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PARTICLE DISPERSION MODELS AND DRAG COEFFICIENTS FOR PARTICLES IN TURBULENT FLOWS*

by

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INTRODUCTION

The dispersion of particles in turbulence is fundamental to a variety of mass and energy transfer processes. The dispersion of particles in jets is important to the combustion process and the design of propulsion systems. The separation of particles by electrostatic precipitation is important in many applications from the power industry to clean-room technology. Understanding the basic phenomena underlying particle transport in turbulence and establishing viable models is important to the development of new technologies for advanced propulsion systems.

This paper reviews some the concepts underlying particle dispersion due to turbulence. The paper addresses the traditional approaches to particle dispersion in homogeneous, stationary turbulent fields and reviews recent work on particle dispersion in large scale turbulent structures. The paper also reviews the state of knowledge on particle drag coefficients in turbulent gas-particle flows.

MECHANISMS FOR PARTICLE DISPERSION

Basically two mechanisms have been used to physically model particle dispersion in turbulence. In homogeneous turbulent flows, the most common approach is to regard dispersion as a stochastic process. On the other hand, dispersion in large scale turbulent structures appears to be more influenced by the ordered motion. Both approaches are treated separately.

Particle Dispersion in Homogeneous, Stationary Turbulence

The traditional approach to treating particle dispersion is turbulence is to regard the process as a gradient diffusion (Fickian) process in which the diffusional velocities are proportional to the concentration gradient, the constant of proportionality being the diffusion coefficient.

$$V_F = -D\frac{\partial c}{\partial x_i} \qquad 1)$$

The earliest work that related the diffusion coefficient to properties in a homogeneous, stationary and isotropic flow was that of Taylor (1921) who provided the following relationship,

$$D = \overline{u'^2} T_L \qquad 2)$$

where $\overline{u'^2}$ is the mean square of the fluctuating velocity in the direction transverse to the main flow direction and T_L is the lagrangian integral time scale. Although Taylor's analysis was done for a fluid point, Synder and Lumley (1971) showed it was applicable to a particle provided the lagrangian time scale corresponded to the particle trajectory.

The earliest study of particle motion in a turbulent field was reported in a PhD thesis by Tchen (1947) who integrated the Basset, Boussinesq, Oseen equation for a particle in a homogeneous, stationary turbulent field. He assumed that the particle remained always in the same turbulent eddy. By so doing the long-time diffusion of the particle was equal to that of a fluid particle.

Many researchers after Tchen have strived to improve Tchen's model by relaxing the assumptions made by Tchen. Peskin (Soo, 1967) solved a nonlinear stochastic equation for the motion of a particle which did not deviate far from the initial coincident turbulent eddy. Unlike Tchen's analysis, Peskin assumed that only Stokes drag acted on the particle. He predicted that the diffusivity decreased uniformly with an inertial parameter which related the aerodynamic response time of the particle to the time scale for turbulence. The physical argument underlying this result was the larger the aerodynamic response time of the particle is compared to the eddy life time, the less a particle would respond to the unsteady turbulent field. Hinze (1972) used similar time-scale arguments for particle dispersion and concluded that if the particle density ratio is large, only those particles less than one tenth of the dissipation length scale will respond to the turbulent fluctuations.

A more general analysis of particle dispersion in homogeneous, isotropic stationary turbulence has been reported by Reeks (1977). He assumed a linear drag law and body force acted on the particle and obtained an expression for diffusion which depended on time, particle aerodynamic response time and the correlation function for the velocity field. He then utilized Phythian's model (1975) for the turbulent energy spectrum and predicted a particle diffusion coefficient. His results show that the diffusion coefficient for particles with no body force increases with increasing time and, at long times, approaches an asymptotic value. This long-time diffusion coefficient increases with increasing aerodynamic response time and can exceed that of a fluid particle. This result differed from Peskin's model. The reason underlying the trend predicted by Reeks is the fact that the diffusion is jointly dependent on the mean square of the particle velocity fluctuations and lagrangian integral time scale as shown by equation 2. Although the amplitude of the fluctuation velocity of the heavier particle is reduced, the lagrangian time scale is increased proportionately more giving rise to an increased diffusion coefficient.

Another factor controlling particle dispersion in homogeneous, stationary turbulence is the "crossing trajectory" effect first identified by Yudine (1959). If the mean velocity of the particles is different from that of the fluid, such as particles dropping at their terminal velocity through a turbulent field, the particles remain less time in a given eddy. The reduction in fluid-particle interaction time reduces the particle diffusion coefficient. Reeks also predicted a decrease in long-time diffusion coefficient by including a body force in the particle motion equation.

Thus there are two primary factors controlling particle dispersion in homogeneous turbulence, the inertial effect and the crossing trajectory effect. Experimentally it has been difficult to separate these effects since a heavy (not a fluid) particle will have both inertial and crossing trajectory effects. The most convincing experimental evidence that separates the two effects have been provided by Well and Stock (1983). They suspended glass beads by Coulomb forces in a horizontal flow of near-homogeneous, grid-generated turbulence. The crossing trajectory effect was controlled by adjusting the field strength. Their data show that the inertial effects on particle diffusion coefficient are small compared the crossing trajectory effect. The small increase in diffusion with increasing inertial effects predicted by Reeks was not discernible because of the scatter in the experimental data.

Future developments in the analysis of stochastic turbulent flows will address departures from homogeneity and isotropy. Reeks (1981) has initiated work in this direction.

Particle Dispersion in Large Scale, Turbulent Structures

Large scale turbulent structures are encountered in flows generated by a large velocity gradient such as free shear layers and jets. Under these circumstances, large scale turbulent structures are formed which grow and pair with time. These structures were first identified by Brown and Roshko (1974) in flow visualization studies of mixing layers. A typical photograph of a large scale structure is shown in figure 1 . These turbulent flow fields are inhomogeneous, non-stationary and anisotropic but represent important, practical problems in industrial applications such as combustion systems.

Particle dispersion in turbulent flow characterized by large scale structures is mechanistically different than that in homogeneous flows. The particle motion is controlled by the moving structures and not by the fine scale turbulence. Thus, the dispersion process cannot be regarded as gradient transport.

A conceptual model for particle dispersion in large scale structures is the entrapment of particles in the structure and the subsequent centrifuging of the particles beyond the structures. This concept was first suggested by Singamsetti (1966) who observed, experimentally, that particles in a submerged jet could disperse more quickly than a fluid particle. The same trends have been observed by Lilly (1973), Householder 1968), Laats and Frishman (1970) and Subramanian and Ganesh (1984) in the experimental study of particle and droplet laden free jets. Lilly attributed his results to an increase lagrangian time scale but Yuu *et al.* (1978) claim Lilly's results were a manifestation of his experimental set-up. Laats and Frishman noticed this trend only in the early portion of the jet formation and surmised that it was due to a Magnus effect. Goldschmidt *et al.* (1972), in reviewing Householder's data, mentioned the possibility of particle centrifuging by large scale structures but concluded that the mechanism was not viable because it did not explain the observed trends in centrifuging bubbles.

Yule (1981) gave some credence to the mechanism when he observed droplets in jet flows being centrifuged toward the outer flow.

Crowe *et al.* (1985) report an effort to quantify those conditions under which the large scale structures are responsible for dispersing heavy particles beyond fluid particles. They proposed a time scaling argument similar to that used by Hinze (1972) for homogeneous turbulence. The aerodynamic response time of a particle is the time required for a particle released from rest in a uniform flow to accelerate to 63% of the flow velocity. For Stokes flow it is

$$\tau_A = \frac{\rho d^2}{18\mu} \qquad 3)$$

where ρ is the fluid density, d is the particle diameter and μ is the dynamic viscosity of the fluid. It is a measure of the responsiveness of a particle to changes in flow velocity. The characteristic time of the flow is given by

Figure 1. Shadowgraph Visualization of Large Scale Turbulent Structures in Plane Mixing Layer (Brown and Roshko, 1974)

$$\tau_F = \frac{\delta}{\Delta U} \tag{4}$$

where δ is the mixing layer thickness and ΔU is the velocity difference across the layer. Thus the scaling factor is

$$St = \frac{\tau_A}{\tau_F} = \frac{\rho d^2 \Delta U}{18\mu\delta} \tag{5}$$

A schematic diagram showing the effect of Stokes number on particle dispersion is shown in figure 2. For small Stokes numbers, the particles are in near equilibrium with the conveying fluid and the particles will disperse as a fluid particle. At large Stokes numbers, the particles have insufficient time to respond to the structures and they disperse less than the fluid particle. However at intermediate Reynolds numbers, the particles are entrapped by the rotating structure and are centrifuged beyond the structures giving rise to a dispersion exceeding that of a fluid particle.

A preliminary numerical study (Crowe *et al.* 1985) using a simple vortex sheet model proposed by Stuart (1967) showed that particles could be centrifuged beyond the vortex at Stokes number between 0.1 and 1 as shown in figure 3. A subsequent study by Gore et al (1985) using pseudospectral direct simulation for modeling the vortex structures showed the same trend lending support to the model.

Recent experimental studies have shown the importance of large scale structures in the turbulent dispersion process. Kamalu *et al.* (1987) at Washington State University have reported on particle dispersion studies in horizontally oriented plane mixing layer. Both naturally evolving and subharmonically forced mixing layers were studied. The forced mixing layer was generated by a sound source, the effect of which was more ordered vortex structures. Particles, 40 microns in diameter, were released from a source upstream of the layer and photographed. A photograph of the particles in a subharmonically forced mixing layer is shown in figure 4. Shown in the same figure are streaklines generated by smoke with no particles in the flow. One notes the absence of particles in the vortex cores and the accumulation of particles near the edge of the vortex structures.

Laser Doppler measurements of particle velocities by Wen *et al.* (1987) lend more quantitative support on the role of large scale structures in particle dispersion. The measured lateral particle and fluid velocities taken in the same facility used by Kamalu *et al.* are shown in figure 5. The fluid velocities show the expected trend; negative on the top (high speed side) and positive on the bottom (low speed side) which indicates a motion of the fluid towards the mixing region. The particle velocities, on the other hand, are positive on the top and negative on the bottom indicating motion away from the mixing layer.

The importance of large scale structures in particle dispersion has also been verified in recent experimental studies by Kobayashi et al (1987) and Lazaro and Lasheras (1987).

The importance of large scale structures in turbulent dispersion of particles has been established. It represents a demarcation from particle dispersion in homogeneous turbulence because it is more deterministic than stochastic. Therefore it is not reasonable to model particle dispersion as a gradient transport process.

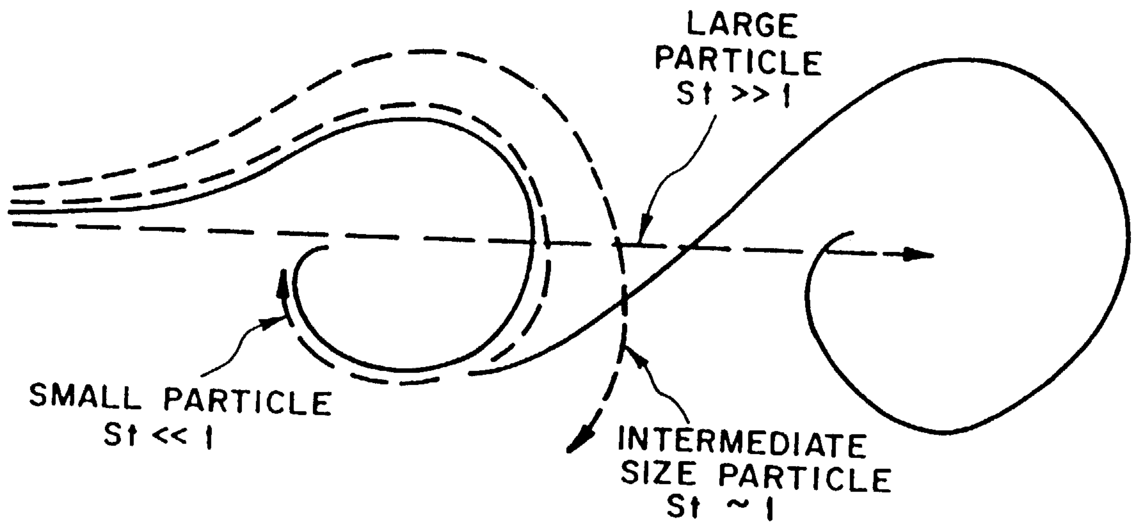

Figure 2. Schematic Diagram Illustrating the Effect of Large Scale Structures on Particle Dispersion

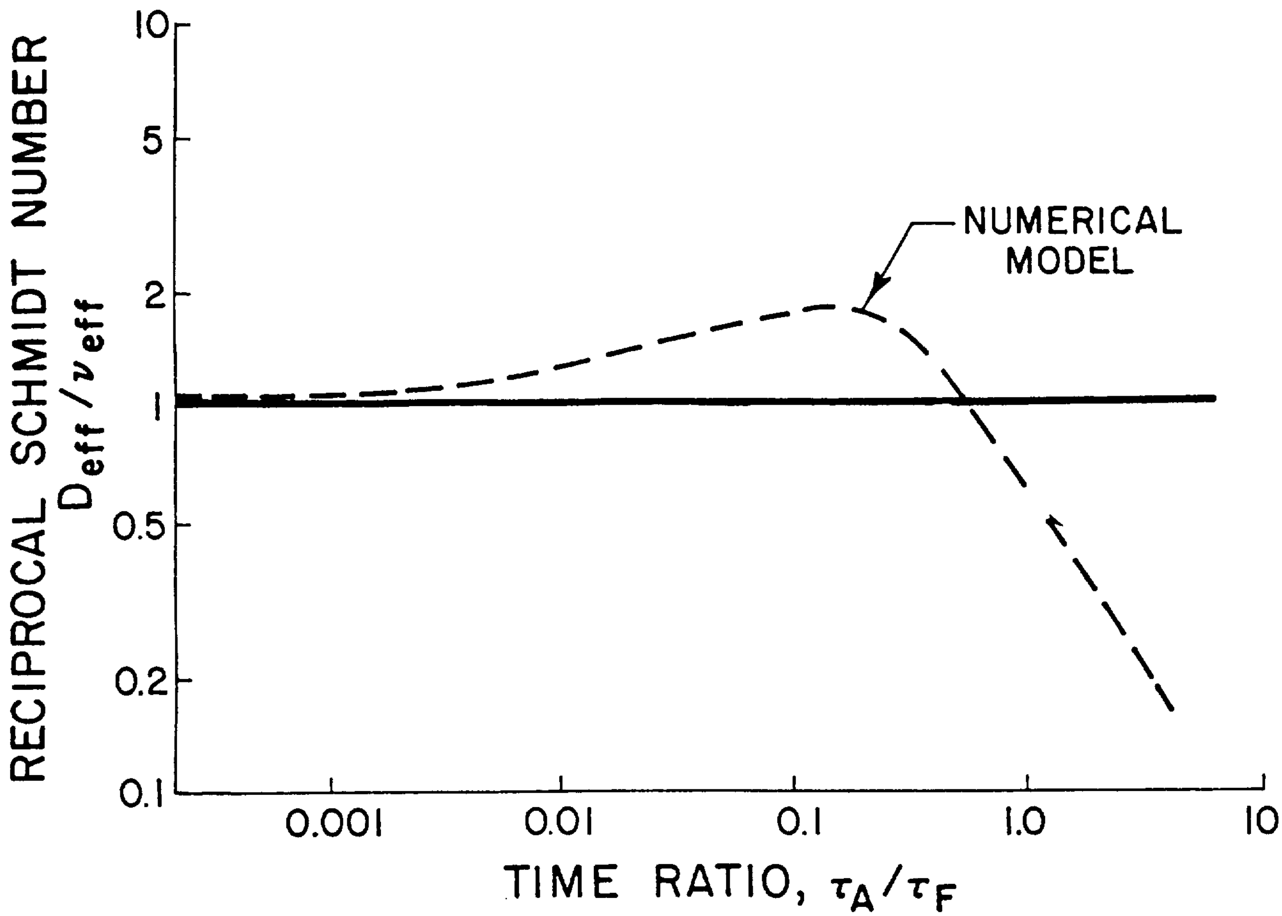

Figure 3. Predicted Particle/Fluid Dispersion Ratio as Function of Stokes Number

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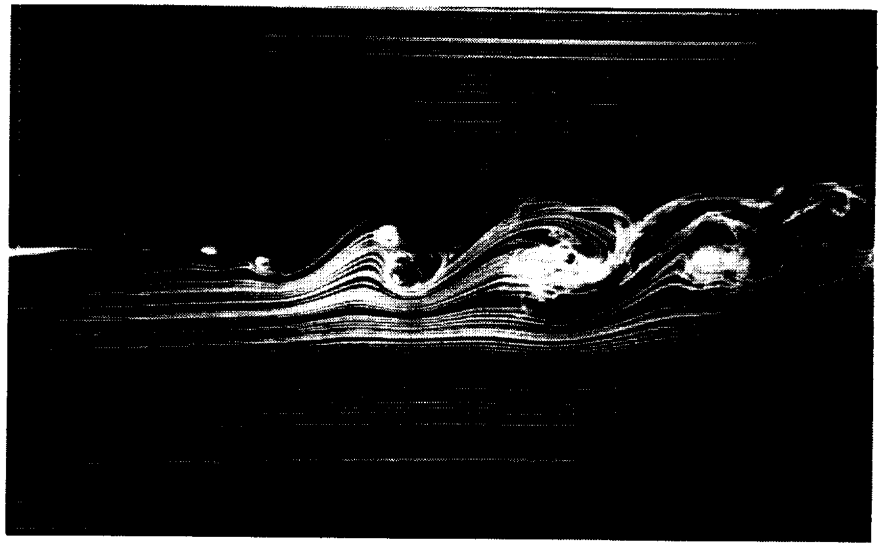
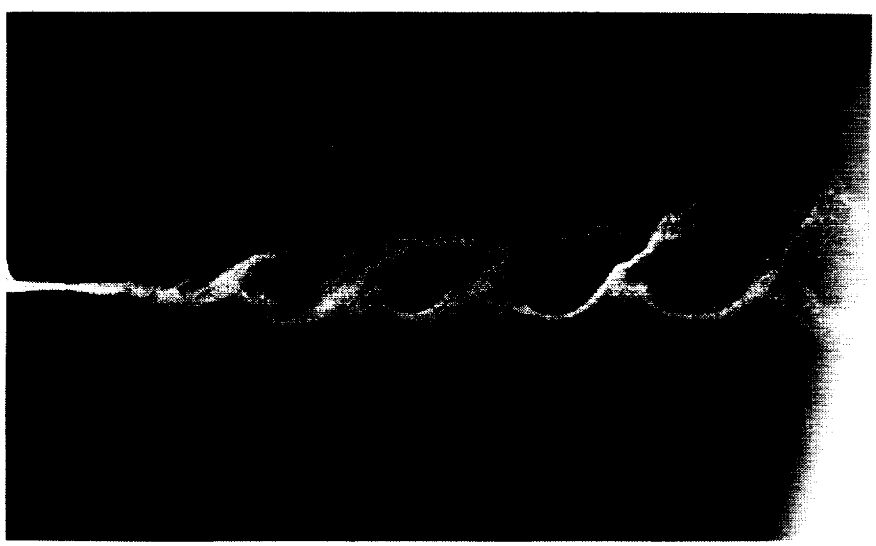


Streaklines in Fluid



Particle Field

Figure 4. Photographs of Fluid Streaklines and Particles in Large Scale Turbulent Structures (Kamalu *et al.*, 1987)

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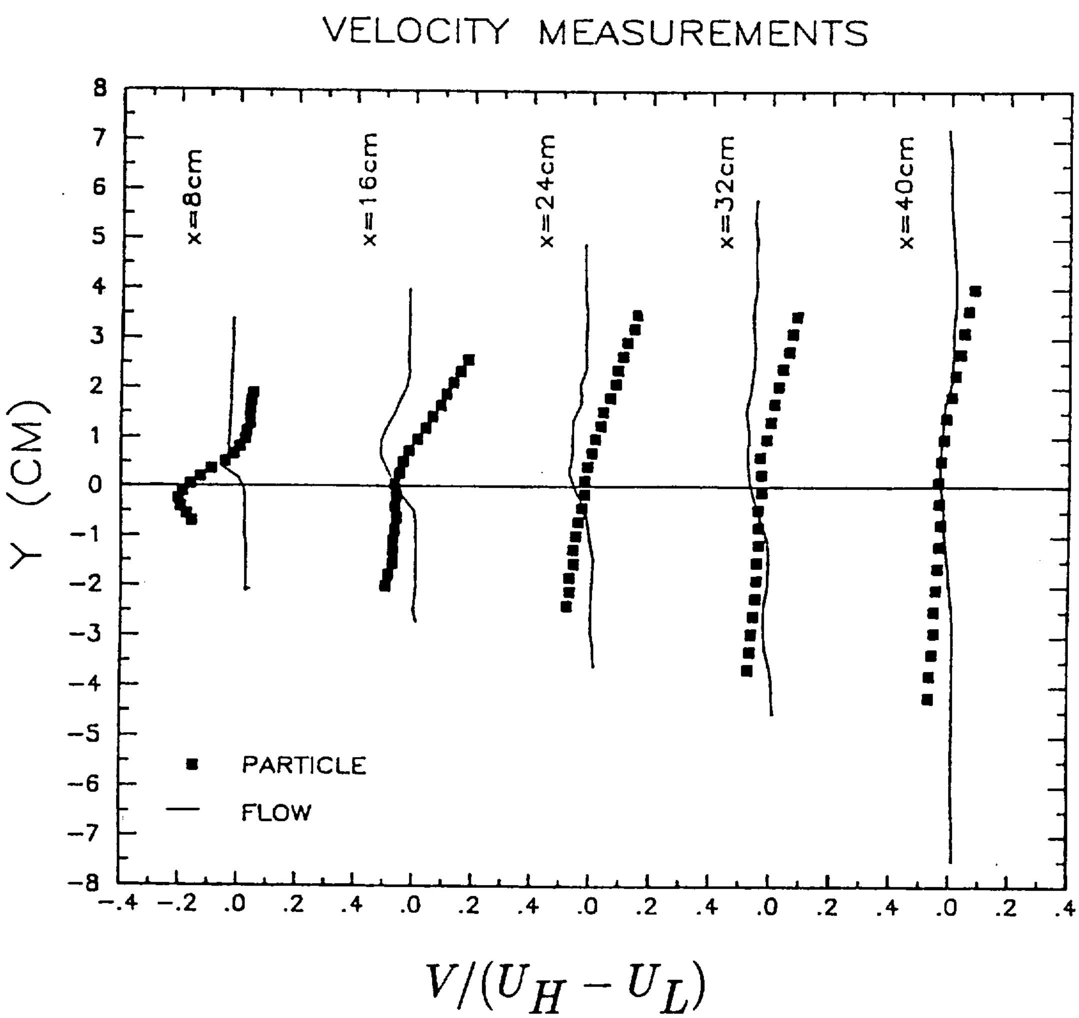

Figure 5. Measured Vertical Velocity Component of Fluid and Particles in Mixing Layer (Wen *et al.*, 1987)

NUMERICAL MODELS FOR PARTICLE DISPERSION

Numerical models for fluid-particle flow can be divided into two categories; two-fluid models and trajectory models. A review of these approaches has been provided by Crowe (1982). In the two fluid model, the particulate phase is regarded as another fluid. In the trajectory model, the particle field is established by integrating particle trajectories through the field.

Gradient Transport Models

It is natural in the two fluid model to treat particle dispersion as a gradient transport process and assign a diffusion coefficient to represent particle dispersion due to turbulence. Elghobashi *et al.* (1983) have considered in detail the two-fluid model with the two-equation model for turbulence and applied it to free jets. They recognize that the gradient transport assumption is valid only if the energy containing eddies are much smaller than the length scale for the transport gradient. They suggest a correction term to Ficks law that represents a convective flux due to flow inhomogeneity and which disappears for homogeneous turbulence. The diffusion coefficient is related to the kinematic viscosity through an effective Schmidt number, the value for which is not provided in the paper. The model requires three additional empirical constants above those required for the $k-\epsilon$ model in single phase flows. They apply their model to the prediction flow properties of a jet studied experimentally by Modarress *et al.* (1984) and claim good agreement between measurements and predictions.

Chen and Wood (1985) also use the two-fluid approach to model a jet. They assume that the particle and fluid phase have the same average velocity and justify this assumption on time scale arguments. They also use a gradient transport assumption for particle dispersion due to turbulence and chose an effective Schmidt number of 0.7 which corresponds to the value for diffusion of a passive scalar in a round turbulent jet. Subramanian and Ganesh (1984) measure an effective Schmidt number of 0.47 for the same configuration. Chen and Wood applied their model to experimental studies reported by Modarress*et al.* (1984) and Wood *et al.* (1984) and noted good agreement between predictions and measurements.

The real difficulty in using the gradient transport model for particle dispersion in connection with the two-fluid model is encountered in wall-dominated flows. Here one has to establish boundary conditions for the particle concentration and velocities. The concentration profile will depend on the interaction of particles with the wall. For example, if the particles stick to the wall a different boundary condition must be used than if the particles rebound elastically from the wall. For inelastic collisions, another assumption must be used.

In addition the particle velocity component parallel to the wall is not zero as in a single phase continuum flows. Chen (1986) utilizes concepts from rarefied gasdynamics and calculates a slip velocity which depends on a fluid-particle interaction length. The normal component of velocity is set equal to zero although this would not be true for an inelastic collision. Other ramifications of the two-fluid versus trajectory models are discussed by Crowe (1986).

The advantage of the gradient diffusion model for particle dispersion is that it can be accommodated directly into the two fluid model. It also does not require excessive computational times. The difficulty is the selection of appropriate Schmidt numbers and other empirical parameters for a given application. The relative advantages and disadvantages of the eulerian and trajectory approaches are discussed by Durst *et al.* (1984).

Monte Carlo Methods

The method most natural to predicting particle dispersion using the trajectory approach is the Monte Carlo method. By this method the turbulent field is represented with a random number generator. The basic idea was first proposed by Hutchinson et al. (1971) and was subsequently used by Gosman and Ioannides (1981) in conjunction with the $k-\epsilon$ turbulence model for sprays. The turbulent fluctuational velocity is selected from a random number generator with a variance proportional to the turbulence energy. The particle motion equation is integrated with this velocity field until it passes from the eddy. The particle-eddy interaction time is established by the characteristic life time of the eddy or by the time for the particle to pass through the eddy. The dissipation length and time scales are chosen as the characteristic size and time and are given by

$$L_e = C_\mu^{3/4} k^{3/2}/\epsilon \qquad 6a)$$

$$T_e = L_e/(2k/3)^{1/2} \qquad 6b)$$

where k is the turbulence energy, ϵ is the dissipation rate and C_μ is an empirical constant arising from the $k-\epsilon$ model. The time to pass through an eddy is approximated by

$$T_p = \frac{L_e}{|U_g - U_p|}$$

where U_p is the particle velocity and U_g is the mean gas velocity. The interaction time is the minimum of the eddy life time and the passage time. If the passage time is small compared to the eddy life time then the crossing trajectory effect is important.

Gosman and Ioannides applied this model as a test case to the experimental study done by Snyder and Lumley (1971) who measured the dispersion of a series of particle types in grid-generated turbulence produced by a vertically oriented wind tunnel. Synder and Lumley found that the heavier copper particles dispersed less than the lighter hollow glass beads. From the current state of knowledge, it is accepted that this trend is due to the crossing trajectory effect. Gosman and Ionaddies report good agreement between the their predictions and Synder and Lumley's results even though their droplet equations do not contain a body force term due to gravity. This was probably an omission in the paper.

The Monte-Carlo technique was used by Chen and Crowe (1984) to model particle dispersion measurements in fully developed pipe flow reported by Arnason and Stock (1984). As with Gosman and Ioannides, they found that the technique worked well for the near isotropic, homogeneous field in Synder and Lumleys experiments. However, it was necessary to tune the model by changing C_μ to achieve the best fit. Applying the model to the pipe flow experiments yielded very poor agreement with the experimental results as shown in figure 6. The model predicted that the larger particles would disperse less than the small particles due to the crossing trajectory effect but the opposite trend was found experimentally. Chen and Crowe rationalized that the turbulence model was too crude for the complex turbulent flow in a pipe. One shortcoming of the model is the lack of information on the lagrangian length scale which should be used for L_e defined above. The above equations provide, at best, an estimate of the lagrangian length scale.

Faeth *et al.* (Solomon *et al.*, 1983; Solomon *et al*, 1985) have used the Monte-Carlo model extensively with their model for particle and droplet laden jet flows and show good agreement with experimental results. The model is calibrated to fit the analytic results of Hinze (1975) for diffusion of fluid particles from a point source in homogeneous, isotropic turbulence.

The general utility of the Monte-Carlo method for particle and droplet dispersion remains to be established. The Monte Carlo method is attractive because of the minimal empiricism needed to model the flow (provided the stochastic representation of the turbulence is reliable) and the simplicity of handling boundary conditions. The primary problem is the large number of trajectories needed to establish a stationary average in a computational cell.

Another dispersion model which attempts to capture the desirable qualities of the gradient transport and trajectory approaches is the "hybrid" model first proposed by Jurewicz (1976) and subsequently used by others. In this approach the trajectory model is first used to calculate particle concentrations in each cell. Then, a diffusional velocity is added to the particle velocity which is proportional to the concentration gradient and diffusion coefficient. Of course, this model requires selection of a diffusion coefficient.

Nonstationary, nonhomogeneous models

The gradient transport models are inadequate to model particle dispersion by large scale structures. Since particle dispersion appears to be controlled by the large scale motion, the numerical model must represent the essence of these flows. Work in this area is just beginning to appear in the literature.

Chein and Chung (1987) report a report a numerical study of particle dispersion in vortex pairs modeled using discrete vorticies. They found that particles with intermediate Stokes numbers (0.5 to 5) are dispersed more than the fluid particles while at larger Stokes numbers the heavy particles disperse less than the fluid particle.

Chien and Chung (1988) also used the discrete vortex method for generating a time dependent two-dimensional mixing layer. Particles released in this flow field show the same general trend as shown by figure 7. At low Stokes numbers, the particles follow the fluid and disperse as a fluid particle. As the Stokes number is increased the particles disperse more and near a Stokes number of unity the vortex core is almost void of particles as they are centrifuged out. In this regime the particles disperse more than the fluid particle. With further increase in Stokes mumber, the particles become unresponsive to the vortices and move in near rectilinear trajectories. The same trend is noted in a numerical study of jets by Chung and Troutt (1988).

Future work in particle dispersion in large scale structures will witness more advanced fluid mechanic models such as vortex models and pseudo-spectral methods as the computational capability is enhanced by future generation computers.

PARTICLE DRAG COEFFICIENTS

Fundamental to the development of numerical models for gas solids or gas-droplet flows is the particle or droplet equation of motion. In general, there are many forces acting on the disperse phase particle such as the virtual mass force, Basset force, pressure gradient force and the steady state drag force. The most widely accepted formulation for

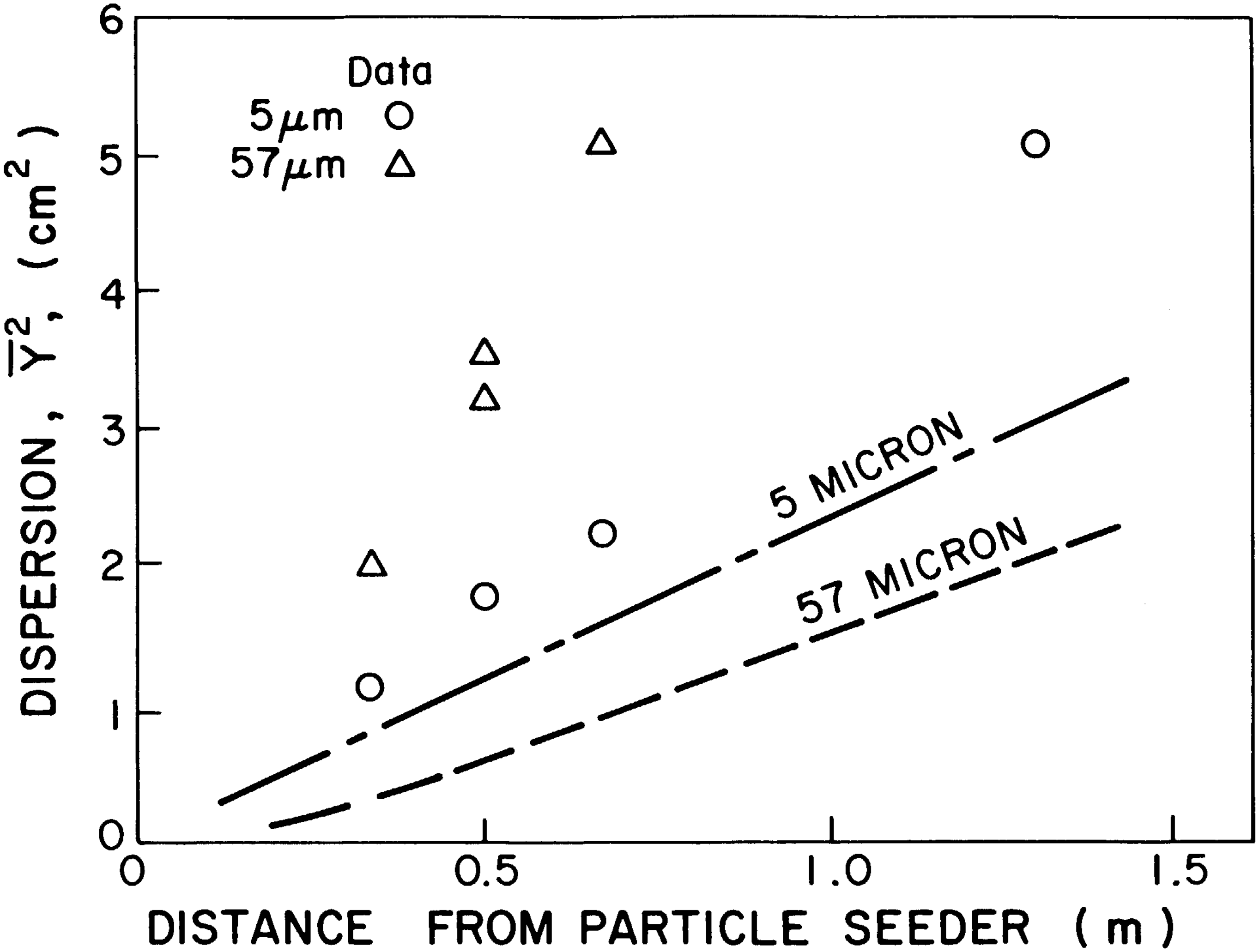

Figure 6. Measured and Predicted Particle Dispersion in Fully Developed Duct Flow (Chen and Crowe, 1984)

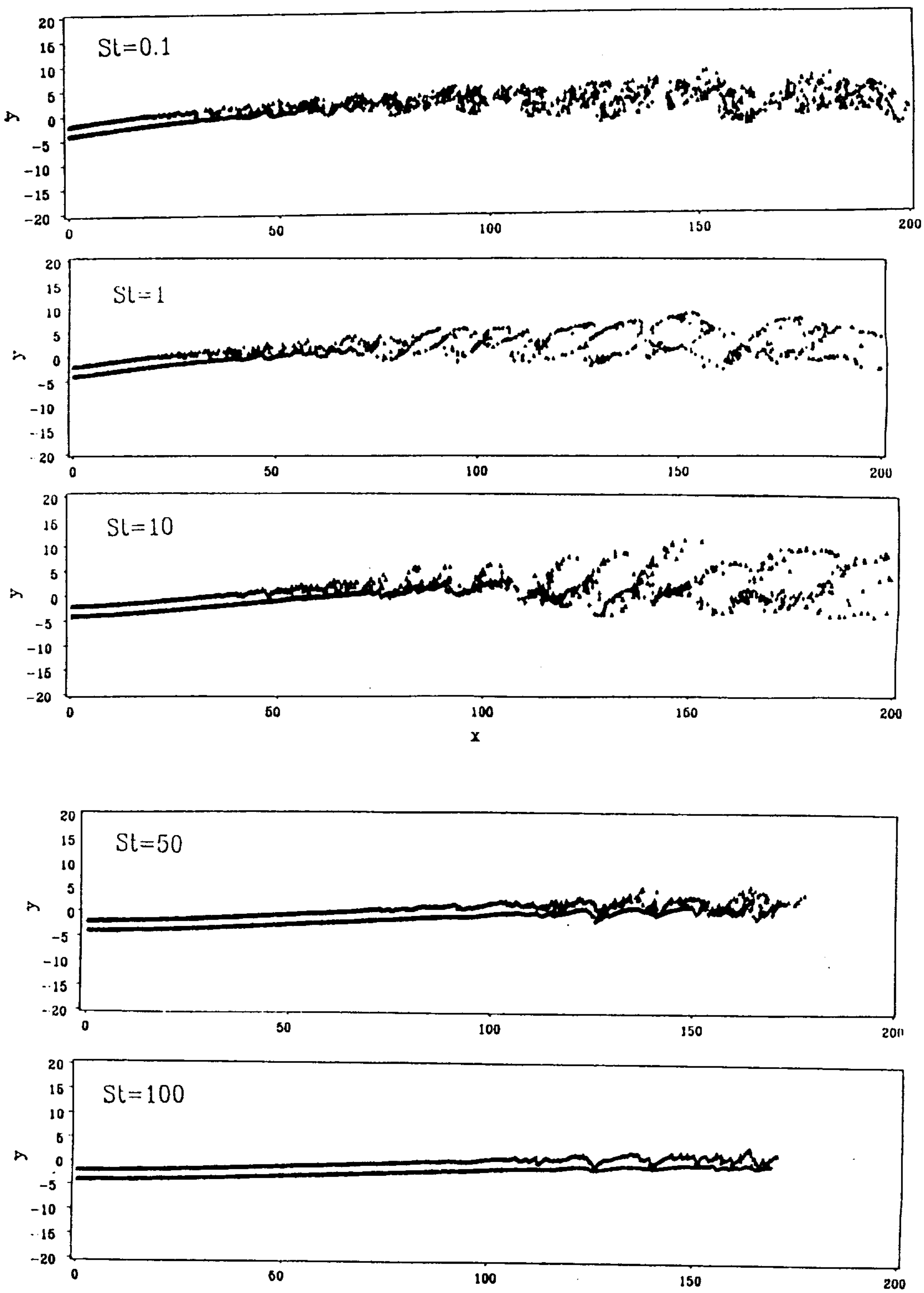

Figure 7. Predicted Particle Field over Range of Stokes Numbers in a Two-Dimensional Mixing Layer (Chien and Chung, 1987)

the equation of motion for low Reynolds number flow is that of Maxey and Riley (1983) who derived the equation from basic principles. For gas-solids flows in which the material density of the particle is three orders of magnitude larger than the conveying phase, the primary force is the steady state drag force which is quantified by the value of the drag coefficient, C_D, and related to the steady state drag by.

$$F_D = \frac{1}{2}\rho A_p C_D (U_g - U_p)|U_g - U_p|$$

where ρ is the gas density, A_p is the projected area and $(U_g - U_p)$ is the relative velocity vector between the fluid and the particle.

There is a plethora of literature available on particle drag coefficient. Most of the data have been obtained for single particles or spheres mounted in an airstream. However, in numerical model development, one is more interested in the drag coefficients of particles in a cloud. The particle drag data show significant discrepancies as shown in figure 8.

Ingebo (1956) published a NACA report on particle drag coefficient which he measured by releasing solid particles in an airstream downstream of a grid. The particles were tracked by a rotating mirror camera and the velocity-distance data were reduced to obtain the acceleration and drag force. Ingebo found the drag coefficient was less than the standard value for a sphere and attributed the discrepancy to the acceleration of the particles. Crowe (1962) suggested that the low value could have been due to a critical Reynolds number effect created by the grid upstream of the particle injection location. Arrowsmith (1973) suggested that the local air velocity in the cloud was less than the tunnel speed affecting the calculation of the relative velocity. The discrepancy has yet to be resolved.

Hanson (1952) measured the deceleration of hexane droplets issuing from an atomizer into an air flow. The spray was photographed and the droplet deceleration reduced from the photographs. He assumed that the local gas velocity was constant throughout the chamber. Hanson's results for C_D are very low. Hanson attributed the low drag coefficient to evaporation but this explanation seems unlikely.

Rudinger (1969) injected particles into a vertically oriented shock tube and passed a shock wave through the particle cloud. He used a rotating drum camera to record particle motion. The drag coefficients he reduced were significantly higher than the "standard" curve. Rudinger hypothesized that the turbulence generated by the particles created zig-zag motion which made the particles appear to have an higher "effective" drag coefficient. However the same trend would have been noticed in Ingebo's results.

Crowe (1962) reported measurements on the drag coefficient of burning gun powder. The burning powder was subjected to a shock wave in a vertical shock tube in a manner similar to Rundinger's experiment. Particle motion was measured with a high speed camera. The data lie slightly above the standard curve but well below Rudinger's.

Briffa (1981) has reported on the measurement of droplet drag in sprays. A water spray was photographed to yield a triple exposure of a droplet. The local air velocity was measured by photographing the motion of Lycopodium dust. The reduced drag coefficients were smaller than the standard curve. Briffa attributed this trend to the Basset term in the equation of motion but it is unlikely that the Basset term would be so predominant.

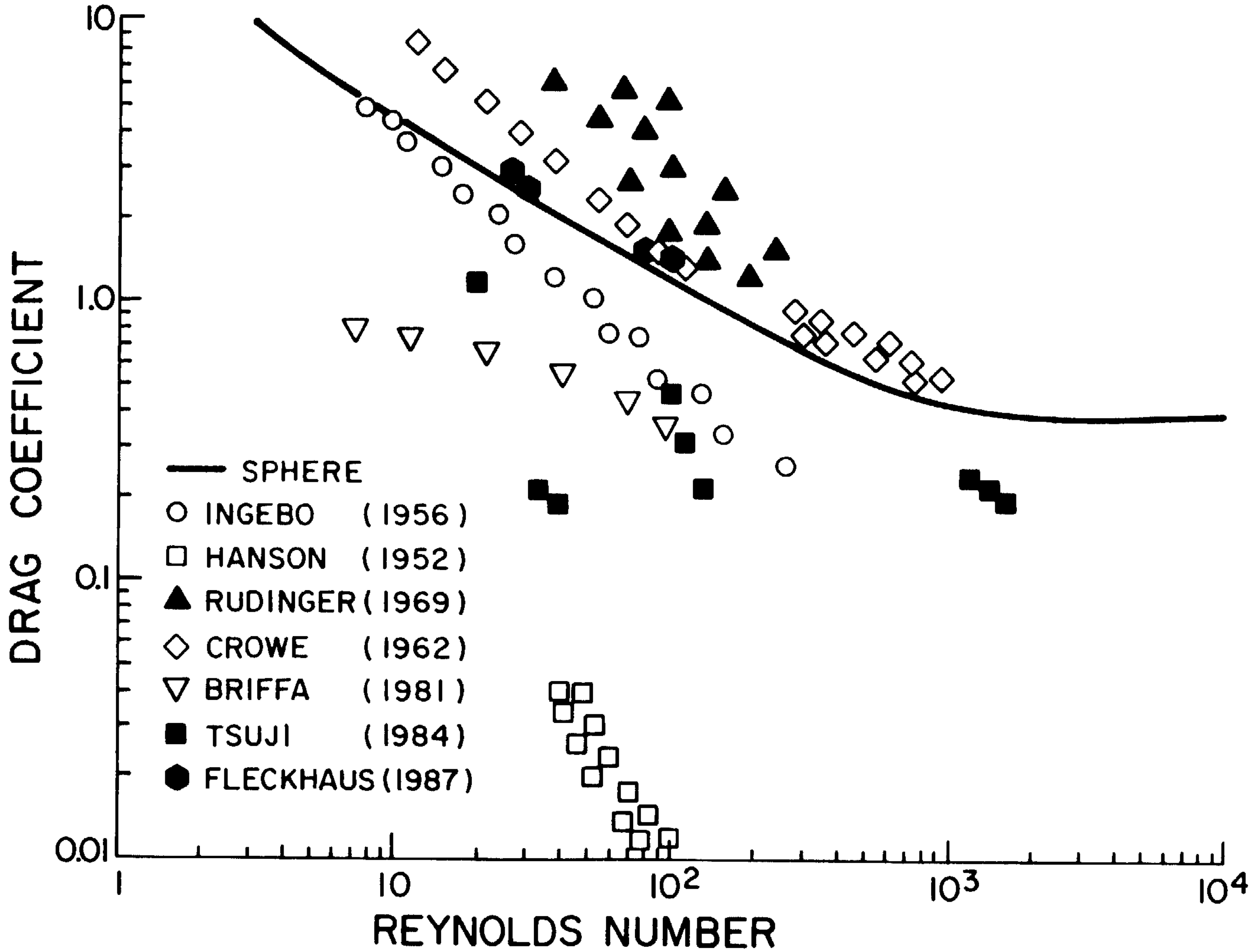

Figure 8. Particle Drag Coefficients for Particles in Gas- Particle Mixtures

Tsuji *et al.* (1982) generated a stationary array of particles and measured the drag on one particle in the array using the pendulum method. Two configurations were tested; side by side particles and one particle in the wake of another. They found that the drag of the particle in the wake was reduced for separation distances of less than 10 diameters. The difference in drag coefficient does not explain the discrepancies observed in figure 8.

Tsuji *et al.* (1984) reported on the LDV measurements of particle and air velocities of 200, 500 and 3000 micron particles in a vertical pipe. Small tracer particles were used to measure the gas flow velocity and the signals from the test particles and tracer particles were separated by a special signal discrimination device. The drag coefficients resulting from Tsuji *et al.*'s experiments have been reduced by Lee (1987). The data fall below the standard drag curve but demonstrate significant scatter. Lee correlates the data with particle volume fraction, Froude number, Reynolds number (based on turbulent fluctuational velocity) and density ratio (particle to fluid material density ratio). By so doing, he was able to fit the data on a single curve. Still, extension of the empirical results to other conditions is tenuous because one would not anticipate that the aerodynamic drag would depend on the density ratio.

Very recently, Fleckhaus *et al.* (1987) have reported measurements of particle velocities and concentrations in a jet with a two- dimensional LDA system. They also had tracer particles in the jet to obtain the gas-phase velocity. By fitting cubic splines to their velocity measurements, they were able to reduce particle accelerations. The drag coefficients were obtained by knowing the particle (glass beads) size and relative velocity. Their drag data lie above, but close to, the standard curve.

There is a need to establish a valid drag coefficient for particles in a turbulent flow and to resolve the many discrepancies apparent in the data. It would be appropriate to repeat some of the earlier experiments using modern instrumentation to either validate the data or indicate the reason for the observed trends. Until more specific information is available, one is advised to use the standard drag curve for an isolated particle.

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