Expansion Tube Test Time Predictions

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ABSTRACT
The interaction of an interface between two gases and a strong expansion is investigated and the effect on flow in an expansion tube is examined. Two mechanisms for the unsteady pitot-pressure fluctuations found in the test section of an expansion tube are proposed. The first mechanism depends on the Rayleigh-Taylor instability of the driver-test gas interface in the presence of a strong expansion. The second mechanism depends on the reflection of the strong expansion from the interface. Predictions compare favourably with experimental results. The theory is expected to be independent of the absolute values of the initial expansion tube filling pressures.
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### TABLE OF CONTENTS

1. **INTRODUCTION**  
2. **THE EXPANSION TUBE**  
   2.1 The Ideal Expansion Tube  
   2.2 Boundary Layer Entrainment Effect  
   2.3 Real Gas Effects  
   2.4 Experimental Results from Expansion Tubes  
3. **LITERATURE REVIEW**  
   3.1 Turbulence at the Interface and Development of Mixing  
   3.2 Rayleigh-Taylor Instability  
   3.3 Conditions for Rayleigh-Taylor Instability in Shock Tubes  
4. **MECHANISMS CAUSING EARLY PRESSURE FLUCTUATIONS**  
   4.1 Equations of Motion of a Minimum Density Blob  
   4.2 Reflection of Waves from the Contact Surface  
5. **IMPLEMENTATION OF SOLUTION**  
   5.1 Basic Assumptions  
   5.2 Computer Program  
   5.3 Verification of Computer Code and Truncation Error  
6. **COMPARISON OF COMPUTATIONS WITH EXPERIMENT**  
   6.1 Shock Speed  
   6.2 Langley Results  
   6.3 U.Q. Argon Driver Results  
   6.4 U.Q. Helium Driver Results  
   6.5 U.Q. Air Driver Result  
7. **CONCLUSIONS**  
8. **REFERENCES**  
9. **FIGURES**  
A. Complete Set of Finite Difference Equations  
B. Program Listing
1. INTRODUCTION

An expansion tube is a facility for producing high-enthalpy short-duration hypersonic gas flows. The principle of operation is to use an unsteady expansion for the purpose of expanding the test gas, rather than a nozzle as in a shock tunnel. A facility built at NASA Langley (Moore, 1975) was expected to outperform conventional shock tunnels due to total-enthalpy multiplication (Trimpi, 1962). Experimental experience in the Langley expansion tube (Moore, 1975; Miller, 1977; Miller, 1978; Shinn and Miller, 1978) indicated that the duration of useful test gas flow was much less than expected. Evidence for this was primarily in the form of pitot-pressure time-histories measured at the test section. The pitot pressure time-histories indicated two unexpected phenomena. Firstly, the region of constant pressure test flow was found to be disturbed by large pitot pressure perturbations, and, secondly, the magnitude of the pitot pressure was seen to 'dip' under some circumstances (Miller, 1977; Miller, 1978).

This work is aimed at explaining the first mentioned phenomenon, that is the pitot-pressure perturbations. It is expected that explanation of the basic phenomenon, or phenomena, will enable a range of useful test conditions to be established for expansion tubes. The theory formulated here will be applicable to free-piston driven expansion tubes such as at the University of Queensland.

The chapters in this report have been arranged in the following order: firstly, a description of the expansion tube (ideal and real); secondly, a review of the literature relating to the basic mechanisms causing reduction in expansion tube test times; thirdly, the new theory and computer implementation; fourthly, comparison to experiment; and fifthly, the conclusions.
2. THE EXPANSION TUBE

2.1 The Ideal Expansion Tube

The expansion tube in which the experimental data was obtained is the small 'TQ' expansion tube in the Department of Mechanical Engineering at the University of Queensland, Brisbane, Australia. The major difference in operation between this facility and the Langley facility is the free-piston driver (Stalker, 1967). The first advantage of this type of driver is that higher driver temperatures can be achieved than with a conventional driver. Secondly, the temperature and pressure of the driver gas can be varied over a wide range by different choice of diaphragm rupture pressure and filling pressures. Thirdly, the driver is at approximately constant pressure during the shock/expansion tube flow rather than the driver being a constant volume.

Figure 1 shows the wave diagram for a free-piston driven expansion tube. The flow is in three stages. In the first stage the piston is driven down the compression tube by air at high pressure thus compressing the driver gas. The driver gas is chosen to have a high speed of sound. When the piston has imparted most of its energy to the driver gas the pressure of the gas is enough to rupture the primary diaphragm.

In the second stage the hot, high pressure driver gas flows into the shock tube causing a strong shock wave to be propagated down the tube through the test gas. As driver gas flows out of the driver tube the piston velocity is chosen to match this flow-rate and hence to maintain the driver pressure at an approximately constant level. An interface, or contact surface, separates the driver and test gases.

Upon the primary shocks arrival at the secondary diaphragm, which initially separates the test gas from the low pressure acceleration gas, the third stage of flow is initiated. The secondary diaphragm bursts and a strong shock wave propagates through the acceleration gas. An second interface separates the test gas and the acceleration gas. A shock wave may be reflected at the secondary diaphragm. The test gas expands through the strong isentropic centred expansion wave generated by the low gas pressure in the expansion tube, thus acquiring kinetic energy. This expanded test gas arrives at the end of the tube and flows into the test section.
Figure 2 shows the ideal pitot pressure time history at the test section. The acceleration tube flow causes the initial step in pitot pressure and the test gas causes the second much greater step (the magnitude of the step is greater because the temperature of the test gas is significantly less than the acceleration gas). The test period continues until the arrival of the tail of the strong expansion when the pitot pressure begins to ramp up (due to the decrease in Mach number).

2.2 Boundary Layer Entrainment Effect

The effect on shock tube flow of unsteady boundary layers which develop behind the primary shock wave have been studied by Mirels (1963) and (1964) for laminar and turbulent boundary layers. The effect of the boundary layer is to entrain fluid from the region between the primary shock and the interface (see Figure 3). This causes the shock wave and the interface to approach each other, reaching a maximum separation if the tube is long enough. It can be seen that the flow between the shock and the interface is non-uniform in shock-fixed coordinates. When the limiting separation has been reached the free-stream flow has a finite subsonic speed after processing by the (fixed) shock but the contact surface is stationary. Therefore the flow between the shock and the contact surface is non-uniform. As a first approximation the free-stream flow can be assumed to be uniform. This will be true exactly for strong shocks as the shock speed approaches infinity. To find the separation of the shock and the contact surface as a function of distance the approximation of a uniform free-stream can be made and the flow is further assumed to be steady at each instant. The shock is assumed to be strong with constant speed and hence each gas particle undergoes the same increase in entropy as it is processed by the shock. Mirels has derived expressions for the limiting separations and the separation function with distance for both laminar and turbulent boundary layers for a range of real and ideal gases.

This effect has important ramifications on expansion tube flow since it means that the time interval between incident shock and tail of expansion wave arrival at the test section will be decreased (Figure 4).

2.3 Real Gas Effects

Since high enthalpies are expected behind strong shock waves such as those generated in an expansion tube (up to 5 km/s in T2 acceleration tube section and about 2 km/s in shock tube section with helium driver - Paul, Stalker and Stringer, 1988) real gas effects such as vibrational excitation, dissociation and relaxation are expected to occur. However,
according to Trimpi (1962), less dissociation would be expected to occur than in a reflected shock tunnel. There is the possibility of the flow freezing while being expanded but this should not be significant due to the fact that, except for near the centre of the expansion wave, the expansion is spread over a significant proportion of the acceleration tube length as opposed to the relatively short length of a nozzle in a shock tunnel. Hence it would be expected that there would be time for the gas to relax to equilibrium.

Moore (1975) used two real air model to predict the wall static pressure and pitot-pressure at the test section of the Langley expansion tube as a function of interface velocities. The interface velocity was inferred from measurements of the incident shock wave and by using the theory of Mirels. The two models of air were firstly, thermodynamic equilibrium and secondly, vibrational and chemical freezing. The reflected shock wave from the secondary diaphragm was assumed to lie between the limits of being degenerate or of standing at the secondary diaphragm station. The measured wall static pressures agreed closely with the equilibrium model while the pitot-pressures were between the equilibrium and the frozen predictions. However, Miller (1975) found that predictions assuming equilibrium expansion for air with no reflected shock wave gave the best comparison with experiment.

2.4 Experimental Results from Expansion Tubes

Unsteadiness in Test Section Pitot-Pressure

Results from the Langley and the TQ expansion tubes both reveal unsteady pitot pressure effects showing variation of the acceleration tube pressure (Figure 5). The flow conditions in the University of Queensland facility were chosen to duplicate the Reynolds number, based on shock tube diameter, at the same shock velocities as in the Langley tube (Paul, Stalker and Stringer, 1988). The pitot pressure traces are similar except for the 'dip' phenomenon observed in the Langley tube (Moore, 1975; Miller, 1977 and Miller, 1978). It can be seen from the experimental results that when the acceleration tube pressure is increased, for a constant shock tube pressure, that the frequency of the pressure fluctuations increases. This suggests that there could be more than one mechanism causing fluctuations.

Shock Generated by Secondary Diaphragm Rupture

Ideally the secondary diaphragm which initially separates the test and acceleration tube gases, should be light and rupture instantaneously. However experiments by Shinn and Miller (1978) indicated that these
Conditions were very often not met in practice. They obtained from tube wall pressure transducers evidence that a shock wave was reflected from the secondary diaphragm and traveled upstream against the oncoming test gas flow. Subsequently the shock wave reflected from the interface between the driver and test gases. In some cases, the shock overtook the acceleration tube incident shock thus increasing wall pressures (see Figure 6). This effect was more pronounced when the secondary diaphragm was of greater thickness and when helium was used as a test gas. In the case of air and carbon dioxide test gases the shock wave was not strong enough to travel upstream and consequently was swept downstream by the oncoming test gas flow (Miller, 1975).

Boundary Layer Transition Effect

It was shown by Shinn and Miller (1978) that the reason for the dip in the pitot pressure of the Langley tube is due to the transition of the boundary layer behind the incident shock wave in the acceleration tube section.
3. LITERATURE REVIEW

3.1 Turbulence at the Interface and Development of Mixing Region

The interface between the driver and test gases in a shock tunnel is expected to be a region of high turbulence (Hooker, 1961) partly explained by non-ideal diaphragm rupture (White, 1958) and Rayleigh-Taylor instability (Taylor, 1950; Lewis, 1950; Lin and Fyfe, 1961). This turbulence leads to mixing of the driver and test gases. Because of mixing, less test gas will be available for expansion through the nozzle into the test section since the interface becomes a mixing region. This phenomenon is also relevant to the driver-test gas interface in an expansion tube since less test gas will be available for processing by the strong expansion and hence the test time will be shortened.

An early analysis to determine the conditions under which a mixing region developed was by White (1958). White considered equal amounts of driver and test gas (volume V/2), at different temperatures (T_D and T_B), mixing at the interface at constant pressure. Taking the limit where the temperature ratio across the interface, \( N = T_D / T_B \), was large, the change in volume of the interface could be determined. Making the assumption that the driver gas had a smaller molar specific heat, \( C_{P_D} \), (i.e. a monatomic gas) than the test gas, \( C_{P_B} \), an increase in volume was obtained when the driver gas was cooler than the test gas at the interface. The change in volume is given by,

\[
1 + \frac{\Delta V}{V} = \frac{1 + N}{2} \left( \frac{1 + C_{P_D}/C_{P_B}}{N + C_{P_D}/C_{P_B}} \right)
\]  

(1)

and for \( N \gg 1 \),

\[
1 + \frac{\Delta V}{V} = \frac{1}{2} \left( 1 + \frac{C_{P_D}/C_{P_B}}{} \right)
\]  

(2)

This situation occurs in conventional shock tubes where there is no preheating of the driver gas, and in free-piston driven facilities for some conditions. It should be noted that the higher the primary shock Mach number the hotter the test gas in relation to the driver gas and hence the more spread out the mixing region. The flow between the incident shock wave and the interface will be affected by this change in contact region volume, which can be thought of as an increase in effective "piston" velocity. In
the other limit where the expanded driver gas is much hotter than the test gas a decrease in volume of the mixing region would be expected.

Lin and Fyfe (1961) showed by dimensional arguments that the eddy diffusivity, which controls the spreading rate of the mixing region, was proportional to primary diaphragm diameter.

3.2 Rayleigh-Taylor Instability

Taylor (1950) and Lewis (1950) showed theoretically and experimentally that "...when two superposed fluids of different densities are accelerated in a direction perpendicular to their interface, this surface is stable or unstable according to whether the acceleration is directed from the heavier to the lighter fluid or vice-versa." The amplification and suppression of interface instability is shown in Figure 7. This phenomenon is known as Rayleigh-Taylor instability of accelerated interfaces and is applicable in shock and expansion tube flow to the driver/test gas interface.

3.3 Conditions for Rayleigh-Taylor Instability in Shock Tubes

An analysis was carried out by Levine (1970) who assumed that Rayleigh-Taylor instability of the driver/test gas interface caused a reduction in available test gas in a shock tube. A density gradient was produced by the mixing of cold driver gas with hot test gas at the interface in different proportions assuming constant pressure. A minimum density was found since the driver gas has a smaller average molecular weight than the test gas. This meant that the density of some of the gas in the mixing region was less than the hot gas sample and the driver gas. The acceleration field required to accelerate the less dense gas was provided by relaxation effects in an ionized monatomic test gas behind a strong shock wave. The test gas ionised a certain time after being processed by the primary shock wave, resulting in a reduction in temperature and an increase in density and hence, by continuity, an acceleration (Figure 8).

Levine used a semi-empirical approach to determine the mixing rate at the interface and hence the minimum density and the resulting test gas sample size. He derived an equation of motion for a 'blob' of light gas projected ahead of the contact surface in the presence of a heavier test gas. A simplifying assumption was made that the ratio of less to more dense gas remained constant during the period of the shock tube flow. From this he determined whether a test gas sample was likely to accumulate or not for given shock tube conditions. Gas at density $\rho_{test}$ is buoyant in fluid of
density $\rho_{ax}$ under pseudo-gravitational field $g$ where $v_p$ is the velocity at which fluid is propelled ahead of the contact surface. The equation is,

$$\rho_{to}: \frac{dv_p}{dt} = (\rho_{ax} - \rho_{p:} - g) \quad (3)$$

Houwing, Hornung and Sandeman (1981) and Houwing and Sandeman (1983) investigated Rayleigh-Taylor instability of an interface in shock tube flow similar to the case of Levine. They showed that less dense "blobs" can occur under two conditions. Firstly when the driver gas was less dense than the test gas or, as in the case of Levine, when the driver and test gases were mixed. Density profiles as a function of the proportion of driver gas are shown in Figure 9 and are reproduced from Houwing, Hornung and Sandeman (1981). In both cases the test gas temperature was greater than that of the driver gas. Houwing and Sandeman make the statement that if the ratio of the minimum density to the test gas density is calculated using the same method as Levine it is approximately equal to the ratio of average molecular weights across the interface.

Houwing, Hornung and Sandeman considered acceleration fields caused firstly by relaxation effects, due to vibrational non-equilibrium and dissociation behind the primary shock wave, and secondly from boundary layer mass entrainment effects. Only the mass entrainment effect is considered here since real gas effects are not expected to be as significant in expansion tube flow and will not be taken into account in this analysis.

Houwing et al. (1981) and (1983) derived a more complete equation of motion for the blobs than Levine by including the virtual mass of the buoyant sphere. The equation of motion follows that derived by Batchelor (1967) and is reproduced from Houwing and Sandeman (1983),

$$M \frac{du_p}{dt} = M_1 \frac{du_2}{dt} - \frac{1}{2} M_2 \frac{d(u_p - u_2)}{dt} \quad (4)$$

where $\rho_1$ is the density and $u_2$ is the velocity of a non-deforming sphere in a frictionless accelerating fluid of density $\rho_2$ and velocity $u_2$. Here $M$ is the mass of the sphere and $M_1$ is the mass of the fluid displaced. The blobs are assumed to be typical of a large number of particles which comprise the mixing region. When the sphere distorts to conform to the enveloping streamlines, as in the actual flow, the buoyant gas acts like a continuum. The equation of motion is then integrated to obtain the blob velocity as a
function of distance downstream of the diaphragm station with the lower limit that the blobs have the same velocity as the contact surface immediately after diaphragm rupture. It is assumed that the flow is steady and that the free-stream velocity decreases monotonically with distance from the shock wave.

Boundary layer entrainment will cause the interface to accelerate due to removal of gas from the region of flow behind the primary shock wave. Hence if blobs which are less dense than the test gas have been generated by interface mixing then a mechanism exists for accelerating some of the interface gas more than the test gas.
4. MECHANISMS CAUSING EARLY PRESSURE FLUCTUATIONS

4.1 Equations of Motion of a Minimum Density Blob

This section discusses pitot-pressure fluctuations caused by blobs of light gas. Due to Rayleigh-Taylor instability of accelerated interfaces blobs of gas, of a lower density than the test gas, can be generated by mixing at the interface. These blobs tend to accelerate more rapidly than the surrounding test gas, in the direction of the acceleration. Hence in the acceleration field of the strong expansion they overtake the test gas and have the potential to arrive at the test section during the period of useful test flow causing pressure fluctuations (see Figure 10). As mentioned above there are two ways of generating lower density blobs. Firstly if the driver gas is less dense than the test gas blobs of driver gas will be buoyant in the test gas; and secondly by mixing in different proportions a cold monatomic driver gas with a hot diatomic test gas, where the driver gas has a smaller average molecular weight than the test gas, a blob with a density less than that of both gases can be produced.

The mechanism is implemented in three stages. Firstly the driver and test gases mix generating less dense blobs of gas. Secondly the blobs separate from the contact surface in the shock tube flow region, due to Rayleigh-Taylor instability, and are propelled forward of the test gas by the boundary layer entrainment effect in the shock tube region. Thirdly the blobs are propelled forward by the strong expansion in the acceleration tube region. The mixing model of Levine was used for the generation of the blobs at the interface. In the shock tube the equations used were similar to those of Hauwing and Sandeman. New equations are developed for flow in the strong expansion region.

Generation of Density Minimum

The minimum density due to mixing at the interface is derived below.

- Conservation of Energy

\[ m_d h_d + m_c h_c = mh \]  
\[ \alpha \left( \frac{5}{2} R_d T_d + (1 - \alpha) \frac{9}{2} R_t T_t \right) = h \]  
\[ \alpha = \frac{m_d}{m_d + m_c} \]
\[ R_d = \frac{R}{R_t} \]
where
\( m \) = denotes driver gas
\( \dot{m} \) = denotes test gas
\( m \) = mass
\( h \) = static enthalpy
\( R \) = engineering gas constant
\( T \) = static temperature
\( \sigma \) = driver mass fraction
\( \sigma_0 \) = universal gas constant
\( \mu \) = molecular weight

-Enthalpy of Mixture at Interface (average translational and rotational kinetic energy)

\[
\frac{h_m}{m} = \frac{n_1 \left( \frac{3}{2}RT + 3R \tau \right) + n_2 \left( \frac{3}{2}RT \right)}{m_1 + m_2}
\]

\[
= \sigma \frac{5}{2} R_\sigma T + (1 - \sigma) \frac{7}{2} R_\sigma T
\]

\[
T = \frac{5\sigma R_\sigma T_\sigma + 7(1 - \sigma) R_\tau T_\tau}{5\sigma R_\sigma + 7(1 - \sigma) R_\tau}
\]

-Equation of State

\[
\rho = \frac{\frac{m_2 + m_1}{V}}{\left( \frac{\sigma R_\sigma + (1 - \sigma) R_\tau}{T} \right)}
\]

\[
\rho = \frac{p \left( (5R_\sigma - 7R_\tau) \sigma + 9R_\tau \right)}{(R_\sigma - R_\tau) \alpha + R_\tau \left( (5R_\sigma T_\sigma - 7R_\tau T_\tau) \alpha + 7R_\tau \tau \right)}
\]

where
\( \rho \) = static pressure
\( \rho_1 \) = static density

-Density Ratio

\[
\frac{\rho}{\rho_1} = \frac{\left( \frac{\mu_1}{\mu_2} - \gamma \right) \alpha + 7}{\left( \frac{\mu_1}{\mu_2} - 1 \right) \alpha + 1 \left( (5 \frac{T_\sigma}{T_\tau} \mu_1 \mu_2 - \gamma) \alpha + 7 \right)}
\]

when \( \alpha \to 0 \) then \( \frac{\rho}{\rho_1} \to 1 \)

when \( \alpha \to 1 \) then \( \frac{\rho}{\rho_1} \to \frac{T_\sigma}{T_\tau} \frac{\mu_2}{\mu_1} \)
Hence for an ideal gas the density minimum depends on the ratio of molecular weights and the temperature ratio across the interface, assuming monatomic driver gas and diatomic test gas.

-Minimum Density Ratio, obtained by differentiating (12),

\[
\alpha = -\frac{b}{a} = \sqrt{\left(\frac{b}{a}\right)^2 - \frac{bd}{ac} - \frac{bf}{ae} - \frac{df}{ce}}
\]

where

\[
a = 5 \frac{W_1}{W_2} - 7
\]

\[
b = 7
\]

\[
c = \frac{W_1}{W_2} - 1
\]

\[
d = 1
\]

\[
e = 5 \frac{T_2}{T_1} \frac{W_1}{W_2} - 7
\]

\[
f = 7
\]

In a real gas an increase in \(C_v\) due to vibration and dissociation will produce a lower minimum density than for an calorically ideal gas.

- Acceleration of Blobs and Interface during Expansion Tube Flow

It is assumed that no heat is transferred to the blobs from the test gas during their flight. An analysis was performed to determine the maximum blob size which could be heated significantly during the period of the shock tube flow. The heated blobs were found to be too small to be important with a diameter less than one thirtieth of the expansion tube diameter.

The blobs are assumed to be in mechanical equilibrium with the test gas during the period of their flight through the test gas, i.e. at the same pressure. Thus when the test gas pressure changes due to the expansion wave the blob properties change accordingly, assuming no heat transfer, to keep them at the same pressure as the surrounding test gas.

Following Batchelor,

\[
M \frac{U}{2} = -M_c \frac{U}{2} (U - V) + M_c V
\]

where \(M\) is the mass of the sphere, \(M_c\) is the mass of the fluid displaced by the sphere, \(U\) is the velocity of the sphere, and \(V\) is the velocity the
surrounding fluid would have had if the sphere was not present. The first term represents the acceleration of the sphere, the second represents the acceleration reaction of the displaced fluid on the sphere and the third represents the buoyancy force. Rearranging and taking differentials one obtains,

$$\frac{dU}{V} = \frac{\frac{3}{2} M_c}{M + \frac{1}{2} M_c} \frac{dV}{V}$$

for a small change in the velocity of the sphere as a function of a small change in velocity of the surrounding fluid. It can be seen that when $M/M_c < 1$ that $dU > dV$ and hence if this model was applied to blobs of less dense gas generated at the interface then they would accelerated more quickly than the surrounding test gas. The equation of motion can be integrated one mesh step at a time taking local values of $V$ and $M/M_c$.

**Effect on Pitot Pressure**

The effect of blobs on the test section pitot-pressure, is expected to be fluctuations due to the difference in temperature and density of the blobs compared to the test gas. The frequency of the fluctuations is expected to relate to the most probable blob size. Thus the presence of blobs means the pitot-pressure trace will display a contact region spread over a considerable time period instead of a sharply defined interface.

**4.2 Reflection of Waves from the Contact Surface**

Another possible mechanism for producing pitot-pressure perturbations is now discussed. Under some circumstances the strong expansion through which the test gas expands, after reflecting from the driver-test gas interface, can arrive at the test section during the test period. Since the interface is expected to be a region of high turbulence due to non-ideal diaphragm rupture there is the potential for unsteady pressure perturbations to be propagated along the characteristics of the reflected expansion and hence to disrupt conditions at the test section during the test period. The effect of the reflected expansion on the pitot pressure trace is shown in Figure 11. It can be seen that the pitot pressure falls, until the arrival of the contact surface, rather than rises as in the case where the reflected expansion does not arrive at the test section. This is due to the reversal of the velocity gradient. Unsteady effects which exist at the interface can then be propagated along the characteristics of the reflected expansion. It should be noted that the trajectory of the reflection of the head of the strong expansion can be determined analytically.
If small perturbations of the flow properties, generated at the contact surface, are assumed this is equivalent to having another two families of physical characteristics and another two families of state characteristics corresponding to the perturbations of the gas properties. Mirels and Braun (1962) solved the problem of the propagation of small perturbations in uniform and self-similar flows. In their cases the physical characteristics were coincident for both the perturbed and unperturbed components of the state properties. Hence the magnitude of the perturbations of the state-variables could be integrated along characteristics in the expansion wave, since it was self-similar, and the pitot pressure fluctuations calculated. The magnitudes of the fluctuations depended on the turbulence at the interface. However in this analysis only the time of arrival of pressure perturbations is sought so the magnitude of the perturbations is not required.

As found from the Langley experiments an upstream propagating shock wave can be generated by the rupture of the secondary diaphragm. An estimate of the effect of this shock wave on the test section flow can be obtained by noting that the trajectory of a very weak shock wave is the same as that of the reflected head of the strong expansion (Figure 12). Thus an approximation to the time of arrival of such a shock wave can be gained by finding the time at which the reflected head of the strong expansion arrives at the test section.

Another possible effect of the reflected shock wave is that after it has been transmitted through the driver-test gas interface bifurcation may occur. Bifurcation occurs when the tube wall boundary layer stagnation pressure is not great enough to allow it to be decelerated by a normal shock and hence oblique shocks form and gas collects at the foot causing it to grow with time (Figure 13). This means that a jet of gas can be generated on the walls of the tube, formed by the oblique shock waves, which has a greater velocity towards the test section end of the tube than does the gas processed by the normal shock wave. Thus driver gas can arrive at the test section earlier than expected. This mechanism has been examined by Davies and Wilson (1969) and others. It will not be pursued here.

It should be noted that no pitot-pressure perturbations occurred in the Langley tube without the presence of a secondary diaphragm (Shinn and
Miller, 1978). Hence the secondary diaphragm must be important in the generation of pitot-pressure fluctuations.
5. IMPLEMENTATION OF SOLUTION

The method of characteristics for unsteady flow in one dimension has been used to predict the flow in the expansion tube assuming perfect gases. The effect of boundary layer entrainment has been included approximately by calculating new trajectories for the driver-test and test-acceleration gas interfaces. The effect of the entrainment on the free-stream flow has not been considered; this is known as the uniform free-stream approximation. The pitot pressure has been predicted as a function of time at the test section by the Rayleigh pitot pressure formula with an empirical correction being employed to account for the higher predicted shock speeds than those measured in experiment.

5.1 Basic Assumptions

The gases are all assumed to be thermally and calorifically perfect and in thermodynamic equilibrium. In the expansion tube flow ideal diaphragm rupture has been assumed. The free-piston driver is treated as a constant pressure reservoir with the conditions calculated using isentropic compression of the driver gas. The Mirels boundary layer entrainment effect has been included assuming the uniform free-stream approximation for the contact surface trajectories. Primary shock waves have been assumed to have constant velocity and hence no entropy variation exists for different particles of gas. The latter two assumptions are both applicable for strong shock waves. At the interface mixing occurs adiabatically and isobarically in an initial thin contact surface. The blobs of low density gas generated are small, non-deforming spheres in mechanical equilibrium with the surrounding gas flow and are typical of a large number of such which make up the mixing front. The test section flow is assumed to be quasi-steady for the pitot pressure determination.

5.2 Computer Program

The finite difference equations for the method of characteristics for one-dimensional unsteady flow are given in Appendix A. The method was implemented on a Apple Macintosh Plus Personal Computer in compiled BASIC. The method uses a combined graphical-numerical approach. The computer implementation is interactive and the procedure is similar to that required if the wave diagram were to be constructed on graph paper, except the machine does all the calculations and the 'house-keeping'. A flow chart of the program logic is shown as Figure 14. The program waits for the user to select from the menu the next type of point he wishes to calculate, for example: 'Interior', 'Contact', or 'Expansion'. Once the user has defined this he then selects, using the mouse, the existing points from which he
wants the new point to be calculated. The computer then calculates the new point and displays its location on the screen. The properties at a point can be perused at any time by the user. A database is generated on disc as calculation proceeds so that the solution can be regenerated or added to at a later date. The program listing can be found in Appendix B.

When calculating the wave diagram it becomes necessary to refine the mesh if flow properties are changing rapidly. In this case the program has a facility for 'splitting' the mesh by linear interpolation of properties between known points. This raises the problem of how to save the data for each point in the database such that it can be retrieved and the flowfield reconstructed correctly. The storage of data adopts a method of interrelating records known as linked records. Stored with the values of the properties at each point are two numbers. These numbers give the numbers of the records where the properties of the two upwind points on which the point depends are stored. It is easy therefore to split the mesh and to change the way the records are linked when a new intermediate point is created.

5.3 Verification of Computer Code and Truncation Error

The computer code was checked by calculating the trajectory of the contact surface through the expansion fan when the same gas at the same conditions is on either side. This is the same as calculating a particle path. The three families of characteristics give,

\[
\frac{dt}{dx} = \frac{1}{u - a}
\]

\[
\frac{u}{2} \left( \frac{a}{\gamma - 1} \right) = \frac{u_i}{2} + \frac{a_i}{\gamma - 1} = \frac{u_j}{2} + \frac{a_j}{\gamma - 1}
\]

\[
\frac{dt}{dx} = \frac{1}{u_i}
\]

\[
x = \frac{t_i}{1 - \gamma} \left[ a \left( 1 + \gamma \left( \frac{t_i}{t_j} \right)^{\gamma} - 2 \left( a_i + \frac{\gamma - 1}{2} u_i \right) \right) \right]
\]

The numerical solution to the wave diagram is given as Figure 15. The analytical solution for the path line is exactly coincident to the numerical solution to the resolution of the diagram.

The compatibility relations of the method of characteristics depend on the mesh and so approximations must be made in computing flow properties. Prior
to use of this procedure, the point properties are assumed to vary in a polynomial fashion along characteristics between the known and unknown points. The order of the polynomial variation can be selected according to the desired accuracy required of the solution. A method of improving these inherent approximations is to use a mesh size which is appropriate for the level of accuracy required. The average value of the properties was used for calculation of the physical characteristics hence the accuracy of the mesh is of the order of \((Ax)^3\) and \((At)^3\). For calculations of flow properties on the contact surface average values were also used but iteration was required hence the maximum accuracy expected, after convergence, is of the order of \((Au)^3\) and \((Ap)^3\). The calculation of flow properties at other points is exact.
6. COMPARISON OF COMPUTATIONS WITH EXPERIMENT

6.1 Shock Speed
The predicted shock speeds are up to thirty percent higher than the measured ones. (All the following experimental results are taken from Paull, Stalker and Stringer, 1988.) This was accounted for in the pitot-pressure prediction by the use of an empirical correction factor.

6.2 Langley Results
As the acceleration tube pressure is increased the model predicts that unsteady effects, due to the reflected expansion, should arrive earlier. Blobs are predicted but they arrive very much later than in the useful test time and so are not relevant. There is evidence of another unsteady effect at the lower acceleration tube pressures possibly due to waves being reflected from the walls of the tube. The dip noted in the case with the highest shock tube pressure is due to boundary-layer transition in the acceleration tube.

The reflected expansion trends compare favourably to reflected shock trends as determined by wall pressures measurements (Shinn and Miller (1978)). Hence the reflected head of the expansion predicts the reflected shock behaviour at least qualitatively.

6.3 U.Q. Argon Driver Results
As the acceleration tube pressure is increased the model predicts that unsteady effects, due to the reflected expansion, should arrive earlier. Blobs are not predicted. There is evidence of another unsteady effect at the lower acceleration tube pressures possibly due to waves being reflected from the walls of the tube.

No blobs are predicted for any case with an argon driver. (For an ideal gas the density minimum depends on the ratio of molecular weights and the temperature ratio across the interface, assuming monatomic driver and diatomic test gas).

The absence of the dip phenomenon can be explained by the fact that boundary layer transition would not be expected from Reynolds number calculations based on the acceleration tube length of TQ.

6.4 U.Q. Helium Driver Results
Taking the column of results for which the acceleration tube pressure is approximately 120 mm it can be seen for lower shock tube pressures the
reflected expansion arrives before the blobs while for the higher shock tube pressures the blobs arrive before the reflected expansion. The blobs arrive latest for the central case ($p_1 = 13.8 \text{ kPa}$), while the reflected expansion arrives latest for the $p_1 = 101 \text{ kPa}$ case. It can also be seen that the blobs tend to produce large scale pitot pressure fluctuations while the reflected expansion causes fluctuations on a smaller scale.

Considering holding shock tube pressure constant while varying the acceleration tube pressure, an increase in acceleration tube pressure causes both the blobs and the reflected expansion to arrive earlier. This agrees with the Langley and argon driver predictions (for the reflected expansion). These effects can be seen by considering either the top row or the bottom row of the array.

(It should be noted that for the case in the extreme upper right corner of the array that the expansion reflected from the driver-test gas interface is predicted to further interact with the test-acceleration gas interface. This effect was not included in the model and hence this prediction is less certain. What is certain is that the reflected expansion arrives very early.)

6.5 U.Q. Air Driver Result

There were no blobs predicted for the case with an air driver and although the reflected expansion is predicted to arrive reasonably early the fluctuations are not sufficient to degrade to a serious extent the relatively long period of test flow found in this case.

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7. CONCLUSIONS

The model developed here explains some of the previously unexplained features of expansion tube flow tolerably well. It also indicates that the two mechanisms considered are pressure independent, except for a small pressure dependence of the boundary layer entrainment effect. Therefore either scaling the initial pressure filling ratios either up or down should produce flow with the same characteristics. Hence the initial filling pressure ratios that produce the longest period of test flow can be obtained. Therefore no additional work is required to determine the best pressure ratios for higher absolute pressure conditions.
8. REFERENCES


9. FIGURES
Figure 1: Wave diagram of ideal expansion tube.

- Driver gas
- Contact surface
- Primary shock wave
- Unsteady expansion
- Test gas
- Primary diaphragm
- Secondary diaphragm
- Driver section
- Shock tube section
- Acceleration tube section
- Distance
Figure 2: Ideal pitot-pressure time-history at the test section.
Figure 3: Mirel's boundary layer entrainment effect.
Figure 4: Entrainment effect on expansion tube flow.
Figure 5: Typical measured pitot-pressure time-histories.
Figure 7: Rayleigh-Taylor instability of accelerated interfaces.
Figure 8: Wave diagram of development of mixing region.
Figure 9: Profiles showing the minimum density in the mixing region.
Figure 10: Wave diagram of time of arrival of blob at test section.
Figure 11: Ideal pitot-pressure trace showing effect of reflected head of expansion.
Figure 12: Wave diagram of reflected head of expansion.
Figure 13: Reflected shock bifurcation.
Figure 14: Computer program flow chart.
Figure 15: Analytical particle trajectory.
Langley Expansion Tube

$P_i = 3.4 \text{kPa},$

Helium driver.

Figure 16 (a) & 17 (a): Langley (helium) and TC (argon)
pitot-pressures and predictions. (Note ‘RE’ = arrival time
of reflected expansion).

TQ Expansion Tube

$P_i = 13.7 \text{kPa},$

Argon driver.

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Figure 16 (a) & 17 (a): Langley (helium) and TQ (argon) pitot-pressures and predictions. (Note 'RE' = arrival time of reflected expansion).
Figure 16 (b): Langley wave diagram, $p_i = 15 \mu\text{Hg}$. 
Figure 16 (c): Langley wave diagram, $p_i = 60 \mu\text{Hg}$.
Figure 16 (d): Langley wave diagram. P$_i$ = 180 $\mu$m Hg.
Figure 17 (c): TQ wave diagram, argon. \( p_1 = 3.5 \) kPa,

\[ P_2 = 250 \mu \text{m Hg}. \]
Figure 17 (d): TQ wave diagram, argon, $P_0 = 13.7$ kPa.

$P_i = 500 \mu\text{m Hg.}$
Figure 17 (e): TC wave diagram, argon, $p_i = 13.7$ kPa,

$P_i = 2000$ $\mu$Hg.
Helium driver, $\lambda = 29$, $P_r = 34.5$ MPa.

Figure 18 (a): TQ pilot pressures and predictions for helium test gas.
Figure 18 (a): TC pitot-pressures and predictions for helium test gas.
Figure 18 (c): TG wave diagram, helium. p. = 101.0 kPa.

p.1 = 30 μm Hg.
Figure 18 (e): TC wave diagram, helium, $p_i = 3.5$ kPa.

$P_{it} = 120 \mu$Hg.
Figure 18 (f): TO wave diagram, helium, \( p_i = 7.0 \) kPa,
\( p_f = 120 \) \( \mu \)torr Hg.
Figure 18 (g): 70 wave diagram, helium, $p_i = 13.8$ kPa,

$p_e = 120$ $\mu$Hg.
Figure 18 (b): TZ wave diagram, helium, $P_1 = 101.0$ kPa,
$P_2 = 150 \mu$Hg.
Figure 18 (j): TC wave diagram, helium, $p_i = 101.0$ kPa,
$P_i = 2010$ $\mu$Hg.
$\text{P}_{\text{air}}$ = 0.055 atm

\text{RE} = \text{REFLECTED EXPANSION}

Figure 19 (a): TO pilot-pressures and predictions for air test gas.
Figure 19 (b): TQ wave diagram, air.
A. Complete Set of Finite Difference Equations

A.1 Non-Dimensionalisation of Variables

The reference conditions chosen for the wave diagram are the acceleration tube length, the diaphragm rupture pressure and the speed of sound in the driver gas prior to expansion.

A.2 Equations for Ideal Expansion Tube Flow

Shock Tube Section Flow

References: Stalker (1964)
Liepmann and Roskho (1957)

\[
\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \left[ \frac{\gamma + 1}{2} \sqrt{\frac{(\gamma - 1)(\frac{A_r}{A_d})(\frac{p_2}{p_1} - 1)}{\gamma(2\gamma + (\gamma + 1)(\frac{p_2}{p_1} - 1)}}} \right]^{\frac{-2\gamma}{\gamma - 1}}
\]

\[
\frac{\rho_1}{\rho_2} = \frac{p_2}{p_1} \frac{\gamma - 1}{\gamma + 1} \frac{P_1}{P_2}
\]

\[
\frac{T_1}{T_2} = \frac{1}{\gamma} \left( \frac{P_1}{P_2} \right)^{\frac{\gamma - 1}{\gamma}}
\]

\[
\frac{T_1}{T_2} = \frac{1 + \frac{\gamma - 1}{\gamma + 1} \frac{P_1}{P_2}}{1 + \frac{\gamma - 1}{\gamma + 1} \frac{P_1}{P_2}}
\]

\[
M_2 = \frac{1}{\gamma} \left( \frac{P_2}{P_1} - 1 \right) \left[ \frac{P_2}{P_1} \left( \frac{\gamma + 1}{2\gamma} \right) + \frac{\gamma - 1}{\gamma} \frac{P_1}{P_2} \right]^{\frac{\gamma - 1}{\gamma}}
\]

\[
M_3 = \frac{2}{\gamma - 1} \left[ \frac{P_2}{P_1} \frac{\gamma - 1}{\gamma} \sqrt{\frac{2}{\gamma} \left( \frac{\gamma + 1}{2} - 1 \right)} \right]
\]

\[
A_2 = \sqrt{\gamma R_1 T_2}
\]

\[
A_3 = \sqrt{\gamma R_4 T_3}
\]

\[
U_2 = M_2 a_2
\]

\[
U_3 = M_3 a_3
\]

\[
\frac{p_2}{p_1} = \frac{T_2}{T_1} \frac{\rho_2}{\rho_1}
\]

\[
\frac{p_2}{p_1} = \frac{T_3}{T_2} \frac{\rho_2}{\rho_1}
\]
A.3 Mirels Effect for Laminar or Turbulent Boundary Layers

Laminar

Reference: Mirels (1963)

The acceleration tube flow is laminar for TQ and partly laminar for Langley. Therefore assume that the maximum separation of the shock and the contact surface has been reached. This only has a cosmetic effect on the wave diagram in the acceleration tube region. It does not affect the results. The effect on test gas, and blobs, is difficult to determine.
due to the expansion wave thickness and the complex nature of the boundary layer (see Mirels and Mullen, 1964).

\[
\frac{\ln 2}{L_2} = \left(1 + \frac{1}{4k}\right)^2 \frac{P_{20}}{P_{21}} \frac{Pr_{20}/P_{21}}{\rho_{20}/\rho_{21} - 1} M_{2zC} \frac{Pr_{20}/Pr_{21}}{\mu_{20}/\mu_{21}} \left(\frac{d}{L_2}\right)^2 \frac{p_{20}}{p_{21}} \frac{p_c}{p_2} \frac{p_c}{p_2}
\]

\[
M_{2zC} = \frac{u_{2zC}}{c_2} \sqrt{\frac{\gamma_{21} R_{20} T_2}{\gamma_{21} R_{21} T_1}}
\]

<table>
<thead>
<tr>
<th>(M_2)</th>
<th>(\beta_1) (Air)</th>
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<tr>
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</table>

\[-\frac{X_2}{2} = \ln (1 - T_2^n) + T_2^n, \quad n = \frac{1}{2}\]

\[
X_2 = \frac{u_{2zC} T_2}{(P_{20}/P_{21}) L_{2n}}
\]

\[
T_2 = \frac{I_2}{L_{2n}}
\]

\[
\ln \left[1 - \left(\frac{u_{2zC} \left(T_{20} - \frac{1}{u_{2z2}} \frac{L_2}{L_1}\right) - 1}{I_{2n}}\right)^{1/2}\right] + \left[u_{2zC} \left(T_{20} - \frac{1}{u_{2z2}} \frac{L_2}{L_1}\right) - 1\right]^{1/2} + \frac{u_{2zC} \left(T_{20} - \frac{1}{u_{2z2}} \frac{L_2}{L_1}\right) \rho_{21}}{2 \frac{I_{2n}}{\rho_{21}}} = 0
\]

The limiting separation approximation used in the acceleration tube is given by,

\[
T_{20} = \frac{1}{u_{2z2}} + \frac{1}{u_{2z2}} \frac{L_2}{L_1}
\]

Turbulent

Reference: Mirels (1964)
The shock tube flow is turbulent. The Mirels effect also effects the blob trajectory. The limiting separation is not reached in the shock tube length.

\[ \frac{1}{L_2} = \left( \frac{1}{4\beta_1} \right)^{5/4} \frac{P_0}{P_1} \frac{P_2/P_1}{\mu_1} M_2^{1/4} \left( \frac{P_2/P_3}{\mu_2} \right)^{1/4} - 4 \quad \frac{\omega}{L_2} \left( \frac{P_0}{P_1} \right)^{5/4} \left( \frac{P_2/P_3}{\mu_2} \right)^{1/4} \]

\[ M_2 = u_{s2} \sqrt{\frac{\gamma R_1 T_1}{\gamma R_2 T_2}} \]

\[ \beta_1 = \beta_0 \left( \frac{(P_2/P_1)^2 + 1.25(P_2/P_1) - 0.80}{(P_2/P_1) ((P_2/P_1) - 1)} \right) \]

<table>
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\[ - \frac{X}{2} = \frac{5}{8} \left( \ln \frac{1 - T_0}{1 + T_0} - 2 \arctan T_0 + 4T_0 \right), \quad n = \frac{1}{5} \]

\[ X = \frac{u_{s2} T}{(P_2/P_1) L_2} \]

\[ T = \frac{1}{L_1} \]

\[ \left( \frac{5}{8} \ln \left\{ \frac{1 - \left[ \frac{X_0}{L_1} \left( \frac{u_{s2}}{U_2} - A_2 - 1 \right) \right]}{\left[ 1 + \frac{X_0}{L_1} \left( \frac{u_{s2}}{U_2} - A_2 - 1 \right) \right]} \right\} - 2 \arctan \left[ \frac{X_0}{L_1} \left( \frac{u_{s2}}{U_2} - A_2 - 1 \right) \right] \right)^{1/5} \]

\[ + 4 \left[ \frac{X_0}{L_1} \left( \frac{u_{s2}}{U_2} - A_2 - 1 \right) \right]^{1/5} + \left[ \frac{\rho_2}{\rho_1} \left( \frac{u_{s2}}{U_2} - A_2 \right) \right] L_1 = 0 \]

\[ t_0 = \frac{X_0}{U_2 - A_2} + \frac{1}{u_{s2}} \frac{L_1}{L_2} \]
A.4 Blob Trajectories including Mirels Effect

\[ x = \frac{u_s t'}{(\rho_2/\rho_1)^{1/5}} \left( \frac{3R}{2 + R} \right)^{1/5} \]

\[ T = \frac{1}{\lambda_m} \left( \frac{3R}{2 + R} \right)^{1/5} \]

\[ R = \frac{\rho}{\rho_{min}} \]

\[ v = \left( \frac{3R}{2 + R} \right)^{5/4} \]

\( l' \) = mixing front separation from shock wave

\[ \frac{5}{8} \ln \left\{ \frac{1 - \left[ \frac{X_{GB} V}{\lambda_m} \left( \frac{u_{s2}}{U_2 - A_2} - 1 \right) \right]^{1/5}}{1 + \left[ \frac{X_{GB} V}{\lambda_m} \left( \frac{u_{s2}}{U_2 - A_2} - 1 \right) \right]^{1/5}} \right\} - 2 \arctan \left[ \frac{X_{GB} V}{\lambda_m} \left( \frac{u_{s2}}{U_2 - A_2} - 1 \right) \right]^{1/5} + 4 \left[ \frac{X_{GB} V}{\lambda_m} \left( \frac{u_{s2}}{U_2 - A_2} - 1 \right) \right]^{1/5} \left[ \frac{p_1}{\rho_2} \frac{u_{s2} V}{2\lambda_m} \left( \frac{X_{GB}}{U_2 - A_2} + \frac{1}{u_{s2} L_2} \right) \right] = 0 \]

\[ t_{GB} = \frac{X_{GB}}{U_2 - A_2} + \frac{1}{u_{s2} L_2} \]

A.5 The Unsteady Method of Characteristics

Reference: Ferri (1961)

\[ \frac{\delta p}{\delta x} + u \frac{\delta p}{\delta x} + \rho a^2 \frac{\delta u}{\delta x} = 0 \]

\[ \frac{\delta p}{\delta x} + \rho u \frac{\delta u}{\delta x} + \rho \frac{\delta u}{\delta x} = 0 \]

\[ \Delta S = 0 \]

equation of state

\[ p = \rho RT \]

definition of speed of sound for a perfect gas

\[ a = \sqrt{\left( \frac{\delta p}{\delta p} \right)_T} = \gamma RT \]
physical characteristics

along first family, \( \frac{dx}{dt} = u + a \)

along second family, \( \frac{dx}{dt} = u - a \)

state characteristics

along first family, \( \frac{dp}{dt} + \rho a \frac{du}{dt} = 0 \)

along second family, \( \frac{dp}{dt} - \rho a \frac{du}{dt} = 0 \)

Interior Points

\[
x_3 = \frac{t_1 - t_2 + x_3(u_2 - a_2 + u_3 - a_3) - x_3(u_1 + a_1 + u_3 + a_3)}{2(u_2 - a_2)(u_3 - a_3) - \frac{u_1 + a_1 + u_3 + a_3}{2(u_2 - a_2)(u_3 - a_3) - 2(u_1 + a_1)(u_3 + a_3)}}
\]

\[
t_3 = \frac{x_1 - x_2 + \frac{2t_2(u_2 - a_2)(u_3 - a_3) - 2t_2(u_1 + a_1)(u_3 + a_3)}{2(u_2 - a_2)(u_3 - a_3) - \frac{2(u_1 + a_1)(u_3 + a_3)}{2(u_2 - a_2)(u_3 - a_3) - 2(u_1 + a_1)(u_3 + a_3)}}}{2(u_2 - a_2)(u_3 - a_3) - \frac{2(u_1 + a_1)(u_3 + a_3)}{2(u_2 - a_2)(u_3 - a_3) - 2(u_1 + a_1)(u_3 + a_3)}}
\]

\[
u_3 = \frac{u_1 + a_1 + a_2 - a_3}{\gamma - 1}
\]

\[
a_3 = \frac{(\gamma - 1)(u_1 - u_2)}{4} + \frac{a_1 + a_2}{2}
\]

For driver point:

\[
P_3 = P_1 \left( \frac{a_3}{a_1} \right)^{\frac{2\gamma}{\gamma - 1}}
\]

\[
T_3 = a_3^2
\]

\[
\rho_3 = \frac{P_3}{T_3}
\]

For test gas point:

\[
P_3 = P_1 \left( \frac{a_3}{a_1} \right)^{\frac{2\gamma}{\gamma - 1}}
\]

\[
T_3 = a_3^2 \frac{\gamma P_3}{\rho_3 R_1}
\]
Expansion Wave Points

\[ c_3 = \frac{-(\theta + \mu)}{100} \left( \arctan \left( \frac{1}{U_2 - A_2} \right) - \arctan \left( \frac{1}{U_5 - A_5} \right) \right) \]

\[ u_2 = \frac{2}{(\gamma + 1)\tan c_3} + \frac{(\gamma - 1)U_2}{\gamma + 1} \]

\[ a_2 = \frac{(\gamma - 1)(U_2 - u_2)}{2} + A_2 \]

\[ p_2 = p_1 \left( \frac{a_2}{a_1} \right)^{\gamma - 1} \]

\[ p_2 = p_1 \left( \frac{a_2}{a_1} \right)^{\gamma - 1} \]

\[ x_2 = 0 \]

\[ t_2 = \frac{x_2 - x_1}{u_1 + a_1} + t_1 \]

\[ x_3 = \frac{t_1 - t_2 + \frac{x_2(u_2 - a_2 + u_1 - a_1)}{2(u_2 - a_2)(u_3 - a_3)} - \frac{x_1(u_1 + a_1 + u_3 + a_3)}{2(u_1 + a_1)(u_3 + a_3)}}{2(u_2 - a_2)(u_3 - a_3)} \]

\[ t_3 = \frac{x_3 - x_2 + \frac{2t_2(u_2 - a_2)(u_3 - a_3)}{u_2 - a_2 + u_3 - a_3} - \frac{2t_1(u_1 + a_1)(u_3 + a_3)}{u_1 + a_1 + u_3 + a_3}}{2(u_2 - a_2)(u_3 - a_3)} \]

\[ p_3 = p_1 \left( \frac{a_3}{a_1} \right)^{\gamma - 1} \]

\[ T_3 = a_3 \frac{X_R a}{\gamma R_1} \]

\[ p_3 = \frac{P_3 R_i}{T_3 R_i} \]

Contact Surface Points (velocities and pressures equal)

\[ (u_3)_1 = u_1 \]

\[ (a_3)_1 = \frac{(\gamma - 1)(u_2 - u_3)}{2} + a_2 \]

\[ (a_3)_1 = (a_3)_1 \]
\( \begin{align*}
(u_1) &= \frac{(u_3) + u_4}{2} + \frac{(a_{1d}) - a_4}{\gamma - 1} \\
(a_1) &= \frac{(\gamma - 1)((u_3) - u_4) + (a_{1d})}{2} \\
(x_3) &= \frac{t_4 - t_2 + \frac{x_5(u_3 - a_2 + (u_3)_{r-1} - (a_{1d})_{r-1})}{2(u_3 - a_2)((u_3)_{r-1} - (a_{1d})_{r-1})}}{2(u_3 - a_2) + (u_3)_{r-1} - (a_{1d})_{r-1} - (u_3)_{r-1} - u_4} \\
(t_3) &= \frac{x_4 - x_2 + \frac{2t_4(u_3 - a_2 + (u_3)_{r-1} - (a_{1d})_{r-1})}{2(u_3 - a_2)((u_3)_{r-1} - (a_{1d})_{r-1})}}{2(u_3 - a_2) + (u_3)_{r-1} - (a_{1d})_{r-1} - (u_3)_{r-1} - u_4} \\
(x_1) &= \left\{ (x_3) - t_4 + \frac{x_4(u_4 - a_{1d} + u_5 - a_4)}{2(u_4 - a_{1d})(u_5 - a_4)} \\
&\quad - \frac{(x_5)_{r-1} + (a_{1d})_{r-1} + (u_3)_{r-1} + (a_{1d})_{r-1}}{2((u_3)_{r-1} + (a_{1d})_{r-1} + (u_3)_{r-1} + (a_{1d})_{r-1})} \right\} \\
(t_2) &= \left\{ (x_3) - x_4 + \frac{2t_4(u_4 - a_{1d})(u_5 - a_4)}{u_4 - a_{1d} + u_5 - a_4} \\
&\quad - \frac{2(t_4)((u_3)_{r-1} + (a_{1d})_{r-1} + (u_3)_{r-1} + (a_{1d})_{r-1})}{(u_3)_{r-1} + (a_{1d})_{r-1} + (u_3)_{r-1} + (a_{1d})_{r-1}} \right\} \\
(u_4) &= \frac{u_5 - (u_4)_{r-1} + (x_5 - (x_4)_{r-1})^2}{\sqrt{(t_3 - t_4)^2 + (x_5 - x_4)^2}} \\
(a_{1d}) &= a_{4d} \sqrt{(t_5 - (t_3)_{r-1})^2 + (x_5 - (x_4)_{r-1})^2} \\
(p_1) &= p_3 \sqrt{(t_5 - (t_4)_{r-1})^2 + (x_5 - (x_4)_{r-1})^2} \\
(p_2) &= p_3 \sqrt{(t_5 - (t_4)_{r-1})^2 + (x_5 - (x_4)_{r-1})^2} \\
(p_3) &= p_4 \left( \frac{(a_{1d})_{r-1}}{a_{4d}} \right)^{1/2} \\
(p_{3d}) &= \frac{(p_3)_{r-1}}{(a_{1d})_{r-1}} \\
\end{align*} \)
\[
(p_{3e})_i = \frac{\gamma_i (p_{3e})_{i-1}}{\gamma_i (a_{3e})_{i-1}}
\]

\[
(u_1)_i = \frac{p_1 - p_2 + u_1 (p_{3e})_1 + (p_{3d})_1 (a_{3d})_{i-1} + u_2 (p_{3e})_2 + (p_{3d})_2 (a_{3d})_{i-1}}{2} + \frac{u_1 (p_{3e})_2 + (p_{3d})_2 (a_{3d})_{i-1}}{2}
\]

\[
(p_{3d})_i = \frac{u_1 - u_2 + \frac{2p_1}{p_{3e} + (p_{3d})_1 (a_{3d})_{i-1}} + \frac{2p_2}{p_{3e} + (p_{3d})_2 (a_{3d})_{i-1}}}{p_{3e} + (p_{3d})_1 (a_{3d})_{i-1} + p_{3e} + (p_{3d})_2 (a_{3d})_{i-1}}
\]

\[
(a_{3d})_i = a_{4d} \left( \frac{p_{3d}}{p_4} \right)^{\gamma_i - 1} / 2\gamma_i
\]

\[
(a_{3e})_i = a_{4e} \left( \frac{p_{3e}}{p_4} \right)^{\gamma_i - 1} / 2\gamma_i
\]

\[
T_{3e} = \frac{\chi R_4 a_{3e}^2}{\chi R_5}
\]

\[
T_{3d} = a_{3d}^2
\]

**Blob Point**

\[
(x_1)_i = x_6
\]

\[
(x_2)_i = x_3
\]

\[
(t_2)_i = \frac{(t_6 - t_3)(x_2)_{i-1} - x_6 + t_6}{x_6 - x_3}
\]

\[
(u_2)_i = \frac{u_6 \sqrt{(t_3 - t_2)^2 + (x_3 - (x_2)_{i-1})^2} + u_3 \sqrt{(t_6 - t_2)^2 + (x_6 - (x_2)_{i-1})^2} + \sqrt{(t_6 - t_3)^2 + (x_6 - x_3)^2}}{2}
\]

\[
(p_{3d})_i = \frac{p_6 \sqrt{(t_3 - t_2)^2 + (x_3 - (x_2)_{i-1})^2} + \sqrt{(t_6 - t_3)^2 + (x_6 - x_3)^2}}{2}
\]

\[
(W_2)_i = \frac{2((u_2)_i - u_1)}{2R + 1} + W_i
\]

\[
(x_2)_{i-1} = \frac{2W_i (W_2)_i + ((t_2)_i - t_1)}{W_i + (W_2)_i} + x_2
\]

**Blob Expansion Point**

\[
c_3 = \frac{-(\theta + \mu)}{100} \left( \text{arctan} \left( \frac{1}{U_2 - A_2} \right) - \text{arctan} \left( \frac{1}{U_5 - A_5} \right) \right) + \text{arctan} \left( \frac{1}{U_i - \Theta} \right)
\]

\[
u_3 = \frac{2}{(\gamma_i + 1) \tan c_3} + \frac{(\gamma_i - 1) U_i}{\gamma_i + 1}
\]

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\[ a_3 = \frac{\gamma}{2} \left( \frac{1}{2} \left( U_2 - u_3 \right) + A_2 \right) \]

\[ T_3 = a_3 \frac{\gamma P_0}{\gamma - 1} \]

\[ \rho_3 = \rho_1 \left( \frac{T_1}{T_3} \right)^{\frac{1}{2}} \]

\[ W_3 = \frac{3}{4 \rho_1} \left( \frac{u_3 - u_1}{u_3} \right) + W_1 \]

\[ x_3 = \frac{1}{u_3} \left( \frac{L_1}{L_2} - \frac{t_1}{u_3} \right) + \frac{X_1 (W_1 + W_2)}{2 W_1 W_2} \frac{1}{u_3 - a_3} \]

\[ t_3 = \frac{x_3}{u_3 - a_3} \frac{1}{u_3} \frac{L_1}{L_2} \]

Boundary Conditions

1. Moving piston - gas remains in contact with piston.
2. Supersonic outflow through open-ended duct - both families of characteristics travel in same direction and both exit, same as interior points.

A.6 The Pitot Pressure

\[ \frac{P_1}{P_0} = \left( \frac{2\gamma}{\gamma + 1} \right)^{\frac{1}{2}} \left( \frac{M_1^2 - \frac{\gamma - 1}{\gamma + 1}}{\left( \frac{\gamma + 1}{2} \right)^{\frac{\gamma}{\gamma - 1}}} \right)^{\frac{1}{2}} \]
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82
The interaction of an interface between two gases and a strong expansion is investigated and the effect on flow in an expansion tube is examined. Two mechanisms for the unsteady pitot-pressure fluctuations found in the test section of an expansion tube are proposed. The first mechanism depends on the Rayleigh-Taylor instability of the driver-test gas interface in the presence on a strong expansion. The second mechanism depends on the reflection of the strong expansion from the interface. Predictions compare favourably with experimental results. The theory is expected to be independent of the absolute values of the initial expansion tube filling pressures.