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# Nonlinearities in Spacecraft Structural Dynamics

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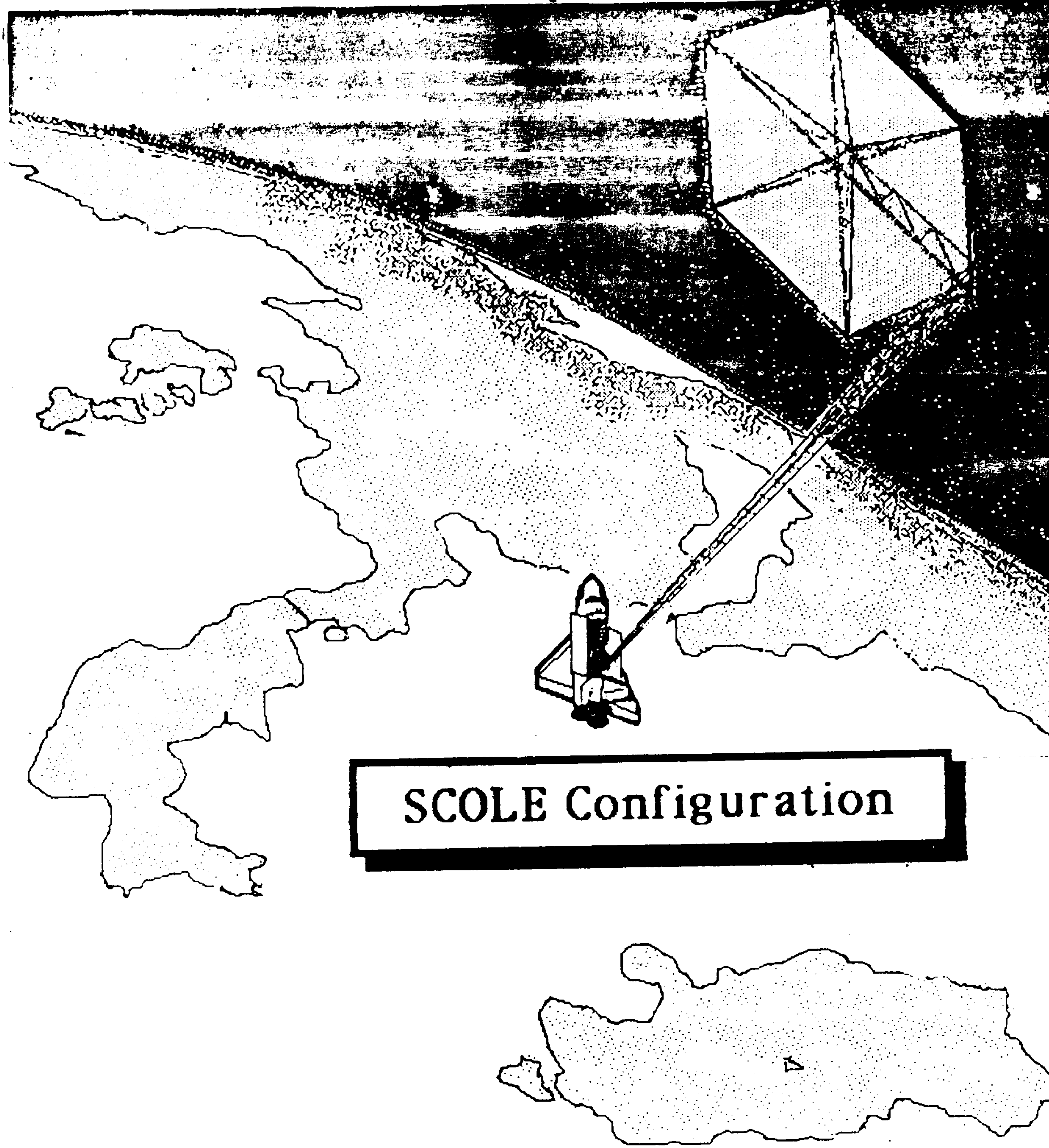
# OUTLINE

- SCOLE Configuration – Equations of Motion
- Modeling Error Sources
- Approximate Solutions
- Comparison of Model Accuracy
- Linear & Nonlinear Damping
- Experimental Results
- Future Work



# SCOLE Configuration

# Equations of Motion

**Shuttle (and Reflector) Body**

$$\dot{\varpi}_1 = -I_1^{-1}(\tilde{\varpi}_1 I_1 \varpi_1 - M_1 - M_{1,Beam})$$

$$\dot{v}_1 = (F_1 + F_{1,Beam})/m_1$$

$$\dot{T}_1^T = -\tilde{\varpi}_1 T_1^T$$

**Roll (and Pitch) Beam Bending**

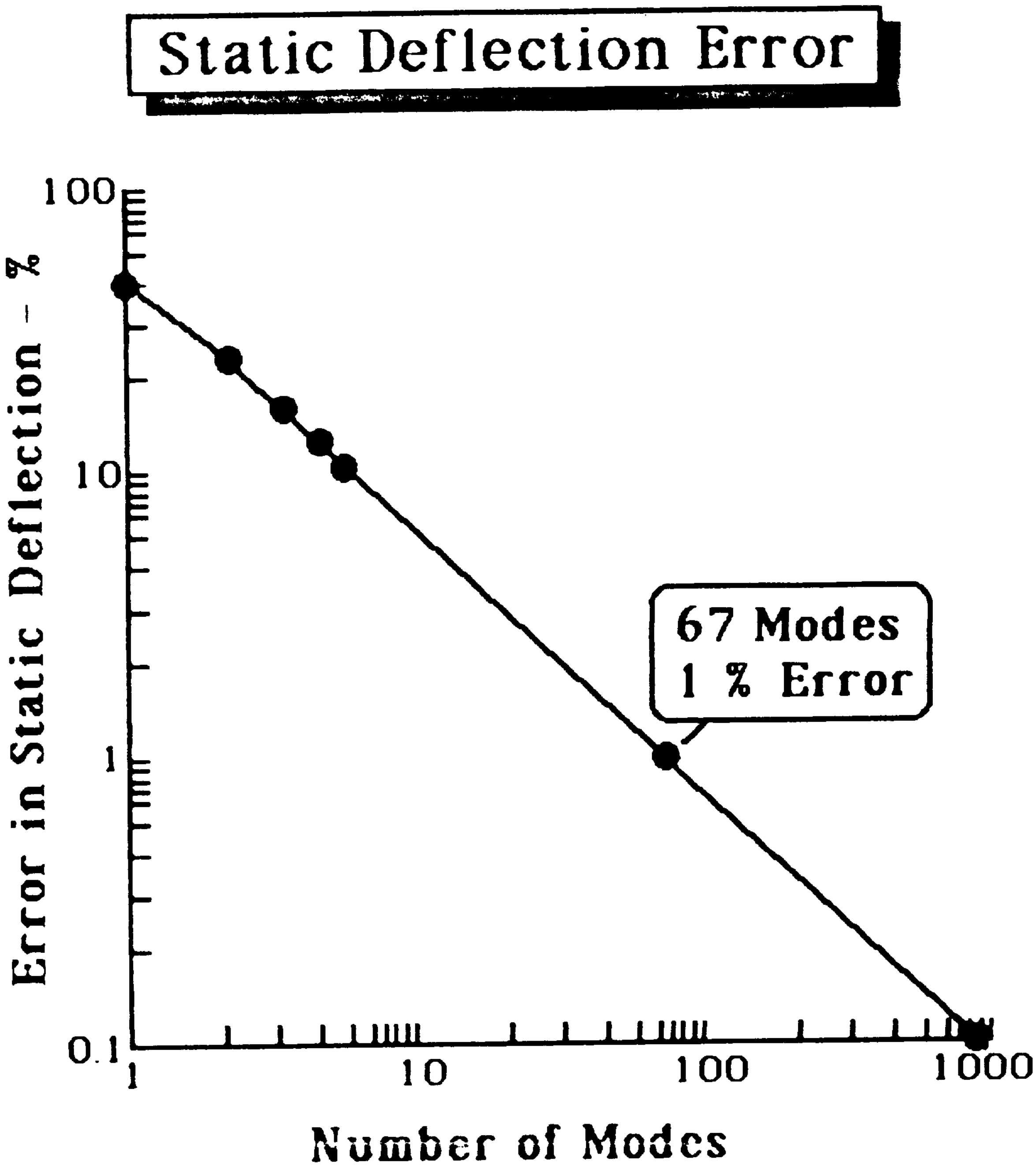
$$\rho A_\phi \frac{d^2 u_\phi}{dt^2} - CI_\phi \frac{d^3 u_\phi}{ds^2 dt} + EI_\phi \frac{d^4 u_\phi}{ds^4} = \sum_{n=1}^{4} [f_{\phi,n} \delta(s-s_n) + g_{\phi,n} \frac{d\delta}{ds}(s-s_n)]$$

**Yaw Beam Torsion**

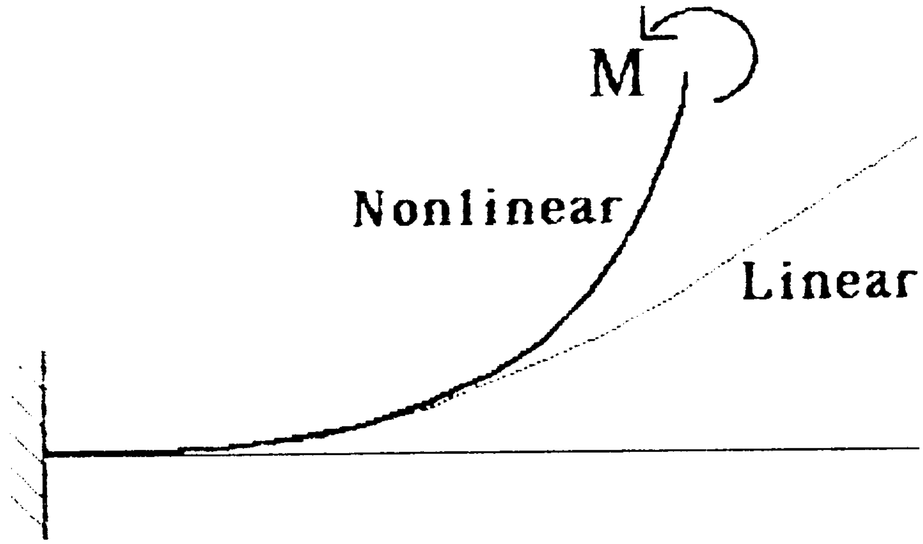
$$\rho I_\psi \frac{d^2 u_\psi}{dt^2} + CI_\psi \frac{d^3 u_\psi}{ds^2 dt} - GI_\psi \frac{d^2 u_\psi}{ds^2} = \sum_{n=1}^{4} g_{\psi,n} \delta(s-s_n)$$

**Beam Elongation**

$$\rho A \frac{d^2 u_z}{dt^2} + C_z A \frac{d^2 u_z}{ds dt} - EA \frac{d^2 u_z}{ds^2} = \sum_{n=1}^{4} f_{z,n} \delta(s-s_n)$$


Static Deflection Error
100
10
1
0.1
Error in Static Deflection - %
67 Modes
1 % Error
1
10
100
1000
Number of Modes

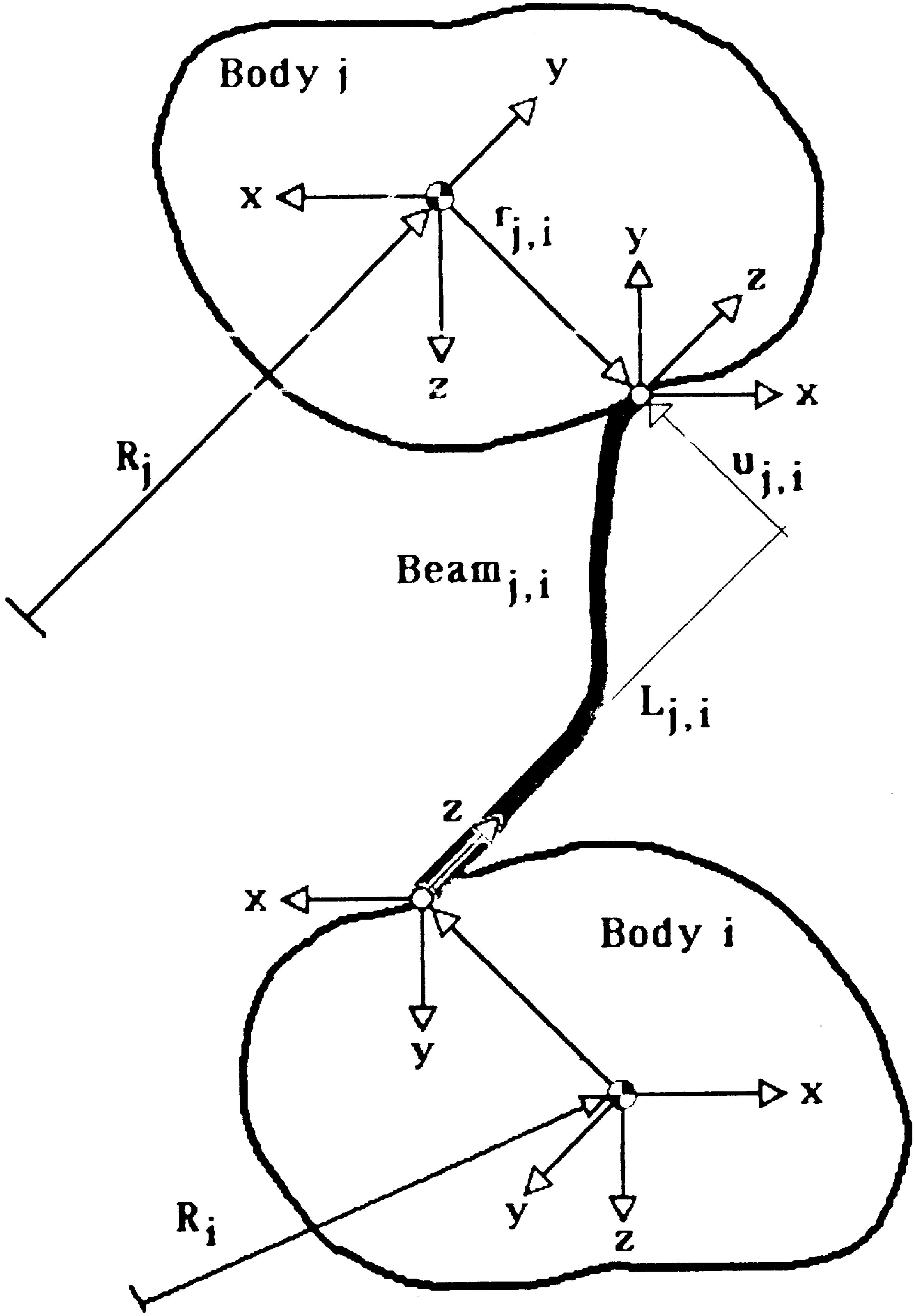
# Large Amplitude Deflection Effects




| Deflection, y/L | Error, e/y |
|---|---|
| .05 | .17 % |
| .10 | .67 % |
| .20 | 2.7 % |
| .30 | 6.0 % |
| .40 | 10.6 % |
| .50 | 16.4 % |

## Lumped-Mass Model

- Exact Static Deflection
- Approximates Low-Frequency Modes
- Nonlinear Kinematics
- Linearized State Space, Modal Model
- Classical Damping(Working Proportional)
- Extended to n-Body Network



Body j
y
x
$r_{j,i}$
y
z
z
x
$R_j$
$u_{j,i}$
$Beam_{j,i}$
$L_{j,i}$
z
x
Body i
y
x
y
$R_i$
z

## Stiffness Matrices

$$M_{u'} = \begin{bmatrix} -\frac{4EI}{L} - \frac{2WL}{15}^{*} & 0 & 0 \\ 0 & -\frac{4EI}{L} - \frac{2WL}{15}^{*} & 0 \\ 0 & 0 & -\frac{GJ}{L} \end{bmatrix}$$

$$M_{u} = \begin{bmatrix} 0 & \frac{6EI}{L^2} + \frac{W}{10}^{*} & 0 \\ \frac{6EI}{L^2} + \frac{W}{10}^{*} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* Gravity Effect

# Stiffness Matrices

$$F_u = \begin{bmatrix} -\frac{12EI}{L^3} - \frac{6W}{5L}^{*} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} - \frac{6W}{5L}^{*} & 0 \\ 0 & 0 & -\frac{EA}{L} \end{bmatrix}$$

$$F_{u'} = \begin{bmatrix} 0 & \frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* Gravity Effect

# Asymptotic Approximation

- Motion Approaches Clamp-Clamped System as Mode Number Increases
- Accuracy Increases with Increasing Mode Number
- Explicit Expressions for Modal Frequencies and Mode Shapes
- First Variation Approximation for Motion of End Bodies
- Singular Perturbation Technique can be used to Improve Approximate Solutions

# Comparison of Modal Frequencies

| MODE NO | EXACT REF.5&6 | FINITE ELEMENT REF. 9 | FINITE ELEMENT REF. 10 | FINITE ELEMENT REF. 11 |
|---|---|---|---|---|
| 1 | .278025624 | .278 | .277 | .2740 |
| 2 | .313776751 | .317 | .314 | 3229 |
| 3 | .812326353 | .726 | .805 | .7494 |
| 4 | 1.18366347 | 1.226 | 1.175 | 1.244 |
| 5 | 2.05047101 | 2.069 | 2.028 | 2.052 |
| 6 | 4.75561758 | 4.77 | 4.617 | |
| 7 | 5.51248431 | 5.52 | 5.388 | |
| 8 | 12.2598619 | 12.4 | 11.782 | |
| 9 | 12.8877037 | 13.0 | 12.513 | |
| 10 | 23.5359367 | 24.2 | 14.670 | |
| 11 | 24.2568205 | 24.7 | 22.968 | |
| 12 | 26.4794890 | 26.2 | 23.490 | |
| 13 | 38.9199260 | 45.4 | 37.568 | |
| 14 | 39.4643489 | 45.9 | 38.146 | |
| 15 | 45.1313668 | 56.3 | 44.653 | |
| 16 | 57.90 | | 45.161 | |
| 17 | 57.92 | | | |
| 18 | 80.72 | | | |
| 19 | 80.72 | | | |

16% ERROR

# Comparison of Modal Frequencies

| MODE NO. | EXACT REF. 5&6 | Lumped Mass | Asymptotic |
|---|---|---|---|
| 1 | .278025624 | .258 | |
| 2 | .313776751 | .370 | |
| 3 | .812326353 | .926 | |
| 4 | 1.18366347 | 1.79 | |
| 5 | 2.05047101 | 2.57 | |
| 6 | 4.75561758 | | 4.23885 |
| 7 | 5.51248431 | | 4.23885 |
| 8 | 12.2598619 | | 11.88805 |
| 9 | 12.8877037 | | 11.88805 |
| 10 | 23.5359367 | | 23.313674 |
| 11 | 24.2568205 | | 23.313674 |
| 12 | 26.4794890 | | |
| 13 | 38.9199260 | | 38.534998 |
| 14 | 39.4643489 | | 38.534998 |
| 15 | 45.1313668 | | |
| 16 | 57.90* | | 57.455629 |
| 17 | 57.92* | | 57.455629 |
| 18 | 80.72* | | 80.24802 |
| 19 | 80.72* | | 80.24802 |

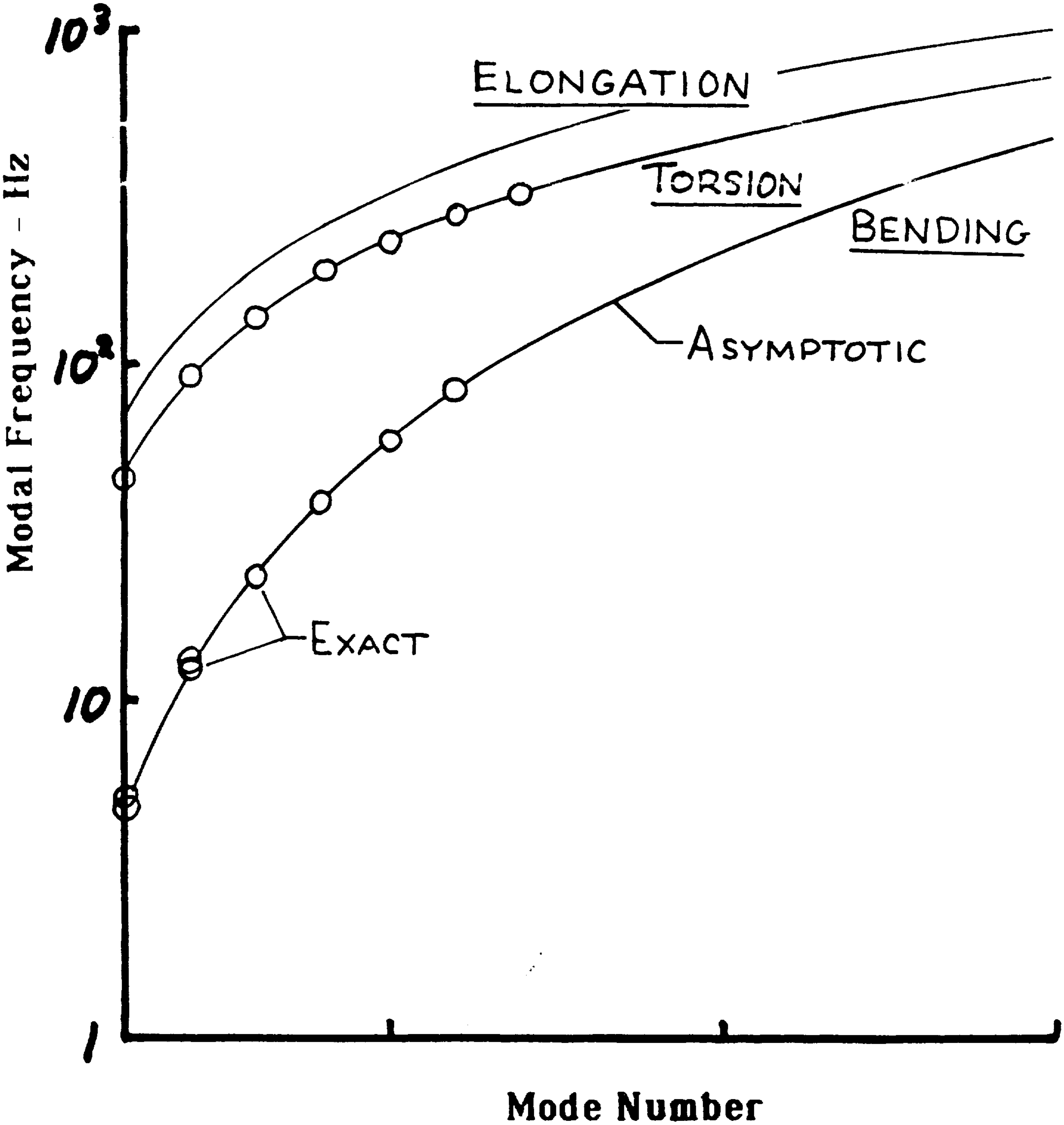
51 % ERROR (mode 4, Lumped Mass)

30 % ERROR (mode 7, Asymptotic)

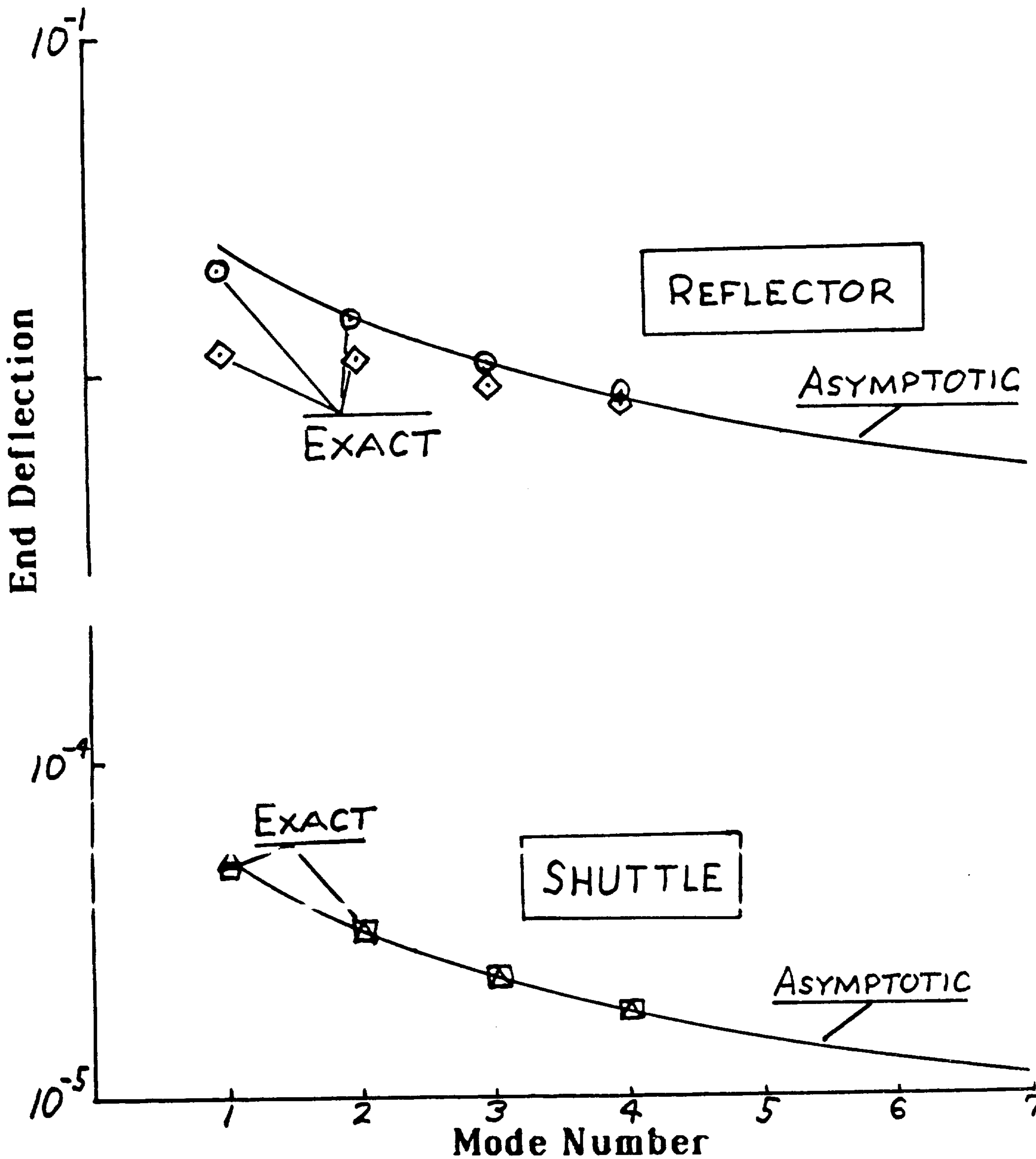
.6 % ERROR (modes 18–19, Asymptotic)

* - Uncoupled (Reference 3).

## Comparison of Modal Frequencies

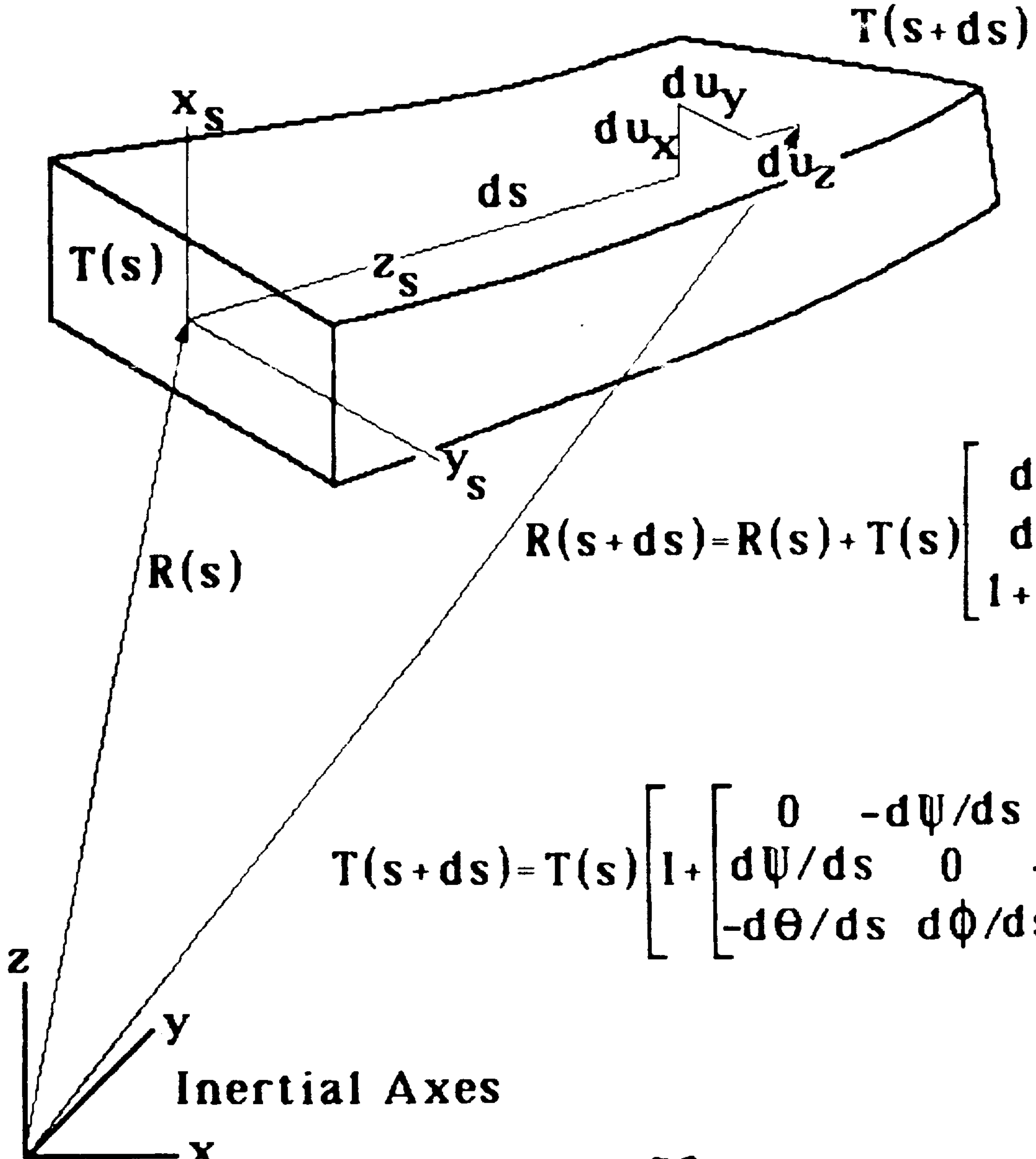


# Comparison of Deflections



# A 3-Dimensional Beam Equation

- Includes Nonlinear Kinematics
- Makes No Small Angle Approximation
- Bending(2 Directions), Torsion & Elongation




$$R(s+ds)=R(s)+T(s)\begin{bmatrix} du_x/ds \\ du_y/ds \\ 1+du_z/ds \end{bmatrix}ds$$

$$T(s+ds)=T(s)\left[1+\begin{bmatrix} 0 & -d\psi/ds & d\theta/ds \\ d\psi/ds & 0 & -d\phi/ds \\ -d\theta/ds & d\phi/ds & 0 \end{bmatrix}ds\right]$$

It follows that the deflection vector of the beam is:

$$R(s) = \int_0^s T(s')\begin{bmatrix} du_x/ds' \\ du_y/ds' \\ 1+du_z/ds' \end{bmatrix} ds'$$

and the direction cosine of the cross section axes is given by:

$$\frac{dT(s)}{ds} = T(s)\begin{bmatrix} 0 & -d\psi/ds & d\theta/ds \\ d\psi/ds & 0 & -d\phi/ds \\ -d\theta/ds & d\phi/ds & 0 \end{bmatrix}$$

The forces and moments applied to the beam are related to the beam deformations by:

$$\begin{matrix} dFx \\ dFy \\ dFz \\ dMx \\ dMy \\ dMz \end{matrix} = \left[\begin{array}{ccc|ccc} -k'GA & & & & & \\ & -k'GA & & & & \\ & & -EA & & & \\ \hline & & & -EI_{xx} & & \\ & & & & -EI_{yy} & \\ & & & & & -EI_{zz} \end{array}\right]\begin{bmatrix} du_x/ds \\ du_y/ds \\ du_z/ds \\ d\phi/ds \\ d\theta/ds \\ d\psi/ds \end{bmatrix} ds$$

$$dF = F_u du + F_e de$$
$$dM = M_u du + M_e de$$

Where

$$du = \begin{bmatrix} du_x/ds \\ du_y/ds \\ du_z/ds \end{bmatrix} ds \qquad de = \begin{bmatrix} d\phi/ds \\ d\theta/ds \\ d\psi/ds \end{bmatrix} ds$$

The incremental force can be related to deformation of the beam.

$$\frac{dR}{ds} = \frac{R(s+ds)-R(s)}{ds} = T\begin{bmatrix}0\\0\\1\end{bmatrix} + T\begin{bmatrix}du_x/ds\\du_y/ds\\du_z/ds\end{bmatrix}$$

$$\begin{bmatrix}du_x/ds\\du_y/ds\\du_z/ds\end{bmatrix} = T^T\frac{dR}{ds} - \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\frac{dF}{ds} = \frac{F(s+ds) - F(s)}{ds} = F_u T^T\frac{dR}{ds} - F_u\begin{bmatrix}0\\0\\1\end{bmatrix} + F_e\frac{de}{ds}$$

Similarly, for the incremental moment..

$$\frac{dM}{ds} = M_u T^T\frac{dR}{ds} - M_u\begin{bmatrix}0\\0\\1\end{bmatrix} + M_e\frac{de}{ds} + \frac{dF}{ds}$$

The equations of motion for the beam element are:

$$m\frac{d^2R}{dt^2} = T\frac{d}{ds}\left[\frac{dF}{ds}\right] + TF$$

and

$$I_o\frac{d^2e}{dt^2} = -\widetilde{\frac{de}{dt}}I_o\frac{de}{dt} + \frac{d}{ds}\left[\frac{dM}{ds}\right] + M$$

The equations of motion for the beam element become:

$$m\frac{d^2R}{dt^2} = T\frac{d}{ds}\left[F_u\left(T^T\frac{dR}{ds} - \begin{vmatrix}0\\0\\1\end{vmatrix}\right) + F_e\frac{de}{ds}\right] + T\,F$$

$$I_o\frac{d^2e}{dt^2} = -\frac{\widetilde{de}}{dt}I_o\frac{de}{dt} + \frac{d}{ds}\left[M_u\left(T^T\frac{dR}{ds} - \begin{vmatrix}0\\0\\1\end{vmatrix}\right) + M_e\frac{de}{ds}\right]$$

$$+ F_u\left(T^T\frac{dR}{ds} - \begin{vmatrix}0\\0\\1\end{vmatrix}\right) + F_e\frac{de}{ds} + M$$

Where

$$e = \text{the arc direction cosine}[T] = \begin{bmatrix}\phi\\ \theta\\ \psi\end{bmatrix}$$

$$\frac{de}{dt} = \begin{bmatrix}d\phi/dt\\ d\theta/dt\\ d\psi/dt\end{bmatrix}$$

$$\frac{dT}{dt} = T\frac{\widetilde{de}}{dt}$$

$$\frac{dT}{ds} = T\frac{\widetilde{de}}{ds}$$

$m$ = the mass per unit length

$I_o$ = the moment of inertia per unit length

$$= \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz}\\ -I_{xy} & I_{yy} & -I_{yz}\\ -I_{xz} & -I_{yz} & I_{zz}\end{bmatrix}$$

For the case in which deflections are small and the end of the beam is aligned with the inertial axes:

$$T \approx I + \tilde{e} = \begin{bmatrix} 1 & -dU & d\theta \\ d\psi & 1 & -d\phi \\ -d\theta & d\phi & 1 \end{bmatrix}$$

$$\frac{dR}{ds} = T\begin{bmatrix} du_x/ds & + & 0 \\ du_y/ds & + & 0 \\ du_z/ds & & 1 \end{bmatrix} \approx \begin{matrix} du_x/ds \\ du_y/ds \\ du_z/ds \end{matrix} + \begin{bmatrix} \theta \\ -\phi \\ 1 \end{bmatrix}$$

$$R \approx R_o + u + \int_o^s \begin{bmatrix} \theta \\ -\phi \\ 1 \end{bmatrix} ds$$

The linearized equations become:

$$m\frac{d^2R}{dt^2} = \frac{d}{ds}\left[F_u\left(\frac{dR}{ds} - \begin{vmatrix} \theta \\ -\phi \\ 1 \end{vmatrix}\right) + F_e\frac{de}{ds}\right] + F + \tilde{e}F$$

$$I_o\frac{d^2e}{dt^2} = \frac{d}{ds}\left[M_u\left(\frac{dR}{ds} - \begin{vmatrix} \theta \\ -\phi \\ 1 \end{vmatrix}\right) + M_e\frac{de}{ds}\right] + F_u\left(\frac{dR}{ds} - \begin{vmatrix} \theta \\ -\phi \\ 1 \end{vmatrix}\right) + F_e\frac{de}{ds} + M$$

For bending only, in a single plane the equations of motion become those for the Timoshenko beam.

$$m\frac{d^2R_x}{dt^2} = \frac{d}{ds}\left[kGA\left(\frac{dR_x}{ds} - \theta\right)\right] + F_x$$

$$I_{xx}\frac{d^2\theta}{dt^2} = \frac{d}{ds}\left[EI_{xx}\frac{dR_x}{ds}\right] - kGA\left(\frac{dR_x}{ds} - \theta\right) + M_x$$

If rotary inertia effects are neglected the result is the Bernoulli-Euler beam.

$$m\frac{d^2R_x}{dt^2} = -\frac{d^2}{ds^2}\left[EI_{xx}\frac{d^2R_x}{ds^2}\right] + F_x$$

# Kelvin-Voight Damping

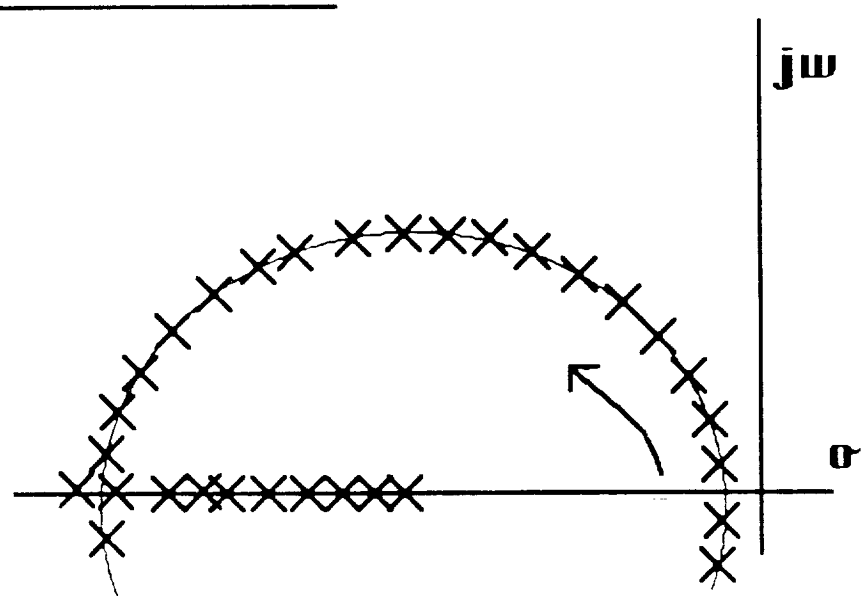
Bernoulli-Euler Beam Equation with Kelvin-Voight Damping

$$EIu'''' + C\dot{u}'''' + m\ddot{u} = 0$$

Allows Separation of Variables

Theoretical Basis for Damping

Locus of Modal Characteristics




EXCESSIVE DAMPING AT HIGH MODE NUMBERS !!!!

# Proportional Damping

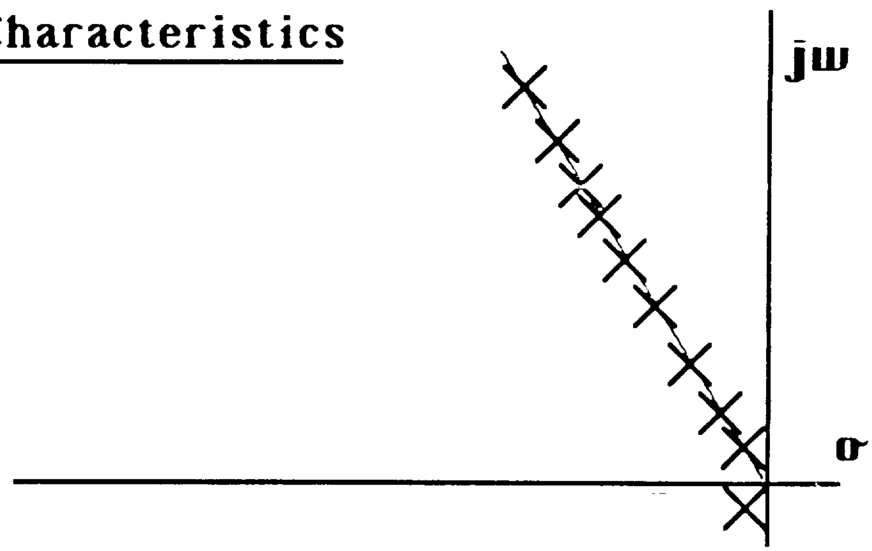
Bernoulli-Euler Beam Equation with Proportional Damping

$$EIu'''' + C\dot{u}'' + m\ddot{u} = 0$$

Allows Separation of Variables for Pinned and Infinite End Conditions

Lacks Theoretical Basis for Damping

Locus of Modal Characteristics




REASONABLE DAMPING AT HIGH MODE NUMBERS

# Piano-Wire Damping

## Viscous Damping Ratio

Smaller Mass $\xi = .0015$

Larger Mass $\xi = .0013$

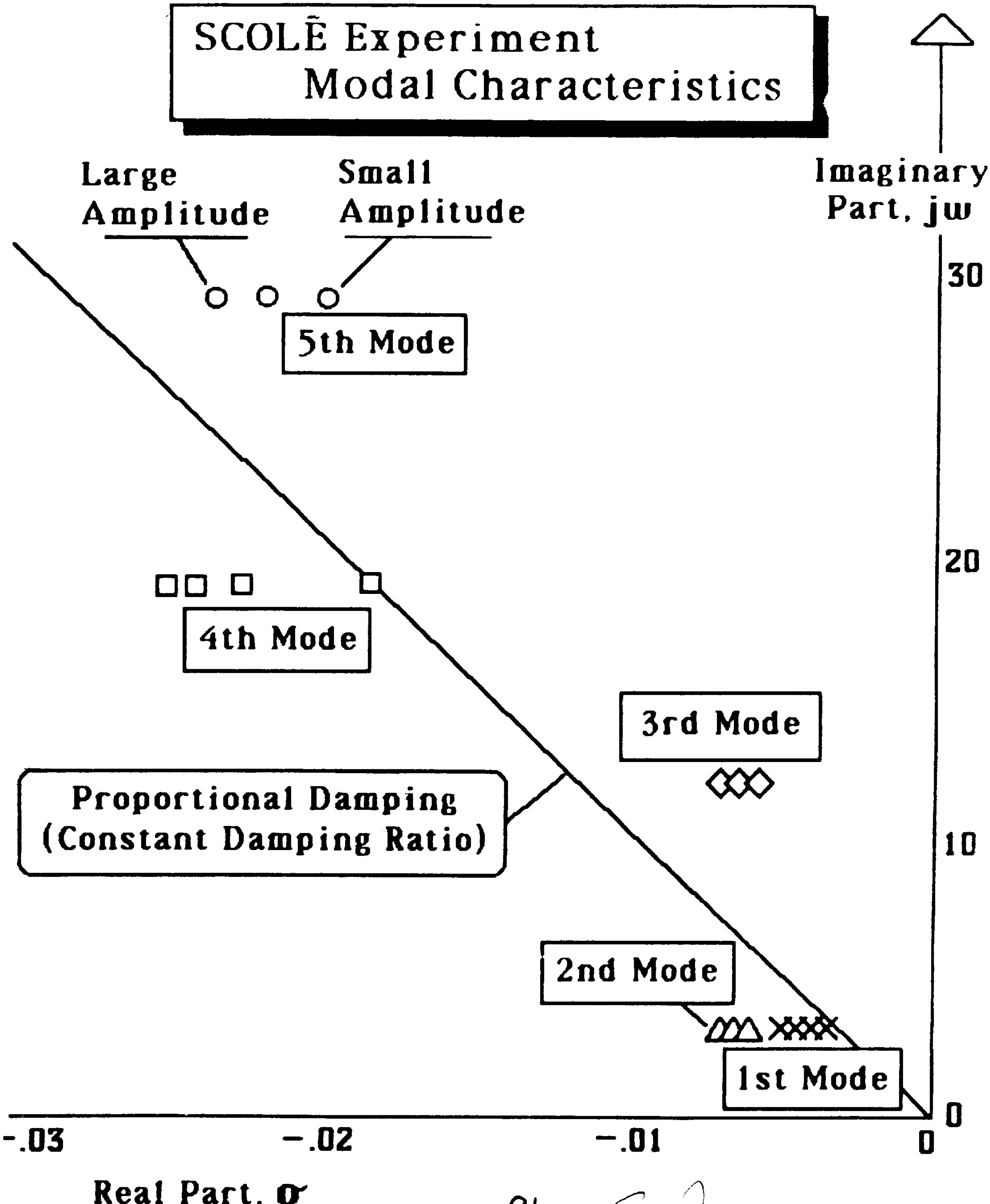
General Mass $\xi = .0015\sqrt{\frac{m_1}{m}}$

## Nonlinear Damping

$$m\ddot{x} = -c_1\dot{x} - c_2|\dot{x}|\dot{x} - kx$$

$$A_{n+1} = A_n - A_n(.00138)2\pi - (.0012)A_n^2$$

Determined to be Air Damping


SCOLE Experiment
Modal Characteristics
Large Amplitude
Small Amplitude
Imaginary Part, jω
30
5th Mode
20
4th Mode
3rd Mode
Proportional Damping
(Constant Damping Ratio)
10
2nd Mode
1st Mode
0
-.03
-.02
-.01
0
Real Part, σ

91

# SCOLE DAMPING

**Viscous Damping Ratio**

| Mode | Configuration # 1 | Configuration # 2 |
|---|---|---|
| 1 | .0016 | .0013 |
| 2 | .0011 | .0009 |
| 3 | .00058 | |
| 4 | .0011 | |
| 5 | .00084 | |

Nonlinear Damping is Evident for Large Amplitude Motion. Analysis is Underway.

# Nonlinear Damping

## Mass, Spring, Nonlinear Damper

$$m\ddot{x} = -c|x|^{a}|\dot{x}|^{b}\dot{x} - kx$$

## Considering Only Light Damping ...

$$\omega = \sqrt{k/m}$$

## For Free Decay

$$x(t) = A(t)\sin(\omega t)$$

$$\dot{A} = -\frac{c}{m}|x|^{a}|\dot{x}|^{b} = -\frac{c}{m}\omega^{b}A^{a+b}$$

## Solving

$$dt = \frac{m\,dA}{c\,\omega^{b}A^{a+b}}$$

$$t + t_0 = \frac{m}{c(a+b-1)\,\omega^{b}A^{a+b-1}}$$

$$A(t) = \left[\frac{m}{c(a+b-1)\,\omega^{b}(t + t_0)}\right]^{\frac{1}{a+b-1}}$$

# Nonlinear Damping

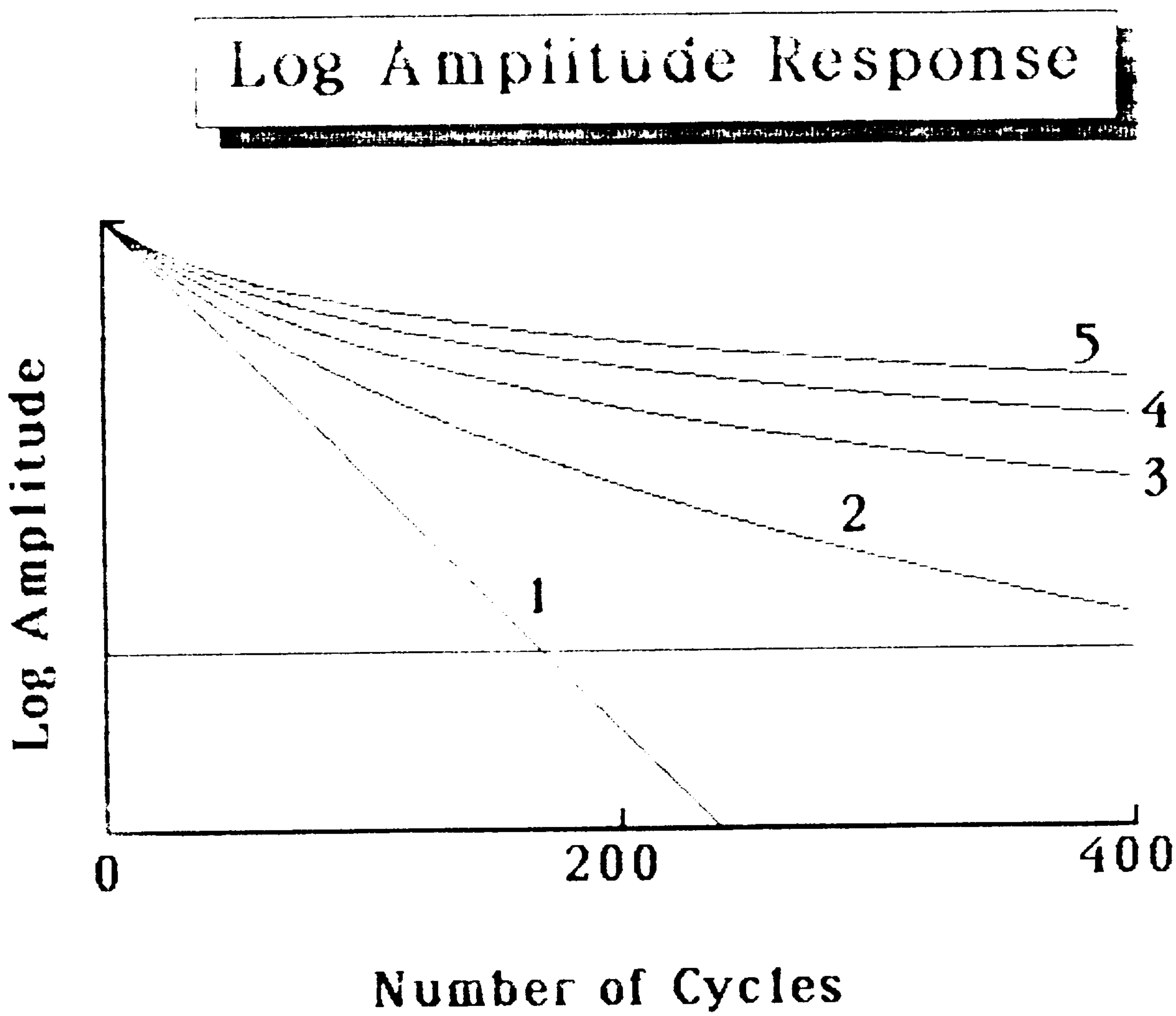
$$A(n) = \left[\frac{m}{c\,(a+b-1)\,\omega^{b}\,n}\right]^{\frac{1}{a+b-1}}$$

Where $t + t_0 = n\frac{2\pi}{\omega}$ , $\omega = [k/m]^{1/2}$

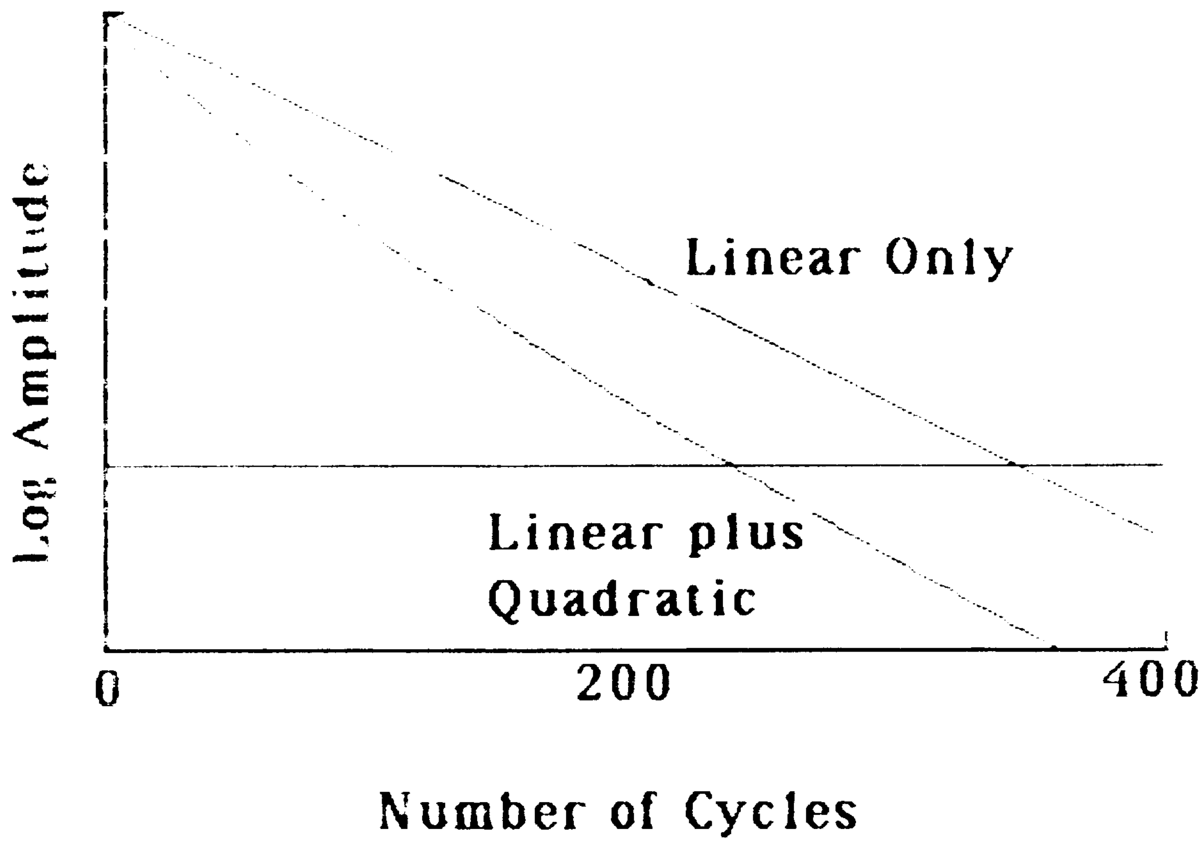
$$A(n) = \left[\frac{m^{(b+1)/2}}{c\,(a+b-1)\,k^{(b-1)/2}\,2\pi n}\right]^{\frac{1}{a+b-1}}$$

**For Example**

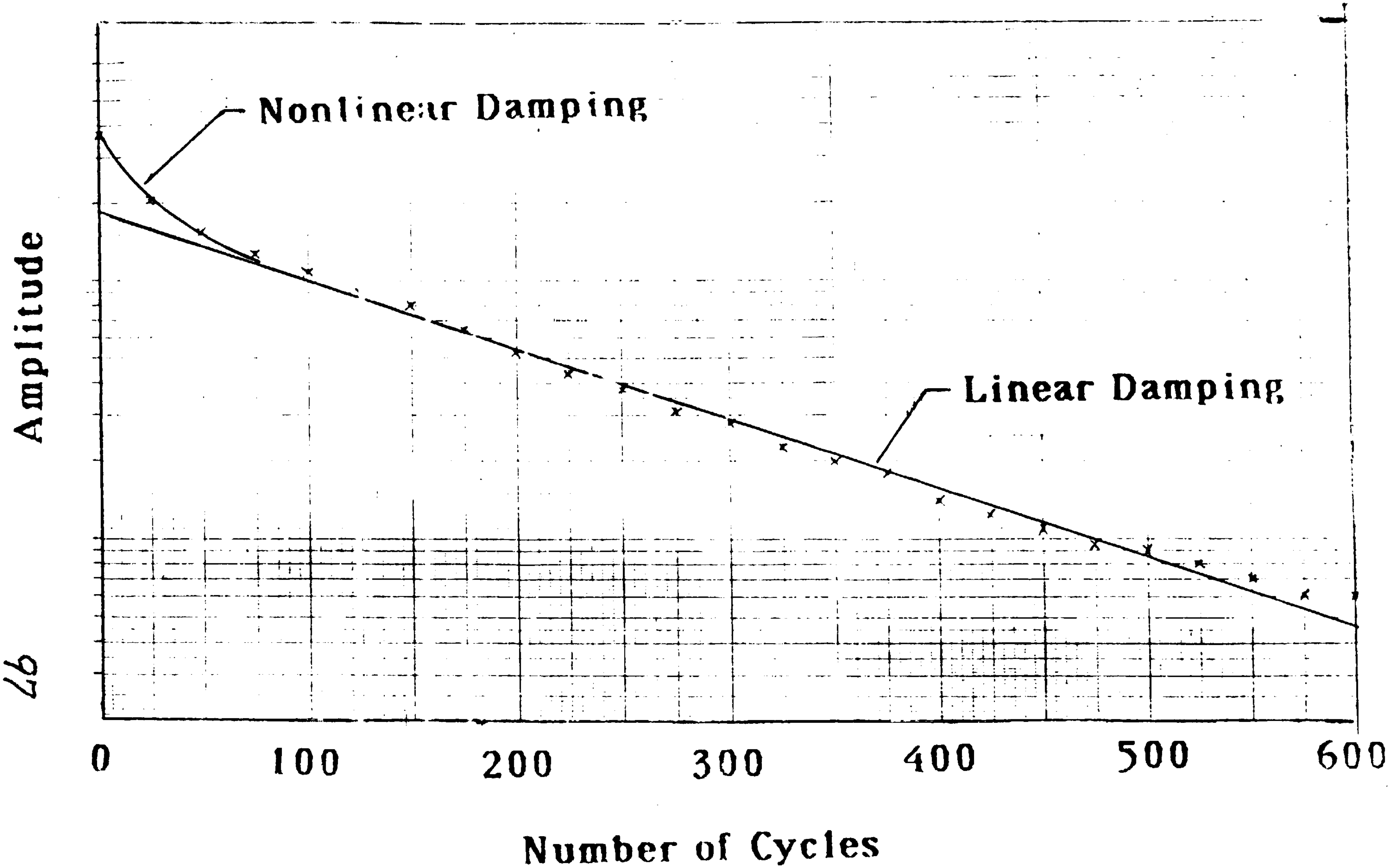
- $\frac{c}{m}\dot{x}$    $a = b = 0$    $\frac{dA}{A} = 2\pi\zeta = \frac{c\pi}{\sqrt{km}}$
- $\frac{c}{m}|\dot{x}|\,\dot{x}$    $a = 0$, $b = 1$    $\frac{dA}{A} = \frac{4cA\omega}{3m} = \frac{4cAk^{1/2}}{3m^{3/2}}$
- $\frac{c}{m}|x|\,\dot{x}$    $a = 1$, $b = 0$    $\frac{dA}{A} = \frac{2cA}{3m}$


Log Amplitude Response
Log Amplitude
5
4
3
2
1
0
200
400
Number of Cycles

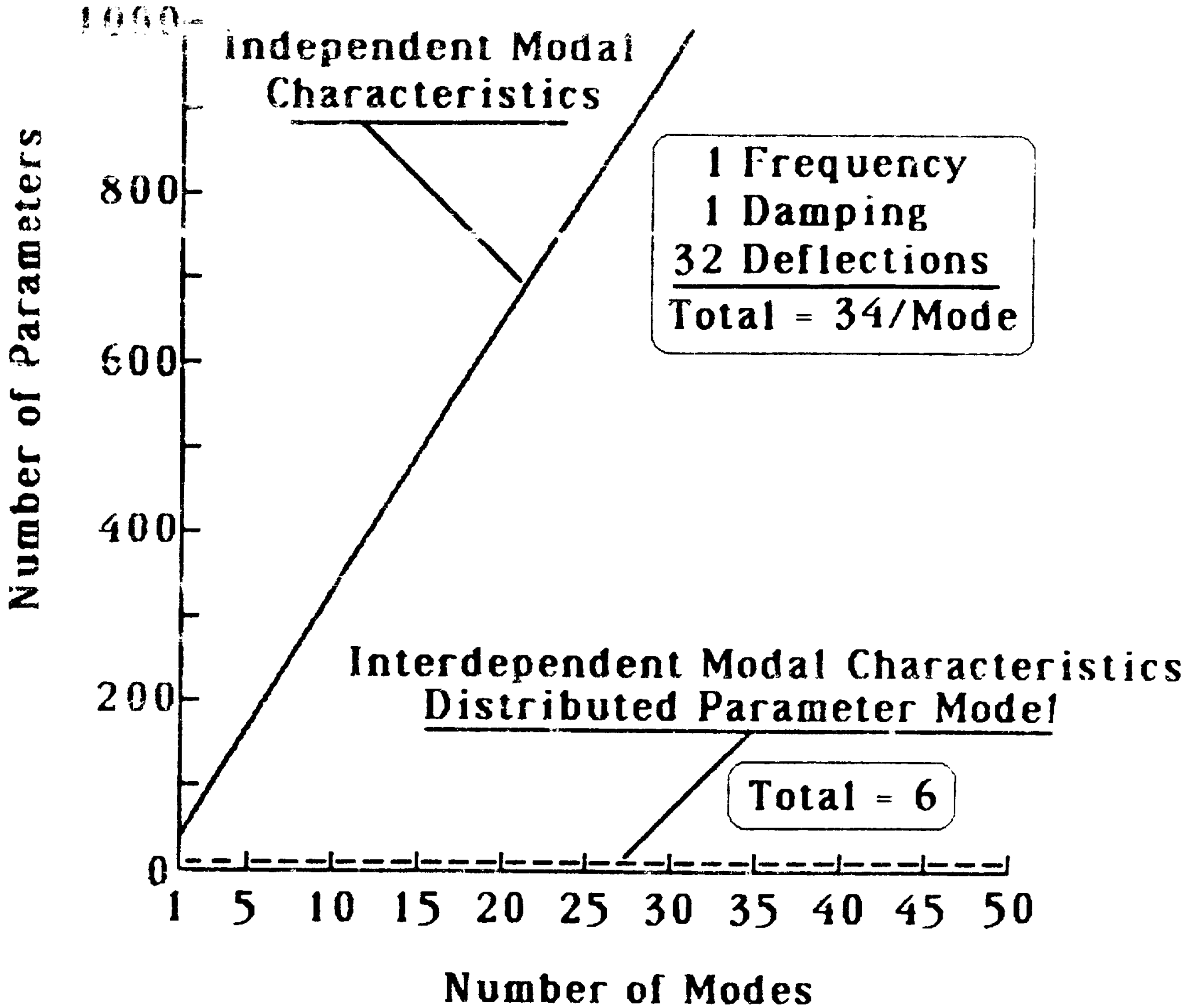
## Log Amplitude Response



# SCOLE DAMPING



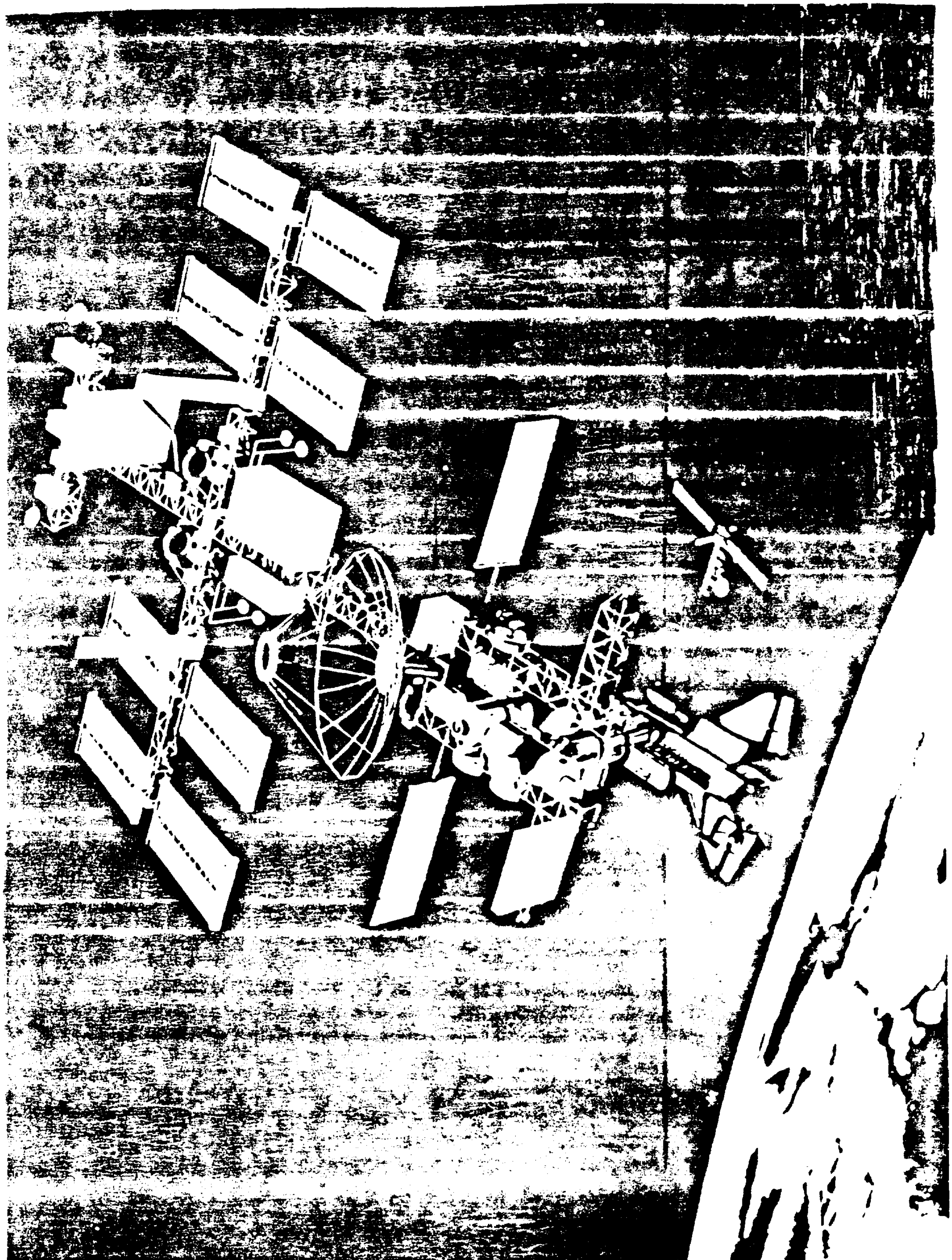
# The Curse of Dimensionality



# Concluding Remarks

- The Accuracy of the Modal Characteristics of the SCOLE Configuration were Examined Using Exact and Approximate Solutions.
- Sixty-Seven Modes are Required for a Static Deflection Error of less than 1 %. SCOLE Model Requires Hundreds of Modes.
- Exact Solutions Encounter Numerical Difficulties.
- Asymptotic Solutions in Combination with Limited Exact Solutions Enable Generation of a Proof Model with the Required Accuracy.
- Damping Must be Incorporated into the Model from the Start. Proportional Damping is Not Adequate.

ORIGINAL PAGE IS
OF POOR QUALITY



"It will never be possible to have the absolute conviction before flight that a valid mathematical model has been devised for a space vehicle.

....we surely must make every effort to ensure that failures do not come from inadequate analysis of the best models available."

Peter Likins
1971