A Study of the Use of Linear Programming Techniques to Improve the Performance in Design Optimization Problems

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A STUDY OF THE USE OF LINEAR PROGRAMMING TECHNIQUES TO IMPROVE THE PERFORMANCE IN DESIGN OPTIMIZATION PROBLEMS.

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ABSTRACT

There are two purposes of this project. One is to determine whether linear programming techniques can improve the performance in handling design optimization problems with a large number of design variables and constraints relative to the feasible directions algorithm. The second purpose is to determine whether using the Kreisselmeier-Steinhauser (KS) function to replace the constraints with one constraint will reduce the cost of the total optimization. Using the software program, CONMIN, reference cases are run with both the linear and non-linear options. Next the same test case is run using the linear programming subroutine, LINPR1, from the math library. Comparisons are then made between the solutions obtained from both subroutines, CONMIN and LINPR1.

PROCEDURE

A simple problem of a hub with 12 spokes was used as the test example (see Appendix). This problem has 12 design variables and 24 constraints. The calculations were done on a DEC MICROVAX II workstation using code written in FORTRAN 77.

Using CONMIN, results were obtained using the non-linear and linear options. Next the 24 constraints were replaced by one, a KS-based cumulative constraint, and results obtained, again for both the non-linear and linear options (figure 1).

Since the value of rho in the KS function influences the result, different values of rho were used in the KS function. The comparisons between the linear and non-linear CONMIN solutions using the 24 constraints and one constraint (when using the KS function) are in figure 2.

After obtaining the results using CONMIN, the non-linear problem was turned into a linear programming one to be solved using the linear programming subroutine LINPR1. This is a library routine from the math library and uses the simplex method. The results using this routine are compared to the CONMIN results in figure 3.

CONCLUSIONS

The optimal value of the objective function was
consistently lower than the one obtained from the linear programming routine, regardless of the use of the KS

CONMIN VS LINPR1

![Bar chart comparing CONMIN and LINPR1](image)

**Figure 3 CONMIN vs. LINPR1**

function (figure 3). This is an unexpected and important finding of this study.

In terms of efficiency, all the runs were comparable in the number of function evaluations needed. However, there is a reduction of memory required by CONMIN when the constraints are replaced by a single cumulative constraint using the KS function.

**REFERENCES**

3. Mathematical and Statistical Software at Langley Documentation; LINPR1, Part I, Section H1.1, 03/01/87.

APPENDIX

**Problem Formulation:**

![Diagram](image)

**Nomenclature**

- **NS**: number of spokes
- **NLC**: number of loading cases
- **$\alpha_{i+1}$**: $\alpha_i + 2\pi/NS$
- **$E$**: Young's modulus
- **$A_i$**: cross-sectional area of rod $i$
- **$R$**: radius of the circle = length of each rod
- **$u_j^x$**: displacement of the hub along $x$ for $j$th loading case
- **$u_j^y$**: displacement of the hub along $y$ for $j$th loading case
- **$P_{ij}^x, P_{ij}^y$**: load components along $x, y$ for $j$th loading case
- **$b_i$**: $= EA_i R$
- **$\sigma_{at}$**: allowable tension stress
- **$\sigma_{ac}$**: allowable compression stress

**Details of the Analysis**

$$k_{11} = \sum_{i=1}^{NS} b_i \cos^2 \alpha_i$$  \hspace{1cm} (1)

$$k_{12} = \sum_{i=1}^{NS} \frac{1}{2} b_i \sin 2\alpha_i$$  \hspace{1cm} (2)

$$k_{22} = \sum_{i=1}^{NS} b_i \sin 2\alpha_i$$  \hspace{1cm} (3)

Displacements for loading cases $j$

$$\text{DET} = k_{11} k_{22} - k_{12}^2$$  \hspace{1cm} (4)
\[ u'_x = \frac{(P^1_{x,k22} - P^2_{y,k12})}{\text{DET}} \quad (5) \]
\[ u'_y = \frac{(P^1_{y,k11} - P^2_{x,k12})}{\text{DET}} \quad (6) \]

Strain in rod \( i \) for loading case \( j \)
\[ \varepsilon'_i = \left( -u'_x \cos \alpha_i - u'_y \sin \alpha_i \right) / R \quad (7) \]

Stress in rod \( i \) for loading case \( j \)
\[ s'_i = \varepsilon'_i E \quad (8) \]

There are \( N_S \cdot N_L \) stresses \( \sigma'_i \)

Material volume
\[ V = R \cdot \sum_{i=1}^{N_S} A_i \quad (9) \]

DETAILS OF THE OPTIMIZATION USING LINPR1

Turning the Optimization Problem into a Linear Programming One

Introduce a new variable
\[ X_i = \frac{1}{A_i} \quad (10) \]
Compute derivatives
\[ \frac{\partial V}{\partial X_i} \text{ and } \frac{\partial g_m}{\partial X_i} \quad (15) \]

Because of 14, equation 15 becomes
\[ \frac{\partial V}{\partial X_i} = \frac{\partial g_m}{\partial X_i} = (-1/X^2_i) \quad (16) \]

Using equation 9 put \( \frac{\partial V}{\partial A_i} = R \) in equation 16 then
\[ \frac{\partial V}{\partial X_i} = R(-A^2_i) \quad (18) \]

Approximate Linear Optimization Problem

Let \( V^0, g^m \) be the values at the initial \( X_i = X^0_i \)
Approximate \( V(X_i), g_m(X_i) \) by extrapolation
\[ V = V^0 + \frac{\partial V}{\partial X_i}(X_i - X^0_i) \quad (19) \]
\[ g_m = g^m + \frac{\partial g_m}{\partial X_i}(X_i - X^0_i) \quad (20) \]
The approximate problem is:
\[ \min(V^0 + \frac{\partial V}{\partial X_i}(X_i - X^0_i)) \quad (21) \]

STOC
\[ g^m + \frac{\partial g_m}{\partial X_i}(X_i - X^0_i) \leq 0 \quad (22) \]

where
\[ \beta X_{i0} \leq (X_i - X_{i0}) \leq (1 + \beta)X^0_i \quad (23) \]
equation 23 is a move limit that does not allow \( X_i \) to move too far from \( X^0_i \). Initially \( \beta = .2 \).
Abstract

There are two purposes of this project. One is to determine whether linear programming techniques can improve the performance in handling design optimization problems with a large number of design variables and constraints relative to the feasible directions algorithm. The second purpose is to determine whether using the Kreisselmeier-Steinhauser (KS) function to replace the constraints with one constraint will reduce the cost of the total optimization. Comparisons are made using solutions obtained using linear and non-linear methods. The results indicate that there is no cost saving using the linear method or in using the KS function to replace constraints.