I. Introduction

Recently, there has been an increasing interest in the use of stick propellant charges in large-caliber guns systems. Stick propellants offer many advantages over conventional randomly packed multi-perforated granular propellant charges. The regular geometry of stick propellants allows a higher loading density, flexibility in charge design, and easier charge loading. The higher charge density is preferable for low vulnerability ammunition (LOVA) propellants, which require a higher propellant mass to produce an equivalent performance. It has been observed by Robbins et al. (1,2) that flow resistance through the charge of a stick propellant bundle is lower than that through packed beds of granular propellants, thus enabling faster and more reproducible flame spreading through the stick propellant charge. The lower flow resistance also reduces considerably the phenomena of high pressure gradients and severe pressure waves in the gun tube, as indicated by Minor (3).

A number of studies (1-10) on various aspects of stick propellant combustion have been reported to date. The NOVA code, developed by Gough (11,12) for ballistic performance of granular charges, was used with some modifications (1,4) to predict the performance of a stick propellant charge. Results obtained were in good agreement with experimental data for multiperforated granular NACO propellants and single-perforated slotted stick bundles (1). However, the same is not true for the case of single-perforated, unslotted stick propellants.

The structure mechanics consideration in the continuum modeling of unslotted stick charge combustion of the modified NOVA code (4) is rather crude, due to the application of a steady-state relationship between radial and hoop stresses and internal and external pressures. Although pressure distributions in the internal perforation were calculated, only the external pressure was used in evaluating the axial stress component, which is in turn related to the intra-granular stress. Grain deformation and fracture of unslotted long sticks are mainly due to the radial expansion and attainment of a critical hoop stress.

Rupture of stick propellants during combustion in a gun barrel was observed by Robbins and Horst (2). Grain fracture can lead to high peak pressures due to increase in total burning surface area. The pressure difference across the web of a stick propellant can cause the grain to deform prior to fracture and alter the flow-channel width and the distance between opposite burning surfaces, thereby changing the combustion process. Hence, the prediction of grain deformation and rupture should be given due importance in the overall interior ballistic cycle.

The proposed model is based on a relatively new method, previously applied to two-phase reacting flow problems such as spray combustion of liquid droplets (14). In the separated-flow approach, the continuous phase (gas phase) is treated by a Eulerian approach, while the condensed phase media (stick propellants) are divided into several representative groups and tracked, using a Lagrangian approach, as they move in the continuous phase. The interstitial gas-phase region in the present problem is treated in a similar fashion to the continuous phase in spray combustion, while the internal gas-phase regions in the grain perforations are treated separately. The flame-spreading and combustion phenomena inside the perforation are similar to those in a cylindrical side-burning rocket motor grain. The ignition transient analysis developed by Peretz et al. (15) is therefore adapted to model the flame spreading and combustion processes in the perforation. This two-phase separated flow approach is convenient because the
stack propellants have identical dimensions and
symmetry about their own axes as well as a central
axis. The number of representative sticks in a
stick propellant bundle is relatively low, there-
fore making it possible to study some of the
detailed flow and combustion phenomena of these
representative stick propellant grains.

All of the gun interior ballistic codes and
analyses developed so far do not consider the dis-
tribution of chemical species (16-18). The heat
release is assumed to occur at the same location as
that of pyrolysis of the propellant. However, for
EODA or other types of modern propellants, some
chemical species could be pyrolyzed from the pro-
pellant surface, be carried by the flow during the
ignition transient interval, and then react at some
downstream location. Therefore, in general, the
heat release could occur at a different position
than the site of initial pyrolysis. These complicate-
d phenomema, which were simplified in previous
analyses, are included in the present model.

Objectives of the current research are to for-
mulate a combined Eulerian-Lagrangian model for the
combustion of a bundle of stick propellants inside
a gun chamber. It is intended to cover many as-
psects of realistic simulation of stick propellant
combustion characteristics so that the model is ca-
pable of predicting the phenomena of ignition,
flame spreading, and combustion of stick propel-
lants. Development of the model is also intended
to help explain the experimental observations from
test firings being conducted by the authors in a
simulated gun system. The data to be obtained will
be used for model validation.

II. Method of Approach

1. Physical Model

Figure 1 shows a bundle of stick propellants
loaded in the combustion chamber of a gun. For
the present model, only unslotted long stick propellant
grains are considered. In the theoretical
formulation, the combustion chamber is divided into five
separate regions: 1) lumped parameter region near
the base pad; 2) internal perforation region; 3)
external interstitial gas-phase region; 4) solid
propellant region; and 5) lumped parameter region
near the base of the projectile. Although the
solid propellant region consists of many stick pro-
pellants, only a few representative sticks are
required to be modeled due to the similarity of the
sticks in the same family. Each region has a sep-
arate set of governing equations which are coupled
through various boundary conditions. The erosive
burning of the stick propellants under cross-flow
conditions is also taken into account. Since frac-
ture phenomena of stick propellants under dynamic
loading conditions are under investigation, the
present model is limited to the time period before
one or more stick propellants rupture as a result of
high pressure differential across the web of the
stick propellant. After the onset of rupture, a
stick propellant could become partially slotted,
broken, and/or highly deformed. It is then diffi-
cult to distinguish the internal versus external
surface. These phenomena are beyond the scope of
the present model.

2. Mathematical Formulation

A. Basic Assumptions:

(1) Assumptions used in the gas-phase regions are:
   a. no body forces;
   b. bulk viscosity is negligible;
   c. Soret and Dufour effects are negli-
gible;
   d. gases obey Noble-Abel equation of
      state;
   e. all binary diffusion coefficients are
equal;
   f. Fick's law of diffusion is valid;
   g. flow is one-dimensional transient
      (properties are uniform in r and 8 di-
      rections);
   h. total flow area in the external gas-
      phase region is assumed to be h A
      [Dupuit-Forchheimer hypothesis (19)];
   i. turbulence correlations of flow prop-
      erties in the axial direction are consid-
      ered negligible in comparison with the
      product of their mean flow properties.
      However, turbulence effects in the
      transverse direction are embedded in
      the empirical correlations.

(2) Assumptions used for the solid propellant
are:
   a. burning in the inner and outer surfaces
      of a stick is axisymmetric;
   b. density of the solid is constant;
   c. any subsurface heat release occurs very
      close to the surface and therefore can
      be lumped onto the surface;
   d. torsion and rotation are negligible;
   e. each stick propellant is locally axi-
      symmetric;
   f. burning at the end surfaces is uniform;
   g. end surfaces are perpendicular to the
      axis of the stick;
   h. propellant material behaves as a linear
      viscoelastic material in shear and
      elastic material in bulk deformation.

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Fig. 1 Stick propellant Charge in a Cartridge
of a Large Caliber Gun.
### Overall Model

#### External gas phase
- **Basic assumptions**
- **Governing equations (based upon Eulerian coordinate system)**
- **Empirical correlations**
  - Heat transfer between gases and solid surfaces
  - Viscous drags between gases and solid surfaces
  - Mass, momentum equations for $M$ and $u_e$
- **Propellant burning rate**
- **Initial and boundary conditions**
- **Ignition criterion**
- **Conductive heat transfer**
  - Convective heat transfer coefficient
- **Skin friction coefficient**

#### Internal gas phase
- **Basic assumptions**
- **Governing equations (based upon Eulerian coordinate fixed to the stick propellant)**
- **Equation of state**
- **Initial and boundary conditions**
- **Propellant burning rate**

#### Transport properties
- **Gas-phase reaction kinetics**
- **Transport properties**

#### Solid phase
- **Basic assumptions**
- **Governing equations (based upon Lagrangian coordinate fixed to the moving stick)**
- **Empirical correlations and constants**
  - Dynamic structural analysis for grain deformation
  - Energy equation
  - Initial and boundary conditions
  - Ignition criterion
  - Thermal properties
  - Burning rate law

#### Thermal properties

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**Fig. 2 Overall Structure of the Theoretical Model**

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In order to give an overview of the structure of the mathematical model, Fig. 2 was constructed. It shows the major components and subcomponents of the formulation for each region in the physical model. To simplify the diagram, the cross-coupling lines between each region are not shown. However, it is important to note that the physicochemical processes in these regions are closely coupled. Mathematical representations of each component and subcomponent are given in the following sections.

C. Governing Equations for the External Gas-Phase Region

The external gas-phase region occupies the whole interstitial space in the cartridge, excluding the shaded region shown in Fig. 3. By using the control volume analysis (reference to Eulerian coordinate), the following governing equations are obtained.

\[
\begin{align*}
\text{Continuity Equation:} & \quad \frac{\partial \rho e}{\partial t} + \frac{\partial (\rho e u)}{\partial x} + \frac{\partial (\rho e v)}{\partial y} + \frac{\partial (\rho e w)}{\partial z} = 0 \\
\text{Momentum Equation:} & \quad \frac{\partial (\rho e u)}{\partial x} + \frac{\partial (\rho e v)}{\partial y} + \frac{\partial (\rho e w)}{\partial z} = 0 \\
\text{Energy Equation:} & \quad \frac{\partial (\rho e \varepsilon)}{\partial t} + \frac{\partial (\rho e u \varepsilon)}{\partial x} + \frac{\partial (\rho e v \varepsilon)}{\partial y} + \frac{\partial (\rho e w \varepsilon)}{\partial z} = 0 \\
\text{Species Continuity Equation:} & \quad \frac{\partial (\rho e Y_i)}{\partial t} + \frac{\partial (\rho e u Y_i)}{\partial x} + \frac{\partial (\rho e v Y_i)}{\partial y} + \frac{\partial (\rho e w Y_i)}{\partial z} = 0
\end{align*}
\]

where \(Y_i\) is the mass fraction of the \(i\)th species defined as

\[
h_i = \rho e \Delta h_{i,1} + \int_0^T C_i p_i dT
\]

and \(\rho e\) is the sum of the internal and kinetic energy of the gas phase in the control volume. The energy transfer due to molecular species diffusion has been neglected because of a high degree of turbulence.

Species Continuity Equation:

\[
\frac{\partial (\rho e Y_i)}{\partial t} + \frac{\partial (\rho e u Y_i)}{\partial x} + \frac{\partial (\rho e v Y_i)}{\partial y} + \frac{\partial (\rho e w Y_i)}{\partial z} = 0
\]

where the source term \((\omega_i)\) consists of contributions cause by surface pyrolysis and gas-phase reactions, i.e.

\[
\omega_i = (\omega_i)_{es} + (\omega_i)_{eg}
\]

where the surface pyrolysis part can be expressed as

\[
(\omega_i)_{es} = \rho_{be} \dot{A} \dot{A} Y_i
\]

where \(Y_i\) represents the mass fraction of the \(i\)th species pyrolyzed from the solid propellant before mixing with ambient gases. Following the flame model proposed by Wu et al. (20) in their study of erosive burning of homogeneous propellants, the solid propellant pyrolyzes into three groups of species:

\[
\begin{align*}
\text{Solid Propellant} & \quad \text{Oxidizer-rich gases (O)} \\
& \quad \text{Fuel-rich gases (F)} \\
& \quad \text{First group of species} \\
& \quad \text{with delayed reaction (D)}
\end{align*}
\]

Under low cross flow conditions, the flame structure adjacent to a burning homogeneous propellant...
surface exposed to a large cavity can be assumed to have three stages, as shown in Fig. 4a. This flame structure is based upon the erosive burning study in a rocket motor by Wu et al. (20). In the case of large-caliber guns densely loaded with stick propellants, the void spaces adjacent to burning surfaces are relatively small, and the species pyrolyzed from the surface can be entrained by the high-velocity gases flowing along the axis. The heat release in the final flame generated by the chemical reaction of pyrolyzed species from a specific location occurs at a downstream location, as shown in Fig. 4b. To determine the gas phase reaction rate and heat-release rate, the same chemical reaction mechanism, proposed in Ref. 20, is adopted. This mechanism can be represented by three overall chemical steps:

\[
\begin{align*}
V_{DF} + V_{DO} & \rightarrow V_{DR2} \\
V_{DR1} + V_{P} & \rightarrow V_{P} \\
V_{DR2} & \rightarrow V_{P}
\end{align*}
\]

Oxidizer-rich gases (O) can be regarded as NO species, fuel-rich gases (F) as a group of aldehydes (CH2O), and other species generated by surface pyrolysis as one group of delayed reaction species (DRI). Based upon the work of Fifer (21) and Kubota (22), chemical reaction in the Fizz zone is largely due to the reaction involving reduction of NO2 to NO. After the delay in the dark zone, the reactions in the final flame can be assumed to occur at high activation energies, associated with reactions (11) and (12) to form the final products from DRI and the second group of delayed reaction species (DR2). In the dark and final flame zones, chemical reactions result in oxidation of CO, and perhaps some H2 by NO. More detailed discussions of kinetic parameters and mechanisms are given in Refs. 20-22.

It is useful to point out that the chemical reaction mechanism proposed in Ref. 20 can be followed for highly turbulent cross-flow situations. The O and F species pyrolyzed from the solid propellant originate at the same place, and flow together in a tortuous path to form DR2 and final products (P). The delayed reaction species of DRI can also be considered to flow together in the process to form product species P. Gases in the cross-flow can transfer heat to these species and alter the delay times required to form product species P. The stoichiometric coefficients \(V_{DF}, V_{DO}, V_{DR1}, V_{P}, V_{DR2}\) are therefore functions of propellant ingredients only. The method for calculating these parameters is based upon the original molecular structure of the propellant and is discussed in Ref. 20.

In Ref. 20, the rate of production of species \(i\) was based upon the chemical reaction rate as well as the eddy-break-up rate (23) controlled by the turbulence intensity and concentration gradient. Turbulence intensity is so high in the gun situation because of base-pad ignition and combustion of propellants that the species diffusion term can be regarded as extremely short. Therefore, the rate of consumption or production of species is determined solely from chemical reaction time.

The rate of chemical reaction of species F, DRI, O, and DR2 are given below in the same form as in Ref. 20.

\[
\begin{align*}
\omega_{DF} &= -A_{DF1} \exp\left(-E_{DF}/RT\right)\rho \sqrt{\nu_{DF} e_{DF}} \sqrt{V_{DF}} \\
\omega_{DO} &= -A_{DO1} \exp\left(-E_{DO}/RT\right)\rho \sqrt{\nu_{DO} e_{DO}} \sqrt{V_{DO}} \\
\omega_{DR2} &= -A_{DR2} \exp\left(-E_{DR2}/RT\right)\rho \sqrt{\nu_{DR2} e_{DR2}} \sqrt{V_{DR2}}
\end{align*}
\]

Equation of State:
The Noble-Abel dense gas law is used.

\[
P = \frac{1}{\gamma} \rho \frac{\gamma - 1}{\gamma} = \frac{\gamma}{\gamma - 1} \rho
\]

The initial and boundary conditions as well as empirical correlations for the external gas-phase region are given in a later section.

D. Governing Equations for the Stick Propellant

The following governing equations for a representative stick are derived based upon the Lagrangian coordinate.

1) Mass and Momentum Equations:

The instantaneous mass of the representative stick can be calculated from
The instantaneous values of stick propellant length, \( L(t) \), and the local inner and outer radii, \( r_i(t,\xi) \) and \( r_o(t,\xi) \), are determined by integrating the following first order differential equations.

\[
\frac{dr_i(t,\xi)}{dt} = r_{b1}(t,\xi) + V_{ri}(t,\xi) \tag{19}
\]

\[
\frac{dr_o(t,\xi)}{dt} = -r_{be}(t,\xi) + V_{ro}(t,\xi) \tag{20}
\]

\[
dL(t) = -r_{b1}(t) - r_{b2}(t) + V_{slB}(t) - V_{SLB}(t) \tag{21}
dt
\]

where \( V_{ri} \) and \( V_{ro} \) represent the radial velocities of the inner and outer surfaces of the stick propellant due to mechanical deformation with respect to the centerline of the stick. \( V_{slB} \) and \( V_{SLB} \) represent the rate of mechanical deformation at the right and left boundary surfaces with respect to the geometric center of the stick. The equation of motion is formulated according to the following momentum balance principle.

\[
d\left( M_t u_s \right) = \Sigma F_c + \left( \text{net rate of momentum flux flowing into the control volume encompassing the stick propellant} \right) \tag{22}
\]

which gives (see Fig. 5,)

\[
d\left( M_t u_s \right) = -2\pi \left[ \begin{array}{c} \frac{L}{2} \left[ \frac{P \sin\theta \sin\theta}{s} \right]_1 \xi - \frac{P \sin\theta \sin\theta}{s} \right] \frac{d\xi}{d\xi} - (r_{pp} + r_{pw}) \nonumber \right. \\
+ 2\pi \left[ \frac{L}{2} \left[ \frac{P \sin\theta + \cos\theta}{s} \right]_1 \xi - \frac{P \sin\theta + \cos\theta}{s} \right] \frac{d\xi}{d\xi} \\
+ 2\pi \left[ \frac{L}{2} \left[ \frac{r_{2r} \sin\theta - r_{2r} \sin\theta}{s} \right]_1 \xi - \frac{r_{2r} \sin\theta - r_{2r} \sin\theta}{s} \right] \frac{d\xi}{d\xi} \\
- \pi \left[ \left( r_{r2} - r_{r1} \right)(p + \frac{\rho g}{r}) \right]_{\xi = \frac{L}{2}} \\
+ \pi \left[ \left( r_{r2} - r_{r1} \right)(p + \frac{\rho g}{r}) \right]_{\xi = -\frac{L}{2}} \tag{23}
\]

where \( F_p \) represents the net force acting between the stick propellant and chamber wall, and \( F_{pp} \) is the force between adjacent propellants.

\[
\text{Fig.5 Momentum fluxes and Pressure Forces Acting on the Entire Surface of a Single Perforated Stick Propellant.}
\]

2) Transient Heat Conduction Equation:

To determine the instantaneous temperature distribution in the stick propellant, a heat conduction equation must be considered. The equation, which takes into account the subsurface radiation absorption for translucent propellants, has the following form.

\[
\frac{3(\rho c_p)}{s} \frac{dL(t)}{dt} = \frac{1}{r} \frac{3T}{s} \left( \frac{3T}{s} \right)_{\xi = \frac{L}{2}} + \left( \frac{1}{r} - \frac{E_b}{s} \right) \tag{24}
\]

where \( E_b \) is the black-body emissive power evaluated as \( \frac{1}{s} \), and "a" represents the flux absorption coefficient of the propellant. This equation is based upon a two-flux model which assumes that the radiation fluxes are dominant in the radial directions (inward and outward). The source terms represent the net rate of energy absorbed due to radiant energy fluxes. The outward and inward radiant fluxes, \( I_e \) and \( J_e \), can be determined from the following flux-transport equations.

\[
\frac{d(I_e)}{dr} = -(s+a)I_e + ar_{E_b} + J_e + \frac{1}{r} \frac{d}{dr} \left( I_eJ_e \right) \tag{25}
\]

\[
\frac{d(J_e)}{dr} = (s+a)J_e - ar_{E_b} + J_e - \frac{1}{r} \frac{d}{dr} \left( I_eJ_e \right) \tag{26}
\]

These two equations were used by Cosman and Lockwood. (24)

3) Dynamic Structural Analysis

As a result of the different ignition and flame spreading processes occurring in the internal perforation and external gas-phase regions, a pressure differential exists across the web of a representative stick. A finite element analysis is needed to compute the resulting transient viscoelastic deformation of the stick propellant and to predict the attainment of a critical condition for grain fracture. Regression of the boundary as a result of pyrolysis and burning should also be taken into account. Furthermore, the mechanical properties of the stick propellant must be specified.

The propellant material can be considered as linear viscoelastic in shear and elastic in bulk deformation. This is a commonly accepted practice for solid propellant. (25, 26) The elastic bulk behavior is assumed to follow

\[
\sigma_1 = 3K\varepsilon_1 \tag{27}
\]

where \( K \) is the bulk modulus. The deviatoric behavior is taken as

\[
\sigma_2 = \int_0^t G_1(t-t') \frac{\partial e_2(t')}{\partial t'} dt' \tag{28}
\]

where the shear relaxation modulus \( G_1(t) \) is assumed to be of the form

\[
G_1(t) = G_0 + (G_0 - G_0)e^{-8t} \tag{29}
\]
where $G_\infty$ is the long-time shear modulus, $G_0$ is the short-time shear modulus, and $\delta$ is the decay constant. Since a closed-form solution for the axisymmetric dynamic problem is not possible, a well-established finite element code "HONDO-II" (26) is employed for computations of grain deformation. The code utilizes the principle of virtual work for the solution. It states that at all the points along the path of motion, the differential virtual work $\delta w = 0$ must vanish for all variations $\delta x_k$ satisfying the imposed displacement boundary conditions (26). $\delta w$ is defined as

$$\delta w = \int \left[ \left( \frac{\partial f}{\partial \delta x_k} \right) \delta x_k \right] \, dv + \int \left[ \left( \frac{\partial f}{\partial \delta x_k,m} \right) \delta x_k \right] \, dv - \int \left[ \left( \frac{\partial f}{\partial \delta x_k} \right) \delta x_k \right] \, ds = 0 \quad (30)$$

where $\delta w$ is surface traction and $\delta x_k,m$ is Cauchy stress tensor. The stick propellant is divided into a number of elements over the cross-sectional area of the web. The HONDO-II code uses four mode bilinear isoparametric elements. The basic equation of motion, viz., the minimization of virtual work, is then considered for each of the nodes in the stick propellant. Thus, the equation of motion for a node becomes

$$\delta w = \int \left[ \left( \frac{\partial f}{\partial \delta x_k} \right) \delta x_k \right] \, dv + \int \left[ \left( \frac{\partial f}{\partial \delta x_k,m} \right) \delta x_k \right] \, dv - \int \left[ \left( \frac{\partial f}{\partial \delta x_k} \right) \delta x_k \right] \, ds = 0 \quad (31)$$

where $\delta w = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ and $\delta f$ are the bilinear interpolation functions. In Eq. (31), $N$ is the total number of elements surrounding the node in question. The calculations are carried out on an element-by-element process to get the final equations of motion. The time integration of these equations gives the positions of nodes at the new time step. A central difference method is used for time integration in the HONDO-II code.

### E. Governing Equations for the Internal Gas-Phase Region

Conservation equations for the control volume in the perforation of a stick propellant are similar to those given in Ref. 15.

**Continuity Equation:**

$$\frac{\partial (\rho \Delta V)}{\partial t} + \frac{\partial (\rho \Delta V \cdot \Delta v)}{\partial x} = r_b \delta t \rho b \quad (32)$$

where $\Delta L$ is the distance measured from the center of the stick, and $\Delta V$ is the velocity of the gas relative to the velocity of the stick propellant. $\Delta V$ is related to the absolute velocity $u_{gi}$ by

$$v_{gi} = u_{gi} - u$$

**Momentum Conservation Equation:**

$$\begin{align*}
\frac{\partial (\rho \Delta V \cdot \Delta V)}{\partial t} &+ \frac{\partial (\rho \Delta V \cdot \Delta V)}{\partial x} = - \Delta \frac{\partial p}{\partial x} - \rho \Delta V \cdot \Delta f
\end{align*} \quad (33)$$

Energy Conservation Equation:

The energy conservation equation written for the stored total energy (internal and kinetic) per unit mass, $E_{gi}$, is

$$\frac{\partial (\rho \Delta V E_{gi})}{\partial t} + \frac{\partial (\rho \Delta V \cdot \Delta V E_{gi})}{\partial x} = 3 \left( \frac{\partial (\rho \Delta V F_{gi})}{\partial x} \right)$$

Species Continuity Equation:

$$\frac{\partial (\rho \Delta V Y_i)}{\partial t} + \frac{\partial (\rho \Delta V \cdot \Delta V Y_i)}{\partial x} = 2 \left( \frac{\partial (\rho \Delta V Y_i)}{\partial x} \right)$$

Species continuity equation for the internal gas-phase is similar to that for the external gas-phase, with the void fraction taken as one and the absolute velocity replaced by the relative velocity.

$$\frac{\partial (\rho \Delta V Y_i)}{\partial t} + \frac{\partial (\rho \Delta V \cdot \Delta V Y_i)}{\partial x} = 2 \left( \frac{\partial (\rho \Delta V Y_i)}{\partial x} \right)$$

**F. Heat Losses to the Walls of the Combustion Chamber**

In order to consider heat losses from the combustion zone to the cartridge chamber, the gun tube, and the projectile, temperature profiles in these metal components are required. For a test rig with a blowout diaphragm and short barrel, the transient ignition and combustion phenomena occur in an extremely short time interval and the heat loss to the surrounding walls can be considered negligible. However, if the simulation is made for the full ballistic cycle occurring in a long gun barrel, the loss to the walls must be considered. The heat conduction to the gun tube can be given as

$$\frac{\partial T}{\partial t} = \frac{1}{\alpha} \nabla \cdot \left( k \nabla T \right) + \frac{1}{\rho} \frac{1}{c_p} \frac{\partial E}{\partial t}$$

The transient heat conduction equation for the wall on the breech end will follow the one-dimensional form:

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The equation for the projectile is:
\[
\rho \frac{\partial \left( C_w T \right)_{w,w}}{\partial t} = \frac{3}{2z} \left( k \frac{\partial T}{\partial z} \right)_{w} \tag{38}
\]

The solution of these equations are coupled to the external gas-phase region through the boundary conditions on the wall surfaces.

G. Initial and Boundary Conditions

For each of the above governing equations, there is a set of boundary and/or initial conditions required to complete the formulation. The initial conditions can be specified readily, since the gas and stick propellant velocities are zero and the pressure and temperature of gas in the cartridge is at room conditions. The stick propellant charge with known geometry and its surrounding chamber are also at room temperature. Since the amount of air in the initial loading of the cartridge is extremely small in comparison with the gases generated from combustion, it can be treated as any one of the five species discussed above. In view of the fact that air contains oxygen, it is treated as oxidizer-rich gas (O).

A number of important boundary conditions are given in the following.

1). Boundary Conditions for the stick Propellant

The boundary condition for Eq. (24) at outer surface of the stick propellant can be written as
\[
k_s \frac{\partial s}{\partial r} \bigg|_{r=0} = \frac{\theta_s}{c_s} (T(T_s - T_e)) + \rho_s r_s \beta_e (Q_s)_{chem} + \left( \frac{q_{rad}}{c_s} \right)_s + \left( \frac{q_{C/F}}{c_s} \right)_s \tag{40}
\]

where \( \left( \frac{q_{C/F}}{c_s} \right)_s \) represents the rate of energy input due to the deposit of condensed phase igniter products onto the external surface and \( \left( \frac{q_{rad}}{c_s} \right)_s \) the net radiative heat flux to the surface can be expressed as
\[
\left( \frac{q_{rad}}{c_s} \right)_s = \frac{\theta_s}{c_s} \left[ r = r_o \right] + \varepsilon_s J \left|_{r = r_o} \right. - \varepsilon_s F \frac{\partial T}{\partial r} \bigg|_{r = r_o} \tag{41}
\]

Similarly, the boundary condition for the inner surface of the stick propellant can be written as
\[
-k_s \frac{\partial s}{\partial r} \bigg|_{r=1} = \frac{\theta_s}{c_s} (T(T_s - T_e)) + \rho_s r_s \beta_e (Q_s)_{chem} + \left( \frac{q_{rad}}{c_s} \right)_s + \left( \frac{q_{C/F}}{c_s} \right)_s \tag{42}
\]

where
\[
\left( \frac{q_{rad}}{c_s} \right)_s = \frac{\theta_s}{c_s} \left[ r = r_s \right] + \varepsilon_s J \left|_{r = r_s} \right. - \varepsilon_s F \frac{\partial T}{\partial r} \bigg|_{r = r_s} \tag{43}
\]

The pressure distributions along the internal perforation and external interstitial regions solved from the gas-phase equations are used as boundary conditions for the solid-phase dynamic structural analysis.
Energy balance:

\[ \frac{dT}{dt} = \frac{1}{(C_v \rho)_{LCV}} \left\{ C_v T \frac{\partial T}{\partial t}_{LCV} + \left[ \left( \frac{\rho}{\rho} \right)_{LCV} \frac{\partial \rho}{\partial t}_{LCV} \right] \right\} \]

Equation of state:

\[ P_{LCV} = \frac{RT}{\left( \frac{\rho}{\rho} \right)_{LCV}} \]  

H. Empirical Correlations and Constants

Several empirical correlations are used in the model. For the internal gas-phase region, these comprise of the correlations for the burning rate law including erosive burning effect, the convective heat-transfer coefficient, and viscous drag coefficient. The latter two correlations are the same as those used in Ref. 15. New erosive burning correlation being developed by authors and coworkers in parallel to this study will be used. For the external gas-phase region, correlations for the convective heat-transfer coefficient, and viscous drag coefficient between the gas-phase and solid surfaces are needed. These correlations are similar to those for the internal perforation region except the fact that they are based on a hydraulic diameter \( D_H \) defined as

\[ D_H = \frac{2( R_c^2 - N r_2^2) \ln(1 - \beta)}{2(r_c^2 - N r_0^2) \ln(1 - \beta)} \]

The burning rate law for the external region will be the same as that for the internal perforation region.

For the stick propellants, correlations for \( P_d \) and \( P_i \), the ignition criterion, and mechanical properties are needed. At the present time, not all of this information is available; especially \( P_d \) and \( P_i \) need to be characterized.

4. Summary of Differences Between the Present and Conventional Formulation

In order to bring out the major differences between the present formulation and the conventional interior ballistic predictive models (4), a summary table is given below.

As summarized by Robbins and Einstein (29), there are differences between the measured and calculated pressures from the NOVA code or its extensions for long unslotted stick propellants. Also, the measured muzzle velocities are higher than those calculated for slotted stick propellants. A number of improvements and considerations suggested in the workshop (29) are incorporated in the present model.
<table>
<thead>
<tr>
<th>Subject under Consideration</th>
<th>Present Formulation</th>
<th>Conventional Formulation (4)</th>
</tr>
</thead>
</table>
| Typical grain configuration  | * Simulation of a number of typical full-length grains in a bundle of stick propellants.  
                                  * Each stick is modeled as a separate tube with deformable and combustible walls. | * Simulation of an average grain in a spatial location along a packed bed of granular propellants.  
                                  * Each bundle is modeled as a continuum characterized by the velocity and stress in the sticks. |
| Averaging of flow properties  | * The external flow properties are averaged over the cross-sectional flow area of interstitial voids, while the internal flow properties are averaged over the flow area of each stick propellant. | * Flow properties in both external and internal regions are averaged over their respective flow areas. |
| Grain deformation and fracture| * Simulated by the unbalanced pressure forces between the internal perforation and external interstitial void region.  
                                  * Linear viscoelastic constitutive law is used.  
                                  * Employs dynamic finite-element structure mechanics computational code. | * The process of grain deformation and fracture are not addressed, except the longitudinal stresses are considered in the solid-phase momentum equation.  
                                  * Linear elastic constitutive law is used.  
                                  * Employs a steady-state relationship between stresses (radial and hoop) and pressures (internal and external). |
| Grain displacement and acceleration | * The kinematics of the full-length grain is determined from the summation of all forces exerted on the grain. | * The bulk properties of the grains are determined from local momentum balance. |
| Radiative heat transfer       | * Subsurface radiation penetration is allowed and treated by a two-flux model. | * No subsurface radiation penetration is considered. |
| Type of formulation and frame of reference | * Kinematics and grain deformation are formulated by following the stick (Lagrangian approach) while the gas-phase properties for internal and external regions are determined from a fixed frame of reference (Eulerian approach). | * Both the gas-phase and solid-phase properties are all determined from the conservation equations formulated based upon a fixed frame of reference (Eulerian approach). |
| Species distribution and location of heat release | * Five groups of species are considered.  
                                  * Heat release does not have to occur at the site of pyrolysis. | * Gas phase is made of combustion products from ignition and propellant.  
                                  * Heat release occurs locally at the site of pyrolysis. |
### Nomenclature

#### Letter Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>flux model absorption coefficient, (m^{-1})</td>
</tr>
<tr>
<td>(A)</td>
<td>cross-sectional area of the gun barrel, (m^2)</td>
</tr>
<tr>
<td>(A_i)</td>
<td>exponential factor of the (i^{th}) species, (m^3/m\text{mol-s})</td>
</tr>
<tr>
<td>(A_p)</td>
<td>cross-sectional area of the perforation, (m^2)</td>
</tr>
<tr>
<td>(A_S)</td>
<td>specific area of the external surfaces of the stick propellant, (m^2)</td>
</tr>
<tr>
<td>(b)</td>
<td>convolume of the Noble-Abel equation of state, (m^3/kg)</td>
</tr>
<tr>
<td>(C_p)</td>
<td>constant pressure specific heat, (J/kg-K)</td>
</tr>
<tr>
<td>(C_S)</td>
<td>specific heat of stick propellant, (J/kg-K)</td>
</tr>
<tr>
<td>(C_v)</td>
<td>constant volume specific heat, (J/kg-K)</td>
</tr>
<tr>
<td>(C_{v0})</td>
<td>group of species pyrolyzed from propellant surface having delayed reactions</td>
</tr>
<tr>
<td>(C_{v1})</td>
<td>delayed reaction species generated from (O) and (P) species</td>
</tr>
<tr>
<td>(D_v)</td>
<td>viscous drag force per unit area, (N/m^2)</td>
</tr>
<tr>
<td>(d_{ij})</td>
<td>binary diffusion coefficient, (m^2/s)</td>
</tr>
<tr>
<td>(e_{kk})</td>
<td>deviatoric strain</td>
</tr>
<tr>
<td>(E)</td>
<td>total stored energy (internal plus kinetic) per unit mass, (J/kg)</td>
</tr>
<tr>
<td>(E_a)</td>
<td>activation energy, (J/m\text{ol})</td>
</tr>
<tr>
<td>(E_b)</td>
<td>black-body emissive power, (=T^4, J/m^2-s)</td>
</tr>
<tr>
<td>(F)</td>
<td>fuel rich species pyrolyzed from propellant surface</td>
</tr>
<tr>
<td>(F_{c})</td>
<td>external force exerted on the solid propellant in the axial direction, (N)</td>
</tr>
<tr>
<td>(h)</td>
<td>specific enthalpy, (J/kg)</td>
</tr>
<tr>
<td>(h_{c})</td>
<td>convective heat-transfer coefficient, (W/m^2-K)</td>
</tr>
<tr>
<td>(h_{j})</td>
<td>specific enthalpy of the (j^{th}) species, (J/kg)</td>
</tr>
<tr>
<td>(h_{t})</td>
<td>total heat-transfer coefficient, (W/m^2-K)</td>
</tr>
<tr>
<td>(\Delta h_f, i)</td>
<td>standard enthalpy of formation of the (i^{th}) species, (J/kg)</td>
</tr>
<tr>
<td>(I_r)</td>
<td>outward radiation flux in the positive radial direction, (W/m^2)</td>
</tr>
<tr>
<td>(J_r)</td>
<td>inward radiation flux in the negative radial direction, (W/m^2)</td>
</tr>
<tr>
<td>(K)</td>
<td>bulk modulus of the propellant material, (N/m^2)</td>
</tr>
<tr>
<td>(M_g)</td>
<td>total mass of gas in control volume, (kg)</td>
</tr>
<tr>
<td>(M_s)</td>
<td>instantaneous total mass of a single stick propellant, (kg)</td>
</tr>
<tr>
<td>(O)</td>
<td>oxidizer species pyrolyzed from propellant pressure, (N/m^2)</td>
</tr>
<tr>
<td>(P)</td>
<td>final product species</td>
</tr>
<tr>
<td>(P_b)</td>
<td>perimeter of the internal perforation, (m)</td>
</tr>
<tr>
<td>(Q_{rad})</td>
<td>radiative heat flux per unit time absorbed by the solid propellant surface, (W/m^2)</td>
</tr>
<tr>
<td>(Q_{chem})</td>
<td>surface heat release due to pyrolysis, (J/kg)</td>
</tr>
<tr>
<td>(Q_{chem})</td>
<td>rate of heat loss to the tube wall, (W/m^2)</td>
</tr>
<tr>
<td>(r_b)</td>
<td>propellant burning rate, (m/s)</td>
</tr>
<tr>
<td>(r_1)</td>
<td>inner radius of the perforation, (m)</td>
</tr>
<tr>
<td>(r_o)</td>
<td>outer radius of the stick propellant, (m)</td>
</tr>
<tr>
<td>(R)</td>
<td>gas constant, (J/kg-K)</td>
</tr>
<tr>
<td>(R_c)</td>
<td>radius of the combustion chamber, (m)</td>
</tr>
<tr>
<td>(R_u)</td>
<td>universal gas constant, (J/m\text{ol-K})</td>
</tr>
<tr>
<td>(S)</td>
<td>flux model scattering coefficient, (1/m)</td>
</tr>
<tr>
<td>(S_{ij})</td>
<td>deviatoric stress tensor, (N/m^2)</td>
</tr>
<tr>
<td>(T)</td>
<td>surface area, (m^2)</td>
</tr>
<tr>
<td>(T_t)</td>
<td>time, (s)</td>
</tr>
<tr>
<td>(T_f)</td>
<td>temperature, (K)</td>
</tr>
<tr>
<td>(U)</td>
<td>adiabatic flame temperature of the solid propellant, (K)</td>
</tr>
<tr>
<td>(V)</td>
<td>absolute velocity, (m/s)</td>
</tr>
<tr>
<td>(V_{c})</td>
<td>gas velocity relative to the solid propellant, (m/s)</td>
</tr>
<tr>
<td>(W)</td>
<td>volume, (m^3)</td>
</tr>
<tr>
<td>(W_k)</td>
<td>molecular weight of the (i^{th}) species, (kg/m\text{ol})</td>
</tr>
<tr>
<td>(X_k)</td>
<td>coordinate axis in (k^{th}) direction, (m)</td>
</tr>
<tr>
<td>(\ddot{x}_k)</td>
<td>acceleration in (k^{th}) direction, (m/s^2)</td>
</tr>
<tr>
<td>(\delta_{kk})</td>
<td>deformation gradient tensor</td>
</tr>
<tr>
<td>(\dfrac{\partial}{\partial t})</td>
<td>mass fraction of (i^{th}) species, (i) could represent (F, O, DR1, DR2, P)</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>number of molecules of (i^{th}) species</td>
</tr>
<tr>
<td>(\xi)</td>
<td>Lagrangian axial coordinate, (m)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>dynamic viscosity of the gas, (N-s/m^2)</td>
</tr>
<tr>
<td>(\nu_{i})</td>
<td>virtual work, (J)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density, (kg/m^3)</td>
</tr>
<tr>
<td>(\sigma_{kk})</td>
<td>Stefan-Boltzmann constant, (W/m^2-K^4)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>stress tensor, (N/m^2)</td>
</tr>
<tr>
<td>(\tau_{kk})</td>
<td>viscous shear stress, (N/m^2)</td>
</tr>
<tr>
<td>(\tau_{kk})</td>
<td>surface traction in (k^{th}) direction, (N/m^2)</td>
</tr>
<tr>
<td>(\tau_w)</td>
<td>tube wall shear stress, (N/m^2)</td>
</tr>
<tr>
<td>(\tau_{w})</td>
<td>normal viscous stress, (N/m^2)</td>
</tr>
<tr>
<td>(\phi_{s})</td>
<td>mass fraction of combustion product in the condensed phase from the igniter</td>
</tr>
<tr>
<td>(\phi_{i})</td>
<td>rate of production of (i^{th}) species, (kg/m^3-s)</td>
</tr>
</tbody>
</table>

#### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_{kk})</td>
<td>dilatory strain</td>
</tr>
<tr>
<td>(\varepsilon_{s})</td>
<td>surface emissivity of solid propellant</td>
</tr>
<tr>
<td>(\theta_{k})</td>
<td>angle measured (ccw) from axis to the tangent to the inner surface of the stick propellant, (rad)</td>
</tr>
<tr>
<td>(\theta_0)</td>
<td>angle measured (ccw) from axis to the tangent to the outer surface of the stick propellant, (rad)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>number of molecules of (i^{th}) species</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Lagrangian axial coordinate, (m)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>virtual work, (J)</td>
</tr>
<tr>
<td>(\rho)</td>
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</tbody>
</table>

#### Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e)</td>
<td>external interstitial region</td>
</tr>
<tr>
<td>(g)</td>
<td>gas-phase, internal or external region</td>
</tr>
<tr>
<td>(i)</td>
<td>internal perforation region</td>
</tr>
</tbody>
</table>
REFERENCES


Editor's Narrative:

Round-Table Discussion: Propulsion Applications of Mixing and Demixing Processes of Multiphase Flows

Several points were made either explicitly or implicitly in the presentations and discussions on the second day concerning mixing processes in multiphase systems and the modeling of these systems. First, there are situations where transport in turbulent flows can be in a direction other than along the gradient of the property being transported. This comes about through coupling of density inhomogeneities with pressure gradients, the former being a consequence of the large temperature differences in combustion environments. Hence, in turbulent flames, some transport processes are "counter gradient" or more correctly, "non-gradient" in nature. Errors due to neglect of these processes are not understood, but could possibly be significant in some combustion systems. Modelers are in the process of learning to incorporate these ideas into practical codes.

Second, the weakest element in modeling of turbulent multiphase flows is probably the turbulence modeling itself. The treatment of the coupling between the turbulence and the non-continuously distributed dispersed phase is also a critical area and includes the question of whether, for numerical purposes, to treat the dispersed particulate (or droplet) phase as a continuum or as an assembly of separate particles.
Third, it is possible to treat a very complex problem in
great detail by using appropriate levels of approximation and by
coupling separate regions together which are best treated with
different models or with different levels of approximation.

Fourth, while continuum approaches to multiphase flow calcu-
lations are not particularly illuminating in revealing details of
the physical processes at work (due to the necessity of averaging
quantities before solving the governing equations), there are
cases where they can provide useful results for a given problem.
The two-fluid approach works best where the dispersed particulate
phase is monodisperse, and where the flows are non-reacting.

Sixth, advantages and disadvantages of Favre averaging
approaches were discussed. An advantage is that the conservation
equations for variable density with Favre averaging are much like
the standard constant density equations. Disadvantages are that
there is some difficulty in obtaining molecular terms and com-
puted and measured quantities are more difficult to compare.

Finally, physical situations for which multiphase flow mod-
els are appropriate generally contain a very broad range of
length and time scales within the same problem. This means that
when computational approaches to these problems are used, some
levels of approximation will need to be made in order to deal
with these large ranges of scale. The point in the analysis at
which approximation is made is a key difference between a two-
fluid model and an Eulerian-Lagrangian model of a dispersed par-
ticulate phase in a fluid phase.
A workshop on transport processes in multiphase flows was held at the Marshall Space Flight Center on February 25 and 26, 1988. This document provides the program, abstracts, and text of the presentations made at this workshop. In addition, the content of two open discussion sessions are presented in narrative form. The objective of the workshop was to enhance our understanding of mass, momentum, and energy transport processes in laminar and turbulent multiphase shear flows in combustion and propulsion environments.