LLOFX

User and Technical Documentation

Eagle Engineering, Inc.

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1.0 Introduction

The program LLOFX calculates in-plane trajectories from an Earth-orbiting space station to Lunar orbit in such a way that the journey requires only two delta-v burns (one to leave Earth circular orbit and one to circularize into Lunar orbit). The program requires the user to supply the space station altitude and Lunar orbit altitude (in kilometers above the surface), and the desired time of flight for the transfer (in hours). It then determines and displays the trans-Lunar injection (TLI) delta-v required to achieve the transfer, the Lunar orbit insertion (LOI) delta-v required to circularize the orbit around the Moon, the actual time of flight, and whether the transfer orbit is elliptical or hyperbolic. Return information is also displayed. Finally, a plot of the transfer orbit is displayed.

2.0 Principle Behind the Program

Calculation of the trajectory takes advantage of the fact that the Moon travels at great velocity in orbit about the Earth (1.02 kilometers per second). The vehicle’s circular orbit about the Earth is turned into an elliptical transfer orbit that intercepts the Moon’s orbit. This transfer orbit is rotated ahead of the Earth-Moon line in such a way that, as the vehicle enters the Moon’s sphere of action (SOA) ahead of the Moon, the high velocity of the Moon in the direction of the vehicle causes the vehicle to appear to be headed back toward the Moon (from a Lunar point of view). This program identifies the eccentricity, size, and rotation of the transfer ellipse or hyperbola that causes the velocity vector of the vehicle (in Lunar coordinates) to correspond to an orbit passing in front of the Moon with a perigee at the Lunar orbit altitude supplied by the user.

3.0 Description of the Process

Given the altitude of the space station circular orbit, the program calculates circular velocity. Through a process of iteration, velocity is added to this in small increments (delta-v) so that the orbit becomes elliptical or hyperbolic. Assuming that the burn
occurs at the far side of the Earth on the Earth-Moon line, such an ellipse/hyperbola will be symmetrical along the Earth-Moon line, as defined at the time of SOA penetration (see Figure 1). The velocity is increased until this trajectory orbit's apogee is beyond the Moon's SOA. (A hyperbolic orbit meets this condition by definition).

Initially, the program identifies the vehicle's velocity vector (in Lunar coordinates) at a point on the transfer orbit such that the distance from the Earth to that point equals the distance from the Earth to the Moon's SOA, measured along the Earth-Moon line. Note that because of the Moon's motion around the Earth, and because we are using Lunar coordinates, the velocity vector points away from the transfer orbit.

The position on the SOA is determined at which the velocity vector identified above would correspond to an orbit passing in front of the Moon with a perigee equal to the user supplied lunar orbit altitude (see Figure 2). The position is identified by an angle centered at the Moon (LUGAMMA), measured up from the Earth-Moon line. The ellipse/hyperbola is rotated through an angle called the coast angle (CSTANG) such that it intersects the SOA at this position. (The vehicle physically performs this by causing the TLI burn to occur at an angle past the Earth-Moon line equal to the coast angle. See Appendix D for a description of how the coast angle is calculated.)

This point of intersection of the SOA and the transfer orbit occurs further along the transfer orbit than the point at which the initial velocity vector was identified. The velocity vector at this new point is different from the originally calculated velocity vector (see Figure 3). This new velocity vector (B in the diagram) is not pointing in a direction that will allow interception with Lunar orbit perigee. A new position on the SOA must be determined to allow this condition to be met (B' in the diagram). Figure 4 shows the calculations required to determine this new position (LUGAMMANEW) given
the velocity vector's x- and y-components and the flight path angle (ANGMOMGAM). (See Appendix E for a description of how ANGMOMGAM is calculated).

The steps described in the previous paragraph must be iterated until LUGAMMANEW converges (to within 0.02 radians). The resulting coast angle describes the point in Earth orbit at which to perform the TLI burn.

The program then adds the time of flight between SOA and Lunar orbit to the time of flight between TLI and SOA to get total time of flight. If the total flight time does not fall within one hour of the desired time of flight, TLI delta-v is adjusted by 1/10th the percentage of error between actual and desired time, and the entire process is begun anew. Otherwise, the program displays the transfer orbit properties.
A. Velocity vector as it would have been, if distance to SOA had not changed.

B. New velocity vector, accounting for increased distance to SOA.

B'. Desired position of new velocity vector, to intercept orbit perigee.
4.0 Program Execution Instructions

The following instructions describe the steps to be taken by the user to execute this program.

A. Obtain access to the DEC VAX minicomputer and sign on with user identification.

B. At the $ prompt, type SETDRV 415 if using a Techtronix 4115 terminal, or SETDRV 407 if using a Techtronix 4111 terminal. If a terminal different from these is being used, consult the system administrator for the correct device driver code.

C. At the next $ prompt, type SETTEK LLOFX if using a Techtronix terminal, or RUN LLOFX if using a different terminal.

D. When prompted by the program, enter the following information:

1. Earth-orbiting space station altitude (kilometers above the Earth's surface).
2. Desired orbital altitude above the Lunar surface (kilometers).
3. Earth-Moon distance at time of Lunar intercept (kilometers). This can range from 359,856 to 405,970 kilometers. The average distance is 384,400 kilometers.
4. Beginning reference point for a screen plot of the transfer orbit (kilometers). The Earth appears at the left side of the screen, and is centered at zero kilometers. The Moon appears at the right side of the screen, and is centered at the range specified in (3) above. The beginning reference point defines the left boundary of the plot. A negative beginning reference point ensures the entire Earth orbit is included in the plot.
5. Ending reference point for the screen plot of the transfer orbit (kilometers). This defines the right boundary of the plot. It must be at least as large as the range specified in (3) above if the Moon is to be included in the plot.
6. Aspect ratio (Y:X ratio) of the screen during the plot of the transfer orbit. This allows adjustment for the fact that a screen pixel is longer than it is wide, and removes the resulting distortion. The valid range for aspect ratio is 0.55 to 0.75.

E. After the aspect ratio has been entered, the program will execute. The following information is displayed upon completion of execution:

1. A message describing whether the transfer orbit is hyperbolic or elliptical.
2. Trans-Lunar injection delta-v.
3. Lunar orbit insertion delta-v.
4. Trans-Earth injection delta-v.
5. Earth orbit insertion delta-v.

6. Total delta-v.

7. Actual time of flight for the outbound leg (Earth to Moon). Return time is the same.

8. A screen plot of the transfer orbit.

F. To re-execute the program with new parameters, begin again at step (C) above.
Appendix A - Program Flow Chart
Initial values

Calculate preliminary variables

Update variables

In light time loop

A. iterations

Calculate Tmar

B. iterations

Recalculate velocity

Collision properties

IN0 Increase Translunar injection Delta-V

Save eccentricity

Lunar orbit injection Delta-V

Save lunar pwitlon as old value

IPASS = IPASS + 1

Save velocity at SO1 from earth, range to SOA

TLDIV. LOIDV. and CAYYASOA

Set values for lunar hyperbolic entry orbit

Calculate position angle for lunar

Hyperrn

Calculate total flight time

End

Set values for lunar hyperbolic entry orbit

End

Is orbit elliptical?

Yes

No

Calculate elliptical properties

Is orbit elliptical?

Yes

No

Increase Trans-lunar injection Delta-V

Does orbit reach moon?

Yes

No

Save eccentricity

Calculate preliminary variables

Update variables

In light time loop

START
Hyperprop

Calculate hyperbolic properties

RETURN

SCACALCS

Calculate distance to penetration point

Is orbit elliptical?

NO

Calculate hyperbolic time

YES

Calculate elliptical time

Is IPASS = 1?

YES

Coast angle = PI - theta

NO

Calculate coast angle and velocities

Calculate GAMMA at SOA to get lunar perisaphe

RETURN

A-3
Appendix B - Program Code Listing
PROGRAM
THIS PROGRAM PROPAGATES ORBIT TO MOON AND BACK
IN METRIC VECTOR UNITS

PROGRAM WAS:
   PROPOSED BY GUS BABB
   WRITTEN BY CHRIS VARNER AND MIKE D'ONOFRIO
   DOCUMENTED BY STEVE ERICKSON
   FOR NASA's ADVANCED SPACE TRANSPORTATION SYSTEM
   CONTRACT NO. NAS 9-17878
   EAGLE ENGINEERING INC. 1988

FIRST DECLARE VARIABLES REAL EXCEPT VARIABLES
BEGINNING WITH 'I'

IMPLIED REAL*4 (A-H,J-Z)
REAL*4 X(250),Y(250),XE(250),YE(250),XL(250),YL(250)

OPEN GRAPHICS SUBROUTINES

CALL JBEGIN
CALL JDINIT(1)
CALL JDEVON(1)
CALL JIENAB(1,4,1)

INPUTS
WRITE (5,5)
FORMAT ('1INPUT ALL NUMBERS AS REAL VALUES')
WRITE (5,10)
FORMAT ('OINPUT SPACE STATION ALTITUDE km')
READ *,H
WRITE (5,15)
FORMAT ('0INPUT LUNAR ALTITUDE km')
READ *,HL
WRITE (5,17)
FORMAT ('1INPUT EARTH-MOON DISTANCE avg.=384400')
READ *,EMRANGE
WRITE (5,20)
FORMAT ('OINPUT FLIGHT TIME IN HOURS')
READ *,OFTHR WRITE (5,23)
FORMAT ('OINPUT END X POINT TO VIEW',/,' FROM 0 TO 500000')
READ *,XMAX WRITE (5,24)
FORMAT ('OINPUT ASPECT RATIO OF SCREEN (Y/X)',/,' FROM .55 TO .75')
READ *,AR
XMAX1=.2*(XMAX-XMIN)+XMAX
XMIN1=XMIN-.2*(ABS(XMAX-XMIN))
YMIN=(XMIN-XMAX)/2.0
YMAX=(XMAX-XMIN)/2.0
YMIN1=(XMIN1-XMAX1)/2.0
YMAX1=(XMAX1-XMIN1)/2.0
FUDGE FACTOR COMPENSATING FOR ASPECT RATIO OF SCREEN

YMAX = AR * YMAX
YMAX1 = YMAX1 * AR
ECCE = 0.0
OBTALC = 1000000.0
TIMEERROR = 3601.0
TLIDV = 3.07

FUDGE FACTOR COMPENSATING FOR ASPECT RATIO OF SCREEN

OBLTLM = OTHR * 3600.
RTNFLTM = RFTHR * 3600.
GAMAIN = 0.0
PI = 3.1415926535
RO = 6378.1
ROL = 1738.0
MUE = 398603.0
MUL = 4902.97
R = R + RO
VOM = 1.02
RPE = R
V = SQRT(MUE/RPE) + TLIDV
ICOUNT = 0
RSOA = EMRANGE / 10.017
LOOPFLAGS$ = 'ON'

START THE FLIGHT TIME LOOP

DO 1400 WHILE (LOOPFLAGS$.EQ. 'ON')
   GAIN = 1. / (OBTALC * 10.0)
   TLIDV = TLIDV + TIMEERROR * GAIN
   MU = MUE
   ICOUNT = ICOUNT + 1
   R = R + RO
   RPE = R
   RPL = ROL + HL
   GAMMA = PI * GAMAIN / 180.

   IS THE ORBIT ELLIPTICAL OR HYPERBOLIC?

   IF (ICOUNT.LT.10000) THEN
      V = SQRT(MUE/RPE) + TLIDV
   END IF
   IF (V**2.0.LE.(2.*MU/R)) THEN
      ORBIT IS ELLIPTICAL
      Q = R * V**2.0 / MU
      RP = (1.0 - SQRT(1.0 - Q**2.0) * (COS(GAMMA)**2)) * R / (2.0 - Q)
      RA = (1.0 + SQRT(1.0 - Q**2.0) * (COS(GAMMA)**2)) * R / (2.0 - Q)
      A = (RA + RP) / 2.
      ECC = (RA - RP) / (RA + RP)
      P = (1.0 + ECC) * RP
      NU = SQRT(MU/A**3)
   END IF
COFO=(P/R-1.)/ECC
FO=ATAN(SQRT(1.0001-COFO**2)/COFO)
IF (FO.LT.0.) FO=PI+FO
IF (GAMMA.LT.0.) FO=-FO
EO=2.*(ATAN(SQRT((1.-ECC)/(1.+ECC)))*TAN(FO/2.))
THETAO=FO
TAUO=(EO-ECC*SIN(EO))/NU
TIMEO=TAUO

DOES THE ELLIPSE REACH THE MOON?
IF (RA.LT.(EMRANGE+RSOA)) THEN

NO
TLIDV=TLIDV + .01
GO TO 250
ENDIF
ELSE

ORBIT IS HYPERBOLIC
CALCULATE HYPERBOLIC PROPERTIES
CALL HYPERPROP(PI,A,B,MU,R,V,GAMMA,P,ECC,RP,
+ VP,COTHETA,THETAO,TIMEO)
ENDIF
LUGAMMA=0.0
IPASS=1
CALL SOACALCS(PI,VXL,IPASS,VYL,VT,GAMMAT,VPL,NU,
+ ANGMOMGAM,RSOA,TFROMPER,RT,MUL,RPL,ECC,P,MU,A,VOM,
+ LUGAMMA,LUGAMMANEW,EMRANGE,TAUO,CSTANG)
ITERATE
IPASS=IPASS+1
SAVE THE LUNAR POSITION ANGLE AS AN OLD VALUE
LUGAMMA=LUGAMMANEW
CALL SOACALCS(PI,VXL,IPASS,VYL,VT,GAMMAT,VPL,NU,
+ ANGMOMGAM,RSOA,TFROMPER,RT,MUL,RPL,ECC,P,MU,A,VOM,
+ LUGAMMA,LUGAMMANEW,EMRANGE,TAUO,CSTANG)
IF (ABS(LUGAMMANEW-LUGAMMA) .GE. .02) GOTO 200
THEN DO ANOTHER ITERATION
SAVE VELOCITY AT SOA, TIME FROM EARTH TO SOA, RANGE TO SOA
TLIDV, AND LOIDV, AND GAMMASOA AS OLD VALUES
TTSOA=TFROMPER
RESOA=RT
TLIDV=V-SQRT (MUE/(H+RO))
LOIDV=VPL-SQRT (MUL/RPL)
AE=A
NUE=NU
PE=P
ECCE=ECC
VESOA=VT
GAMMASOA=GAMMAT

SET VALUES FOR LUNAR HYPERBOLIC ENTRY ORBIT
MU=MUL
V=SQRT(VYL**2+VXL**2)
GAMMA=ANGMOMGAM
R=RSOA
CALL HYPERPROP(PI,A,B,MU,R,V,GAMMA,P,ECC,RP,VP,
+ COTheta,THETAO,TIMEO)

CALCULATE THE POSITION ANGLE FOR LUNAR PERIGEE
THETALP=LUGAMMA+THETAO
CALCULATE THE TOTAL FLIGHT TIME AND
TIME INSIDE SOA
TIMESOA=TIMEO
OBTCALC= (ABS(TTSAOA)+ABS(TIMESOA=TIMEO))

CHECK TO SEE IF TIME IS WITHIN REQUESTED BOUNDS
TIMEERROR=OBTCALC-OBFLTTM
IF (ABS(TIMEERROR).LE.600.) THEN
   OUTPUTS
   IS IT HYPERBOLIC?
   IF (ECCE.GE.1.0) THEN
      YES
      WRITE(5,50)
      FORMAT('---------HYPERBOLIC ORBIT---------')
   ELSE
      WRITE(5,55)
      FORMAT('---------ELLIPICAL ORBIT---------')
   ENDIF
   WRITE(5,60) TLIDV,LOIDV
   FORMAT('OUTBOUND PROPERTIES',/,' TLI DELTA-V, km/sec.=',
+ F6.2,/,' LOI DELTA-V, km/sec.=',F6.2)
   WRITE(5,62) LOIDV,TLIDV
   FORMAT('INBOUND PROPERTIES',/,' TEI DELTA-V= ',F6.2,/,
+ ' EO1 DELTA-V= ',F6.2)
   WRITE(5,65) TLIDV+LOIDV,(ABS(TTSAOA)+ABS(TIMESOA=TIMEO))/3600.
   FORMAT('TOTAL DELTA-V = ',F7.2,/,' FLIGHT TIME = ',F10.3)
   ECCM=ECC

GRAPHICS
IT=200
ECC=ECCE
A=AE
P=PE
MU=MUE
NU=NUE
RI=6371.23
CALL GATTRI (1,0,1.0)
CALL GATTRI (2,0,1.0)
CALL GATTRI (3,0,1.0)
CALL GATTRI (4,5,1.0)
CALL GATTRI (5,5,1.0)
CALL GATTRI (6,5,1.0)
CALL GATTRI (7,5,1.0)
CALL GATTRI (11,5,1.0)
CALL GATTRI (12,5,1.0)
CALL GCHART (1,5,'EARTH-MOON ORBIT',16)
CALL GCHART (12,5,1.0)
' 'DISTANCE WITH RESPECT TO EARTH CENTER',
37,YMIN1,YMAX1,0,'KM',2)
DO 1209 IE=1,3
ICO=0
DO 1009 IN=1,IT
INM1=IN-1
X(IN)=RI*COS(FLOAT(INM1)*10.*PI/180.)
Y(IN)=RI*SIN(FLOAT(INM1)*10.*PI/180.)
IF (IE.NE.1) X(IN)=X(IN)+EMRANGE
IF (X(IN).GT.XMIN.AND.X(IN).LT.XMAX) THEN
ICO=ICO+1
Y(ICO)=Y(IN)
X(ICO)=X(IN)
ENDIF
CONTINUE
GRAPH
CALL JOPEN
CALL JCOLOR (6)
IF (IE.EQ.2) CALL JCOLOR(1)
IF (IE.EQ.3) THEN
CALL JCOLOR(0)
CALL GCURVE (X,Y,ICO,-1,0,0)
ENDIF
IF (IE.NE.3) CALL GCURVE(X,Y,ICO,0,0,0)
CALL JCLOSE
IF (IE.EQ.1) RI=1739.35
IF (IE.EQ.2) RI=RSOA
CONTINUE
ITIMESE=200
IXE=0
DO 1309 I=0,ITIMESE
IPI=I+1
RT=(RO+H+1.0)+FLOAT(I)*(RESOA-RO-H)/FLOAT(ITIMESE)
CALL ORBITPROP (ECC,RT,P,PI,MU,A,THETA,TFROMPER,
TAUO,GAMMAT,NU,VT)
XE(IPI)=RT*(-1.0)*COS(CSTANG+THETA)
YE(IPI)=RT*(-1.0)*SIN(CSTANG+THETA)
IF (XE(IPI).GT.XMIN.AND.XE(IPI).LT.XMAX) THEN
IXE=IXE+1
XE(IXE)=XE(IPI)
YE(IXE)=YE(IPI)
ENDIF
CONTINUE
CALL JOPEN
CALL JCOLOR(2)
CALL GCURVE(XE,YE,IXE,0,0,0)
CALL JCLOSE
ECC=ECCM
MU=MUL
V=SQRT(VYL**2+VXL**2)
GAMMA=ANGMOMGAM
R=RSOA
CALL HYPERPROP(PI,A,B,MU,R,V,GAMMA,P,ECC,RP,
+ VP,COTHTHETA,THTAO,TIMO)
THETALP=LUGAMMA+THETAO
TIMESOA=TIMEO
ITIMES=31
ITIMESM1=ITIMES-1
IXL=0
DO 1359 I=0,ITIMESM1
   IP1=I+1
   RT=RSOA-FLOAT(I)*(RSOA-RP)/(FLOAT(ITIMESM1))
   CALL ORBITPROP(ECC,RT,PI,MU,A,THETA,TFROMPER,
+ TAUO,GAMMAT,NU,VT)
   XL(IP1)=EMRANGE-RT*COS(THETALP-THETAO)
   YL(IP1)=RT*SIN(THETALP-THETAO)
   IF(XL(IP1).GT.XMIN.AND.XL(IP1).LT.XMAX) THEN
      IXL=IXL+1
      XL(IXL)=XL(IP1)
      YL(IXL)=YL(IP1)
   ENDIF
CONTINUE
CALL JOPEN
CALL JCOLOR(4)
CALL GCURVE(XL,YL,IXL,0,0,0)
CALL JCLOSE
INBOUND
DO 1379 I=1,IXE
   YE(I)=YE(I)*(-1.0)
CONTINUE
DO 1389 I=1,IXL
   YL(I)=YL(I)*(-1.0)
CONTINUE
CALL JOPEN
CALL JCOLOR(2)
CALL GCURVE(XE,YE,IXE,0,0,0)
CALL JCOLOR(4)
CALL GCURVE(XL,YL,IXL,0,0,0)
CALL JCLOSE
LOOPFLAG$='OFF'
ENDIF
ELSE
PRINT *,'******************LOOPING ERROR OCCURED**********'
PRINT *,'PROBABLY CAUSED BY EXCESSIVE FLIGHT TIME'
PRINT *,'MAXIMUM FLIGHT TIME IS APPROX 75 HRS AT LUNAR'
PRINT *,'PERIGEE AND 120 HRS AT LUNAR APOGEE'
SUBROUTINE HYPERPROP (PI, A, B, MU, R, V, GAMMA, P, ECC, RP, VP, COTHETA, THETAO, TIME)

IMPLICIT REAL*4 (A-Z)
A = MU * R / (R**2 - 2.0 * MU)
B = SQRT (R**3 * V**2 * COS(GAMMA)**2 / (R * V**2 - 2.0 * MU))
P = R * V**2 * COS(GAMMA)**2 / MU
ECC = SQRT (P / (A + 1.0))
RP = A * (ECC - 1.0)
VP = SQRT (MU / P) * (1.0 + ECC)
COTHETA = (P / R - 1.0) / ECC
THETAO = ATAN (SQRT (1.0001 - COTHETA**2) / COTHETA)
IF (THETAO.LT.0.0) THETAO = PI + THETAO
IF (GAMMA.LT.0.0) THETAO = -THETAO
TIMEO1 = SQRT (P**3 / MU) / (ECC**2 - 1.0)
TIMEO2 = ECC * SIN (THETAO) / (1.0 + ECC * COTHETA)
TIMEO3 = (1.0 / SQRT (ECC**2 - 1.0)) * LOG ((ECC + COTHETA + SQRT (ECC**2 - 2.0)) * SIN (THETAO) / (1.0 + ECC * COTHETA))
TIMEO = TIMEO1 * (TIMEO2 - TIMEO3)
RETURN
END

SUBROUTINE SOACALCS (PI, VXL, IPASS, VYL, VT, GAMMA, P, NU, ANGMO, GAM, RSOA, TFROMPER, RT, MUL, RPL, ECC, P, MU, A, VOM, LUGAMMA, LUGAMMANEW, EMRANGE, TAUO, CSTANG)

IMPLICIT REAL*4 (A-H, J-Z)
DL = RSOA * COS (LUGAMMA)
DE = EMRANGE - DL
RT = SQRT (DE**2 + RSOA**2 - DL**2)
CALL ORBITPROP (ECC, RT, P, PI, MU, A, THETA, TFROMPER, TAUO, GAMMA, NU, VT)
RY = RSOA * SIN (LUGAMMA)
CSTANG1 = ATAN (RY / DE)
CSTANG = PI + CSTANG1 - THETA
VXE = VT * SIN (GAMMA + PI - CSTANG - THETA)
VYE = VT * COS (GAMMA + PI - CSTANG - THETA)
VXL=VXE
VYL=VYE-VOM

CALCULATE GAMMA AT SOA TO GET LUNAR PERIAPSIS

VSOAL2=VXL**2 + VYL**2
VPL2= VSOAL2-(2.0*MUL/RSOA)+(2.0*MUL/RPL)
VSOAL=SQRT(VSOAL2)
VPL=SQRT(VPL2)
ANGMOMGAM=ATAN(SQRT(VSOAL2*RSOA**2-VPL2*RPL**2)/(VPL*RPL))
LUGAMMANEW=ATAN(-VYL/VXL)-ANGMOMGAM +PI/2.
RETURN
END

SUBROUTINE ORBITPROP(ECC,RT,P,PI,MU,A,THETA,TFROMPER,
TAUO,GAMMAT,NU,VT)

IMPLICIT REAL*4(A-H,J-Z)
IF(ECC.LT.1.0) THEN

ELLIPtical ORBIT
PROPAGATE ELLIPTICAL ORBIT FORWARD TO RT OUTBOUND

F=ATAN((SQRT(ECC**2*RT**2-(P-RT)**2))/(P-RT))
IF(F.LT.0.0) F=F+PI
VT=SQRT(MU*(2.0/RT-1.0/A))
GAMMAT=ATAN(ECC*SIN(F)/(1.0+ECC*COS(F)))
ETH=2.0*(ATAN(SQRT((1.0-ECC)/(1.0+ECC))*TAN(P/2.0)))
TAUTH=(ETH-ECC*SIN(ETH))/NU
TFROMPER=TAUTH-TAUO
THETA=F
ELSE

HYPERBOLIC ORBIT
ORBIT PROPAGATION FOR HYPERBOLIC ORBIT
COTHETA=(P/RT-1.0)/ECC
THETA=ATAN(SQRT(1.0001-COTHETA**2)/COTHETA)
IF(THETA.LT.0.0) THETA=PI+THETA
VT=SQRT(MU*(2.0/RT+1.0/A))
COGAMAT=SQRT(P*MU)/(RT*VT)
GAMMAT=ATAN(SQRT(1.0001-COGAMAT**2)/COGAMAT)
IF(THETA.LT.0.0) GAMMAT=-GAMMAT
TFROMPER1=SQRT(P**3/MU)/(ECC**2-1.0)
TFROMPER2= ECC*SIN(THETA)/(1.0+ECC*COTHETA)-(1.0/SQRT(ECC**2-1.0))
+ ECC**2-1.0)*LOG((ECC+COTHETA+SQRT(ECC**2-1.0)
+ *SIN(THETA))/(1.0+ECC*COTHETA))
TFROMPER=TFROMPER1*TFROMPER2
ENDIF
RETURN
END
Appendix C - Program Variables
<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Semimajor Axis of the Transfer Orbit (km)</td>
</tr>
<tr>
<td>AE</td>
<td>Semimajor Axis of the Transfer Orbit (km), Earth coordinates. Stored Value of &quot;A&quot; Before Switching to Lunar Coordinates.</td>
</tr>
<tr>
<td>AR</td>
<td>Aspect Ratio of Screen Plot</td>
</tr>
<tr>
<td>ANGMOMGAM</td>
<td>Flight Path Angle at Sphere of Action Penetration Point for the Lunar Hyperbolic Orbit (rad)</td>
</tr>
<tr>
<td>B</td>
<td>Hyperbolic Asymptotes Parameter</td>
</tr>
<tr>
<td>COFO</td>
<td>Cosine of the True Anomaly (dimensionless)</td>
</tr>
<tr>
<td>COGAMAT</td>
<td>Cosine of the Transfer Orbit Flight Path Angle at the SOA (non-dimensional)</td>
</tr>
<tr>
<td>COTHETA</td>
<td>Cosine of the True Anomaly for Hyperbolic Orbits (non-dimensional)</td>
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<td>CSTANG</td>
<td>Actual Coast Angle (rad)</td>
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<tr>
<td>CSTANG1</td>
<td>Angle from the Earth-Moon Line to the Sphere of Action Penetration Point (rad)</td>
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<tr>
<td>CSTANGO</td>
<td>Coast Angle Initial Guess (rad)</td>
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<tr>
<td>DE</td>
<td>X Component of Distance from the Earth to Sphere of Action Penetration Point (km)</td>
</tr>
<tr>
<td>DL</td>
<td>X Component of Distance from the Moon to Sphere of Action Penetration Point, Measured Positive in the Negative X Direction (km)</td>
</tr>
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<td>ECC</td>
<td>&quot;New&quot; Eccentricity of the Transfer Orbit (dimensionless)</td>
</tr>
<tr>
<td>ECCE</td>
<td>&quot;Old&quot; Eccentricity of the Transfer Orbit (dimensionless)</td>
</tr>
<tr>
<td>ECCM</td>
<td>Eccentricity of the Hyperbolic Lunar Fly-By Orbit</td>
</tr>
<tr>
<td>EMRANGE</td>
<td>Earth-Moon Range (km)</td>
</tr>
<tr>
<td>EO</td>
<td>Initial Eccentric Anomaly Along the Transfer Orbit (rad)</td>
</tr>
<tr>
<td>ETH</td>
<td>Eccentric Anomaly for the Point of Sphere of Action Penetration Along the Transfer Orbit (rad)</td>
</tr>
</tbody>
</table>
F  True Anomoly for the Intersection Point of the Transfer Orbit and the Lunar Sphere of Action (rad)
FO  Initial True Anomaly Along the Transfer Orbit (rad)
GAIN  Error Multiplier for Adjusting Initial Velocity Change for a New Flight Time (delta-v/s)
GAMAIN  Inertial Flight Path Angle at Transfer Orbit Perigee (deg)
GAMMA  Inertial Flight Path Angle at Transfer Orbit Perigee (rad)
GAMMASOA  Final Inertial Flight Path Angle of the Transfer Orbit at the Sphere of Action Penetration Point (rad)
GAMMAT  Inertial Flight Path Angle of the Transfer Orbit at Sphere of Action Penetration (rad)
H  Space Station Orbital Altitude (km)
HL  Lunar Orbit Altitude (km)
I  Iteration Counter Used During Transfer and Lunar Fly-By Orbit Plots
ICO  Plot Matrix Subscript Used to Store Circle Plot Position Values Within the User-Specified Range
ICOUNT  Iteration Counter
IE  Identifier Used During Circle Plotting to Determine If Circle is the Earth, the Moon, or the SOA
IN  Iteration Counter Used During Circle Plotting Routines
INM1  IN Minus 1
IP1  I Plus 1
IPASS  Iteration Counter
IT  Constant. Maximum Value of IN
ITIMES  Number of Plot Positions in Lunar Fly-By Orbit
ITIMESE  Number of Plot Positions in Transfer Orbit
ITIMESM1  ITIMES Minus 1
IXE  Plot Matrix Subscript Used to Store Transfer Orbit Plot Position Values Within the User-Specified Range
IXL  Plot matrix Subscript Used to Store Hyperbolic Lunar Fly-By Orbit Plot Position Values Within the User-Specified Range
LOIDV
Lunar Orbit Injection Velocity Change (km/s)

LOOPFLAG
Do-While Flag That Limits the Number of Iterations Seeking Correct Flight Time

LUGAMMA
Angle Between the Earth-Moon Line and the "Old" Sphere of Action Penetration Point (rad)

LUGAMMANEW
Angle Between the Earth-Moon Line and the "New" Sphere of Action Penetration Point (rad)

MU
Local Gravity Constant (km^3/s^2)

MUE
Gravitational Constant for the Earth (km^3/s^2)

MUL
Gravitational Constant for the Moon (km^3/s^2)

NU
Mean Motion of the Transfer Orbit (rad/s)

NUE
Stored Value of NU. Recalled During Plotting Routines

OBFLTTM
Outbound Flight Time (secs)

OBTCALC
Total Time of Flight from Earth Orbit to Lunar Orbit (sec)

OFTHR
Outbound Flight Time (hrs)

P
Semi-Latus Rectum of the Transfer Orbit (km)

PE
Stored Value of P. Recalled During Plotting Routines

Q
Vis-Viva Parameter (dimensionless)

R
Orbital Radius of the Space Station (km)

RA
Radius to the Apogee of the Transfer Orbit (km)

RESOA
Final Distance to the Sphere of Action (km)

RFTHR
Return Flight Time (hrs)

RI
Various Circle Radii, Used During Plotting Routines

RO
Radius of the Earth (km)

ROL
Radius of the Moon (km)

RP
Radius to the Perigee of the Transfer Orbit (km)

RPE
Perigee Radius at the Earth (km)

RPL
Perigee Radius at the Moon (km)
RSOA Radius of the Lunar Sphere of Action (km)
RT Range to Lunar Sphere of Action (km)
RTNFLTTM Return Flight Time (secs)
RY Y Component of the Distance from the Moon to the Sphere of Action Penetration Point (km)
TAUO Initial Time from Perigee (sec)
TAUTH Time of Sphere of Action Penetration from Transfer Orbit Perigee (sec)
TFROMPER Time of Sphere of Action Penetration from TLI (sec)
TFROMPER1 Intermediate Calculation for TFROMPER
TFROMPER2 Intermediate Calculation for TFROMPER
THETA Position Angle for the Sphere of Action Penetration Point, Earth Reference (rad)
THETALP Lunar Perigee Position Angle (rad)
THETA0 Initial Position Angle, and True Anomaly for Hyperbolic Orbits (rad)
TIMEERROR Difference Between Total Time of Flight and Requested Time of Flight (sec)
TIMEO Initial Time From Perigee (sec)
TIMEO1 Intermediate Calculation for TIMEO
TIMEO2 Intermediate Calculation for TIMEO
TIMEO3 Intermediate Calculation for TIMEO
TIMESOATLOI Time from Sphere of Action Penetration to Lunar Orbit Injection (sec)
TLIDV Trans Lunar Injection Velocity Change (km/s)
TMAX Approximate Maximum Flight Time Allowed (hrs)
TTSOA Final Time to the Sphere of Action (sec)
V Velocity (km/s)
VESOA Inertial Velocity at the Sphere of Action Penetration Point (km/s)
VOM Velocity of the Moon (km/s)
VP  Hyperbolic Velocity at Perigee (km/s)
VPL  Lunar Relative Velocity at the Lunar Orbit Radius (km/s)
VPL2  Square of the Lunar Relative Velocity at the Lunar Orbit Radius (km^2/s^2)
VSOAL  Lunar Relative Velocity at the Sphere of Action (km/s)
VSOAL2  Square of the Lunar Relative Velocity at the Sphere of Action (km^2/s^2)
VT  Velocity at Sphere of Action Penetration (km/s)
VXE  X Velocity Component (Earth Coordinates: Inertial)
VXL  X Velocity Component (Lunar Coordinates)
VYE  Y Velocity Component (Earth Coordinates: Inertial)
VYL  X Velocity Component (Lunar Coordinates)
X  X-Component of Plot Position For Circles
XE  X-Component of Plot Position For Transfer Orbit
XL  X-Component of Plot Position For Hyperbolic Lunar Fly-By Orbit
XMAX  High X-Boundary Value Supplied By User For Screen Plot
XMAX1  Additional 20% Added to XMAX For Plot Border
XMIN  Low X-Boundary Value Supplied By User For Screen Plot
XMIN1  Additional 20% Subtracted From XMIN For Plot Border
Y  Y-Component of Plot Position For Circles
YE  Y-Component of Plot Position For Transfer Orbit
YL  Y-Component of Plot Position For Hyperbolic Lunar Fly-By Orbit
YMAX  High Y-Boundary Value For Screen Plot, Adjusted For Aspect Ratio
YMAX1  Preliminary High Y-Boundary Value For Screen Plot
YMIN  Low Y-Boundary Value For Screen Plot

C-6
Appendix D - Calculation of CSTANG

D-1
The coast angle (CSTANG) is the angle past the Earth-Moon line at which a vehicle performs a TLI burn, allowing it to intercept the Moon's SOA at the proper point (see Figure D-1).

The radial vector $\vec{r}_A$ locates the point on the transfer ellipse such that this vector's magnitude equals the distance from the Earth to the SOA at the SOA's closest point. $F_0$ is defined to be the true anomaly of $\vec{r}_A$ (see Figure D-2).

The radial vector $\vec{r}_B$ locates the SOA impact point. $F_1$ is defined to be the true anomaly of $\vec{r}_B$. $\Delta F$ is defined to be the difference between the anomalies.

$$\Delta F = F_1 - F_0.$$ 

The initial coast angle (CSTANG0) is the angle between the major axis of the transfer ellipse and $\vec{r}_A$.

$$\text{CSTANG0} = \pi - F_0.$$ 

The angle of SOA impact (CSTANG1) is the angle between the Earth-Moon line and $\vec{r}_B$.

$$\text{CSTANG1} = \tan^{-1}\left(\frac{RSOA \times \sin(LUGAMMA)}{EMRANGE - RSOA \times \cos(LUGAMMA)}\right)$$

From Figure D-1 it can be seen that

$$\text{CSTANG} = \text{CSTANG0} + \text{CSTANG1} - \Delta F$$

$$= (\pi - F_0) + \text{CSTANG1} - (F_1 - F_0)$$

$$= \pi + \text{CSTANG1} - F_1.$$
Appendix E - Calculation of ANGMOMGAM
The flight path angle ($\gamma$) of the vehicle at Lunar SOA is calculated in terms of the vehicle's angular momentum ($\vec{h}$). The angular momentum is defined to be the cross-product of the position vector ($\vec{r}$) and the linear momentum vector ($m\vec{v}$), where $\gamma$ is in the $\vec{r}, \vec{v}$ plane and $\vec{h}$ is perpendicular to this plane (see Figures E-1 and E-2).

\[
\vec{h} = \vec{r} \times m\vec{v} = h \cdot \hat{u} = (m \cdot r \cdot v \cdot \cos \gamma) \cdot \hat{u}
\]

Angular momentum is constant along a given orbit. Therefore, the angular momentum at perigee is the same as the angular momentum at the sphere of action. If the magnitude of the velocity vector is known at the sphere of action, then the velocity at a specified perigee ($v_p$) can be calculated.

\[
v_p^2 = \sqrt{v^2 - v_{esc(SOA)}^2 + v_{esc(perigee)}^2}
\]

\[
v_p^2 = \sqrt{v^2 - 2 \cdot (\mu_M/r_{SOA}) + 2 \cdot (\mu_M/r_p)}
\]

where $v$ = magnitude of the velocity vector at SOA
\[
\mu_M = \text{moon's gravitational constant}
\]
\[
r_{SOA} = \text{radial distance to the SOA}
\]
\[
r_p = \text{radial distance to perigee.}
\]

At perigee, the flight path angle is zero by definition, and the magnitude of the orbit's angular momentum is the simple product of the distance and velocity.

\[
h = m \cdot r_p \cdot v_p
\]
Since the angular momentum is the same at the sphere of action, the flight path angle can be calculated.

\[ h = m \cdot r_p \cdot v_p = m \cdot r_{SOA} \cdot v \cdot \cos \gamma \]

\[ \gamma_{SOA} = \cos^{-1} \left[ \frac{r_p \cdot v_p}{r_{SOA} \cdot v_{SOA}} \right] = \text{ANGMOMGAM.} \]
Appendix F - Test Cases
The attached tables show actual Apollo transfer orbit data compared with the outputs of the program LLOFX. The burns shown in the chart include the Trans-Lunar Injection (TLI), Lunar Orbit Insertion (LOI), and Trans-Earth Injection (TEI). The TLI burn puts the vehicle in a hyperbolic or elliptical orbit to get in the sphere of influence about the moon. Next, the LOI burn puts the vehicle in a circular orbit of 60 nautical miles around the moon. When returning to the Earth a TEI burn is made in lunar holding orbit. The Apollo flights directly entered the Earth’s atmosphere to perform a splash down. The program assumes that vehicles will come back to a transportation node in low Earth orbit (i.e. space station). To simulate an Apollo entry, the program LLOFX was run with an Earth holding orbit of 10.16 km. Listed are differences that may cause discrepancies: Apollo used free return trajectories that are not used in LLOFX. Apollo vehicles made plane changes going into transfer orbits and during Lunar Orbit Insertion, while LLOFX does not. Apollo confronted out-of-plane Earth-moon trajectories while LLOFX assumes in-plane. Apollo spacecraft had gravity losses due to long burns versus LLOFX’s assumption of instantaneous burns. Also, Earth-moon perturbations might also affect output.

Although many assumptions are made, the numbers are within requested bounds. The TLI burns are less than 5% off the Apollo data and LOI numbers are within approximately 10% of the Apollo data. Therefore, LLOFX is useful for rough performance estimates for lunar vehicles.
<table>
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<th>Flight #</th>
<th>Earth Moon Distance (km)</th>
<th>Flight Time (hrs)</th>
<th>Earth Orbit Altitude (km)</th>
<th>Lunar Orbit Altitude (km)</th>
<th>TLI Delta-V (km/sec)</th>
<th>LOI Delta-V (km/sec)</th>
</tr>
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<tr>
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<td>351600</td>
<td>73.09</td>
<td>334.7</td>
<td>111.2</td>
<td>3.18</td>
<td>0.89</td>
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<tr>
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<td>351600</td>
<td>73.02</td>
<td>334.7</td>
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REFERENCES


2. Apollo Mission Reports.