MODELING OF TRANSITIONAL FLOWS

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ABSTRACT

With the current resurgence of interest in hypersonic flight, there is a great need to improve methods of predicting skin friction and heating that result from the boundary layers which develop over the vehicle surface. Such predictions are currently hampered by uncertainties in the modeling of turbulent stresses that occur over the lengthy transitional region typical of hypersonic boundary layers.

During the course of this summer an effort directed at developing improved transitional models was initiated. The focus of this work was concentrated on the critical assessment of a popular existing transitional model developed by McDonald and Fish in 1972[1]. The objective of this effort was to identify the shortcomings of the McDonald-Fish model and to use the insights gained to suggest modifications or alterations of the basic model.

In order to evaluate the transitional model, a compressible boundary layer code was required. Accordingly, a two-dimensional compressible boundary layer code was developed. The program was based on a three-point fully implicit finite difference algorithm where the equations were solved in an uncoupled manner with second order extrapolation used to evaluate the non-linear coefficients. Iteration was offered as an option if the extrapolation error could not be tolerated. The differencing scheme was arranged to be second order in both spatial directions on an arbitrarily stretched mesh. A variety of boundary condition options were implemented including specification of an external pressure gradient, specification of a
wall temperature distribution, and specification of an external temperature
distribution.

The boundary layer code and the transition model were coupled to-
gether and a series of test cases run for a flat-plate geometry. Although
the long-term goal of this project is to study transitional boundary layers
at hypersonic speeds, the first test cases were run for incompressible flow.
The primary reason for conducting the initial tests at low speeds is that a
large data base of both experimental and computational results exist for
incompressible flows. From this large data base of transitional data, direct
numerical simulation results generated by Zang\[2\] were used as a base of
comparison for the McDonald-Fish transitional model. Figure 1 shows a
comparison of the evolution of the Reynolds stress profile as computed
by the McDonald-Fish model and the direct numerical simulation. It is clear
that the profile predicted by the McDonald-Fish model differs significantly
from that predicted by the direct simulation. Note that the agreement be-
comes progressively worse as the downstream distance increases. Shown in
Figure 2 is a comparison of the amplification of the peak Reynolds stress
as a function of the local Reynolds number. This figure indicates that the
McDonald-Fish model greatly underpredicts the Reynolds stress spatial
growth rate.

Overall the results of the initial phase of this work indicate that the
McDonald-Fish model does a poor job at predicting the details of the tur-
bulent flow structure during the transition region. If the transitional region
is to be modeled accurately a more sophisticated model must be developed
which has the capability of simulating more of the essential structure of
the developing instabilities. A two-equation $k-\varepsilon$ model is suggested as a
candidate for an improved model.

References

[1] McDonald, H. and Fish, R.W., "Practical Calculations of Transitional

[2] Zang, T.A., Private communication, Mail stop 156, NASA Langley
Research Center, Hampton VA, 23669.
Figure 1: Reynolds stress profiles compared

\[ \text{Re}_x = 5.2 \times 10^5 \]

\[ \text{Flow Direction} \]

\[ \frac{\text{Un}'/\text{Un}'_{\text{max}}}{-} \]

\[ \text{Re}_x = 4.1 \times 10^5 \]

Figure 2: Reynolds stress amplification compared

\[ 10^{-3} \]

\[ 10^{-4} \]

\[ 10^{-5} \]

\[ 4.00 \times 10^{-5} \]

\[ 5.00 \times 10^{-5} \]

\[ 6.00 \times 10^{-5} \]

\[ \text{Direct Numerical Simulation} \]

\[ \text{McDonald-Fish Model} \]

Figure 2: Reynolds stress amplification compared