBI measurement of optically bright radio stars and angularly nearly quasars coupled with comparisons of their optical positions (see Lestrade et al., 1987), and also by the use of the optical positions of quasars (Purcell, 1979). The optical and ephemeris frames are tied by optical observations of the planets Dickey (1988) treats a few of the frame ties in some details; for example, for the connection between the radio and the ephemeris frames. In some cases such as the connections between the optical and radio frames, the highlights are given with reference to a more detailed account.

Dickey (1988) also outlines the future with ongoing and planned efforts in several areas: Improved ephemeris-radio frame ties can be accomplished by VLBI observations of pulsars, additional VLA observations of the outer planets and satellites, and future differential VLBI experiments (such as that with orbiting spacecraft around Jupiter and Saturn). The millisecond pulsar PSR 1937+214, having a period of 1.6 ms, has exceptionally low timing noise. Its position in the ephemeris frame can be measured to ~1 mas. This will allow a radio-planetary frame tie, limited only by the accuracy of an interferometric position measurement. Roughly, a factor of five
Fig. 3 Reference frame connections 1986 (Dickey, 1988).
improvement (down to 0.01) is expected here with the full implementation of VLBI observations. An initial experiment of this type has been executed by R. Linfield and C. Gwinn.

As already mentioned, for optical astrometry, Hipparcos will measure a network of stars over the entire sky with accuracies of ~2 mas (Kovalevsky, 1980), while the Space Telescope will measure small fields with similar differential accuracy. However, the Space Telescope can observe much fainter objects (Jeffreys, 1980) and could observe the optical counterparts of extragalactic radio sources, all but possibly one of which are too faint for Hipparcos. A joint program would produce an accurate stellar network linked to the quasar radio frame by the Space Telescope. The occultations of stars by planets and planetary rings can provide an additional link between the optical and ephemeris frames. Also, optical interferometry offers exciting possibilities with the potential resolution being two or three orders of magnitude finer than that of VLBI (Reasenberg, 1986). More details are given in (Dickey, 1988).

3.32 Ties Between the CTS Frames

Boucher and Altamimi (1987) established relationships between a number of Conventional Terrestrial Reference Frames based on colocated observation stations and eq. (2). The selected sets of station coordinates defining each CTS are as follows:

**VLBI.** Three sets of station coordinates have been selected:

- **SSC(NGS) 87 R01.** The coordinate data are derived from a composite set of Mark III VLBI observations collected under the aegis of project MERIT, POLARIS, and IRIS and conducted between September, 1980, and January, 1987. Westford coordinates were fixed to their initial values. The IRIS terrestrial frame is made more nearly geocentric by applying the BTS 1985 translations (Carter et al., 1987).

- **SSC(GSFC) 87 R01.** The data acquired since 1976 by the NASA Crustal Dynamics Project and since 1980 by the NGS POLARIS/IRIS programs. The terrestrial frame is defined by the position of the Haystack 37-M antenna and the BIH Circular D values for 1980 October 17 (Ma et al., 1987).

- **SC(JPL) 83 R05.** The coordinate data are from the JPL Time and Earth Motion Precision Observations (TEMPO) project, using the DSN radio telescopes. The reference frame solution is tied to the BIH on 20 December 1979 (Eubanks et al., 1984).

**Lunar Laser Ranging.** The coordinate data are from the JPL solution: **SSC(JPL) 87 M01** containing four stations, two at Fort Davis, one at Haleakala (Maui), and one at Grasse. The nominal planetary and lunar ephemeris DE121/LE65 was used in the reduction. The ephemeris uses the equator and equinox of B1950.0. It is on the dynamical equinox and has a zero point consistent with the FK5 catalogue (Newhall et al., 1987).

**Satellite Laser Ranging.** Two sets of station coordinates have been selected:

- **SSC(CSR) 86 L01.** The solution is based on Lageos ephemeris from May, 1976, to September, 1986, using the model Lageos Long Arc 8511. The force model, referred to as the CSR 8511 system, adheres closely to the MERIT standards. The tectonic plate motion model AM1-2 of Minster and Jordan (1978) was used and the epoch of the derived station coordinates is 1983 January 1. The GM value is 398400.4404 km$^3$/s$^2$ (Schutz et al., 1987).

- **SSC(DGFI) 87 L01.** The solution is computed from Lageos observations covering the period 1980 to end 1984 and based on five yearly solutions. By the rates of change of the yearly
solutions, the station coordinates then were related to the same reference epoch 1984.0. The reference frame was defined by the three coordinates (longitude, latitude of Yaragadee (7090) and latitude of Wettzell (7834)) which were held fixed in the five solutions. The GM value is \(3.98600448 \times 10^{14} \text{ m}^3 \text{ s}^{-2}\), initial ERP series were from homogeneous BIH series and other constants from MERIT Standards (Reigber et al., 1987).

**Doppler.** Station coordinates are from DMA Doppler project SSC(DMA) 77 D01 solution, and other Doppler campaigns containing more than 100 station positions. They are determined in the NSWC9Z2 datum by point positioning using Precise Ephemerides.

Three comparisons have been performed to get an idea about the consistency of different solutions and relations between these solutions related to a same technique. Table 5 summarizes these different comparisons.

The first comparison is between twoVLBI solutions SSC(NGS) 87 R01 and SSC(GSFC) 87 R01 containing 12 colocated stations. Note the 1 cm of RMS issued from this comparison. The origin difference between the two solutions is due to the arbitrary choice of the VLBI origin in the definition of the terrestrial frame.

The second comparison is between two SLR solutions SSC(CSR) 86 L01 and SSC(DGFI) 87 L01 containing 37 collocated sites. In this case the RMS is about 12 cm. Note also a rotation of 125 mas about the Z-axis between the two solutions.

The last comparison is between the two last SLR solutions of CSR of 85 and 86 giving an RMS of about 11 cm. Note here that the scale factor has been decreased of about \(1.5 \times 10^{-8}\) from 85 to 86 solution.

The slightly larger scatter (10 cm level) of SLR data is mainly explained by the mixture of good third generation stations (4 cm level) with some older ones (20 to 50 cm).
Table 5 Transformation Parameters Between Different CTS Frames (Boucher and Altamimi, 1987) (the uncertainties are given in the second line)

<table>
<thead>
<tr>
<th>SSC</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>D</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>Col.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>10^{-6}</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>nb./ RMS</td>
</tr>
<tr>
<td>NGS 87R01</td>
<td>/</td>
<td>1.697</td>
<td>-0.998</td>
<td>0.339</td>
<td>0.003</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.003</td>
</tr>
<tr>
<td>GSFC87R01</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>CSR 86L01</td>
<td>/</td>
<td>-0.007</td>
<td>-0.026</td>
<td>0.074</td>
<td>0.015</td>
<td>0.013</td>
<td>-0.009</td>
<td>0.125</td>
</tr>
<tr>
<td>DGFI87L01</td>
<td>0.023</td>
<td>0.023</td>
<td>0.023</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>CSR 86L01</td>
<td>/</td>
<td>-0.080</td>
<td>0.040</td>
<td>0.080</td>
<td>0.015</td>
<td>0.004</td>
<td>-0.003</td>
<td>0.009</td>
</tr>
<tr>
<td>CSR 85L07</td>
<td>0.024</td>
<td>0.023</td>
<td>0.022</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Model: | X2 | X1 | T1 | D | -R3 | R2 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Y2</td>
<td>Y1</td>
<td>T2</td>
<td>+</td>
<td>R3</td>
<td>D</td>
<td>-R1</td>
</tr>
<tr>
<td>Z2</td>
<td>Z1</td>
<td>T3</td>
<td>-R2</td>
<td>R1</td>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

A combination of all above data has also been performed incorporating 51 colocated sites and making use of the plate tectonic absolute motion model AMO-2 derived from the global RM-2 model (Minster and Jordan, 1978).

The adopted origin of the adjusted system is derived from dynamical solutions SSC(JPL) 87 MO1 and SSC(CSR) 86 LO1, and the scale factor is the one of SSC(CSR) 86 LO1 while the orientation is the one of SSC(NGS) 87 RO1.

The same dataset, in addition to the corresponding ERP series, has also been selected for the realization of the BIH terrestrial system for 1986 (see BIH Annual Report for 1986).

Table 6 lists the transformation parameters of the individual system with respect to the global one.
Table 6 Transformation Parameters from the Individual 1984.0 CTS Systems to the “BIH 1986” CTS (Boucher and Altamimi, 1987) (the uncertainties are given on the second line)

<table>
<thead>
<tr>
<th>CTS</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>D</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>m</td>
<td>m</td>
<td>10^{-6}</td>
<td>&quot;&quot;</td>
<td>&quot;&quot;</td>
<td>&quot;&quot;</td>
</tr>
<tr>
<td>NGS 87 R01</td>
<td>-0.009</td>
<td>-0.111</td>
<td>-0.112</td>
<td>0.023</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.035</td>
<td>0.036</td>
<td>0.035</td>
<td>0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>GSFC 87 R01</td>
<td>-1.696</td>
<td>0.862</td>
<td>-0.463</td>
<td>0.020</td>
<td>0.001</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>0.029</td>
<td>0.034</td>
<td>0.032</td>
<td>0.004</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>JPL 83 R05</td>
<td>-0.062</td>
<td>0.234</td>
<td>0.140</td>
<td>0.015</td>
<td>0.001</td>
<td>0.011</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.032</td>
<td>0.036</td>
<td>0.035</td>
<td>0.005</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>JPL 87 M01</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.020</td>
<td>-0.004</td>
<td>0.009</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.017</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>CSR 86 L01</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
<td>0.005</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>DGFI 87 L01</td>
<td>-0.015</td>
<td>0.021</td>
<td>-0.053</td>
<td>-0.015</td>
<td>-0.010</td>
<td>0.014</td>
<td>-0.115</td>
</tr>
<tr>
<td></td>
<td>0.041</td>
<td>0.041</td>
<td>0.040</td>
<td>0.006</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>DMA 77 D01</td>
<td>0.302</td>
<td>0.096</td>
<td>4.645</td>
<td>-0.605</td>
<td>-0.030</td>
<td>-0.005</td>
<td>0.797</td>
</tr>
<tr>
<td></td>
<td>0.219</td>
<td>0.206</td>
<td>0.195</td>
<td>0.026</td>
<td>0.009</td>
<td>0.009</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Table 6 leads to some conclusions about the origin, scale and orientation of the individual CTS’s with respect to the global one:

**Origin.** Knowing that the origin of the adjusted system is from CSR SLR and JPL LLR, the origin of all VLBI solutions remains arbitrary. Note the shift of 5 cm of the DGFI SLR solution.

**Scale.** Note the level of consistency of the scale factor of some $10^{-8}$ for the different solutions. Some variations for VLBI and LLR solutions are due to a relativistic bias in the definition of the terrestrial system (Hellings, 1986; Boucher, 1986).

**Orientation.** The orientation of the individual terrestrial systems is usually realized through BIH values. The differences in orientation of the different solutions are arbitrary and of some mas level.
4. MODELING THE DEFORMABLE EARTH

In this section we will try to highlight the modeling problems associated with the components of transformation between the CIS and CTS mentioned in Section 3.

4.1 Precession (P)

At the XVIth General Assembly in Grenoble in 1976, the IAU adopted a new speed of general precession in longitude of $5029'0966$ per Julian century at the epoch $J2000.0$ (JED 2451545.0). This value when referred to the beginning of the Besselian year $B1900.0$ is $5026'767$ per tropical century, which may be compared to the previously adopted (and presently still used) value of $5025'64$ per tropical century at $B1900.0$. The change was calculated by Fricke [1977] from proper motions of stars in the systems GC, FK3, N30, and FK4. From the results, the correction of +1"10 per century to Newcomb's luni-solar precession in longitude was recommended. This value combined with a correction to Newcomb's planetary precession, due to the improved 1976 IAU values of planetary masses, resulted in the above new precessional constant. Expressions to compute the effect of precession from one epoch to another were developed by Lieske et al. [1977]; and the usual equatorial parameters, $z, \theta, \zeta_0$, to be used in the precession matrix [Mueller, 1969],

$$ P = R_3(-z) R_2(\theta) R_3(-\zeta_0), $$

to and from the epoch $J2000$ were computed by Lieske [1979]. The above matrix allows the currently best transformation between the CIS (say, the FK5 at $J2000.0$) and an interim "Mean Equator and Equinox Frame" of some date.

Recent VLBI observations imply that the value of the precessional constant should be $5028'7966$/Julian century at $J2000$ (Herring et al., 1986).

4.2 Nutation (N)

The nutation story is much more complex. First of all, the nutation matrix is [Mueller, 1969]

$$ N = R_1(-\varepsilon -\Delta \varepsilon) R_3(-\Delta \psi) R_3(\varepsilon), $$

where $\varepsilon$ is the obliquity of the ecliptic, $\Delta \varepsilon$ is the nutation in obliquity, and $\Delta \psi$ the nutation in longitude, computed from a certain theory of nutation. This matrix allows transformation from the aforementioned interim mean frame of date to the (also) interim true frame of the same
date. This part is clear and without controversy. The complexities are in the agreement reached (or still to be reached) on the theory of nutation when computing the above parameters. Kinoshita et al. [1979] give an historical review:

"In astronomical ephemerides, nutation has been computed until now by the formulae which were given by Woolard (1963). The coefficients of the formulae are calculated assuming that the Earth is rigid. However, it has been found in recent analyses of observations ... that some coefficients of actual nutations are in better agreement with values calculated by the non-rigid Earth theory.

"Moreover, Woolard (1953 gave the nutation of the axis of rotation. Therefore, a small and nearly diurnal variation appears in the latitude and time observations, which is the so-called dynamical variation of latitude and time, or Oppolzer terms. In the global reduction of latitude and time observations, such as polar motion or time services, the Oppolzer terms have been until now removed from the data at each station (cf. BIH Rapport Annuel 1977, pA-3) or counted out as a part of the non-polar common z and τ-terms (IPMS Annual Report 1974, p. 11). On the other hand, Atkinson (1973) pointed out that if the (forced) nutation of the axis of figure is calculated instead of rotation axis, such a complicated treatment becomes unnecessary.

"Considering these situations, the IAU investigated the treatment of nutations, together with the system of astronomical constants which should be used in new ephemerides, and set up the 'Working Group of IAU Commission 4, on Precession, Planetary Ephemeris, Units, and Time-Scales'. The results by the Working Group are given in the report of Joint Meeting of Commissions 4, 8, and 31, in Grenoble, 1976 (Duncombe et al. 1976). In the report, the proposal by Atkinson is adopted, and the formula for computing the (forced) nutation of figure axis is shown clearly and in detail, by using the equation-numbers given by Woolard (1953). However, the amendments of coefficients taking account of the non-rigidity of the Earth have not been adopted. In regard to this problem, it was noted that there should be a possibility of making further amendments in Kiev Symposium ....

"At the IAU Symposium No. 78 in Kiev in 1977, the problem with the non-rigid values of nutation was discussed, and a series of new values were recommended which seemed to be based on Moloden-skij's non-rigid theory. In the Symposium, however, it was recommended that the axis for which the nutation should be computed was the axis of rotation. This recommendation reversed the resolution given at Grenoble.

"In accordance with the resolution at the Kiev Symposium, an 'IAU Working Group on Nutation under Commission 4' was set up and is investigating these two problems, in order to prepare a fully documented proposal for the next IAU General Assembly in Montreal in 1979. In the second draft of the Working Group circulated on Nov. 16, 1978, the following conclusions are reported: (1) as for the axis to be referred, the Grenoble resolution is still valid, and (2) as for the coefficients of nutation series, the value in which the non-rigidity of the Earth is taken into account should be adopted as a working standard of
astronomical observations. In the draft, a table of nutation series is given, and the numerical values in the table are based on the rigid theory by Kinoshita (1977), with use of IAU (1976) System of Astronomical Constants, and are modified by Molodenskij's non-rigid theory (Molodenskij 1961).

As we understand it, the Kinoshita theory above is for the nutation of the axis of maximum moment of inertia of the "mean shape of the elastic mantle" (briefly, "mean axis of figure of the mantle"). To add to the history, after the above-quoted Working Group Report was circulated, a new proposal was made by J.M. Wahr and M.L. Smith of CIRES that it would be preferable to adapt the non-rigid earth results of Wahr [1979] for the earth model 1066A developed by Gilbert and Dziewonski [1975]. This model is a rotating, elliptically stratified linearly elastic and oceanless earth with a fluid outer core and a solid inner core. The nutations are computed for the "Tisserand mean figure axis of the surface," which is also a mean mantle fixed axis [Wahr, 1979]. The IAU in Montreal in 1979 considered both proposals and opted for the Kinoshita et al. [1979] series. A few months later in December, 1979, the IUGG in Canberra, in Resolution No. 9 addressed to the IAU, requested reconsideration in favor of the Wahr model.

It should be pointed out that regardless of the fact that in geodetic or geodynamic applications we are only concerned with the total transformation SNP, it is of scientific importance to understand clearly the definition of the interim true equator and equinox frame of date, more specifically, the exact definition and the desirability (from the observability point of view) of the axis for which the nutation is computed.

In order to simplify the discussion, let us start with the rigid model. The motion of each of the axes, i.e., the axis of figure (F) (maximum moment of inertia), of the angular momentum (H), and the instantaneous rotation axis (I) are described by differential equations. If we want to refer to one of these axes we have to consider the complete solution of the differential equations, i.e., the free solution and the forced solution components. Confusion can arise if one refers to only one solution component (forced or free), but still calls it axis of figure, instantaneous rotation axis, etc. It is mandatory to point out which solution component one refers to. Neglecting to do so has been the reason for the by now classical confusing controversy about the Atkinson papers, though Atkinson [1975, p.381] clearly states:

"Accordingly, when we speak of computing the nutations for either axis, we mean here computing the forced motion only, excluding the appropriate fraction of the non-computable Chandlerian wobble."

Unfortunately, he, and others as well, then continue to use the term "axis of figure" sometimes in the sense of the axis of maximum moment of inertia and at other times in the sense of the forced motion of the axis of figure.
A remark concerning the "Eulerian pole of rotation" (E₀) as given by Woolard seems in order also. Quoting once again Atkinson [1976]:

"The wording of the resolution on nutation, and the notes on it, which have been circulated by the Working Group, avoid all explicit mention of the axis of figure, even though they specify that the coefficients which Woolard gives for that axis shall be inserted, and they refer to the "Eulerian pole of rotation" although this cannot ever, in principle, coincide with the celestial pole and really has no more direct connection with the observations than is shown for it in [his] Fig. 2, i.e., none at all."

The difference between the Eulerian pole of rotation (E₀) and the pole which Atkinson talks about is due to a homogeneous solution component. (E₀) is obtained from the complete solution of (I) by subtracting the periodic diurnal body-fixed motions of (I).

Consequently, the point E₀ has no periodic motion with respect to the crust, but it does have such a motion in space which is exactly the free nutation. Although this spatial motion is conceptually insignificant considering the observation technique (fundamental observations at both culminations), one gets another point, which is called the (true) Celestial Pole (C) in [Leick and Mueller, 1979], by subtracting the forced body-fixed motions of (H) from the complete nutation set of (H). The thus obtained axis (C) has no periodic diurnal spatial motion because the homogeneous solution of the angular momentum (H) is constant (zero). Equivalently, one can say that the nutations of (C) correspond to the forced solution of the axis of figure (rigid case, of course). This is the pole which Atkinson talks about and which is called (mistakenly) the "mean axis of figure." There is no doubt that this is the point to which the astronomical observations as well as lunar laser ranging refer, and the nutation should be adopted for this point. As for terminology, the IAU in 1979 named this (C) pole appropriately the Celestial Ephemeral Pole because its motion characteristics, i.e., no periodic diurnal motion relative to crust or space, have always been associated with the concept of the celestial pole. It would be preferred that the word "figure" be dropped entirely for several reasons. First, one intuitively associates the axis of figure with the one for which the moment of inertia is maximum. This is true for the (C) only if the free solution (Chandler) is zero. But this is, generally, not the case. Second, the conceptual definition of (C) can easily be extended to elastic models or models with liquid core (the IAU 1979 case). Moreover, in order to emphasize that the observations take place on the earth surface, it would be useful to denote the actual pole accessible to the fundamental observation techniques by another designation, e.g., (CO), similarly to UTO. The "0" would indicate that the nutations of this pole can in principle be determined only from observations because of the lack of a perfect earth model. Any nutation set based on a model is only an approximation to the nutations of the (CO). In this sense the rigid earth nutations of (I), (H) or (F) are all equivalent. Each of these nutations defines its own pole which has a diurnal motion around the (CO). The purpose of the measuring efforts is to find the corrections to the adopted set of nutations in order to get those of the (CO), the only pole which is observable.
Some have suggested the term "zero excitation figure axis" for what is called above the (CO). The term "zero excitation" would not reduce the confusion. The spatial motion of this axis is computed by adding Atkinson's terms to Woolard's series, but this is equivalent to the forced motion of the axis of figure (rigid case). The observed motion of the (CO) relative to the crust only appears as a motion of zero excitation (free motion) at the first sight. Since the conceptual observation time of one position determination is one day, the observed position of the (CO) will always include effects due to oceans, atmospheric mass redistribution, etc., i.e., the geophysical nutations. These motions are better known as the annual polar motion and the sub-harmonics. Therefore, the zero-excitation pole is not directly observable. On the other hand, the concept of the (CO) can still be used in this case since it is by definition the pole which has no periodic diurnal motions relative to the crust or to space.

There is also the common offset of both the rotation axis and the (CO) caused by the tidal deformation [McClure, 1973]. This is an offset of (I) and (CO) relative to (H) for the perfectly elastic model as compared with the rigid model. We have to remember, again, that the observations refer to the (CO). Therefore, any nutation correction which is derived from observations (based on an adopted set of nutations) will automatically give the corrections to the (CO). Consequently, there is no need for a special consideration of this possible separation, at least not for those harmonic motions whose amplitudes are derived from observations. In fact, the analysis of the observed fortnightly term seems to contradict somewhat the predicted amplitude for the perfectly elastic model.

From the above discussion, it also seems clear that ideas advocating the adoption of nutations for the axis of angular momentum violate the concept of observability. It is true that the direction of (H) in space is the same for the rigid, elastic, or any other reasonable earth model. But this property is not of much interest to the astronomer or geodesist who tries to determine the orientation of the earth. It is conceptually simpler to refer to an axis which is observable.

Returning now to the problem of the IAU 1979 adopted set of nutations, there seems to be little difference whether the Kinoshita series is retained or the Wahr set is adopted. Using more and more realistic earth models is certainly appealing. On the other hand, severely model-dependent developments are liable to change as models improve. A more important point is that whichever series is adopted, it should be for the Celestial Ephemeris Pole (C), which (again) has no periodic diurnal motion relative to the crust (not the mantle!) or the CIS.
The IAU subsequently adopted the Wahr model as the IAU 1980 Theory of Nutation. Recent VLBI observations clearly indicate that corrections are needed at least to some (18 yr, 365, 182 and 14 days) of Wahr's theoretical nutation coefficients (Herring et al., 1986). The largest reliable deviation was found in the retrograde annual nutation term. It seems clear that the recent observations are already sensitive to those physical properties of the Earth's interior which had not been foreseen, and to date none of them have explained the deviations observed.

Regarding the issue of observability of the pole to which the nutation theory refers (through earth rotation parameter (ERP) observations) Capitaine (1986) points out that the available observed polar motion is neither referred to the instantaneous pole of rotation (as it can be conceptually defined) nor to a pole which has no diurnal or quasi-diurnal motion with respect to an Earth-fixed or a space-fixed reference frame (as conceptually defined in the 1980 IAU theory of nutation). This observed motion is, in fact, referred to a conventional pole which depends, at a 0.001 level of precision, in the systematic biases of the methods of observation. In addition, the third ERP, which is conventionally linked to UT1, suffers presently from a lack of a clear corresponding reference concept.

A clarification of these motions are thus necessary in order to intercompare and interpret the observed ERP with a 0.001 level of precision.

4.3 Earth Rotation (S)

The two components of the S matrix [Mueller, 1969],

\[ S = R_2(-x_p) R_1(-y_p) R_3(\phi) \]

are the rotational angle of the first (X) axis of the CTS with respect to the first axis of the interim true equator and equinox frame of date, measured in the equator of the Celestial Ephemeris Pole (or whatever is defined in the N matrix), also known as Apparent Sidereal Time (S), and the polar motion coordinates (x_p, y_p) referred to the same pole and the Z axis of the CTS.

In this connection it should be mentioned that some authors prefer a different "true" frame, which would have no rotation about the Z axis [Guinot, 1979; Murray, 1979; Kinoshita et al., 1979]. It is in such an interim frame where, for example, a nutational theory can be conveniently developed, or satellite orbits calculated [Kozai, 1974]. Such a frame can be obtained from the CIS by a modified NP transformation, where

\[ N = R_1(-\Delta e \cos M + \Delta \psi \sin e \sin M) R_2(\Delta \psi \sin e \cos M + \Delta e \sin M), \]
\[ P = R_3(-z + M) R_2(\theta) R_3(-\zeta_0), \]

where M is the precession in right ascension. In this case the rotation of CTS about the Z axis (\( \phi \)) is the Apparent Sidereal Time from which the general precession and nutation in right ascension are removed. What is left, thus, is the rotational angle of the X axis of the CTS directly with respect to that of the CIS. Such a definition of the sidereal angle would, of course, necessitate the redefinition of UT1, a possibility for controversy. It should be noted also, that the above transformation is independent of the ecliptic, a preference of many astronomers.
Here there is not very much modeling that can be considered really useful. Of course, the rotation rate of the earth could be modeled as constant and possibly in the UTC scale. This would then mean that observed departures could immediately be referenced to that scale, a current practice. If one really wanted to go overboard, polar motion could also be modeled with the Chandlerian cycle of, say, 428 days and a circular movement of radius 0.15", centered at the Z axis of the CTS. More complex models may be developed (e.g., Markowitz, 1976, 1979), but since there are no valid physical concepts yet for the excitation of the amplitude of the Chandler motion, such modeling would not serve much purpose.

4.4 Deformations (L')

The deformations which reasonably can be modeled at the present state of the art are those due to the tidal phenomena and to tectonic plate movements.

4.41 Tidal Deformations. Tides are generated by the same forces which cause nutation; thus models developed for the latter should be useful for the former. One would think that for earth tides it may not be necessary to use the theories based on the very sophisticated earth models: the amplitude of the phenomena being only around 30 cm, an accuracy of 3% should be adequate for centimeter work. This should be compared, for example, with the accuracy of the Wahr nutation model claimed to be at the 0.3% level. However, the tides and nutations differ in one important respect. The nutations hardly depend upon the elasticity and are affected only slightly by the liquid core (this is one reason why modern theories such as those of Wahr and Kinoshita give only slightly different results). Thus, except perhaps for the largest terms, one can depend upon theory when dealing with nutation. The tides, on the other hand, depend intimately upon the internal properties of the earth, and one must use tidal theories with caution [Newton, 1974]. Additional problems are handling the transformation of the potential into physical displacements and on the calculations of regional (ocean loading) or local tidal deformations.

As far as the transformation of the tidal potential into displacement is concerned, the traditional way to do this is through the Love numbers for the solid effect and through "load" numbers for ocean loading. These numbers, however, are spherical approximations which, for the purely elastic earth, are global constants. For more sophistication, elliptic terms can be added, but they will change the results by 1-2% only. A liquid core model produces resonance effects, which will result in a frequency dependency. The actual numbers representative for a given location can be determined only through in situ observations, such as gravity, tilt, deflections, which are all sensitive to certain Love number combinations and frequencies. Difficulties in this regard include the frequency dependence of the Love number. For example, the Love number h for radial (vertical) displacement can be determined locally from combined gravity and tilt meter observations by the analysis of the O1 tidal component, but the real radial motion of geodetic interest is influenced by the M2 and other semidiurnal tidal components.
Tidal loading effects have recently been very successfully computed by Goad [1979] using the 1° square Schwiderski [1978] M2 ocean tide model. Global results show agreement with gravimetrically observed deformation on the 0.5 μgal (5 x 10^{-10}) level. From this it would seem that with good quality ocean tide models and with proper attention to the frequency dependence, this problem is manageable.

Suitable equations for displacement, gravity change, deflection change, tilt and strain calculations due to tides may be found in [Melchior, 1978; Vanicek, 1980] and in [Wahr, 1979] for the elliptic case.

As a conclusion one can reasonably state that the global and regional station movements due to tides can be estimated today within centimeters. Local effects, however, can be sizable and unpredictable, and therefore they are best determined from in situ observations. Thus most of the tidal effect in fact can and should be removed from the observations.

4.42 Plate Tectonic Mass Transfer. The concept that the earth lithosphere is made up of a relatively small number of plates which are in motion with respect to each other is the central theme of global plate tectonics. The theory implies the transfer of masses as the plates move with velocities determined from geologic evidence (see, e.g., [Solomon and Sleep, 1974; Kaula, 1975; or Minster and Jordan, 1978]). Material rises from the asthenosphere and cools to generate new oceanic lithosphere, and the lithospheric slabs descend to displace asthenospheric material (see, e.g., [Chappie and Tullis, 1977]). A good example of how such a theory can be used to estimate the vertical motions of observatories located on the lithosphere (in terms of changes in geoid undulations) is given in [Larden, 1980], based on specific models constructed in [Mather and Larden, 1978]. The results indicate that changes in the geoid can reach 150 mm/century. Horizontal displacements can be estimated from the plate velocity models mentioned directly with certain possible amendments [Bender, 1974].

4.43 Other Deformations. If one wants to carry the modeling further, it is possible to estimate seasonal deformations due to variations in air mass and groundwater storage, for which global data sets are available [Van Hylckama, 1956; Stolz and Larden, 1979; Larden, 1980]. A more esoteric effect would be the expansion of the earth (e.g., [Dicke, 1969; Newton, 1968]). The rate of possible expansion is estimated to be 10 - 100 mm/century.

One could continue with other modeling possibilities, but there is a real question on the usefulness of modeling phenomena of this level of magnitudes and uncertainties. As a general philosophy, one could accept the criteria that modeling should be attempted only if reliable and global data is available related to the phenomena in question, and if the magnitudes reach the centimeter per year level or so.

One last item which should be brought up is the fact that the issue of referencing observations and/or geodynamic phenomena is not exhausted by the establishment of reference frames of the Cartesian types discussed in this paper. An outstanding issue is still the geoid as a reference surface. Though it is true that three-dimensional advanced geodetic ob-
servational techniques do not need the geoid as a reference, there are still others, such as spirit leveling, which are used in the determination of crustal deformations in the local scale. In addition, the geoid is needed to reference gravity observations on a global scale (one should remember that a 1 cm error in the geoid corresponds to a 3 μgal error in the gravity reduction, which is (or soon will be) the accuracy of modern gravimeters). Further, in connection with the use of satellite altimetry for the determination of the departures of sea surface topography from the equipotential geoid (a topic of great oceanographic interest), there is a requirement for a geoid of at least 10 cm accuracy. The determination of such a geoid globally, or even over large areas, is a very difficult problem, which, however, is not the subject of the present paper.

4.44 Current (1988) Practice

Some of the above effects can be modelled with good accuracy. A review of current models can be found in MERIT Standards (Melbourne, 1983). Two models are of particular interest for terrestrial frames (Boucher, 1987):

The solid earth tide correction for ground station positions. Especially important is the vertical component:

\[ \Delta h = -0.121 \left( \frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) \text{m} \]

the permanent tidal deformation, where \( \phi \) is the latitude of the station.

Tectonic plate motion correction \( \Delta \hat{X} \) for the horizontal components. The usual one, such as the series of Minster-Jordan models, are defined through a set of angular velocity vectors \( \Omega_p \), one for each plate, and expressed in the terrestrial system, so that the velocity of a point of coordinate \( \bar{X} \) is

\[ \dot{\bar{X}} = \Omega_p \wedge \bar{X} \]

Two absolute motion models are usually adopted in data analysis:

AMO-2, derived from the RM-2 model by applying a "no global rotation" condition,

AM1-2, which minimizes the motion of a set of hot spots, also derived from RM-2 (Minster and Jordan, 1978).

AMO-2 depends only on the adopted contour of plate boundaries, whereas AM1-2 depends on the selection of the hot spots which are more subject to uncertainties. On the other hand, AMO-2 corresponds to the type of law of evolution one wants to give to terrestrial frames and has been consequently adopted by MERIT Standards (Update 1, December 1985). Nevertheless, AM1-2 leads to a system linked to the mantle which is needed to express a geopotential model without secular variations due to a residual rotation of the system. It is therefore favoured by groups which perform dynamical analysis of satellite tracking data.
5. The International Earth Rotation Service

5.1 The MERIT-COTES Programs

The acronyms MERIT and COTES refer to two international programs that were started independently, but which developed together. MERIT refers to an international program to monitor the earth's rotation and intercompare the techniques of observation and analysis with a view to making recommendations about the form of a new international service. On the other hand, the objective of the COTES program was to provide a basis for recommendations on the establishment and maintenance of a new conventional terrestrial reference system for the specification of positions on or near the earth's surface. The two programs were linked when it became clear that the observational campaign planned for MERIT and the new earth rotation service would provide results that could be used for COTES. In particular, in order to determine the earth rotation parameters to high accuracy, it is necessary to establish the positions of the observing sites (or "stations") in a worldwide network that provides a suitable basis for a new terrestrial reference system. The observational data and results that have been obtained in the course of these programs have been collected together for further analysis and for use in current and future scientific studies and practical applications.

Project MERIT was conceived in 1978 at IAU Symposium No. 82 on "Time and the Earth's Rotation." The Symposium recommended the appointment of a "working group to promote a comparative evaluation of the techniques for the determination of the rotation of the earth and to make recommendations for a new international program of observation and analysis in order to provide high quality data for practical applications and fundamental geophysical studies." Two years later, in 1980, the participants in IAU Colloquium No. 56 on "Reference Coordinate Systems for Earth Dynamics" recommended the setting up of a working group "to prepare a proposal for the establishment and maintenance of a Conventional Terrestrial Reference System." Information discussions at the First MERIT Workshop in 1981 were followed eventually by the merging of the two groups and the production of a Joint Summary Report (Wilkins and Mueller, 1986). This report describes briefly the development of the programs of observation and analysis and gives recommendations for new terrestrial and celestial reference systems and for the setting up of a new International Earth Rotation Service (IERS); this report also includes references to earlier reports that describe the techniques used, the organizational arrangements and the programs of the activities, and that give the principal results and references to relevant papers.

The MERIT and COTES programs have been very successful in stimulating the use and development of new techniques of observations using laser ranging and radio interferometry; they also led to improvements in the results from optical astrometry and the Doppler (radio) tracking of satellites, which were in regular use before 1978. Coordinators were appointed for each technique and for certain associated activities, such as the operation of a Coordinating Center for the combination and dissemination of results, the preparation of MERIT Standards, and the collocation of equipment of different techniques.

The quantities measured by each of the techniques that were used in the programs are as follows:

Doppler tracking of satellites: The Doppler shifts (range-rates) in the radio transmissions from Transit navigation satellites.

Satellite laser ranging: The time for pulses of laser light to travel to and from geodetic satellites carrying retroreflectors.

Lunar laser ranging: Time of flight for pulses of laser light to travel to and from retroreflectors on the surface of the moon.
Optical astrometry: Directions to stars measured with respect to local reference frames.

Connected-element radio interferometry, and
Very long baseline radio interferometry: Differences between the travel times of the radio emission from quasars to two or more radio telescopes.

Organizational arrangements for the regular transmission and processing of data already existed for optical astrometry and Doppler tracking, but for the other techniques it was necessary to set up both operational centers and analysis centers. The operational centers coordinated the observations, collected the observational data, computed earth rotation parameters on a rapid-service basis from "quick-look data," and distributed the observational data (perhaps after some processing) to the analysis centers, which determined both earth rotation parameters and station coordinates from all the available data.

There were several designated periods when all stations were requested to make observations and send them as quickly as possible to the operational centers. The first was the MERIT Short Campaign from 1 August to 31 October 1980. This was primarily a test of the technical and organizational arrangements, but it also produced much valuable data and showed clearly the potential of the new techniques. The MERIT Main Campaign covered the 14-month period from 1 September 1983 to 31 October 1984 and included the first COTES Intensive Campaign, which ran from 1 April until 30 June 1984. The data were analyzed independently at two or more analysis centers for each technique, and many excellent series of earth rotation parameters and sets of station coordinates were obtained. These data are still being studied to determine, for example, the systematic differences between the reference systems of the various techniques. The results have established beyond doubt the very close correlation between the short-period variations in the length of day and in the angular momentum of the atmosphere. The pole of rotation has been shown to move much more smoothly than had earlier been thought, but there is still controversy about the sources of excitation of the 14-month term in the motion.

5.2 The International Earth Rotation Service

By the end of the MERIT Main Campaign it had become clear that laser ranging and radio interferometry were able to provide more precise estimates of polar motion, universal time and length of day than could optical astrometry and the Doppler tracking of satellites, which were the prime contributors to the international services in 1978. This conclusion has since been substantiated by the more detailed analyses of the data that have been reported at the MERIT Workshop and Conference held at Columbus, Ohio, on 29 July - 2 August 1985 (Mueller, ed., 1985). The accuracy of the regular determination of the coordinates of the poles by SLR and VLBI is about 5 cm, compared with 30 cm by optical astrometry and Doppler tracking, while for UT and excess length of day the accuracy is about 0.2 ms and 0.06 ms, compared with 1 ms and 0.2 ms.

It must be realized, however that other factors besides precision had to be taken into account before recommendations about the future international services could be formulated. Perhaps the most important factor was whether it is reasonable to expect that the organizations concerned are likely to continue to make and process observations at an appropriate level and to make the results available to the international community without restriction. The MERIT Main Campaign was a period of special activity, and it cannot be assumed that any technique would provide results of the same high quality (as judged by the combination of precision, accuracy, frequency, reliability and promptness) on a long-term basis.

The International Latitude Service was initially set up a set of five dedicated stations, but it was eventually replaced by the International Polar Motion Service which relied on receiving data from a much larger number of instruments which provided local services and data for other scientific purposes as their prime justification. It is to be expected that any new International Earth
Rotation Service will also have to depend largely on the use of observations and results that are obtained for other national and international programs.

In particular it must be recognized that an important application of the Service will be the establishment and maintenance of the new conventional terrestrial reference system. The permanent stations used for monitoring earth rotation will comprise a primary geodetic network of large scale and high precision that will be densified, partly by the use of mobile systems using the same techniques, but mainly by the use of other geodetic techniques, such as the use in radio interferometric mode of signals for navigation satellites.

The choice of the techniques to be used in the new service depends on the subjective evaluation of many factors and not merely on a comparison of the potential quality of the determination of each rotation parameters. Although it is conceivable that a single VLBI network could provide an adequate international earth rotation service, the general conclusions of the discussions in the MERIT and COTES working groups is that the new service should be based on both laser ranging and VLBI and should also utilize any other appropriate data that are made available to it.

The three recommendations given in Appendix 1 were adopted at a joint meeting of the MERIT Steering Committee and the COTES Working Group that was held at Columbus, Ohio, on 3 August 1985. Earlier drafts had been subject to critical review at the MERIT Workshop on 30 July and by interested participants in the Conference on Earth Rotation and Reference Systems held 31 July to 2 August. The joint meeting also adopted a draft resolution for consideration by a Joint Meeting of the IAU Commissions 19 and 31 on 22 November 1985 during the XIXth General Assembly of the IAU at New Delhi. Amended versions of this resolution were adopted by the Joint Meeting and subsequently by the Union on 28 November 1985. A further recommendation concerning the assignment of responsibility within the IAU for matters relating to the celestial and terrestrial reference systems was adopted by the MERIT/COTES meeting on 3 August and served to stimulate a discussion within the IAU, but no decision was announced.

The final version of the IAU resolution on the MERIT/COTES program and recommendations is given in Appendix 2. In effect the resolution endorsed this report and the principal recommendations on concepts, organization and interim arrangements. As a consequence the MERIT and COTES Working Groups were replaced by a Provisional Directing Board for the new International Earth Rotation Service which was to come into operation on 1 January 1988. The IAU resolution was endorsed by the Executive Committee of the International Association of Geodesy in March, 1986 (Mueller and Wilkins, 1986). The recommendations of the Provisional Directing Board were considered and adopted by the IUGG during its XIXth General Assembly in Vancouver, B.C., in August, 1987 (Appendix 3).

With this last action, after ten years of preparation the new International Earth Rotation Service became a reality.

Organization of the Service

For each technique of observation (VLBI, SLR and LLR), prospective host organizations were invited to submit proposals for participation in one or more of the following ways:

- as a coordinating center,
- as an observing station or a network of stations,
- as a data collection (and distribution) center for quick-look and/or full-rate observational data. Such a center could, if appropriate, also process the data to form normal point data for use in analyses, or the task could be carried out by separate centers,
- as a quick-look operational center that would provide rapid service results,
as a full-rate analysis center that would determine ERP's, station coordinates and other parameters to a regular schedule.

Several of these activities might be carried out by one center, and the actual organization would differ according to the number of observing stations and networks and to the nature of the processing required. There will be no need for associate analysis centers in the formal structure, although it is expected that many groups will wish to analyze data provided by the Service. Offers of the deployment of mobile systems for use in improving the terrestrial reference system would be welcomed.

The principal tasks of the Central Bureau are specified in Recommendation B in Appendix 1, and some of them would be carried out by sub-bureaus. There is a need also for separate centers for relevant data from other fields, such as data on atmospheric angular momentum (AAM) and appropriate geodetic data (e.g., GPS results). The former might prove to be useful in predicting the variations in the rate of rotation of the earth, while the latter would be useful in the establishment and maintenance of the terrestrial reference system.

The initial organization of the IERS as of 1 January 1988 is shown in Fig. 4. The concepts and methods underlying the work of the Central Bureau are included in Appendix 4.

Kovalevsky and Mueller in their 1980 review of the Warsaw Conference listed a number of actions required to assure that the reference system issue be resolved "early and that the uniformity is assured by means of international agreements." There were the following:

Re CTS:
1. Selection of observatories whose catalogue will define the CTS.
2. Initiation of measurements at these observatories.
3. Recommendation on the observational and computational maintenance of the CTS (e.g., permanent versus temporary and repeated station occupations, constraints to be used).
4. Decision on how far and which way the earth deformation should be modeled initially.
5. Plans and recommendations for the establishment of new international service(s) to provide users with the appropriate information regarding the use of the CTS frame.

Re CIS:
6. Selection of extragalactic radio sources whose catalogue will define the CIS.
7. Improvement of the positions of these sources to a few milliseconds (arc).
8. Final decision on the IAU series of nutation and to assure that it describes the motion of the Celestial Ephemeris Pole.
9. Early completion of the FK5 and revision of astronomical equations due to the changed equinox (e.g., transformation between sidereal and Universal times).
10. Extension of the stellar catalogues (FK5 and later Hipparcos) to higher magnitudes.
11. Connection of the FK5, and later Hipparcos, reference frames to the CIS frame.

Eight years later it is gratifying to note that significant progress has been made on all items. In fact, with the exception of items 10 and 11, all have been accomplished to the extent possible.
INTER-RELATION

International Earth Rotation Service (IERS)
and
International Union of Geodesy and Geophysics (IUGG), International Astronomical Union (IAU),
Fédération des Services d'Analyse de Données d'Astronomique et de Geophysique (FAGS)

International Union of Geodesy and Geophysics

International Astronomical Union

Union Radio-Scientifique Internationale

UNESCO

INTERNATIONAL EARTH ROTATION SERVICE (IERS)

SUBVENTION

CONTRIBUTION

FAGS

IUGG REPRESENTATIVE
K. Yokoyama (ILGM, JAPAN)

IAU REPRESENTATIVE
G. A. Hilkens (Greenwich Observatory, UK)

LLR CC REPRESENT.

VLBI CC REPRESENT.

SLR CC REPRESENT.

DIRECTING BOARD
6 Persons

IERS Central Bureau

VLBI Coordinating Center

SLR Coordinating Center

LLR Coordinating Center

M. Feissel
(Paris Obs.)

F. E. Carter
(NGS)

B. Schutz
(Center Space Res. UTEx)

C. Veillet
(CERGA, France)

Sub-Bureaus

Analysis/Network Centers

Analysis/Operat. Centers

Analysis/Operat. Centers

Stations

Stations

Stations

VLBI: Very Long Baseline Interferometry
SLR: Satellite Laser Ranging
LLR: Lunar Laser Ranging

Fig. 4

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APPENDIX 1: PRINCIPAL RECOMMENDATIONS OF THE MERIT AND COTES WORKING GROUPS

A. Technical Recommendation on Concepts

The IAU/IUGG MERIT and COTES Joint Working Groups recommend that the following concepts be incorporated in the operation of an international earth orientation service:

(1) The Conventional Terrestrial Reference System (CTRS) be defined by a set of designated reference stations, theories and constants chosen so that there is no net rotation or translation between the reference frame and the surface of the earth. The frame is to be realized by a set of positions and motions for the designated reference stations.

(2) The Conventional Celestial Reference System (CCRS) be defined by a set of designated extragalactic radio sources, theories and constants chosen so that there is no net rotation between the reference frame and the set of radio sources. The frame is to be defined by the positions and motions of the designated radio sources. The origin of the frame is to be the barycenter of the solar system.

(3) This international service should provide the information necessary to define the Conventional Terrestrial Reference System and the Conventional Celestial Reference System and relate them as well as their frames to each other and to other reference systems used in the determination of the earth rotation parameters. The information should include, but not be limited to, pole positions, universal time, precession, nutation, dynamical equinox, positions of the designated reference stations and radio sources, and crustal deformation parameters.

B. Recommendation for the Organization of a New International Earth Rotation Service

The IAU/IUGG MERIT and COTES Joint Working Groups recommend that IAU and IUGG establish a new international service within IAGS for monitoring the rotation of the earth and for the maintenance of the Conventional Terrestrial Reference System to replace both the International Polar Motion Service (IPMS) and the Bureau International de l'Heure (BIH) as from 1 January 1988.

The new service will be known as the International Earth Rotation Service (IERS) and will consist of a Directing Board, a Central Bureau, coordinating centers and observatories. The Central Bureau, the centers and the observatories will be hosted by national organizations.

The Directing Board will exercise organizational, scientific and technical control over the activities and functions of the Service including such modifications to the organizational structure and participation in the Service as are appropriate to maintain an efficient and reliable service while taking full advantage of advances in technology and theory. The voting membership of the Directing Board will consist of one representative each of the IAU, the IUGG, the Central Bureau, and each of the coordinating centers. Additional nonvoting members may be appointed to advise the Board on complex technical and scientific issues.

The Central Bureau will combine the various types of data collected by the Service to derive and disseminate to the user community the earth rotation parameters in appropriate forms, such as predictions, quick-look and refined solutions, and other information relating to the rotation of the earth and the associated reference systems. The Central Bureau will conduct research and analysis to develop improved methods of processing and interpreting the data submitted. The Central Bureau may include sub-bureaus that carry out some of the specific tasks of the Central Bureau.

Coordinating centers will be designated for each of the primary techniques of observation to be utilized by the Service as well as for other major activities which the Directing Board may deem appropriate. Initially, there will be three centers for (1) very long baseline interferometry (VLBI), (2) satellite laser ranging (SLR), and (3) lunar laser ranging (LLR). Additional coordinating centers may be designated for the improvement of the determination of the earth rotation parameters and the maintenance of the conventional reference system by other techniques and to ensure that relevant data on the atmosphere, oceans and seismic
events are available.

The coordinating centers will be on the same level as the Central Bureau in the organizational structure of the Service and will be responsible for developing and organizing the activities by each technique to meet the objectives of the Service. Associated with the coordinating centers there may be network centers for subsets of observatories that may, for reasons of geometry or system compatibility, work more efficiently as autonomous units. There may also be associated analysis centers to process the observational data regularly or for special applications and studies. These centers may submit their results directly to the Central Bureau.

National Committees for the International Unions for Astronomy and for Geodesy and Geophysics will be invited to propose before 1 January 1987 national organizations and observatories that will be willing to host the Central Bureau or one of the centers and/or to provide observational data for use by the Service.

It is essential that the new service have redundancy throughout the organizational structure to insure the uninterrupted timely production of consistent, accurate, properly documented earth orientation and reference frame parameters, even in the event that one of the host national organizations should terminate its participation. A widespread distribution of observatories that regularly make high precision observations by one, or preferably more, modern space techniques by fixed and/or mobile equipment will be needed for this purpose, and national organizations are urged to provide appropriate resources.

APPENDIX 2: RESOLUTION OF INTERNATIONAL ASTRONOMICAL UNION (1985)

The following resolution was adopted at the XIXth General Assembly of the International Astronomical Union at New Delhi on 28 November 1985.

The International Astronomical Union recognizing the highly significant improvement in the determination of the orientation of the earth in space as a consequence of the MERIT/COTES program of observation and analysis, and recognizing the importance for scientific research and operational purposes of regular earth orientation monitoring and of the establishment and maintenance of a new Conventional Terrestrial Reference Frame,

thanks all the organizations and individuals who have contributed to the development and implementation of the MERIT and COTES programs and to the operations of the International Polar Motion Service and the Bureau International de l'Heure,

endorses the final report and recommendations of the MERIT and COTES Joint Working Groups;

decides

(1) to establish in consultation with IUGG a new International Earth Rotation Service within the Federation of Astronomical and Geophysical Services (FAGS) for monitoring earth orientation and for the maintenance of the Conventional Terrestrial Reference Frame; the new Service is to replace both the IPMS and the BIH as from 1 January 1988,

(2) to extend the MERIT/COTES program of observation, analysis, intercomparison and distribution of results until the new service is in operation,

(3) to recommend that an optical astrometric network be maintained for the rapid determination of UT1 for so long as this is recognized to be useful,

(4) to set up a Provisional Directing Board to submit recommendations on the terms of reference, structure and composition of the new service, and to serve as the Steering Committee for the extended MERIT/COTES program,

invites National Committees for the International Unions for Astronomy and for Geodesy and Geophysics to submit proposals for the hosting of individual components of the new service by national organizations and observatories, and

urges the participants in Project MERIT to continue to determine high precision data on earth rotation and reference systems and to make the results available to the BIH until the new service is in operation.
RESOLUTIONS OF THE UNION

RESOLUTION 1

The International Union of Geodesy and Geophysics

Noting that the improved determination of the Earth's orientation parameters resulting from the MERIT and COTES programmes of observation and analysis is highly significant,

considering the importance for scientific research and operational purposes of regularly monitoring the Earth's orientation and of establishing and maintaining a new conventional terrestrial frame of reference,

approving the replacement of the International Polar Motion Service (IPMS) and of the Bureau International de l'Heure (BIH) by the International Earth Rotation Service (IERS) which will be responsible both for earth rotation and for the associated conventional frames of reference, and

recognizing that organisations in many countries have indicated their willingness to participate in such a new service,

endorses the recommendations of its Provisional Directing Board on the terms of reference, structure and composition of the new service,

decides to establish, in cooperation with the International Astronomical Union, the International Earth Rotation Service within the Federation of Astronomical and Geophysical Data Analysis Services (FAGS) as from 1 January 1988, and

thanks all organisations and individuals who have helped to develop and implement the MERIT and COTES programmes, all who have operated IPMS and BIH in the past and all who have indicated their willingness to participate in the new Service.
CONCEPTS AND METHODS OF THE CENTRAL BUREAU OF THE INTERNATIONAL EARTH ROTATION SERVICE

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FOREWORD

The International Earth Rotation Service (IERS) was set up by the International Union of Geodesy and Geophysics and the International Astronomical Union. It started operation on 1988 January 1. Its responsibilities and activities are described in the Geodesist's Handbook (1988). The Central Bureau of IERS is run by a scientific team established in cooperation by Observatoire de Paris, Institut Géographique National and Bureau des Longitudes. This team was selected in 1987 on the basis of the present document which describes in some detail the concepts and methods for establishing and maintaining celestial and terrestrial reference frames for Earth orientation monitoring (polar motion, universal time, precession/ nutation angles). The work of the Central Bureau is based on these concepts and methods, not withstanding future evolution made possible by the improvements in observations and theories.
INTRCDUCriCN

Observatoire de Paris. Institut Géographique National (IGN) and Bureau des Longitudes (BDL) propose to set up and maintain a group to act as the Central Bureau of the International Earth Rotation Service (IERS). This group is based on the present Earth rotation section of Bureau International de l'Heure (BIH), enlarged to the participation of experts in terrestrial systems (IGN) and celestial systems (BDL).

The operation and working procedures of the proposed Central Bureau are defined in agreement with the MERIT/COTES Recommendations. The Central Bureau will work in close cooperation with the Coordinating Centres and the Directing Board of IERS. Its operation implies also direct relationship with the analysis centres.

It is proposed to gather the results obtained by the analysis centres of IERS pertaining to the orientation of the Earth in space (celestial and terrestrial frames, time series of Earth orientation parameters), to analyse and combine them in order to derive the IERS results on Earth rotation and the reference systems, and to distribute them to the world community.

In addition to the standard services, attention will be given to the provision of special services, in order to stimulate the uses of the IERS works. Potential uses are listed hereafter.

Conventional Celestial Reference System: Celestial reference frame for interplanetary navigation, orientation of terrestrial geodetic networks, connexion of celestial reference frames, dynamical study of the solar system, kinematic and dynamical studies of the galaxy, extragalactic kinematics.

Conventional Terrestrial Reference System: Connexion of geodetic datums, unification of station coordinates for various networks (e.g. orbitographic tracking networks, precise positions for time comparisons by satellite link, geodetic control networks), plate motions, local deformations, absolute monitoring of the global mean sea level and applications (e.g. CO, CO2 contents of the atmosphere, eustasy).

Earth orientation time series: orientation of the Earth in space for real time applications in space geodesy, space navigation, astrophysics, implementation of the UTC time scale, monitoring of the global angular momentum of the atmosphere and application to climatic studies, study of effects due to the Earth interior.

Complementary data necessary for the realisation of the tasks of the Central Bureau will be collected and analysed. Specific actions will be taken for two types of data.

a- relative geodetic coordinates (local surveys, GPS, etc)
In order to connect the various reference frames, it is necessary to collect most extensive and accurate colocation data. These data are basically differences of tridimensional cartesian coordinates between tracking instruments or local reference marks. They are derived either by high accuracy local tridimensional surveys performed by terrestrial techniques or by satellite radio positioning in differential mode, typically obtained using the Global Positioning System (GPS).

Institut Geographique National will take the specific responsibility of these data, ensuring:

- data collection among foreign agencies,
- coordination of terrestrial or GPS surveys which would improve the system,
- upon requirement, realization of such works, for field survey and/or computation both for conventional and GPS.

b- atmospheric angular momentum (AAM)

AAM will be used as a backup for rapid solution and prediction of the ERP. Scientific interaction with the group that will be selected to handle the AAM in the IERS will be maintained. The level of interaction will be adapted to the status which will be given to this group in the IERS.

PROPOSED PROCEDURES

1. ALGORITHMS

1.1. Concepts

It is useful to recall a set of concepts through a now widely used terminology:

a) an ideal reference system is an euclidian orthogonal frame in which positions of points or components of vectors can be expressed. One can in particular consider its origin, scale and orientation;

b) in a selected physical model which connects measurements to estimable parameters, the underlying ideal reference systems are unambiguously expressed in such a way that coordinates in these systems are actually estimable (datum definition). Such an underlying system is therefore called conventional reference system. It is described by the adopted conventions together with all relevant constants and models;
c) the previous realisation of a system being rather implicit, it is necessary to select a set of points (or vectors) for which one determines and monitors in time their coordinates, producing by this way a materialisation called conventional reference frame.

According to the MERIT/COTES Recommendations, the IERS should maintain a Conventional Terrestrial System on the basis of designated reference sites and a Conventional Celestial System on the basis of designated extragalactic radio sources, and provide the corresponding time series for the orientation of the Earth.

Each of the observation methods on which the operation of the IERS is based, VLBI, LLR and SLR realises to some extent the above definitions, with limitations due to the nature of the method itself or to technical constraints. Each technique provides an essential part of the ensemble. e.g., VLBI observes directly the extragalactic radio sources, LLR relates a geocentric terrestrial system to a dynamical celestial system, SLR realises the densest geocentric terrestrial system.

It is assumed that in the operation of the IERS as described in the Call for proposals, each analysis centre specialised in one of the different techniques elaborates a consistent set of a celestial frame, a terrestrial frame and a series of ERP. This proposal aims at realising at the Central Bureau of the IERS the intercomparison and combination of these sets of systems and time series in order to derive the Conventional Celestial and Terrestrial Systems, and a series of Earth Rotation parameters consistent with them. The proposed procedures are outlined hereafter. They are based on studies on the concepts and realisation of reference frames published in the recent years, in particular by Moritz (1979), Kovalevsky and Mueller (1981), Aoki et al. (1982), Williams et al. (1983), Zhu and Mueller (1983), Quinot (1984), Boucher and Feissel (1984), Capitaine et al. (1986).

1.2. Definition and maintainance of the set of conventional references

We define the IERS System as the consistent ensemble of a Celestial System, a Terrestrial System, and a series of Earth Orientation Parameters (Earth rotation and orientation in space). For maintaining this System, it is necessary to keep the stability of the components as well as the internal consistency of the ensemble.

The main difficulties expected for insuring the stability of the components are listed hereafter. The detailed proposed procedures are given in sections 2 to 4.

Celestial System:
- initial definition, in continuity with the BIH System,
- relationship between individual Celestial System having different physical realisations,
- intercomparison of individual radio sources catalogues,
- influence of the existence and evolution of the source structures,
- introduction and deletion of radio sources.
Terrestrial System:
- initial definition, in continuity with the BIH System,
- intercomparison of individual Terrestrial frames,
- influence of the general plate motion,
- monitoring of the local motions (deformations),
- introduction and deletion of colocation sites.

Earth orientation:
- initial definition consistent with those of the Celestial and Terrestrial Systems,
- influence of modelling errors in the individual series (precession-nutation or others),
- realisation of the best possible stability of the time series at all frequencies, to cover the complete spectrum of the irregularities of the Earth rotation,
- implementation of quick-look solutions based on incomplete data and of predictions.

The proposed method is based on the data flow which follows from the organisation of the Service.

At the Analysis Centres level as well as for the Central Bureau, various types of solution do exist, which are intermediate between two extreme types:
- scientific solutions, obtained over several years of measurements in a global adjustment of the celestial and terrestrial frames, the series of the ERP and the parameters of some models: precession-nutation, physical models necessary to the reduction of the observations, statistical models, etc,
- operational solutions of the ERP, based on a priori values for the reference frames and a part of the models.

The master solution for the Earth rotation and the reference systems will be implemented yearly on the basis of the scientific solutions received from the analysis centres. It is expected that complete information on the models, standards and procedures used at the analysis centres will be made available. Conversely, the Central Bureau will make available, the corresponding information on their data treatment. The basic parameters and constraints considered in the proposed treatment are as follows.

(1) the parameters of the conventional references:
- two coordinates for a set of extragalactic radio sources in the Conventional Celestial System,
- three coordinates for a set of sites in the Conventional Terrestrial System,
- the corresponding time series of the Earth Rotation Parameters for the whole period under consideration.

(2) the parameters of the individual references:
- rotations from the individual celestial frames to the Conventional Celestial System (when applicable),
- translation, rotation, scale factor from the individual terrestrial frames to the Conventional Terrestrial System,
- parameters of the correction model for the time series of Earth Rotation Parameters.
The constraints used to ensure the stability of the set of systems will include the consideration of

- no net rotation condition for the Celestial System,
- no net translation or rotation condition for the Terrestrial System,
- continuity of the combined series of Earth Rotation Parameters,
- the effect of radio source structures,
- the ties between the different individual celestial frames,
- local geodetic ties in the colocation sites.

The weighting of the data contributing to the global adjustment is based on the formal uncertainties provided with the data. Calibration of the formal uncertainties will be applied when necessary, pending on the statistical assessment of the data error spectrum.

The analysis and treatment of the operational solutions for Earth rotation provided weekly or monthly to the Central Bureau, is described in section 1.4.

1.3. Use of complementary data

In all activities of the Central Bureau, use can be made of data which are collected and managed by other services, within FAGS or other organisations, in order to strengthen the scientific solution.

Celestial System:

Physical information on the radio source structure.

Ties between celestial reference frames
(in connection with the Centre de Données Stellaires and space astrometry projects, e.g., Space Telescope, HIPPARCOS).

Terrestrial System:

Local ties (in connection with geodetic agencies, NASA Crustal Dynamics project, European Space Station Locations data base, or others).

Earth and Oceanic tides (in connection with the International Centre of Earth Tides).

Global deformation data (in connection with the NASA Crustal Dynamics project, or others).

Absolute gravimetry (in connection with the Bureau Gravimétrique International).

Earth Orientation:

Geophysical measurements related to the excitation of the Earth rotation (e.g., World meteorological centres).

Nutation Studies (in connection with the International Centre of Earth Tides).

Dynamical measurements of the Earth rotation (e.g., supraconducting gravimeter), or others (e.g., ring laser).
CONVENTIONAL CELESTIAL REFERENCE SYSTEMS AND FRAMES

The primary VLBI celestial reference frame is based on the extragalactic system as recommended in the MERIT Standards. In addition, there are other celestial reference frames used for a variety of applications. The FK4 and FK5 are conventional stellar reference frames widely used for optical astrometry, and the HIPPARCOS catalogue will eventually be available for milliarcsecond precision optical astrometry. The planetary and lunar ephemeris reference frames of JPL (DE) and MIT (PEP) are adjusted on a very large set of observations; they realise the primary dynamical system. Secondary planetary dynamical reference frames have been developed as, for example, the ephemerides of Bureau des Longitudes which are adjusted on the JPL solution. Finally, trajectories of Earth orbiting satellites are computed in another dynamical reference system based mainly on the gravitational potential of the Earth. These four individual celestial reference systems (extragalactic, stellar, planetary-dynamical and Earth-dynamical) are all unrelated in the sense that they have no object in common. They also encompass largely different spatial scales in the universe, cosmological, galactic and solar system scales; their relations might be complicated by the effects of the gravitational field dominating at each of these scales. Various techniques (VLBI, optical astrometry, LLR, satellite laser ranging and Doppler tracking) are used to monitor the orientation of the Earth. Data analyses are carried out in these unrelated individual celestial reference systems. Hence, comparisons of the Earth orientation parameters yielded by these techniques or combination of their results into a global solution require a clear understanding of the relation between the individual celestial systems and frames in order to eliminate any discrepancies arising from differences in their relative orientations (see Williams et al. 1983, Arias et al. 1986).

An important distinction between these individual celestial reference systems can be emphasised here. The FK4 and FK5 are constructed with the classical astrometric techniques and a change in the ill-determined luni-solar precession constant is compensated by a change in the proper motions of the stars, so that these reference frames might not be inertial. In fact, the introduction of the time varying equinox correction by Fricke (1981) to the FK4 right ascensions in order that the equinox of the FK5 catalogue coincides with the dynamical equinox is the symptom that the FK4 is not inertial. Instead, VLBI observations and radio ranging to planets or lunar laser ranging (LLR) are directly sensitive to the inertial system so that drifts which appear in UT1 are only limited by the noise in the data (0.001"/yr for LLR, Williams et al. 1983) rather than by systematic errors. The reason is that VLBI reference frames are constructed with positions of extragalactic radio sources which have no measurable proper motions and that the planetary-lunar ephemeris reference frame is a materialisation of the underlying inertial reference system of the laws of dynamics.

In the operation of IERS, each of the analysis centres which process VLBI data for the determination of the Earth rotation realises an individual celestial reference frame. They use conventions which insure alignment of their axes with the present celestial system of the BIH within a few 0.001". The corresponding frames are realised by sets of radio source coordinates at some epoch. The available VLBI catalogues include 25 to 150 sources, of which 20 to 50 sources are common to the different catalogues.
The Central Bureau will realise a Conventional Celestial System by determining the coordinates of all sources of the individual celestial frames in a common system, consistently with the realisation of the Conventional Terrestrial System and the series of ERP. This determination will take into account the epoch of the individual frames, the source structures, and other information pertaining to the accuracy of the individual coordinates.

The link to the celestial frames of different physical natures (dynamical, stellar) will be studied on the basis of the Earth rotation determinations when applicable, or by use of complementary data, such as VLBI positions of objects in the solar system or in the Galaxy.

These tasks require the management of data bases which will include

- VLBI astrometric catalogues,
- VLBI surveys and documentation on extragalactic radio sources,
- VLBI positions of objects in the solar system and in the Galaxy.

CONVENTIONAL TERRESTRIAL REFERENCE SYSTEM

In the case of a terrestrial system, the concepts defined in section 1.1. are selected in this way:

a) The ideal system is geocentric, scaled to the SI unit of length and oriented in such a way that it follows the diurnal motion of the Earth. More specifically it is a Tisserand frame for the deformable crust.

b) Each analysis centre which processes data relevant to reference systems and Earth rotation defines its own individual terrestrial reference system. They use conventions which follow point (a) and give a common orientation using alignment techniques on the presently available BIH system.

The Central Bureau will have to implement a model to combine results of the various individual analysis centers and therefore to produce a common Conventional Terrestrial System, with its relations with the individual systems.

c) Each analysis centre also produces an individual terrestrial reference frame which is for all modern space techniques a set of selected instruments with their cartesian (or equivalent) coordinates at some epoch.

Similarly, the Central Bureau will produce a Conventional Terrestrial Reference Frame by determining the coordinates of the various instruments in a common system.

This last activity is a major task which will take benefit of two extra sources of information:

- direct connections between instruments can be achieved by terrestrial three-dimensional geodetic surveys for close colocations (below a few tens kilometers) or by space derived baselines such as mobile VLBI or GPS surveys, for regional colocations (a few hundreds kilometers).
- information about crustal deformations, coming from repeated or permanent terrestrial or space geodetic surveys.

The implementation of the combination model by the Central Bureau requires the management of several data bases:

- description of the various sites with their instruments, control marks, epochs of settlement and removal, etc,
- individual terrestrial frames, with full information,
- local surveys and space baselines (VLBI, GPS...), with full information, in particular epoch, covariance matrix, etc,
- global and local deformation models.

EARTH ORIENTATION

The concept of Earth orientation refers to the Earth Rotation Parameters (ERP: coordinates of the pole with respect to an Earth-fixed system, and universal time) and to the orientation of the rotation axis in space, as governed by the precession-nutation torque. The orientation in space of the reference polar axis is currently computed by the IAU 1980 Theory of Nutation (Seidelman 1982). Errors in the model used result in fictitious diurnal polar motion and reflect directly themselves in the determinations of universal time (Zhu and Mueller 1983).

The Central Bureau will derive several forms of series of ERP, referred to a given model of precession/nutation, and to the Conventional Celestial and Terrestrial Systems.

Different forms of series of ERP will be derived.

Yearly: scientific solutions, obtained in the process of maintaining the Conventional Celestial and Terrestrial Systems. Such solutions will be recomputed whenever new or revised results will be received from the analysis centres.

Monthly: standard solution based on predicted correction models (refreshed yearly and when necessary), prediction.

Weekly: advanced solution, prediction.

Scientific solutions (yearly)

In this process, the series of ERP are first transformed into series of normal values at 0.05 year interval, with associated formal uncertainties derived from the formal uncertainties of the original data.

The series selected must be related homogeneously to an individual terrestrial frame and, in the case of VLBI, to an individual celestial frame made available to the Central Bureau. They should also prove themselves to be enough model error free so that their link to the reference systems is kept with time.
The global adjustment of the set of systems provides as by-products the systematic differences of the individual series of ERP with respect to the combined series. These differences are modelled under a form which depends on the series: bias, drift, periodic annual corrections could be used. The modelled correction is then applied to the original series and normal values of the ERP are adjusted on independent time intervals of 1, 3 or 5 days. The observations can include different types of observables related to the ERP, e.g., baseline components, local UTO or variation of latitude, or directly $x$, $y$ and UT1; the correlation coefficients of the data combined are taken into account. The original uncertainties of the data are scaled to give an unbiased estimate of the true uncertainty.

These series of normal values at 1, 3, or 5-day intervals, with associated uncertainties, are intended for scientific interpretations. Conventional operational uses would be best served by a posteriori smoothings, e.g. by the Vondrak algorithm (see Feissel and Lewandowski 1984), with smoothing characteristics adjusted to the error spectrum of the combined series.

**Operational solution (monthly)**

These solutions are essentially advanced forms of the scientific series of normal values at 1, 3, or 5-day intervals, based on the data available at the time of computation, and on correction models which are predicted on the basis of the adjustment described in 4.1. The procedures for splitting the time series of data, for the weighting, the least squares adjustment and the a posteriori smoothing are the same as described in 4.1. At this level, the degrees of smoothing are chosen in a conservative way, to avoid possible spurious variations due to bad points not yet detected or to any kind of anomaly that could stay unnoticed until more results are available.
REFERENCES


