Determination of Soil Moisture Beneath a Stalk or Trunk Dominated Canopy

K. C. McDonald, M. C. Dobson and F. T. Ulaby
Radiation Laboratory
Department of Electrical Engineering and Computer Science
The University of Michigan

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OVERVIEW

1. Radiative transfer model
   - Simplified L-Band MIMICS model
   - $\sigma^{0}_{HH}, \sigma^{0}_{VV}$ and $\Delta \phi_{HH-VV}$

2. Example model computations
   - Dominant backscatter contributions

3. Application of the model to determining soil moisture
   - Inversion of backscatter data
\[ \sigma^0_t = \sigma^0_c + \sigma^0_g + \sigma^0_{gt} + \sigma^0_{gc} + \sigma^0_{gcg} \]
Direct Crown Backscatter

\[ \sigma_c^0 = \frac{\sigma_v \cos \theta}{2\kappa_e} \left[ 1 - \exp \left( -\frac{2\kappa_e H_c}{\cos \theta} \right) \right] \]

\[ \sigma_v = \frac{3}{2} \omega \approx \frac{3}{2} (0.1) \]

\[ \kappa_e^p = \kappa_l + \kappa_t^p \]

\[
\kappa_l = \text{Im} \left[ \frac{4\pi}{\lambda} \sqrt{1 + \frac{v_l}{3} (\epsilon_l - 1) \left( 2 + \frac{1}{\epsilon_l} \right)} \right]
\]

\[
\kappa_t^p = -N \frac{2\lambda}{\pi} \text{Re} \left[ \sum_{n=-\infty}^{\infty} C_n^p (\theta_0) \right]
\]

\[ p = v \text{ or } h \]
- Direct Ground Backscatter

- Small Perturbation Model

\[ \sigma_g^0 = \sigma_{s,p}^0 \exp \left( -2 \kappa_e^p \sec \theta \right) \]

\[ \sigma_{s,p}^0 = 4 (k_s)^2 (k l)^2 \cos^4 \theta |\alpha_{pp}|^2 e^{-(k l \sin \theta)^2} \]

\[ \alpha_{hh} = \frac{\cos \theta - \sqrt{\epsilon_s - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon_s - \sin^2 \theta}} \]

\[ \alpha_{vv} = (\epsilon_s - 1) \frac{\sin^2 \theta - \epsilon_s (1 + \sin^2 \theta)}{\left[ \epsilon_s \cos \theta + \sqrt{\epsilon_s - \sin^2 \theta} \right]^2} \]

<table>
<thead>
<tr>
<th>Validity Conditions</th>
<th>Recommended Conditions</th>
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<tbody>
<tr>
<td>( s \leq 0.05 \lambda )</td>
<td>( l \leq 0.25 \lambda )</td>
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<tr>
<td>( m = \sqrt{2 \frac{s}{l}} \leq 0.3 )</td>
<td>( s \leq 0.05 \lambda )</td>
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<tr>
<td>( l \leq 0.5 \lambda )</td>
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</tbody>
</table>
• **Ground-Trunk (Stalk) Interaction**

\[
\sigma^0_{gt,pp} = 4N e^{-2\kappa_e H_c \sec \theta |R_p(\theta)|^2} \sigma_p^t
\]

\[
\sigma_p^t(\theta) = \frac{16H_t^2}{\pi} \left| \sum_{n=-\infty}^{\infty} (-1)^n C_n^p \right|^2
\]

\[
p = v \text{ or } h
\]

**Validity Conditions:**

- \( H_t \gg \lambda \)

- Specular forward scatter from soil
Backscatter Contributions, $\theta = 50$ Degrees

Soil Volumetric Moisture

$\sigma_{HH}$ (dB)

$\sigma_{VV}$ (dB)

- Total Backscatter
- Stalk Moisture = 0.5
- Direct Crown
- Direct Ground
- Ground-Stalk

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Backscatter Contributions, $\theta = 30$ Degrees

$\sigma_{HH}$ (dB)

$\sigma_{VV}$ (dB)

Soil Volumetric Moisture

- Total Backscatter
- Stalk Moisture = 0.2
- Direct Crown
- Direct Ground
- Ground-Stalk
Polarization Phase Difference

\[ \Delta \phi_{(HH-VV)g_{s}} = 2\Delta \phi_{p} + \Delta \phi_{g,s} + \Delta \phi_{tr} \]
\[ \Delta \phi_{(HH-VV)g} = 2\Delta \phi_{p} + \Delta \phi_{g,b} \]

- Propagation Phase Difference

\[ \Delta \phi_{p} = \frac{2NH_{t}}{k} \tan \theta \left[ \text{Im} \{ A_{h}(\theta, \pi) \} - \text{Im} \{ A_{v}(\theta, \pi) \} \right] \]

\[ A_{p}(\theta, \pi) = \sum_{n=-\infty}^{\infty} C_{n}^{p}(\theta), \quad p = v \text{ or } h \]

- Trunk Scatter Phase Difference

\[ \Delta \phi_{t} = \tan^{-1} \left\{ \frac{\text{Im} [A_{h}/A_{v}]}{\text{Re} [A_{h}/A_{v}]} \right\} \]

\[ A_{p}(\theta, 0^o) = \sum_{n=-\infty}^{\infty} (-1)^{n} C_{n}^{p}(\theta) \]

\[ p = v \text{ or } h \]
• Ground Scatter Phase Difference

  - Specular Scatter

\[ \Delta \phi_{g,s} = \tan^{-1} \left( \frac{\text{Im} \left( \frac{R_h}{R_v} \right)}{\text{Re} \left( \frac{R_h}{R_v} \right)} \right) - 180^\circ \]

  - Backscatter

\[ \Delta \phi_{g,b} = \tan^{-1} \left( \frac{\text{Im} \left( \frac{\alpha_{hh}}{\alpha_{vv}} \right)}{\text{Re} \left( \frac{\alpha_{hh}}{\alpha_{vv}} \right)} \right) \]
Total Phase Difference

\[
\begin{bmatrix}
|S_{VV}|^2 & 0 & 0 & 0 \\
0 & |S_{HH}|^2 & 0 & 0 \\
0 & 0 & \text{Re}(S_{VV}S_{HH}^*) & -\text{Im}(S_{VV}S_{HH}^*) \\
0 & 0 & \text{Im}(S_{VV}S_{HH}^*) & \text{Re}(S_{VV}S_{HH}^*)
\end{bmatrix}
\]

where:

\[
|S_{VV}|^2 = \sigma_{gt,VV}^0 + \sigma_{g,VV}^0
\]

\[
|S_{HH}|^2 = \sigma_{gt,HH}^0 + \sigma_{g,HH}^0
\]

\[
\text{Re}(S_{VV}S_{HH}^*) = \text{Re} \left[ \sqrt{\sigma_{gt,VV}} \sqrt{\sigma_{gt,HH}} e^{i \Delta \phi_{gs}} \right] + \text{Re} \left[ \sqrt{\sigma_{g,VV}} \sqrt{\sigma_{g,HH}} e^{i \Delta \phi_g} \right]
\]

\[
\text{Im}(S_{VV}S_{HH}^*) = \text{Im} \left[ \sqrt{\sigma_{gt,VV}} \sqrt{\sigma_{gt,HH}} e^{i \Delta \phi_{gs}} \right] + \text{Im} \left[ \sqrt{\sigma_{g,VV}} \sqrt{\sigma_{g,HH}} e^{i \Delta \phi_g} \right]
\]
Dielectric Behavior of Constituents

* Vegetation - Ulaby and El-Rayes (1987)

\[ \varepsilon_{mg} = A + B \left( 4.9 + \frac{\varepsilon_s - \varepsilon_{\infty}}{1 + j \frac{f(Hz)}{f_o}} - \frac{22.74}{f(GHz)} \right) \]
\[ + C \left( 2.9 + \frac{55}{1 + \sqrt{j \frac{f(GHz)}{0.18}}} \right) \]

* Soil - Hallikainen et al. (1985)

\[ \varepsilon_{mv} = \varepsilon'_s - j \varepsilon''_s \]

\[ \varepsilon = (a_0 + a_1 S + a_2 C') \]
\[ + (b_0 + b_1 S + b_2 C') m_v \]
\[ + (c_0 + c_1 S + c_2 C') m_v^2 \]

\[ \varepsilon = \varepsilon'_s \text{ or } \varepsilon''_s \]
Canopy Backscatter, $\theta=50^\circ$, HH Pol.

Canopy Backscatter, $\theta=50^\circ$, VV Pol.
Polarization Phase Difference, $\theta = 50^\circ$
Polarization Phase Difference, $\theta = 30^\circ$
Canopy Backscatter, \( \theta=30^\circ \), HH Pol.

Canopy Backscatter, \( \theta=30^\circ \), VV Pol.
Moisture Parameters, $\theta = 30$ Degrees

### Dry Conditions

- $-10.2 \text{ dB}$
- $-5.6 \text{ dB}$
- $-44 \text{ Deg.}$

### Wet Conditions

- $-88 \text{ Deg.}$
- $0.3 \text{ dB}$
- $0.4 \text{ dB}$

Legend:
- $\sigma_{HH}$
- $\sigma_{VV}$
- $\Delta \phi_{HH-VV}$
Aspen Canopy, $\theta = 30$ Degrees

Soil Volumetric Moisture

$\sigma_{HH}$ (dB)

$\sigma_{VV}$ (dB)

- Total Backscatter
- Direct Crown
- Direct Ground
- Ground-Trunk
Backscatter Contours, $\theta = 30$ Degrees

\( \sigma_{HH} \) Families (dB)

\( \sigma_{VV} \) Families (dB)
Phase Difference Contours, $\theta = 30$ Degrees

\begin{align*}
\Delta \phi &= -80^\circ \\
\Delta \phi &= -60^\circ \\
\Delta \phi &= -40^\circ 
\end{align*}
Conclusions

- Technique applies to canopies with large enough trunk/stalk biomass.

- Steep incidence angles are most effective.

- Polarization phase difference determines the trunk/stalk moisture.

- Polarization magnitudes determine soil moisture.

- VV polarization magnitude is more effective than HH.