LEO to L1
Trajectory Program

NASA Contract No. NAS9-17878
Eagle Eng. Report No. 88-219
October 30, 1988
Foreword

This program is an important tool in the study of alternative routes between the Earth and the Moon. Dr. John Alred was the NASA Technical Monitor for the contract under which this program was produced. Mr. Andy Petro was the NASA Task Manager for this particular task. Mr. Bill Stump was the Eagle Project Manager for the contract under which this program was produced. The program was written by Jack Funk, originally in Quick Basic, and translated into Fortran by Mr. Bill Engblom. Mr. Engblom also prepared the documentation.
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1.0 Introduction

The program LP1 calculates outbound and return trajectories between low earth orbit (LEO) and libration point #1 (L1). Libration points (LP) are defined as locations in space that orbit the Earth such that they are always stationary with respect to the Earth-Moon line. L1 is located behind the Moon such that the pull of the Earth and Moon together just cancel the centrifugal acceleration associated with the libration point's orbit.

The outbound flights depart from a circular orbit of any altitude and inclination about the Earth and culminate in a circular orbit about the Earth at libration point #1 within a specified flight time. The flight involves three burns.

First, the departure orbit is made into a more eccentric orbit (ellipse or hyperbola) with an initial ΔV in order to reach the lunar sphere of influence (SOI), a region where the vehicle is near its lowest velocity in the trajectory. The SOI is a spherical region whose surface normally includes all the points at a distance of 11% of the Earth-Moon distance from the Moon’s center. However, in order to simplify the calculations this radius was increased to include L1, enlarging the sphere radius to 15% of the Earth-Moon distance. Note, this change in SOI radius should not change the results significantly. A given flight may penetrate the SOI at a number of points identified by projecting lunar latitude and longitude onto the SOI. For each flight the program will calculate a set of possible trajectories associated with a set of SOI penetration points -- a matrix of longitudes and latitudes.
Next, a second burn (at the SOI) is performed involving a "flyby" of the Moon from the SOI point above the front side of the Moon to L1 behind the back side. Note, the SOI penetration point and L1 will always be the same distance from the center of the Moon. From the geometry of the trajectory it is apparent that the Lunar "flyby" perigee altitude, supplied by the user, will occur midway between the SOI point and L1. Consequently, the geometry of the orbit will force true anomaly, flight path angle, and absolute time to or from perigee passage to be the same for both the SOI and L1 points. There are two paths between these points, posigrade and retrograde. LPI calculates only posigrade "flyby" trajectories since retrograde orbits that pass through the perigee altitude are not always possible while there is always a posigrade solution. The retrograde trajectories warrant more study in future work. Since L1 is constantly rotating with the Moon, this trajectory is iterated until L1 is reached.

The third burn is simply a circularization of the trajectory at L1 about the Earth. The velocity vector is corrected to that of L1. Note, once the SOI to L1 trajectory has been established, the Earth-SOI flight is iterated until the total transfer time, including the transfers from LEO to the SOI, and SOI to L1, match the user's flight time constraint (an input value). This is done for a matrix of SOI penetration points, as mentioned earlier.

The return trajectories, which start at L1 and finish in the specified LEO orbit within the specified flight time, are calculated similarly. For instance, the "flyby" trajectory is calculated first, starting at L1 and finishing at the SOI via a posigrade orbit calculated using the same geometric simplifications described above. Note, the flight profile used in this program to calculate flights between Earth and L1 may not be the optimum with respect to a minimum ΔV.
After the user has defined the trajectory as outbound or return, the Earth orbit altitude and inclination, and the total flight time, LP1 produces matrices which display the total ΔVs, the three component ΔVs (described above), the "fly-by" trajectory inclinations, and the "fly-by" azimuth angle at the SOI for the resulting flight from Earth to L1 for a representative set of SOI points. These points are defined by the user, who provides a starting longitude and latitude, and an increment for each. The matrix is built with 10 longitudes forming the columns and 19 latitudes forming the rows.

Section 2.0 of this document describes the input required from the user to define the flight. Section 3.0 describes the contents of the six reports that are produced as outputs. Section 4.0 includes the instructions needed to execute the program.

A more detailed description of the process used in LP1 has been included as Appendix D (main program), Appendix E (in-program subroutine), and Appendices F, G, H, I, and J (external subroutines). LP1 was derived from the PLANECHG program (also produced under this contract) with the major addition of the FLYBY subroutine. Therefore, the documentation for PLANECHG may be used as a reference for many of the equations, variables, and conventions used in LP1 (except in the FLYBY routine).
2.0 Program Inputs

The following paragraphs discuss the inputs provided by the user. The prompt is the message displayed by the program onto the screen. The input variable is the program variable assigned to the user’s response. The description provides information about how to respond to the prompt.

1. Prompt: INPUT OUTBOUND OR RETURN

   Input variable: MD
   Description: Enter OUTBOUND for Earth-to-L1 trajectory calculations. Enter RETURN for L1-to-Earth trajectory calculations.

2. Prompt: INPUT PERIGEE ALTITUDE OF EARTH ORBIT (NMI)

   Input variable: HPE
   Description: Enter the height above the Earth’s surface of the Earth circular orbit, in nautical miles.

3. Prompt: INPUT PERIGEE ALTITUDE OF LUNAR ORBIT (NMI)

   Input variable: HPM
   Description: Enter the height above the Lunar surface of the Lunar circular orbit, in nautical miles.
4. Prompt: INPUT EARTH DEPARTURE JULIAN DATE
   Input variable: TIMJ
   Description: Enter the origin trans-SOI injection date (Earth departure date for
   outbound trajectories) in Julian day format, where January 1, 2000 is day
   2,451,545. Refer to Section C of "The Astronomical Almanac of the Year
   1988". Day 2451545 is the default value if zero is entered in this field.

5. Prompt: INITIAL LONGITUDE
   Input variable: ALONI
   Description: Enter initial sphere of influence longitude for the output matrices. This
   value will become the heading for column 1 of the matrices.

6. Prompt: INPUT INCREMENT FOR THE MAP
   Input variable: DELLON
   Description: Enter longitude increments for the output matrices. Applied to the initial
   longitude, this value defines the subsequent column headings of the
   matrices. Longitudes for outbound trajectories should be between zero
   and -90 degrees.
7. Prompt: INPUT INITIAL LATITUDE
Input variable: ALATI
Description: Enter initial sphere of influence latitude for the output matrices. This value will become the heading for row 1 of the matrices.

8. Prompt: INPUT INCREMENT FOR THE MAP
Input variable: DELLAT
Description: Enter latitude increments for the output matrices. Applied to the initial latitude, this value defines the subsequent row headings of the matrices.

9. Prompt: INPUT EARTH ORBIT TO LUNAR ORBIT INCLINATION
Input variable: AINCEO
Description: Enter Earth circular orbit inclination, in degrees. This is not what is typically considered inclination (i.e., a measurement taken from the Earth's equatorial plane), but rather the angle between the plane of the low Earth orbit and the plane of the Moon's orbit about the Earth.

10. Prompt: INPUT FLIGHT TIME
Input variable: FTIM
Description: Enter the desired total flight time from LEO to L1, in hours.
3.0 Program Outputs

This section describes the contents of each of the six reports generated by the program. These reports may be found in the output file, LP1.OUT. Samples of all six reports have also been included.

Report #1: Total Delta Velocity Map For Outbound/Return Trajectories

The top section of this report repeats the input values entered by the user. The second section is a 10 X 19 matrix of total velocities (in ft/sec) required to fly the profile described by the inputs. Each cell corresponds to a particular latitude and longitude on the Lunar sphere of influence. A "flyby" burn (transfer from SOI to L1) may occur at any one of these coordinates, and the value of the corresponding cell is the total velocity required for the flight if the "flyby" burn occurs at that location. The total is the sum of the Earth-SOI transfer orbit injection ΔV (from LEO to SOI), the sphere of influence to L1, Lunar "flyby" ΔV (from SOI to L1), and the destination circular orbit injection ΔV (circularization at L1). Note, the longitudes must always be between 0° and -90° for outbound trajectories, and between 0° and +90° for return trajectories. The third section of the report is a summary of key data corresponding to the matrix cell containing the lowest total velocity. This data includes:

- X-, Y-, and Z-components of velocity at the sphere of influence just before and just after the "flyby" burn (VX, VY, VZ).
- Total magnitude of the velocity at the sphere of influence just before and just after the "flyby" burn (VEL).
- Flight path angle at the sphere of influence just before and just after the "flyby" burn (GAMA).
- Azimuth of the sphere of influence point from the Earth and from the Moon (AZM).
- Earth and "flyby" orbit inclinations measured from the Earth-Moon plane (AINC).

- Earth-SO1 transfer trajectory and "flyby" orbit insertion ascending/descending node positions measured from Earth-Moon line at 0° longitude (ANODE).

- Earth-to-SO1 and SOI-to-L1 times of flight (TIME).

- Earth-SO1 transfer trajectory and final L1 orbit insertion ΔV’s (DVCIR).

- Sphere of influence, Lunar "flyby" ΔV (DVPHER).

- Total ΔV (DVTOTAL).
VELOCITY MAP FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION 0
DATE 7-NOV-88 TIME 14:46:12

JULIAN DAY 2451545. PERIGEE ALT (NMI) EARTH = 250. MOON = 10000.
TRANSLUNAR FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON PAGE3

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MINIMUM VELOCITY PRINT NODE OPTION 0
DATE 7-NOV-88 TIME 14:46:12

FLIGHT TIME = 200. LAT =  10.0 LON = -30.0 LUNAR AINC =  10.0 EARTH AINC =  25.0
BODY VX  VY  VZ  VEL GAMA AZM AINC ANODE TIME DVCIR
MOON  100. 2304. -96.  900. -38.3  90.0  10.0  240.3 109.8  1402. DVPHER =  1679.
EARTH  447.  672. -279.  853.  35.3 114.9  25.0   8.5  90.2  10030. DVTOTAL = 13111.
Report #2
Map of Delta Velocity at Sphere of Influence for Outbound/Return Trajectories

This report is a matrix of the delta velocities that occur at the sphere of influence to match the velocity vectors of the Earth-to SOI trajectory and the SOI-to-L1 "flyby" trajectory for each SOI point. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which a burn may occur.
MAP OF DELTA VELOCITY AT SPHERE OF INFLUENCE FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE
OPTION 0

DATE 7-NOV-88 TIME 14:46:12
EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

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Report #3

Map of Inclinations of Flyby Orbits for Outbound/Return Trajectories

This report contains a matrix of the inclinations of the "flyby" trajectories with respect to the Earth-Moon plane, between each SOI point and L1. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which the trajectory originates or culminates.
MAP OF INCLINATION OF FLY-BY ORBIT FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION

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Report #4

Map of Delta Velocity at L1 for Outbound/Return Trajectories

This report is a matrix of the delta velocities that occur at L1 to circularize the trajectory at L1 (outbound flights), or to initiate a posigrade "flyby" trajectory from L1 to a particular SOI point (return flights). Each cell corresponds to a particular latitude and longitude on the sphere of influence through which the "flyby" trajectory originates or culminates.
MAP OF DELTA VELOCITY AT L1 FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION 0
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EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

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Report #5

Map of Delta Velocity at Earth for Outbound/Return Trajectories

This report is a matrix of the delta velocities that occur to initiate a transfer orbit from LEO to the SOI. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which a burn may occur.
**DELTA VELOCITY AT EARTH FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORY**  
**NODE OPTION 0**

**DATE** 7-NOV-88  
**TIME** 14:46:12

**EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0**  
**INCL EARTH = 25.0**  
**INCL MOON = 90.0**

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Report #6

Map of Azimuths of Flyby Orbits at SOI for Outbound/Return Trajectories

This report contains a matrix of the azimuth angles of the "flyby" trajectories at the SOI for each particular SOI point. Each cell corresponds to a particular latitude and longitude on the sphere of influence through which the "flyby" trajectory originates or culminates.
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4.0 Program Execution Instructions

The following instructions describe the steps to be taken by the user to execute this program:

A. Obtain access to the DEC VAX minicomputer and sign on with user identification.

B. At the $ prompt, type RUN LP1.

C. When prompted by the program, enter the program inputs. See section 2.0 for a discussion of the inputs.

D. After the last input has been entered, the program will execute for approximately 5 minutes, during which comments will appear notifying the user which trajectory, by SOI latitude and longitude, is currently being calculated. Upon completion, the message FORTRAN STOP will appear, followed by the $ prompt.

E. The program outputs will be placed in a file named LIBRATE.OUT;### where ### is a system generated number of the report. To print the most recently generated report, type the following at the $ prompt: TYPE LP1.OUT

F. To re-execute the program with new parameters, begin again at step (B) above.
Appendix A. Program Flow Chart
Declare the program matrices

Open the output files

Define the program constants

Read the program inputs

Calculate the distance from Earth's center to Earth's perigee orbit

Calculate distance from Moon's center to flyby orbit perigee

Temporarily store Earth and Lunar orbit inclinations

A
Record the current date and time

Print header section of Report #1

Initialize matrix and report settings

Calculate orbital velocity of the Moon

Convert initial map latitude and longitude from degrees to radians

Initialize minimum \( \Delta V \)

Initialize current \( \Delta V \) hold variables

B
Initialize print line variables

Flyby subroutine

In-program subroutine

Store total \( \Delta V \), Earth departure \( \Delta V \), SOI \( \Delta V \), flyby inclination, circularization at L1, and azimuth angle.
Store the matrix column headings for use in all reports

- Have all matrix columns been processed for this latitude?
  - NO: Increment column counter by 1
  - Increment longitude
  - YES: Has this been the first row of the matrix?
    - NO: Print column headings for Report #1
    - YES: Print matrix row for this latitude for Report #1
Set VELM to the SOI velocity associated with the minimum total $\Delta V$

Increment row counter by 1

Increment latitude

Reset column counter to 1

Reset longitude to initial input longitude

Have all 19 matrix rows been processed?

YES

Set the SOI latitude and longitude to those associated with the minimum total $\Delta V$

In-Program Subroutine

Print Report #2

Print Report #3

F

C

Reset column counter to 1
Define variables for the sine and cosine of the SOI longitude and latitude

Define the angle between the Moon-to-Earth line and Moon-to-SOI line

Define variables for the sine and cosine of the Lunar orbit inclination

Calculate the ratio of Earth-Moon distance to Moon-SOI distance

Determine the distance from the moon to the SOI

Locate the SOI in rectangular coordinates centered at the Earth

Calculate the distance from the Earth to the SOI
A

Identify the angle between the Earth-to-Moon line and Earth-to-SOI line

Determine the Earth-based latitude and longitude of the SOI

Determine the Earth ascending node for the orbit in which the trans-SOI burn will occur

B
Determine the Lunar ascending node

Determine the rectangular coordinate components of the velocity vector at the SOI point for the flyby trajectory

Calculate the via-viva parameter (Q) for the Earth-to-SOI trajectory

Calculate the minimum velocity required, such that apogee is just at SOI

Increase the velocity slightly, so that the calculations will converge

Is the most recently calculated SOI velocity (VELE) the minimum velocity?

Temporary store the Earth-to-SOI time of flight

VELE = minimum velocity
Increase velocity by 10 ft/sec

Determine the ratio of the time-of-flight shortfall to time-of-flight increase due to 10 ft/sec velocity increase

Apply this ratio to the 10 ft/sec to estimate \( \Delta V \) necessary to meet required time-of-flight

Add this \( \Delta V \) to the original velocity

GAMACALC

GAMACALC (determine time-of-flight #1)

GAMACALC (determine time-of-flight #2)
NO

Determine the longitude of the SOI point from the Earth's viewpoint

Determine the latitude of the SOI point

Calculate the azimuth of the SOI point

VELTRANS

Calculate the Δ V at the SOI point

F
Calculate $\Delta V$ at Earth perigee

Store the old total $\Delta V$

Calculate the new total $\Delta V$

Determine the time of arrival at SOI from Earth

Convert the Lunar distance from Earth radial to feet

POSVELMO
Is the Earth-to-SOI time of flight converging on a solution?

- **NO**
  - NO
  - YES

Is the newly-calculated total $\Delta V$ the smallest so far?

- **NO**
  - NO
  - YES

Store characteristics of trajectory associated with minimum total $\Delta V$

- **NO**
  - NO
  - YES

Store minimum SOI $\Delta V$ and corresponding SOI longitude, latitude, and velocity

Return
Determine location of L1 in Lunar relative coordinates

Determine location of SOI point

Calculate true anomaly of flyby trajectory between SOI point and L1

Calculate eccentricity of flyby orbit using general conic equation

Calculate flight path angle of flyby orbit

Calculate semi-latus rectum
Calculate the time of perigee passage for flyby trajectory at SOI point and L1.

Calculate time of flight for flyby.

Calculate azimuth angle and angle between the nodes and the SOI projection for the SOI point.

Calculate azimuth angle for L1.

Has flight time changed only slightly?

Set latitude/longitude position of L1.

D

No

Yes
Calculate flyby time of flight

Calculate semi-major axis

Calculate eccentric anomaly

Calculate the vis-viva parameter

Calculate flyby orbit inclination

Calculate velocity at SOI and L1

Calculate perigee velocity

A

B
Call VELTRANS to establish the velocity of components of L1 in Lunar-based rectangular components.

Set Y-component of velocity for L1 (X and Z components are 0).

Calculate \( \Delta V \) needed to correct the velocity vector at L1.

END.
GAMACALC

Initialize indicators to presume an elliptical orbit

Calculate the vis-viva parameter (Q)

Is Q within one millionth of being >2?

No

Reduce Q to just under 2

Yes

Is orbit hyperbolic?

No

Yes

Set indicators to show hyperbolic orbit. Print screen message announcing hyperbolic orbit.

Calculate orbit's semi-major axis
Calculate perigee velocity
Calculate semi-latus rectum
Calculate flight path angle
Calculate eccentricity
Make orbit hyperbolic
Is orbit trapped near a parabolic trajectory?

B
Calculate true anomaly

Calculate eccentric anomaly

Is orbital radius at SOI > apogee radius?

No

Yes

Display orbital/apogee radius difference, SOI lat. and long., Q, semi-major axis, eccentricity, flight path angle, and perigee velocity

Calculate time of flight

Return
Calculate a fraction of TIME that represents half a second.

\[ \text{TIME1} = \text{TIME} - \frac{1}{2} \text{ second} \]

\[ \text{MOON (TIME1)} \]

\[ \text{TIME2} = \text{TIME} + \frac{1}{2} \text{ second} \]

\[ \text{MOON (TIME2)} \]

Calculate velocity of the moon in the x-direction.

Calculate the average radius of the Lunar orbit during the one second centered on TIME.
Determine the radius of the Lunar orbit from the Earth-Moon barycenter

Calculate the Moon's velocity in the direction of its orbit

Return
Calculate the Moon's semi-diameter

Calculate the Moon's ecliptic longitude

Calculate the Moon's ecliptic latitude

Calculate horizontal parallax

Calculate the semi-diameter of the Moon's orbit

Calculate the distance to the moon in Earth radii

Form the geocentric direction cosines to rotate into geocentric coordinates

Calculate right ascension and declination

Return
Convert the velocity vector into spherical coordinates

Convert the radial velocity component into rectangular coordinates

Convert the latitude velocity component into rectangular coordinates

Convert the longitude velocity component into rectangular coordinates

Sum the X-components of velocity

Sum the Y-components of velocity

Sum the Z-components of velocity

Return
Appendix B. Code Listing
**Libration Point Program (between L1 and Earth)**

*Written in Quick Basic by: Jack Funk*
*Translated by Bill Engblom*
*Documented by Bill Engblom*

```plaintext
MAIN

IMPLICIT REAL*16 (A-H, O-Z)
CHARACTER*10 MD, TRAJ, TRAJM, TRAJE
CHARACTER*8 TIMP
CHARACTER*9 DATP
CHARACTER*26 HEAD
DIMENSION DELV(10,19), VELMOUT(10,19), ALONO(10), ALATP(19),
* PAGE1(10), PAGE2(10,19), PAGE3(10,19), PAGE4(10,19),
* PAGE5(10,19), PAGE6(10,19)

OPEN OUTPUT FILE

OPEN (UNIT = 1, FILE = 'LP1.OUT', STATUS = 'NEW')
OUTPUT TO FILE; IP: OUTPUT TO SCREEN; IS
IP = 1
IS = 5
DEPR = 57.29578
PI = 3.1415926535
CMUE = 1.407647E+16
CMUM = 1.731432E+14
FTNM = 6076.115
REE = 20925741.
REMO = 5.7039E+6
RREM = 207559. * FTNM
FTM = .3048
PRINT *, 'INPUT OUTBOUND OR RETURN'
READ (6, 5) MD

5 FORMAT (A10)
IF (MD .EQ. 'RETURN') THEN
  MOOD = -1
  HEAD = 'L1 TO EARTH FLYBY TRANSFER'
ELSE
  MD = 'OUTBOUND'
  MOOD = 1
  HEAD = 'EARTH TO L1 FLYBY TRANSFER'
ENDIF

CONTINUE
PRINT *, 'INPUT NODE OPTION 1 OR 2'
READ *, NP

PRINT *, 'INPUT PERIGEE ALTITUDE OF EARTH ORBIT (NMI)'
```
READ *, HPE
PRINT *, 'INPUT PERIGEE ALTITUDE OF LUNAR ORBIT (NMI)' 
READ *, HPM
RPE = HPE * FTNM + REE
RPM = HPM * FTNM + REMO
PRINT *, 'INPUT EARTH DEPARTURE JULIAN DATE '
READ *, TIMJ
IF (TIMJ .EQ. 0.) TIMJ = 2451545.
PRINT *, 'LONGITUDE FOR OUTBOUND TRAJECTORIES SHOULD BE
* BETWEEN 0 AND -90 DEG'
PRINT *, 'AND RETURN TRAJECTORIES BETWEEN 0 AND +90 DEG'
PRINT *, 'INITIAL LONGITUDE'
READ *, ALONI
PRINT *, 'INPUT INCREMENT FOR MAP '
READ *, DELLO
PRINT *, 'INPUT INITIAL LATITUDE'
READ *, ALATI
PRINT *, 'INPUT INCREMENT FOR MAP '
READ *, DELLAT
PRINT *, 'INPUT EARTH ORBIT TO LUNAR ORBIT INCLINATION '
READ *, AINCEO
AINCEO = AINCEO / DPR
AINCE = AINCEO
PRINT *, 'INPUT FLIGHT TIME '
READ *, FTIM
CALL DATE (DATP)
CALL TIME(T1MP)
CONTINUE
WRITE (IP,7) HEAD, NP
FORMAT (T12, ' VELOCITY MAP FOR ',A26,' TRAJECTORIES
* NODE OPTION ',I1)
WRITE (IP,17) DATP, T1MP
FORMAT (T27,'DATE',A10,' TIME',A10)
WRITE (IP,27) TIMJ, HPE, HPM
27 FORMAT ('/,' JUlIAN DAY ',' F8.0,' PERIGEE ALT (NMI) EARTH = ',
*F4.0,' MOON = ',' F6.0)
WRITE (IP,37) FTIM, AINCEO * DPR
37 FORMAT (' TRANSLUNAR FLIGHT TIME (HR) = ',F5.1,' INCL
*EARTH ' F5.1,' INCL MOON PAGE3 ')
IPRINT = 0
II = 1
NN = 1
DVMIN = 99999.
YDLO = QSQRT (CMUE / RREM)
ALAT = ALATI / DPR
ALON = ALONI / DPR
C VELM = VELMI
DVSIM = 99999.
21 CONTINUE
VEL = VELM
DELV(NN,II) = 99999.
PAGE1 (NN) = 0.0
PAGE2 (NN,II) = 0.0
PAGE3 (NN,II) = 0.0
PAGE4 (NN,II) = 0.0
PAGE5 (NN,II) = 0.0
PAGE6 (NN,II) = 0.0
PRINT *, ALAT, ALON, VEL, DVT, DVSI, MOON, EARTH
PRINT *, TIMEE, TIMEM, RREM'
CONTINUE

CALCULATE FLYBY TRAJECTORY CHARACTERISTICS
CALL FLYBY(ALON, ALAT, RREM, RPM, GAMAM, VELM, VPM, AINCM, DELVLP1, *AZMM, AVM, TIMM, MOOD) ICALL = 1
GOTO 25

WRITE (IS, 47) ALAT*DPR, ALON*DPR, VELM, DVTOTAL, DELVEL, C TRAJM$, TRAJE$, TIEM, TIMM, RREM, PHIE*DPR
C 47 FORMAT (1X, F5.0, 2X, F5.0, 2X, F6.0, 2X, F7.0, 2X, F6.0, 2X,
C * A10, 2X, A10, 2X, F6.1, 2X, F6.1, 2X, F7.3, 2X, F5.0)
IF (QABS (TIEM) .LE. .0000000000000001) GOTO 23
STORE VALUES FOR OUTPUT
IF (DVTOTAL .LT. DELV(NN,II) ) THEN
DELV (NN,II) = DVTOTAL
PAGE1 (NN) = DVTOTAL
PAGE2 (NN,II) = DELVEL
VELMOUT (NN,II) = VELM
ALATP (II) = ALAT * DPR
PAGE3 (NN,II) = AINCM * DPR
PAGE4 (NN,II) = DELVLP1
PAGE5 (NN,II) = DVCIIEEE
PAGE6 (NN,II) = AZMM * DPR
WRITE (IS, 57) ALAT * DPR, ALON * DPR, VELMOUT (NN,II)
*, DELV(NN,II), PAGE2(NN,II), TRAJM, TRAJE, TIEM,
C * TIMM, RREMER
C 57 FORMAT (1X, F6.1, 1X, F6.1, 1X, F7.0, 1X, F8.1, 1X, F7.1, 1X,
C * A5, 1X, A5, 1X, F6.1, 1X, F6.1, 1X, F8.3)
WRITE (IS, 67) VXE, VXM, VYE, VYM, VZE, VZM
C 67 FORMAT (' VXE ', F7.1, ' VXM ', F7.1, ' VYE ', F7.1, ' VYM ',
C * ' F7.1, ' VZE ', F7.1, ' VZM ', F7.1)
ENDIF

ALONO(NN) = ALON * DPR
IF (NN .EQ. 10 .AND. IPRINT .EQ. 0 ) THEN
WRITE (IP,77) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
* ALONO(5), ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
77 FORMAT ('/','ALON>', ',', 10(2X,F5.1))
 WRITE (IP,87)
87 FORMAT ('ALAT')
ENDIF
IF (NN .LT. 10) THEN
   ALON = (ALONI + DELLON * QFLOAT (NN ) ) / DPR
   NN = NN + 1
   GOTO 21
ENDIF
IPRINT = 1
WRITE (IP,97) ALAT * DPR, PAGE1(1), PAGE1(2),
* PAGE1(3), PAGE1(4), PAGE1(5), PAGE1(6),
* PAGE1(7), PAGE1(8), PAGE1(9), PAGE1(10)
97 FORMAT (1X, F5.1, 2X, 10(1X,F6.0))
IF (II .LT. 19) THEN
   ALAT = (ALATI + DELLAT * QFLOAT (II ) ) / DPR
   II = II+1
   NN = 1
   ALON = ALONI / DPR
   GOTO 21
ENDIF
VELM = VELMMIN
ALAT = ALATMIN
ALON = ALONMIN
ICALL = 2
GOTO 25
2000 CONTINUE
WRITE (IP,107) NP
107 FORMAT ('/','T23,'MINIMUM VELOCITY PRINT NODE OPTION ',I1)
 WRITE (IP,117) DATP, TIMP
117 FORMAT (T27,'DATE',A10,' TIME',A10)
 WRITE (IP,127) FTIM, ALATMIN * DPR, ALONMIN * DPR, AINCMP * DPR,
* AINCEO * DPR
127 FORMAT ('/','FLIGHT TIME = ',F4.0,' LAT = ',F6.1,' LON = ',F6.1,
* ' LUNAR AINC = ',F5.1,' EARTH AINC = ',F5.1)
 WRITE (IP,137)
137 FORMAT ('BODY VX VY VZ VEL GAMA AZM
*AINC ANODE TIME DVCIR')
 WRITE (IP,147) VXMP, VYMP, VZMP, VMAGMP, GAMAMP * DPR,
* AZMMP * DPR, AINCMPP * DPR, ANODEMP*DPR, TIMMP, DELVLP1P, DELVELP
147 FORMAT ('MOON',4(1X,F6.0),1X,F5.1,1X,F6.1,2X,F5.1,2X,F6.1,
* 1X,F5.1,1X,F7.0, ' DVPHER = ',F7.0)
 WRITE (IP,1475) VXEP, VYEP, VZEP, VELEP, GAMAEP*DPR, AZMEP*DPR,
* AINCEP * DPR, ANODEEP*DPR, TIEMP, DVCIREP, DVTOTALP
1475 FORMAT ('EARTH',4(1X,F6.0),1X,F5.1,1X,F6.1,2X,F5.1,2X,F6.1,
* 1X,F5.1,1X,F7.0, ' DVTOTAL = ', F7.0)
 WRITE (IP,157) CHAR(12)
157 FORMAT ('\',A1)
WRITE (IP,167) HEAD,NP
167 FORMAT (T10,'MAP OF DELTA VELOCITY AT SPHERE OF INFLUENCE
*FOR ','A26,' TRAJECTORIES NODE OPTION ','I1)
WRITE (IP,177) DATP,TIMP
177 FORMAT (T27,'DATE',A10,' TIME',A10)
WRITE (IP,187) HEAD,FTIM, AINCEO * DPR, AINCM * DPR
187 FORMAT (' ','A26,' FLIGHT TIME (HR) = ','F6.1,' INCL EARTH ','
*F6.1,' INCL MOON = ','F5.1)
WRITE (IP,197) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
*ALONO(5),ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
197 FORMAT ('/,' ALON> ',10(2X, F5.1))
18 CONTINUE
WRITE (IP,207)
207 FORMAT (' ALAT')
DO 28 NPI = 1 , 19
WRITE (IP,217) ALATP(NPI), PAGE2(1,NPI),PAGE2(2,NPI),
* PAGE2(3,NPI),PAGE2(4,NPI),PAGE2(5,NPI),PAGE2(6,NPI),
* PAGE2(7,NPI),PAGE2(8,NPI),PAGE2(9,NPI),PAGE2(10,NPI)
217 FORMAT (1X,F5.1,1X,10(1X, F6.1))
28 CONTINUE
WRITE (IP,227 ) CHAR(12)
227 FORMAT (' ', A1)
WRITE (IP, 237 ) HEAD , NP
237 FORMAT (T6,'MAP OF INCLINATION OF FLY-BY ORBIT FOR ','A26,
* ' TRAJECTORIES NODE OPTION ','I1)
WRITE (IP, 247 ) DATP, TIMP
247 FORMAT (T27,'DATE',A10,' TIME',A10)
WRITE (IP, 257 ) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
257 FORMAT (' ','A26,' FLIGHT TIME (HR) = ','F6.1,' INCL EARTH ','
*F6.1,' INCL MOON = ','F6.1)
WRITE (IP,267) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
*ALONO(5),ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
267 FORMAT ('/,' ALON> ',10(2X, F5.1))
WRITE (IP,277)
277 FORMAT (' ALAT')
DO 48 NPI = 1 , 19
WRITE (IP,287) ALATP(NPI), PAGE3(1,NPI),PAGE3(2,NPI),
* PAGE3(3,NPI),PAGE3(4,NPI),PAGE3(5,NPI),PAGE3(6,NPI),
* PAGE3(7,NPI), PAGE3(8,NPI),PAGE3(9,NPI),PAGE3(10,NPI)
287 FORMAT (' ',F5.1,1X,10(2X,F5.1))
48 CONTINUE
WRITE (IP,297) CHAR(12)
297 FORMAT (' ',A1)
WRITE (IP, 307 ) HEAD , NP
307 FORMAT (T6,'MAP OF DELTA VELOCITY AT L1
* FOR ','A26,' TRAJECTORIES NODE OPTION ','I1)
WRITE (IP, 317 ) DATP,TIMP
317 FORMAT (T27,'DATE',A10,' TIME',A10)
WRITE (IP, 327 ) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
327 FORMAT (1X, A26, 'FLIGHT TIME (HR) = ', F5.1, ' INCL EARTH ', F5.1, ' INCL MOON = ', F5.1)
WRITE (IP, 337) ALONO(1), ALONO(2), ALONO(3), ALONO(4), ALONO(5),
* ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
337 FORMAT ('/', 'ALON>', ' 10 (2X, F5.1))
WRITE (IP, 347)
347 FORMAT ('ALAT')
DO 58 NPI = 1, 19
WRITE (IP, 357) ALATP(NPI), PAGE4(1, NPI), PAGE4(2, NPI),
* PAGE4(3, NPI), PAGE4(4, NPI), PAGE4(5, NPI), PAGE4(6, NPI),
* PAGE4(7, NPI), PAGE4(8, NPI), PAGE4(9, NPI), PAGE4(10, NPI)
357 FORMAT (1X, F5.1, 2X, 10 (1X, F6.0))
58 CONTINUE
WRITE (IP, 367) CHAR(12)
367 FORMAT ('A1')
WRITE (IP, 377) HEAD, NP
377 FORMAT (T2, 'DELTA VELOCITY AT EARTH FOR ', A26, ' TRAJECTORY
* NODE OPTION ', I1)
WRITE (IP, 387) DATP, TIMP
387 FORMAT (T27, 'DATE ', A10, ' TIME ', A10)
WRITE (IP, 397) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
397 FORMAT ('A26', 'FLIGHT TIME (HR) = ', F6.1, ' INCL EARTH ', F6.1, ' INCL MOON = ', F6.1)
WRITE (IP, 407) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
* ALONO(5), ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
407 FORMAT ('/', 'ALON>', ' 10 (2X, F5.1))
WRITE (IP, 417)
417 FORMAT ('ALAT')
DO 68 NPI = 1, 19
WRITE (IP, 427) ALATP(NPI), PAGE5(1, NPI), PAGE5(2, NPI),
* PAGE5(3, NPI), PAGE5(4, NPI), PAGE5(5, NPI), PAGE5(6, NPI),
* PAGE5(7, NPI), PAGE5(8, NPI), PAGE5(9, NPI), PAGE5(10, NPI)
427 FORMAT (1X, F5.1, 2X, 10 (1X, F6.0))
68 CONTINUE
WRITE (IP, 437) CHAR(12)
437 FORMAT ('A1')
WRITE (IP, 447) HEAD, NP
447 FORMAT (T2, 'MAP OF AZMM FOR ', A26, ' TRAJECTORY
* NODE OPTION ', I1)
WRITE (IP, 457) DATP, TIMP
457 FORMAT (T27, 'DATE', A10, ' TIME', A10)
WRITE (IP, 467) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
467 FORMAT (1X, A26, 'FLIGHT TIME (HR) = ', F5.1, ' INCL EARTH ', F5.1, ' INCL MOON = ', F5.1)
WRITE (IP, 477) ALONO(1), ALONO(2), ALONO(3), ALONO(4), ALONO(5),
* ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
477 FORMAT ('/', 'ALON>', ' 10 (2X, F5.1))
WRITE (IP, 487)
487 FORMAT ('ALAT')
DO 78 NPI = 1, 19
   WRITE(IP, 497) ALATP(NPI), PAGE6(1,NPI), PAGE6(2,NPI),
*   PAGE6(3,NPI), PAGE6(4,NPI), PAGE6(5,NPI), PAGE6(6,NPI),
*   PAGE6(7,NPI), PAGE6(8,NPI), PAGE6(9,NPI), PAGE6(10,NPI)
497   FORMAT(1X,F5.1,2X,10(I1X,F6.0))
78   CONTINUE
   GOTO 3000
25   CONTINUE
   COSALAT = QCOS (ALAT )
   SINALAT = QSIN (ALAT )
   COSALON = QCOS (ALON )
   SINALON = QSIN (ALON )
   COSPHI = COSALAT * COSALON
   COSAINC = QCOS (AINCM / DPR )
   SINALINC = QSIN (AINCM / DPR )
   RMR = COSPHI + QSQR (COSPHI ** 2. - (1. - CMUE / CMUM ))
   RRM = RREM * 0.15
C CALCULATION OF XYZ COORDINATES RELATIVE TO EARTH AT SPHERE
C OF INFLUENCE
C SPHERE OF INFLUENCE HAS BEEN REDEFINED EQUAL TO THE LIBRATION
C POINT #1 RADIUS
   XXM = RRM * COSALAT * COSALON
   XX = - XXM + RREM
   YY = - RRM * COSALAT * SINALON
   ZZ = RRM * SINALAT
   YYM = - YY
C WRITE (IS,507) XX,YY,ZZ
C 507 FORMAT(' XYZ POSITION AT SPHERE ',F11.0,1X,F11.0,1X,F11.0)
   RRE = QSQR (XX**2. + YY**2. + ZZ**2.)
   COSANGA = XX / RRE
   SINANGA = QSQR (YY**2. + ZZ**2.) / RRE
C ANGA = QATAN (SINANGA / COSANGA )
   ALONX = QATAN (YY / XX )
   ALATX = QATAN (ZZ / QSQR (XX**2. + YY**2. ) )
   ANODEE = ALONX - QASIN (QTAN (-ALATX ) / QTAN (AINCE ) )
C END SPHERE OF INFLUENCE CALC
30   CONTINUE
   ANODEM = ALON-AVM
   IF (ANODEM .LT.0. ) ANODEM = ANODEM + 2.*PI
   CALL VELTRANS (VELM,GAMAM,AZMM,ALAT,ALON,VXM,VYM,VZM,VMAGM,DPR)
   VXM = VXM+XDLO
   VYM = VYM+YDLO
   QQEMIN = 2.1*RPE/(RRE+RPE)
   VELEMIN = QSQR (QQEMIN*CMUE/RRE)
   IF (VELE .LT. VELEMIN) VELE = VELEMIN
   TIEMS = TIEM
C TIMEES CHANGED TO TIEMS, TIMEE CHANGED TO TIEM
35   CONTINUE
   CALL GAMACALC (RPE,VELE,RRE,CMUE,COSGAME,VPE,VCIRE,
*TIME1, TRAJE, DPR, ALAT, ALON, FTNM)
VELE2 = VELE + 10.
CALL GAMACALC(RPE, VELE2, RRE, CMUE, COSGAME, VPE, VCIRE,
*TIME2, TRAJE, DPR, ALAT, ALON, FTNM)
DELVELE = 10. / (TIME2 - TIME1) * (FTIM - TIME1 - TIMM)
IF (DELVELE .LT. 500.) THEN
  VELE = VELE + DELVELE
ELSE
  VELE = VELE + DELVELE / QABS (DELVELE) * 500.
ENDIF
CALL GAMACALC(RPE, VELE, RRE, CMUE, COSGAME, VPE, VCIRE, TIEM,
*TRAJE, DPR, ALAT, ALON, FTNM)
IF (TIEM .EQ. 0.) GOTO 60
TMT = TIEM + TIMM
IF (ABS (FTIM - TMT) .GT. 1.) GOTO 35
GAMAE = QFLOAT (MOOD) * QATAN (QSQRT (1. - COSGAME**2.)) / COSGAME
ALONE = 180. / DPR + ALONX
ALATE = QATAN (ZZ / QSQR (XX**2. + YY**2.))
50 CONTINUE
AINCE = AINCE0

C 'CALCULATION OF EARTH ORBIT AZIMUTH AT SPHERE OF INFLUENCE
IF (AINCE .NE. 0.0) THEN
  PHIE = PI - QASIN (ZZ / (RRE * QSIN (AINCE)))
  AZME = QATAN2 (1. / QTAN (AINCE), QCOS (PHIE))
ELSE
  AZME = PI / 2
ENDIF

C CALL VELTRANS (VELE, GAMAE, AZME, ALATE, ALONE, VXE, VYE, VZE, VELE,
  * DPR)
DELVEL = QSQRT ((VXE - VXM)**2. + (VYE - VYM)**2. + (VZE - VZM)**2.)
DVCIRES = VPE - VCIRE
DVE = QSQRT (VCIRE**2. + VPE**2. - 2. * VCIRE*VPE*
  * COS (AINCE - AINCE0))
DVTSAV = DVTOTAL
DVTOTAL = DELVEL + DELVL1 + DVCIRES

C CALCULATE POSITION AND VELOCITY OF MOON
C ITTERATE FOR FLIGHT TIM TO SPHERE OF INFLUENCE
TIM = (TIMJ - 2451545.) / 36525. + TIEM / 876600.
CALL POSVELMO (TIM, RREMER, XDLO, YDLO)
RREM = RREMER*20295741.
IF (ABS (TIEM - TIEMS) .GT. 1.) GOTO 25
IF (DVTOTAL .LT. DVMIN) THEN
  DVMIN = DVTOTAL
  ALONMIN = ALON
  ALATMIN = ALAT
  VELMIN = VELM
  VXMP = VXM
  VYMP = VYM


VZMP = VZM
VMAGMP = VMAGM
GAMAMP = GAMAM
AZMMP = AZMM
AINCMP = AINCM
ANODEMP = ANODEM
TIMMP = TIMM
DELVLPlP = DELVLPl
DELVELP = DELVEL
VXEP = VXE
VYEP = VYE
VZEP = VZE
VELEP = VELE
GAMAEP = GMAE
AZMEP = AZME
AINCEP = AINCE
ANODEEP = ANODEE
TIEMP = TIEM
DVCIREP = DVCIRE
DVTOTALP = DVTOTAL

ENDIF
IF (DELVEL .LT. DVSIM) THEN
  DVSIM = DELVEL
  ALONSIM = ALON
  ALATSIM = ALAT
  VELMSIM = VELM
  OMEGE = PHIE - THETAE
ENDIF

CONTINUE
WRITE (IS, 517) ALON*DPR, ALAT*DPR
FORMAT (' ALON = ',F6.1,' ALAT = ',F6.1)
IF (ICALL.EQ.1) GOTO 1000
IF (ICALL.EQ.2) GOTO 2000

STOP
END

SUBROUTINE FLYBY (ALON, ALAT, RREM, RPM, GAMAM, VELM, VPM, AIM,
  *DELVLPlP, AZMM, AVM, TIEMM, MOOD)
IMPLICIT REAL * 16 (A-H,O-Z)
IS = 5
DPR = 57.29578
PI = 3.141593
CMUE = 1.407647E+16
CMUM = 1.731400E+14
FTNM = 6076.115
REE = 20925741.
REMO = 5.7039E+6
OMEGM = .9582117947E-2
FTM = .3048
R1 = .15 * PREM
PRINT *, ' TIMEM INCL THETAM GAMAM VELM QQ EEM'
CONTINUE
LOCATION OF LP#1 IN LUNAR RELATIVE COORDINATES. LIBRATION
POINT ROTATES WITH THE MOON AND THEREFORE IS EFFECTED BY
FLYBY TIME
X1 = -R1 * QCOS(OMEGM*TIEMM)
Y1 = -R1 * QSIN(OMEGM*TIEMM) * FLOAT(MOOD)
Z1 = 0.
R1 IS LP#1, R2 IS SPACECRAFT AT SPHERE OF INFLUENCE. SPHERE
OF INFLUENCE IS DEFINED EQUAL TO RADIUS OF LP#1. THIS IS
SLIGHTLY LARGER THAN THE ONE USED IN THE PLANE CHANGE PROGRAM.
HOWEVER, THIS CHANGE SIMPLIFIES THE CALCULATION OF THE FLYBY
TRAJECTORY
X2 = R1 * QCOS(ALAT) * QCOS(ALON)
Y2 = R1 * QCOS(ALAT) * QSIN(ALON)
Z2 = R1 * QSIN(ALAT)
CALCULATION OF TRUE ANOMALY (THETA)
DOT = (X1*X2 + Y1*Y2 + Z1*Z2)
COS2THETA = DOT/R1/R1
THETA = PI - QACOS(COS2THETA)/2.
COSTHETAM = QCOS(THETA)
CALCULATION OF ECCENTRICITY OF FLYBY ORBIT FROM RP, R1, AND
THETA
EEM = (RPM/R1 - 1.)/(COSTHETAM - RPM/R1)
COSGAMA = (1.+EEM*COSTHETAM)/QSRT(1.+2.*EEM*COSTHETAM+EEM*EEM)
CALCULATE FLIGHT PATH ANGLE. GAMMA IS POSITIVE AT L1 AND NEGATIVE AT SOI FOR OUTBOUND AND REVERSED FOR INBOUND TRAJECTORIES
GAMAM = QACOS(COSGAMA) * FLOAT(MOOD)
PPM = RPM * (1. + EEM)
VPM = QSRT(CMUM * (2./RPM + (EEM*EEM - 1.)/PPM))
VELM = QSRT(CMUM * (2./R1 + (EEM*EEM - 1.)/PPM))
R1 CROSS R2 TO CALCULATE INCLINATION FROM Z3
X3 = Y1*Z2 - Z1*Y2
Y3 = Z1*X2 - X1*Z2
Z3 = X1*Y2 - Y1*X2
AMAG = QSRT(X3*X3 + Y3*Y3 + Z3*Z3)
AIM = QACOS(Z3/AMAG)
QQM = R1 * VELM*VELM/CMUM
COSAEX = (EEM + COSTHETAM)/(1. + EEM * COSTHETAM)
AAX = R1/(2. - QQM)
TIEMMS = TIEMM
IF (QQM .GT. 2.) GOTO 101
CALCULATE FLIGHT TIME FOR ELLIPTIC ORBITS
IF (ERRRP .GT. 0.) THEN
TIMX = 0.
GOTO 199
ENDIF
AEX = QACOS(COSAEX)
SINAEX = QSQR(1. - COSAEX*COSAEX)
TIMX = AAX*QSQR(AAX/CMUM) * (AEX-EEM*SINAEX) /3600.
GOTO 199
101 CONTINUE
C CALCULATE FLIGHT TIME FOR HYPERBOLIC ORBITS
COSH = COSAEX
SINHF = QSQR(COSH*COSH - 1.)
FFX = QLOG(COSH + QSQR(COSH*COSH - 1.))
TIMX = -AAX*QSQR(-AAX/CMUM) * (EEM*SINHF - FFX)/3600.
199 CONTINUE
TIEMM = TIMX * 2.
C CALCULATION OF AZIMUTH AND ANGLE AV AT R2
ANODE = OMEGM * TIEMM * FLOAT(MOOD)
IF (ALAT*FLOAT(MOOD) .GE. 0.) ANODE = ANODE + PI
AVM = ALON - ANODE
COSPHIM = QCOS(ALAT) * (QCOS(ANODE) * QCOS(ALON) +
* QSIN(ANODE) * QSIN(ALON))
APHIM = QACOS(COSPHIM)
IF (QABS(ALAT) .GT. 89.9/DPR) THEN
AZMM = (PI + PI * ALAT/QABS(ALAT))/2.
GOTO 200
ENDIF
SINAZM = QCOS(AIM)/QCOS(ALAT)
COSAZM = QSIN(AIM) * QCOS(APHIM)/QCOS(ALAT)
AZMM = QATAN2(SINAZM, COSAZM)
200 CONTINUE
C CALCULATE AZIMUTH FOR L1
ALATL1 = 0.
ALONL1 = QATAN2(Y1, X1)
COSPHIL1 = QCOS(ALATL1) * (QCOS(ANODE) * QCOS(ALONL1) +
* QSIN(ANODE) * QSIN(ALONL1))
APHIL1 = QACOS(COSPHIL1)
SINAZM = QCOS(AIM)/QCOS(ALATL1)
COSAZM = QSIN(AIM) * QCOS(APHIL1)/QCOS(ALATL1)
AZMLP1 = QATAN2(SINAZM, COSAZM)
C PRINT DATA AND ITERATE FOR FLIGHT TIME
IF (QABS(TIEMM - TIEMMS) .GT. 0.1) GOTO 10
C WRITE (IS,17) TIEMM, AIM*DPR, THETA*DPR, GAMAM*DPR,
C *VELM, QQM, EEM
C 17 FORMAT (1X,F5.1,3(2X,F5.1),2X,F10.2,2X,F10.4,2X,F10.6)
IF (QABS(TIEMM - TIEMMS) .GT. 0.1) GOTO 10
C LOCATION OF LB#1 IN LUNAR RELATIVE COORDINATES
ALATL1 = 0.
ALONL1 = 180.
CALL VELTRANS(VELM, GAMAM, AZMLP1, ALATL1, ALONL1, VXL, VYL1, VZL1,
*VYL1, VZL1, VMAGL1)
VYC = -OMEGM*R1/3600.
DELVLP1 = QSQR(VXL1**2+(VYL1 - VYC)**2+VZL1**2.)
GAMA is negative at SOI, positive at L1 for outbound trajectories, reversed for inbound flight. Need to change sign for SOI calculations.

GAMAM = -GAMAM
RETURN
END

SUBROUTINE XYZPOS (RRX, ALAT, ALON, XX, YX, ZX)
IMPLICIT REAL * 16 (A-Z)
XX = -RRX*COS(ALAT)*COS(ALON) + RREM
YX = -RRX*COS(ALAT)*SIN(ALON)
ZX = RRX*SIN(ALAT)
RETURN
END

SUBROUTINE POSVELMO (TIM, RRM, XDLO, YDLO)
IMPLICIT REAL * 16 (A-H, O-Z)
DELT = 0.5/36525./24./3600.
T1 = TIM-DELT
CALL MOON(T1, RAM, DECM, RM1)
T2 = TIM+DELT
CALL MOON(T2, RAM, DECM, RM2)
XDLO = (RM2-RM1)*20925741.
RRM = (RM2+RM1)/2.
RRMB = RRM - 7.4127893E-01
YDLO = 200570.2/RRMB
RETURN
END

SUBROUTINE MOON (T, RAM, DECM, RM)
Finds location of moon in equatorial coords. at any tim
REF: '87 Astronomical Almanac
T is Julian centuries since year 2000
LAM is moon's ecliptic longitude
BETA is moon's ecliptic latitude
PIE is horizontal parallax
RM is dist. to moon in earth radii
RAM is rt. ascension of moon
DECM is moon's declination
SD is semidiameter of moon's orbit
IMPLICIT REAL * 16 (A-Z)
PRINT *, ' MOON'
$P = 3.1415926535$
$C = P / 180$

$LAM = C * 218.32 + C * 481267.883 * T + C * 6.29 * QSIN(C * 134.9 + C * 477198.85 * T) - C * 1.27 * QSIN(C * 259.2 - C * 413335.38 * T) + C * 0.66 * QSIN(C * 235.7 + C * 890534.23 * T)$

$LAM = LAM + C * 0.21 * QSIN(C * 269.9 + C * 954397.7 * T) - C * 0.19 * QSIN(C * 357.5 + C * 35999.05 * T) - C * 0.11 * QSIN(C * 186.6 + C * 966404.05 * T)$

$\beta = C * 5.13 * QSIN(C * 93.3 + C * 483202.03 * T) + C * 0.28 * QSIN(C * 228.2 + C * 960400.87 * T) - C * 0.28 * QSIN(C * 318.3 + C * 60003.18 * T) - C * 0.17 * QSIN(C * 217.6 - C * 407332.2 * T)$

$\pi = C * 0.9508 + C * 0.0518 * COS(C * 134.9 + C * 477198.85 * T) + C * 0.0095 * COS(C * 259.2 - C * 413335.38 * T) + C * 0.0078 * COS(C * 235.7 + C * 890534.23 * T) + C * 0.0028 * COS(C * 269.9 + C * 954397.7 * T)$

$SD = 0.2725 * \pi$
$RM = 1. / QSIN(\pi)$

$1 = QSIN(\beta) * QSIN(LAM)$
$M = 0.9175 * QSIN(\beta) * QSIN(LAM) - 0.3978 * QSIN(\beta)$
$n = 0.3978 * QSIN(\beta) * QSIN(LAM) + 0.9175 * QSIN(\beta)$
$RAM = QATAN2(M, 1)$
$DEC = QASIN(n)$
RETURN

SUBROUTINE GAMACALC (RPX, VV, RRX, CMUX, COSGAMX, VPX, VCIRX, TIMX, TRAJ, DPR, ALAT, ALON, FTNM)

IMPLICIT REAL * 16 (A-H, O-Z)
CHARACTER*10 TRAJ
IHYP = 1
TRAJ = 'ELIP'T
QQX = RRX*VV**2 / CMUX
IF(QQX-2. LT. 1.0E-06) QQX = QQX - 1.0E-06
IF (QQX.GT.2.) THEN
IHYP = -1
TRAJ = 'HYPER'
PRINT *, ' **** TRAJECTORY IS HYPERBOLIC '
ENDIF
AAX = RRX/(2.-QQX)
IF (AAX.GT.1.0E12 .OR. AAX.LT.-1.0E12) AAX = -1.0E12
EEX = 1.-RPX/AAX
PPX = AAX*(1. - EEX ** 2.)
COSGAMX = QSQRT(RPX/RRX*(1+EEX)/QQX)
IF (COSGAMX .GT. 1.) THEN
TIMX = 0.
RETURN
ENDIF
GAMAX = QACOS (COSGAMX)
VEX = QSQRT (CMUX* (1+EEX)/RPX)
VCIRX = QSQRT (CMUX/RPX)
COSTHETAX = (PPX/RRX-1)/EEX
THETAX = QACOS (COSTHETAX)
COSAEX = (EEX+COSTHETAX)/(1+EEX*COSTHETAX)
ERRRP = RRX-AAX* (1+EEX)
IF (ERRRP .GT. 0. .AND. AAX .GT. 0.) THEN
WRITE (IS,537) ERRRP/6076.1155, ALAT*DPR, ALON*DPR
537 FORMAT (' RADIUS > APOGEE BY NMI ',F7.5, F8.0, F7.5)
WRITE (IS,547) QQX, AAX/FNTM, EEX, GAMAX*DPR, VPX
547 FORMAT (' QQX = ',F7.5,' AAX = ',F8.0,' EEX = ',F7.5,
* ' GAMAX = ',F5.1,' VPX = ',F7.1)
ENDIF
IF (QQX .GT. 2.) GOTO 101
C
CALC FLIGHT TIM FOR ELLIPTICAL ORBITS
IF (ERRRP .GT. 0.) THEN
      TIMX = 0.
GOTO 199
ENDIF
AEX = QACOS (COSAEX)
SINAEX = QSQRT (1-COSAEX**2.)
TIMX = QSQRT (AAX ** 3. / CMUX) * (AEX-EEX*SINAEX)/3600.
GOTO 199
101 CONTINUE
C
CALC FLIGHT TIM FOR HYPERBOLIC ORBITS
COSHIF = COSAEX
SINHIF = QSQRT (COSHIF**2.-1.)
FFX = QLOG (COSHIF+QSQRT (COSHIF**2.-1.))
TIMX = QSQRT (-1. *AAX*AAX/AAX/CMUX) * (EEX*SINHIF-FFX)/3600.
199 CONTINUE
RETURN
END

SUBROUTINE VELTRANS (VEL, GAMA, AZM, ALAT, ALON, VXX, VYX, VZX, VMAG,
* DPR)
IMPLICIT REAL * 16 (A-Z)
RRD = VEL * QSIN (GAMA )
RPHID = VEL * QCOS (GAMA )
VLON = RPHID * QSIN (AZM )
VLAT = RPHID * QCOS (AZM )
VXR = -RRD * QCOS (ALAT ) * QCOS (ALON )
VYR = -RRD * QCOS (ALAT ) * QSIN (ALON )
VZR = RRD * QSIN (ALAT )
VXLA = VLAT * QSIN (ALAT ) * QCOS (ALON )
VYLA = VLAT * QSIN (ALAT ) * QSIN (ALON )
VZLA = VLAT * QCOS (ALAT )
VXLO = VLON * QSIN (ALON )
VYLO = -VLON * QCOS (ALON )
VZLO = 0.0
VXX = VXR + VXLA + VXLO
VYX = VXR + VYLA + VYLO
VZX = VZR + VZLA + VZLO
VMAG = QSQR (VXX **2. + VYX ** 2. + VZX ** 2. )
RETURN
END
## Appendix C. Program Variables

<table>
<thead>
<tr>
<th>INPUT VARIABLE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>AINCEO</td>
<td>Earth orbit inclination (degrees). This is the angle between the plane of the low Earth orbit and the plane of the Moon's orbit about the Earth.</td>
</tr>
<tr>
<td>ALATI</td>
<td>Initial Sphere of Influence longitude for map (degrees)</td>
</tr>
<tr>
<td>ALONI</td>
<td>Initial Sphere of Influence latitude for map (degrees)</td>
</tr>
<tr>
<td>DELLAT</td>
<td>Incremental latitude for map (degrees)</td>
</tr>
<tr>
<td>DELLON</td>
<td>Incremental longitude for map (degrees)</td>
</tr>
<tr>
<td>FTIM</td>
<td>Flight time for trajectory (hours)</td>
</tr>
<tr>
<td>HPE</td>
<td>Holding perigee altitude of Earth orbit (nautical miles)</td>
</tr>
<tr>
<td>HPM</td>
<td>Perigee altitude of Lunar &quot;flyby&quot; trajectory (nautical miles)</td>
</tr>
<tr>
<td>MD</td>
<td>Leg of trip for which to perform calculations (OUTBOUND or RETURN)</td>
</tr>
<tr>
<td>TIMJ</td>
<td>Earth departure Julian date (where January 1, 2000 is day 2,451,545. Refer to Section C of &quot;The Astronomical Almanac of the Year 1988&quot;).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTANT</th>
<th>VALUE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>PI/180</td>
<td>Degrees per radian (deg./rad.)</td>
</tr>
<tr>
<td>CMUE</td>
<td>1.407647E+16</td>
<td>Gravitational parameter of the Earth (ft²/sec²)</td>
</tr>
<tr>
<td>CMUM</td>
<td>1.731432E+14</td>
<td>Gravitational parameter of the Moon (ft²/sec²)</td>
</tr>
<tr>
<td>DPR</td>
<td>57.29578</td>
<td>Degrees per radian (deg./rad.)</td>
</tr>
<tr>
<td>FTM</td>
<td>0.3048</td>
<td>Meters per foot (m/ft)</td>
</tr>
<tr>
<td>FTNM</td>
<td>6,076.115</td>
<td>Feet per nautical mile (ft/nmi)</td>
</tr>
<tr>
<td>P</td>
<td>3.1415927</td>
<td>( \Pi ) (dimensionless)</td>
</tr>
<tr>
<td>PI</td>
<td>3.141593</td>
<td>( \Pi ) (dimensionless)</td>
</tr>
<tr>
<td>REE</td>
<td>20,925,741</td>
<td>Radius of the Earth (ft)</td>
</tr>
<tr>
<td>REMO</td>
<td>5,703,900</td>
<td>Radius of the Moon (ft)</td>
</tr>
<tr>
<td>RREM</td>
<td>1,261,152,353</td>
<td>Earth-Moon distance of centers (ft)</td>
</tr>
<tr>
<td>VARIABLE</td>
<td>DESCRIPTION</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>AAX</td>
<td>Semi-major axis of one of the transfer orbits</td>
<td></td>
</tr>
<tr>
<td>AEX</td>
<td>Eccentric anomaly of one of the transfer orbits</td>
<td></td>
</tr>
<tr>
<td>AIM</td>
<td>Inclination of &quot;flyby&quot; trajectory</td>
<td></td>
</tr>
<tr>
<td>AINC</td>
<td>L1 orbit inclination in radians (always 0)</td>
<td></td>
</tr>
<tr>
<td>AINCE</td>
<td>Earth orbit inclination in radians</td>
<td></td>
</tr>
<tr>
<td>AINCEP</td>
<td>Earth orbit inclination in radians</td>
<td></td>
</tr>
<tr>
<td>AINCM</td>
<td>&quot;Flyby&quot; orbit inclination (degrees)</td>
<td></td>
</tr>
<tr>
<td>AINCMP</td>
<td>&quot;Flyby&quot; orbit inclination (degrees) for the SOI point associated with minimum ΔV.</td>
<td></td>
</tr>
<tr>
<td>ALAT</td>
<td>Initial latitude for map (radians)</td>
<td></td>
</tr>
<tr>
<td>ALATE</td>
<td>Latitude of SOI point, measured from the Earth</td>
<td></td>
</tr>
<tr>
<td>ALATL1</td>
<td>Latitude of L1, measured from the Moon (0°) (Moon-fixed)</td>
<td></td>
</tr>
<tr>
<td>ALATLP1</td>
<td>Latitude of L1, measured from the Moon (0°)</td>
<td></td>
</tr>
<tr>
<td>ALATMIN</td>
<td>Latitude of SOI point associated with the minimum total ΔV</td>
<td></td>
</tr>
<tr>
<td>ALATP()</td>
<td>ALAT in degrees</td>
<td></td>
</tr>
<tr>
<td>ALATSIM</td>
<td>Latitude of SOI point associated with the minimum SOI ΔV</td>
<td></td>
</tr>
<tr>
<td>ALATX</td>
<td>Latitude of the SOI point, measured from the Earth</td>
<td></td>
</tr>
<tr>
<td>ALON</td>
<td>Initial longitude for map (radians)</td>
<td></td>
</tr>
<tr>
<td>ALONE</td>
<td>Longitude of SOI point, measured from zero longitude at Earth (-X direction)</td>
<td></td>
</tr>
<tr>
<td>ALONL1</td>
<td>Longitude of L1, measured from the Moon (Moon-fixed)</td>
<td></td>
</tr>
<tr>
<td>ALONLP1</td>
<td>Longitude of L1, measured from the Moon (180°)</td>
<td></td>
</tr>
<tr>
<td>ALONMIN</td>
<td>Longitude of SOI point associated with the minimum total ΔV</td>
<td></td>
</tr>
<tr>
<td>ALONO()</td>
<td>Variable that contains the print matrix column headings (longitudes)</td>
<td></td>
</tr>
<tr>
<td>ALONSIM</td>
<td>Longitude of SOI point associated with the minimum SOI ΔV</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>-----------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>ALONX</td>
<td>Longitude of the SOI point, measured from the Earth-Moon line at the Earth</td>
<td></td>
</tr>
<tr>
<td>AMAG</td>
<td>Magnitude of position vector cross-product (R2 X R1) used to calculate &quot;flyby&quot; inclination</td>
<td></td>
</tr>
<tr>
<td>ANGA</td>
<td>The angle between the Earth-to-Moon line and the Earth-to-SOI-point line</td>
<td></td>
</tr>
<tr>
<td>ANODE</td>
<td>The longitude of the ascending or decending node of &quot;flyby&quot; trajectory</td>
<td></td>
</tr>
<tr>
<td>ANODEE</td>
<td>Longitude of the Earth ascending or descending node</td>
<td></td>
</tr>
<tr>
<td>ANODEEP</td>
<td>Longitude of the Earth ascending or descending node associated with minimum ΔV</td>
<td></td>
</tr>
<tr>
<td>ANODEM</td>
<td>Longitude of the Lunar ascending or descending node</td>
<td></td>
</tr>
<tr>
<td>ANODEMP</td>
<td>Longitude of the Lunar ascending or descending node associated with the minimum ΔV</td>
<td></td>
</tr>
<tr>
<td>APHIL1</td>
<td>Angle between line of nodes and L1</td>
<td></td>
</tr>
<tr>
<td>APHIM</td>
<td>Angle between line of nodes and Moon-to-SO1 line</td>
<td></td>
</tr>
<tr>
<td>AVM</td>
<td>Angle between the Lunar node and the projection of the SOI point onto the Earth-Moon plane (u)</td>
<td></td>
</tr>
<tr>
<td>AZM</td>
<td>Azimuth of the SOI point</td>
<td></td>
</tr>
<tr>
<td>AZME</td>
<td>Azimuth (from the Earth) of the SOI point for Earth-SOI trajectory</td>
<td></td>
</tr>
<tr>
<td>AZMEP</td>
<td>Azimuth (from the Earth) of the SOI point for Earth-SOI trajectory associated with minimum ΔV</td>
<td></td>
</tr>
<tr>
<td>AZMLP1</td>
<td>Azimuth of &quot;flyby&quot; orbit at L1</td>
<td></td>
</tr>
<tr>
<td>AZMM</td>
<td>Azimuth (from the Moon) of the SOI point for &quot;flyby&quot; trajectory</td>
<td></td>
</tr>
<tr>
<td>AZMMP</td>
<td>Azimuth (from the Moon) of the SOI point for &quot;flyby&quot; trajectory with minimum ΔV</td>
<td></td>
</tr>
<tr>
<td>BETA</td>
<td>Moon's ecliptic latitude</td>
<td></td>
</tr>
<tr>
<td>CMUX</td>
<td>Earth or Moon gravitational parameter</td>
<td></td>
</tr>
<tr>
<td>COSAEX</td>
<td>Cosine of the eccentric anomaly of one of the transfer orbits</td>
<td></td>
</tr>
<tr>
<td>COSAINC</td>
<td>Cosine of the L1 orbit inclination (always 1)</td>
<td></td>
</tr>
<tr>
<td>COSALAT</td>
<td>Cosine of the SOI point latitude</td>
<td></td>
</tr>
</tbody>
</table>
COSALON  Cosine of the SOI point longitude
COSANGA  Cosine of the angle between the Earth-to-Moon line and the Earth-to-SOI-point line
COSAZM    Cosine of the azimuth angle at the SOI point or L1 for the "flyby" trajectory
COSGAMA   Cosine of flight path angle at SOI for "flyby" trajectory
COSGAME   Cosine of the flight path angle at SOI of the Earth-to-SOI trajectory
COSGAMX   Cosine of the flight path angle at SOI of one of the transfer orbits
COSHFI    Hyperbolic cosine of the eccentric anomaly of one of the transfer orbits
COSPphi   Cosine of the angle between the Moon-to-Earth line and the Moon-to-SOI-point line
COSPphiL1 Cosine of angle between line of nodes and L1
COSPphiM  Cosine of the angle between the line of nodes and the Moon-to-SOI line
COSTHETAM Cosine of the true anomaly of the "flyby" orbit
COSTHETAX Cosine of the true anomaly of one of the transfer orbits at SOI
COS2THETA Cosine of two times the true anomaly of the "flyby"
DATP      Today’s date
DECM      Declination of the Moon
DELT      A fraction of TIM that represents a half-second
DELV(,)   Total ΔV for matrix of SOI longitudes and latitudes
DELVEL    ΔV at the SOI point to patch the "flyby" trajectory with the Earth-to-SOI trajectory
DELVELP   ΔV at the SOI point to patch the "flyby" trajectory with the Earth-to-SOI trajectory associated with minimum ΔV
DELVELE   Estimated ΔV required for the Earth-to-SOI trajectory in order to satisfy the time-of-flight requirement
DELVLP1   ΔV for circularization at L1
DELVLP1P  ΔV for circularization at L1 associated with minimum ΔV
DOT          Dot product of position vectors for L1 and SOI point (R1  R2)
DVCIRE       ΔV between Earth circular orbit and Earth-to-SO1 trajectory
DVCIREP      ΔV between Earth circular orbit and Earth-to-SO1 trajectory associated with minimum ΔV
DVE          Unused plane change variable
DVMIN        Hold variable for minimum total ΔV
DVSIM        Hold variable for minimum SOI ΔV
DVTOTAL      Total ΔV for flight
DVTOTALP     Total ΔV for flight associated with minimum ΔV
DVTSAV       Temporary storage for total ΔV
EEM          Eccentricity of the "flyby" orbit
EEX          Eccentricity of one of the transfer orbits
ERRRP        Difference between orbital range at SOI and apogee range
FFX          Hyperbolic eccentric anomaly
GAMA         Flight path at SOI of one of the transfer orbits
GAMAE        Flight path at SOI of Earth-to SOI trajectory
GAMAEP       Flight path at SOI of Earth-to SOI trajectory associated with minimum ΔV
GAMAM        Flight path at SOI of "flyby" trajectory
GAMAMP       Flight path at SOI of "flyby" trajectory with minimum ΔV
GAMAX        Flight path angle at SOI of one of the transfer orbits
HEAD         Heading that indicates leg of journey (e.g., "Earth to L1 Flyby" for outbound)
ICALL        Flag to track occurrence of call to in-program subroutine.
II           Counter (1-19), representing increments in latitude for the output matrices
IHYPYER       Indicator that describes whether an orbit is hyperbolic
IP           Output number indicating output to file
IPRINT Flag for print block that contains matrix column headers
IS Output number indicating output to screen
L A geocentric direction cosine
LAM Moon's ecliptic longitude
M A geocentric direction cosine
MOOD Leg of journey (+1: outbound)
N A geocentric direction cosine
NN Counter (1-10) representing increments in longitude for the output matrices
NPI Counter for the 19 matrix rows during report printing
OMEGE Argument of perigee for Earth-SOI trajectory
OMEGM Angular velocity of Moon
PAGE1() Total ΔV for a given SOI longitude and latitude. Cell values for Report #1 matrix
PAGE2(,) SOI ΔV for a given SOI longitude and latitude. Cell values for Report #2 matrix
PAGE3(,) Inclination of "flyby" trajectory for a given SOI longitude and latitude. Cell values for Report #3 matrix
PHIE Used to determine the azimuth of the SOI point
PIE Horizontal parallax
PPM Semi-latus rectum of the "flyby" orbit
PPX Semi-latus rectum of one of the transfer orbits
QQEMIN Vis-viva parameter for the Earth-to-SOI-point trajectory
QQM Vis-viva parameter for the "flyby" orbit
QQX Vis-viva parameter for an orbit
R1 Distance of L1 and SOI surface from center of Moon (15% of Earth-Moon distance)
RAM Right ascension of the Moon
RM Distance from the Earth to the Moon in Earth radii
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMR</td>
<td>Ratio of Earth-Moon distance to Moon-SOI distance</td>
</tr>
<tr>
<td>RM1</td>
<td>Distance from the Earth to the Moon in Earth radii</td>
</tr>
<tr>
<td>RM2</td>
<td>Distance from the Earth to the Moon in Earth radii</td>
</tr>
<tr>
<td>RPE</td>
<td>Distance from Earth’s center to Earth perigee orbit</td>
</tr>
<tr>
<td>RPHID</td>
<td>Orbital path component of the velocity vector</td>
</tr>
<tr>
<td>RPM</td>
<td>Distance from Moon’s center to Lunar &quot;flyby&quot; perigee</td>
</tr>
<tr>
<td>RPX</td>
<td>Distance from Earth center to perigee</td>
</tr>
<tr>
<td>RRD</td>
<td>Radial component of the velocity vector</td>
</tr>
<tr>
<td>RRE</td>
<td>Distance from the Earth to the SOI point</td>
</tr>
<tr>
<td>RREMER</td>
<td>Moon’s distance from the Earth (Earth radii)</td>
</tr>
<tr>
<td>RRM</td>
<td>Distance from the Moon to the SOI point, when the Moon is at its mean distance from the Earth</td>
</tr>
<tr>
<td>RRMB</td>
<td>Distance of the Moon from the Earth-Moon baricenter</td>
</tr>
<tr>
<td>RRX</td>
<td>Distance from Earth to SOI</td>
</tr>
<tr>
<td>SD</td>
<td>Semi-diameter of the Moon’s orbit</td>
</tr>
<tr>
<td>SINAEX</td>
<td>Sine of the eccentric anomaly of one of the transfer orbits</td>
</tr>
<tr>
<td>SINAINC</td>
<td>Sine of the Lunar orbit inclination</td>
</tr>
<tr>
<td>SINALAT</td>
<td>Sine of the latitude of the SOI point</td>
</tr>
<tr>
<td>SINALON</td>
<td>Sine of the longitude of the SOI point</td>
</tr>
<tr>
<td>SINANGA</td>
<td>Sine of the angle between the Earth-to-Moon line and the Earth-to-SOI point line</td>
</tr>
<tr>
<td>SINAZM</td>
<td>Sine of the azimuth angle at the SOI point or L1 for the &quot;flyby&quot; trajectory</td>
</tr>
<tr>
<td>SINHF</td>
<td>Hyperbolic sine of the eccentric anomaly of one of the transfer orbits</td>
</tr>
<tr>
<td>T</td>
<td>Number of Julian centuries since the year 2000 AD</td>
</tr>
<tr>
<td>THETA</td>
<td>True anomaly of &quot;flyby&quot; at the SOI</td>
</tr>
<tr>
<td>THETAX</td>
<td>True anomaly of one of the transfer orbits at SOI</td>
</tr>
<tr>
<td>TIEM</td>
<td>Earth-to-SOI time of flight (seconds)</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>TIEMP</td>
<td>Earth-to-SOI time of flight (seconds) associated with minimum ΔV</td>
</tr>
<tr>
<td>TIEMS</td>
<td>Temporary storage for Earth-to-SOI time of flight</td>
</tr>
<tr>
<td>TIM</td>
<td>Time of arrival at SOI point from Earth, in centuries since the year 2000 AD</td>
</tr>
<tr>
<td>TIME1</td>
<td>Earth-to-SOI time of flight (seconds)</td>
</tr>
<tr>
<td>TIME2</td>
<td>Earth-to-SOI time of flight (seconds)</td>
</tr>
<tr>
<td>TIEMM</td>
<td>Time of flight from SOI to L1</td>
</tr>
<tr>
<td>TIEMMP</td>
<td>Time of flight from SOI to L1 associated with minimum ΔV</td>
</tr>
<tr>
<td>TIEMMS</td>
<td>Temporary storage for SOI to L1 time of flight</td>
</tr>
<tr>
<td>TIMP</td>
<td>Time Now</td>
</tr>
<tr>
<td>TIMT</td>
<td>Total time of flight</td>
</tr>
<tr>
<td>TIMX</td>
<td>Time of perigee passage for &quot;flyby&quot;</td>
</tr>
<tr>
<td>TRAJ</td>
<td>Text that describes whether an orbit is hyperbolic or elliptical</td>
</tr>
<tr>
<td>TRAJE</td>
<td>Text that describes whether the Earth-to-SOI trajectory is hyperbolic or elliptical</td>
</tr>
<tr>
<td>TRAJM</td>
<td>Text that describes whether the &quot;flyby&quot; trajectory is hyperbolic or elliptical</td>
</tr>
<tr>
<td>T1</td>
<td>One-half second before TIM</td>
</tr>
<tr>
<td>T2</td>
<td>One-half second after TIM</td>
</tr>
<tr>
<td>VCIRE</td>
<td>Velocity of Earth circular orbit</td>
</tr>
<tr>
<td>VCIRX</td>
<td>Velocity of the Earth circular orbit</td>
</tr>
<tr>
<td>VEL</td>
<td>Velocity at SOI of one of the transfer orbits</td>
</tr>
<tr>
<td>VELE</td>
<td>Velocity at SOI of the Earth-to-SOI trajectory</td>
</tr>
<tr>
<td>VELEP</td>
<td>Velocity at SOI of the Earth-to-SOI trajectory associated with minimum ΔV</td>
</tr>
<tr>
<td>VELE2</td>
<td>Ten feet per second more than VELE</td>
</tr>
<tr>
<td>VELEMIN</td>
<td>Minimum velocity required such that apogee of the trajectory is just at SOI</td>
</tr>
<tr>
<td>VELM</td>
<td>Velocity at SOI of the &quot;flyby&quot; trajectory</td>
</tr>
<tr>
<td>VELMMIN</td>
<td>Velocity at SOI of the &quot;flyby&quot; trajectory associated with the minimum total ΔV</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>VELMOUT</td>
<td>Velocity at SOI of the &quot;flyby&quot; trajectory, for a given SOI longitude and latitude</td>
</tr>
<tr>
<td>VELMSIM</td>
<td>Velocity at SOI of the &quot;flyby&quot; trajectory associated with the minimum SOI ΔV</td>
</tr>
<tr>
<td>VLAT</td>
<td>Latitude component of the velocity vector</td>
</tr>
<tr>
<td>VLON</td>
<td>Longitude component of the velocity vector</td>
</tr>
<tr>
<td>VMAG</td>
<td>Velocity vector magnitude</td>
</tr>
<tr>
<td>VMAGLP1</td>
<td>Velocity vector magnitude at L1 for &quot;flyby&quot; trajectory</td>
</tr>
<tr>
<td>VMAGM</td>
<td>Velocity vector magnitude at SOI for &quot;flyby&quot; trajectory</td>
</tr>
<tr>
<td>VMAGMP</td>
<td>Velocity vector magnitude at SOI for &quot;flyby&quot; trajectory with minimum ΔV</td>
</tr>
<tr>
<td>VPE</td>
<td>Perigee velocity of the Earth-to-SO1 trajectory</td>
</tr>
<tr>
<td>VPM</td>
<td>Perigee velocity of the &quot;flyby&quot; trajectory</td>
</tr>
<tr>
<td>VPX</td>
<td>Perigee velocity for one of the transfer orbits</td>
</tr>
<tr>
<td>VV</td>
<td>Velocity at SOI of one of the transfer orbits</td>
</tr>
<tr>
<td>VVMINE</td>
<td>Velocity at SOI point (apogee) of Earth-to-SO1 trajectory</td>
</tr>
<tr>
<td>VXE</td>
<td>X-coordinate of velocity vector at SOI for Earth-to-SO1 trajectory</td>
</tr>
<tr>
<td>VXEP</td>
<td>X-coordinate of velocity vector at SOI for Earth-to-SO1 trajectory associated with minimum ΔV</td>
</tr>
<tr>
<td>VXLA</td>
<td>X-component of the latitude component of the velocity vector</td>
</tr>
<tr>
<td>VXLO</td>
<td>X-component of the longitude component of the velocity vector</td>
</tr>
<tr>
<td>VXLP1</td>
<td>X-component of the velocity vector at L1 for &quot;flyby&quot;</td>
</tr>
<tr>
<td>VXM</td>
<td>X-coordinate of velocity vector at SOI for &quot;flyby&quot; trajectory</td>
</tr>
<tr>
<td>VXMP</td>
<td>X-coordinate of velocity vector at SOI for &quot;flyby&quot; trajectory with minimum ΔV</td>
</tr>
<tr>
<td>VXR</td>
<td>X-component of the radial component of the velocity vector</td>
</tr>
<tr>
<td>VXX</td>
<td>Total X-component of the velocity vector</td>
</tr>
<tr>
<td>VYCOR</td>
<td>Y-component of velocity for orbit at L1 (no X- or Z- component)</td>
</tr>
<tr>
<td>VYE</td>
<td>Y-coordinate of velocity vector at SOI for Earth-to-SO1 trajectory</td>
</tr>
</tbody>
</table>
VYEP  Y-coordinate of velocity vector at SOI for Earth-to-SO1 trajectory associated with minimum ΔV
VYLA  Y-component of the latitude component of the velocity vector
VYLO  Y-component of the longitude component of the velocity vector
VYLP1 Y-component of the velocity vector at L1 for "flyby"
VYM   Y-coordinate of velocity vector at SOI for "flyby" trajectory
VYMP  Y-coordinate of velocity vector at SOI for "flyby" trajectory with minimum ΔV
VYR   Y-component of the radial component of the velocity vector
VYX   Total Y-component of the velocity vector
VZE   Z-coordinate of velocity vector at SOI for Earth-to-SO1 trajectory
VZEP  Z-coordinate of velocity vector at SOI for Earth-to-SO1 trajectory associated with minimum ΔV
VZLA  Z-component of the latitude component of the velocity vector
VZLO  Z-component of the longitude component of the velocity vector
VZLP1 Z-component of the velocity vector at L1 for "flyby"
VZM   Z-coordinate of velocity vector at SOI for "flyby" trajectory
VZMP  Z-coordinate of velocity vector at SOI for "flyby" trajectory with minimum ΔV
VZR   Z-component of the radial component of the velocity vector
VZX   Total Z-component of the velocity vector
X1    X-component of the distance from the Moon to L1
X2    X-component of the distance from the Moon to the SOI penetration point
X3    X-component of cross-product (R1 X R2)
XDLO  X-coordinate of the velocity of the Moon
XX    Distance in the X-direction from the Earth to the SOI point’s X-coordinate
XXM   Distance in the X-direction from the Moon to the SOI point’s X-coordinate
Y1    Y-component of the distance from the Moon to L1
Y2 Y-component of the distance from the Moon to the SOI penetration point
Y3 Y-component of cross-product (R1 X R2)
YDLO Y-coordinate of the velocity of the Moon
YY Distance in the Y-direction from the Earth-Moon line to the SOI point’s Y-coordinate
YYM Distance in the Y-direction from the Moon to the SOI point’s Y-coordinate
Z1 Z-component of the distance from the Moon to L1
Z2 Z-component of the distance from the Moon to the SOI penetration point
Z3 Z-component of cross-product (R1 X R2)
ZZ Distance in the Z-direction from the Earth-Moon plane to the SOI point
Appendix D. Detailed Program Description

This section is a line-by-line description of the lines of code in the main program of LP1. Refer to Appendix B for the actual code listing, and figures K-1 through K-3 for supporting illustrations.

1. Declare the matrices DELV, VELMOUT, PAGE1, PAGE2, PAGE3, PAGE4, PAGE5, PAGE6, ALONO, and ALATP.

2. Open the output file (LP1.OUT).

3. Define the program constants.

4. Read the program inputs.

   A. MD (leg of trip for which to perform calculations; outbound or return)

   B. HPE (perigee altitude of Earth circular orbit)

   C. HPM (perigee altitude of Lunar "flyby" trajectory)

   D. TIMJ (Earth departure Julian date. Default is 2451545, representing Jan. 1, 2000 AD)

   E. ALONI, DELLON (Initial longitude and increment for output map)

   F. ALATI, DELLAT (Initial latitude and increment for output map)

   G. AINCEO (Angle between the plane of the low Earth orbit and the Earth-Moon plane)

   H. FTIM (flight time)

5. Calculate the distance from the Earth’s center to Earth perigee orbit.
   \[ RPE = HPE \times FTNM + REE \]

6. Calculate the distance from the Moon’s center to Lunar perigee orbit.
   \[ RPM = HPM \times FTNM + REM0 \]

7. Store the inclinations in temporary variables (AINCE and AINC).

8. Record the current date and time (DATP and TIMP).

9. Print the headings for Report #1, "Velocity Map for Inbound/Outbound Trajectories".

10. Echo back the input values onto Report #1.

11. Initialize the velocity map matrix coordinates to 1,1 (II = rows, or latitude increments; NN = columns, or longitude increments). Set the flag IPRINT to zero, which has the
effect of causing the longitude increments to be printed as a subheading. Initialize the minimum ΔV hold variable (DVMIN) to 99999.

12. Calculate the orbital velocity of the Moon.
   \[ YDLO = \mu E / RREM \]

13. Convert the initial map latitude and longitude from degrees to radians (ALAT and ALON).

14. Initialize DVSIM at 99999. This variable will be used to hold the lowest ΔV for the trajectory, selected from all combinations of latitude and longitude.

15. Initialize the current Total ΔV matrix cell (DELV) to 99999.

16. Initialize with zeros the print line variables for the current cell on each report (PAGE1, PAGE2, PAGE3, PAGE4, PAGE5, PAGE6).

17. Call the flyby subroutine to calculate the characteristics for a "flyby" trajectory between the current SOI point and L1. Pass input values for SOI latitude and longitude, Earth-Moon distance, and perigee altitude; and return output values for flight path angle and velocity (same for SOI point and L1), perigee velocity, time of flight, delta velocity needed to circularize at L1 (outbound) or initiate the "flyby" (return), azimuth angle, inclination of the "flyby" trajectory, and the angle between the lunar nodes and the SOI point projection.

18. Call the in-program subroutine given the current SOI conditions for the "flyby" trajectory (latitude, longitude, inclination, flight path, azimuth, and velocity) and iterate for a transfer orbit between LEO and the SOI point (with a velocity vector correction at the SOI) and determine the total ΔV and total flight time needed.

19. Store the total ΔV, SOI ΔV, and "flyby" inclination in arrays for output.

20. If all the matrix columns have been processed for this latitude, and if this has been the first row of the matrix, print the matrix column headings for Report #1.

21. If all ten matrix columns have not been processed for this latitude, increment the column counter by one and increment the longitude by the amount specified by the user in the inputs. Return to step 15 to process the next longitude/latitude combination.

22. If all ten matrix columns have been processed for this latitude, print the matrix row for this latitude for Report #1.

23. If all 19 matrix rows have not been processed, increment the row counter by one, increment the latitude by the amount specified by the user in the inputs, reset the column counter to one, reset the longitude to the initial input longitude, and return to step 15 to process the next longitude/latitude combination.

24. If all 19 matrix rows have been processed, continue with the following steps.

25. Print the bottom section of Report #1, and the remaining five reports.
Appendix E. In-Program Subroutine Description

This section is a line-by-line description of the lines of code in the subroutine that is imbedded in program LP1, beginning at line #25. Refer to Appendix B for the actual code listing, and figures K-2 through K-4 for supporting illustrations.

I. Establish the key distances and angles of the current SOI point from the Earth and Moon.

A. Calculate the distance from the Moon to the SOI point.

1. Define variables for the sine and cosine of the SOI latitude and longitude (COSALAT, COSALON, SINALAT, SINALON).

2. Given a point on the SOI defined by the latitude and longitude, define the angle between the Moon-to-Earth line and the Moon-to-SOI line.
\[ \cos(\theta) = \cos(lat) \times \cos(lon) \]

3. Define variables for the sine and cosine of the Lunar orbit inclination (COSAINC and SINAINC).

   a. \[ RRE^2 = RRM^2 + RREM^2 - 2 \times RRM \times RREM \times \cos(\theta) \]
   b. \[ \mu E/RRE^2 = \mu M/RRM^2 \quad \therefore \quad RRE^2/RRM^2 = \mu E/\mu M \]
      (because \( \mu R^2 = \) gravitational acceleration, and by definition, at the sphere of influence, acceleration towards the Earth equals acceleration towards the Moon).
   c. \[ RRE^2/RRM^2 = \mu E/\mu M = 1 + RREM^2/RRM^2 - 2 \times RREM/RRM \times \cos(\theta) \]
      \[ = 1 + RMR^2 - 2 \times RMR \times \cos(\theta) \]
   d. \[ 0 = - (\mu E/\mu M) + 1 + RMR^2 - 2 \times RMR \times \cos(\theta) \]
      \[ = RMR^2 - 2 \times RMR \times \cos(\theta) + 1 - (\mu E/\mu M) \]
   e. For the quadratic formula:
      \[ a = 1, \quad b = -2 \times \cos(\theta), \quad c = 1 - (\mu E/\mu M) \]
      \[ RMR = (2 \times \cos(\theta) \pm 4 \cos^2(\theta) - 4(1 - \mu E/\mu M)) / 2 \]
      \[ RMR = \cos(\theta) + \cos^2(\theta) - 1 + \mu E/\mu M \]

5. Determine the distance from the Moon to the SOI.
   \[ RRM = RREM / RMR \]
B. Determine an Earth-based rectangular coordinate system such that the X-direction is on the line from the Earth to the Moon; the Y-direction is in the direction of the Moon's orbit; and the Z-direction is perpendicular to X and Y in right-hand coordinates. Project the SOI point onto the X-Y plane (Earth-Moon plane) in order to calculate the X- and Y-coordinates.

1. Determine the distance from the Moon to the X-intercept.
   \[ XXM = RRM \cdot \cos() = RRM \cdot \cosALAT \cdot \cosALON \]

2. Determine the distance from the Earth to the X-intercept.
   \[ XX = RREM - XXM \]

3. Determine the Y-coordinate. Note that longitude increases in the negative Y direction.
   \[ YY = -RRM \cdot \cosALAT \cdot \sinALON \]

4. Determine the Z-coordinate.
   \[ ZZ = RRM \cdot \sinALAT \]

5. Determine the distance from the Moon to the Y-intercept.
   \[ YYM = -YY \]

C. Calculate the distance from the Earth to the SOI point.
   \[ RRE = XX^2 + YY^2 + ZZ^2 \]

D. Identify the angle between the Earth-to-Moon line and the Earth-to-SOI line.

1. \[ \cos(ANGA) = XX / RRE \]

2. \[ \sin(ANGA) = YY^2 + ZZ^2 / RRE \]

3. \[ ANGA = \tan^{-1}(\sin(ANGA) / \cos(ANGA)) \]

E. Determine the Earth-based latitude and longitude of the SOI point.

1. \[ ALONX = \tan^{-1}(YY / XX) \]

2. \[ ALATX = \tan^{-1}(ZZ / XX^2 + YY^2) \]

F. Determine the Earth ascending node for the orbit in which the trans-SOI burn will occur.

1. Given the latitude of the burn at Earth (negative SOI latitude) and the inclination of the Earth orbit, spherical coordinate trigonometry states that the longitude from the node to the burn point is
   \[ \nu = \sin^{-1}(\tan(-ALATX) / \tan(AINCE)) \]

2. Longitude of the burn point (measured from the Earth-Moon line) is ALONX.
3. The node is calculated by subtracting the longitude in (a) above from the longitude in (b).
   \[ \text{ANODEE} = \text{ALONX} - \sin' \left( \tan(-\text{ALATX}) \right) / \tan(\text{AINCE}) \]

II. For the SOI-to-L1 trajectory, calculate Lunar node positions, and velocity vector at the current SOI point with respect to Earth.

A. Determine the node.
   \[ \text{ANODEEM} = \text{ALON} - \text{AVM} \]

B. Determine the components of the velocity vector at the SOI point.

1. Call the subroutine VELTRANS to determine the X-, Y-, and Z-components of the velocity vector, from the viewpoint of the moon.

2. Add the X- and Y-components of the Moon’s velocity to determine the SOI velocity from the viewpoint of the Earth.
   \[ \text{VXM} = \text{VXM} + \text{XDLO} \quad \text{VYM} = \text{VYM} + \text{YDLO} \]

III. For the Earth-to-SO1 trajectory, calculate velocity at the SOI point, \( \Delta V \) at Earth circular orbit, and time of flight.

A. Modify the velocity at SOI, if necessary, to bring the trajectory up to at least the minimum velocity required to intercept the SOI.

1. Calculate the vis-viva parameter \( \text{QQEMIN} \) for the Earth-to-SO1 trajectory.
   \[ \text{QQEMIN} = 2 - \left( \text{RRE} / a \right) \quad \text{where} \quad a = (\text{RPE} + \text{RRE}) / 2 \]
   \[ \text{QQEMIN} = 2 - ((2 * \text{RRE}) / (\text{RPE} + \text{RRE})) \]
   \[ \text{QQEMIN} = 2 * (1 - (\text{RRE} / (\text{RPE} + \text{RRE}))) \]
   \[ \text{QQEMIN} = 2 * (\text{RPE} / (\text{RPE} + \text{RRE})) \]

2. Calculate the minimum velocity required, such that apogee is just at SOI.
   \[ \text{QQEMIN} = (V^2 * \text{RRE}) / \text{CMUE} \]
   \[ \therefore \text{VELEMIN} = \left( \text{QQEMIN} * \text{CMUE} \right) / \text{RRE} \]

3. Increase the velocity slightly so that the calculations converge.

4. Compare the most recently calculated SOI velocity (\( \text{VELE} \)) with the minimum velocity. If it is too low, bring it up to the minimum.

B. Temporarily store (until step IV-D) the calculated Earth-to-SO1 time of flight.

C. Further increase the velocity at SOI, if necessary, to meet the Earth-to-SO1 time of flight requirement (but capping the \( \Delta V \) at 500 ft/sec). Calculate the new SOI flight path angle, \( \Delta V \) at Earth circular orbit, and time of flight.

1. Call the subroutine GAMACALC to determine the time of flight from Earth perigee to SOI (\( \text{TIMEE1} \)), given the velocity \( \text{VELE} \).
2. Increase the velocity by 10 ft/sec, and call GAMACALC again to determine the new time of flight (TIMEE2).

3. For the Earth-to-SO1 leg, determine the ratio of time-of-flight shortfall to the time-of-flight increase that was due to the 10 ft/sec increase in velocity.

\[ \text{Shortfall/Increase} = \frac{(FTIME-\text{TIMEE1}-\text{TIMEM})}{(\text{TIMEE2} - \text{TIMEE1})} \]

Apply this ratio to the increased velocity of 10 ft/sec to estimate the \( \Delta V \) necessary to meet the required time of flight.

\[ \text{DELVELE} = 10 \times \frac{(FTIME - \text{TIMEE1} - \text{TIMEM})}{(\text{TIMEE2} - \text{TIMEE1})} \]

4. Cap this \( \Delta V \) at 500 ft/sec, and add it to the original velocity.

5. Call the subroutine GAMACALC to determine the new Earth-to-SO1 time of flight for the revised velocity.

D. Compare the total Earth-to-Moon time of flight to the required time of flight. If the total is short, return to step III-C.

E. Determine the components of the velocity vector at the SOI point.

1. Calculate the flight path angle at SOI (GAMAE) for the Earth-to-SO1 trajectory. The cosine of the flight path angle will be between zero and one, corresponding to angles between zero and 90. In reality, Moon-to-SO1 flight path angles will range from zero to 90, but SOI-to-Moon angles will range from zero to -90. Gamma must be derived from \( \cos(GAM) \) and multiplied by \(-\text{MOOD}\) (from the input: +1 for outbound and -1 for return) to get the correct flight path angle. The arctan function is used instead of arccos to evaluate gamma because of that function's better debugging diagnostics capabilities.

\[ \text{GAMAE} = -\text{MOOD} \times \tan^{-1}\left( \frac{1 - \cos(GAM)}{\cos(GAM)} \right) \]

2. Determine the longitude of the SOI from the Earth's viewpoint.

\[ \text{ALONE} = \text{ALONX} + 180 \] (since zero longitude is on the Earth-Moon line on the side of the Earth away from the Moon).

3. Determine the latitude of the SOI point.

\[ \text{ALATE} = \tan^{-1}\left( \frac{ZZ}{XX^2 + YY^2} \right) \]

4. Calculate the azimuth of the SOI point

\[ \text{AZME} = \tan^{-1}\left( \frac{1}{\tan(AINCE)}, \cos(PHIE) \right) \]
where \( PHIE = \Pi - \sin^{-1}\left( \frac{ZZ}{RRE} \right) \sin(AINCE) \)

5. Call the subroutine VELTRANS to determine the X-, Y-, and Z-components of velocity at SOI for the Earth-to-SO1 trajectory.
IV. Combine the SOI-to-Moon calculations with the Earth-to-SOI calculations to identify total flight characteristics.

A. Calculate total ΔV

1. Calculate ΔV at the SOI point required to patch the Earth-to-SOI trajectory into the SOI-to-L1 trajectory. (This value is calculated by determining the difference between each of the components of the two trajectories).
   \[ \text{DELVEL} = (V_{XE} - V_{XM})^2 + (V_{YE} - V_{YM})^2 + (V_{ZE} - V_{ZM})^2 \]

2. Calculate ΔV at Earth perigee.
   \[ \text{DVCIRE} = V_{PE} - V_{CIRE} \]

3. Save the old total ΔV.
   \[ \text{DVTSAV} = \text{DVTOTAL} \]

4. Calculate the new total ΔV.
   \[ \text{DVTOTAL} = \text{DELVEL} + \text{DELVLPI} + \text{DVCIRE} \]

B. Determine the distance from the Moon to the Earth for this trajectory just calculated. Use this value in subsequent iterations of the in-program subroutine, if necessary.

1. Determine the time of arrival at SOI from Earth, in centuries since the year 2000 AD.
   \[ \text{TIM} = \text{Departure date} + \text{Time of flight} - 2000 \]
   where Departure date = TIMJ / 36525 days per century
   Time of flight = TIEM hrs / 876600 hrs per century
   The year 2000 = Day 2451545 / 36525 days per century.

2. Call the subroutine POSVELMO to determine the Moon's distance from the Earth and velocity at the time of arrival at SOI from the Earth.

3. Convert Lunar distance from Earth radii to feet.
   \[ \text{RREM} = \text{RREMER} * 20295741 \text{ ft/radii.} \]

C. Compare the value of Earth-to-SOI time of flight saved in step III-B above with the new value of time of flight calculated in step III-C above. If the difference has not yet converged (to less than 0.1 hour), reiterate this in-program subroutine beginning at step I-A. Otherwise, continue.

D. If the newly calculated total ΔV is the smallest ΔV calculated so far, then store it in the variable DVMIN. Also store all the significant orbital characteristics of the trajectory for later use in the bottom section of Report #1.

E. If the newly calculated SOI ΔV is the smallest calculated so far, then store it in the variable DVSIM. Also store its corresponding SOI longitude (ALONSIM), SOI latitude (ALATSIM), and SOI velocity for the SOI-to-Moon trajectory (VELMSIM).
Appendix F. Subroutine FLYBY

The subroutine FLYBY receives the parameters SOI longitude (ALON) and latitude (ALAT), Earth-Moon distance, "flyby" perigee radius (RPM), and flight mode (MOOD). FLYBY contains the same list of constants as in the main program, plus the radius of the SOI (R1) and the mean angular speed of the Moon (OMEGM). It calculates the magnitude of the velocity at the SOI and at L1 (VELM), the flight path angle at the SOI and at L1 (GAMAM), the inclination of the "flyby" trajectory (AIM), the circularization ΔV needed at L1 (DELVLPL1), the azimuth angle at the SOI (AZMM), the angle between the Lunar node and the SOI projection (AVM), and the flight time between SOI and L1 (TIEMM).

As mentioned briefly in the introduction, the FLYBY routine takes advantage of one simplification. By enlarging the SOI radius to include L1 from 11% of the Earth-Moon distance to 15%, the problem can be simplified considerably without significant error. Since the SOI penetration point is at the same distance from the Moon as L1, perigee passage will occur exactly half-way between the two points for any orbit containing them. Consequently, the true anomaly, flight path angle, and velocity will be the same at both the SOI and L1.

1. Initialize constants.
2. Determine location of L1 in Lunar relative coordinates. Remember, L1 rotates with the Moon, and therefore is affected by "flyby" time. X-axis is measured from Lunar center to Earth. Y-axis is measured in the opposite direction of the Moon's velocity.
   \[
   \begin{align*}
   X1 &= -R1 \cdot \cos(OMEGM \cdot TIEMM) \\
   Y1 &= -R1 \cdot \sin(OMEGM \cdot TIEMM) \cdot MOOD \\
   Z1 &= 0
   \end{align*}
   \]
3. Determine location of SOI point.
   \[
   \begin{align*}
   X2 &= R1 \cdot \cos(ALAT) \cdot \cos(ALON) \\
   Y2 &= R1 \cdot \cos(ALAT) \cdot \sin(ALON) \\
   Z2 &= R1 \cdot \sin(ALAT)
   \end{align*}
   \]
4. Calculate true anomaly.
   A. Dot product of position vectors for SOI point and L1.
   \[
   DOT = X1 \cdot X2 + Y1 \cdot Y2 + Z1 \cdot Z2
   \]
   B. From geometry:
   \[
   \cos(2\Theta) = DOT/R1/R1
   \]
   C. It follows:
   \[
   \Theta = \arccos(\cos(2\Theta))/2
   \]
5. Calculate eccentricity of "flyby" orbit using general conic equation.
   \[
   EEM = (RPM/R1 - 1)/(\cos(\Theta) - RPM/R1)
   \]
6. Calculate flight path angle.
   \[
   \begin{align*}
   \cos(\gamma) &= (1 + EEM \cdot \cos(\Theta))/(1 + 2 \cdot EEM \cdot \cos(\Theta) + EEM^2) \\
   \gamma &= \arccos(\cos(\gamma)) \cdot MOOD
   \end{align*}
   \]
7. Calculate semi-latus rectum.
   \[ PPM = RPM \times (1 + EEM) \]

8. Calculate perigee velocity.
   \[ VPM = (\mu_m \times \left(\frac{2}{RPM} + \frac{(EEM^2 - 1)}{PPM}\right)) \]

   \[ VELM = (\mu_m \times \left(\frac{2}{R1} + \frac{(EEM^2 - 1)}{PPM}\right)) \]

10. Calculate inclination by crossing position components (R2 X R1).
    \[ \begin{align*}
    X3 &= Y2 \times Z1 - Z2 \times Y1 \\
    Y3 &= Z2 \times X1 - X1 \times Y2 \\
    Z3 &= X2 \times Y1 - X1 \times Y2 \\
    MAG &= (X3 \times X3 + Y3 \times Y3 + Z3 \times Z3) \\
    IAM &= \arccos(Z3/MAG)
    \end{align*} \]

11. Calculate the vis-viva parameter.
    \[ QQM = R1 \times VELM \times VELM/\mu_m \]

12. Calculate eccentric anomaly.
    \[ \begin{align*}
    \cos AEX &= \frac{(EEM + \cos(\Theta))/(1 + EEM \times \cos(\Theta))}{1 + EEM} \\
    AEX &= \arccos(\cos AEX)
    \end{align*} \]

13. Calculate semi-major axis.
    \[ AAX = \frac{R1}{2 - QQM} \]

14. Store old "flyby" time of flight.
    \[ TIEMMS = TIEMM \]

15. Calculate the time of perigee passage.
    A. If the orbit is elliptical:
       \[ TIMX = \left(\frac{AAX^3}{\mu_m}\right) \times (AEX - EEM \times \sin(AEX)). \]
    B. If the orbit is hyperbolic:
       \[ TIMX = \left(-\frac{AAX^3}{\mu_m}\right) \times (EEM \times \sinh(EEM) - \log(\cosh(EEM) + \sinh(EEM))). \]

16. Calculate time of flight for "flyby".
    \[ TIEMM = TIMX \times 2 \]

17. Calculate longitude of nodes.
    \[ \text{ANODE} = \text{OMEGM} \times \text{TIEMM} \\
    \text{IF} \ ((\text{ALAT} \times \text{MOOD}) \geq 0) \text{ then ANODE} = \text{ANODE} + \pi \]

18. Calculate longitude angle between nodes and the SOI projection.
    \[ AVM = \text{ALON} - \text{ANODE} \]
19. Calculate angle between nodes and SOI projection.
\[
\text{COSP}HIM = \cos(\text{ALAT}) \times (\cos(\text{ALODE}) \times \cos(\text{ALON}) + \sin(\text{ANODE}) \times \\
\sin(\text{ALON}))
\]
\[
\text{APHIM} = \arccos(\text{COSP}HIM)
\]

20. Calculate azimuth angle for R2 (at SOI).
\[
\text{SINAZM} = \cos(\text{AIM})/\cos(\text{ALAT})
\]
\[
\text{COSAZM} = \sin(\text{AIM}) \times \cos(\text{APHIM})/\cos(\text{ALAT})
\]
\[
\text{AZMM} = \arctan2(\text{SINAZM}, \text{COSAZM})
\]

21. Calculate position for L1 (X-Y axes are fixed at Moon)
\[
\text{ALATL1} = 0
\]
\[
\text{ALONL1} = \arctan2(\text{Y1}, \text{X1})
\]

22. Calculate angle between nodes and L1.
\[
\text{COSP}HIL1 = \cos(\text{ALATL1}) \times (\cos(\text{ALODE}) \times \cos(\text{ALONL1}) + \sin(\text{ANODE}) \times \\
\sin(\text{ALONL1}))
\]
\[
\text{APHIL1} = \arccos(\text{COSP}HIL1)
\]

23. Calculate azimuth angle for R1 (at L1).
\[
\text{SINAZM} = \cos(\text{AIM})/\cos(\text{ALATL1})
\]
\[
\text{COSAZM} = \sin(\text{AIM}) \times \cos(\text{APHIL1})/\cos(\text{ALATL1})
\]
\[
\text{AZMLP1} = \arctan2(\text{SINAZM}, \text{COSAZM})
\]

24. If new "flyby" flight time is not very close to the saved value go back to step 2.

25. Set latitude/longitude position of L1 (X-Y axes are rotating)
\[
\text{ALATLP1} = 0^\circ
\]
\[
\text{ALONLP1} = 180^\circ
\]


27. Set Y-component of velocity for L1 (X- and Z- components are 0).
\[
\text{VYCOR} = -\text{OMEGM} \times \text{R1}/3600.
\]

28. Calculate the $\Delta V$ needed to correct the velocity vector at L1 to that of L1’s orbit (outbound) or the "flyby" to the SOI (return).
\[
\text{DELVLP1} = (\text{VXLP1}^2 + (\text{VYLP1} - \text{VYCOR})^2 + \text{VZLP1}^2)
\]

29. Sign of flight path angle must be changed for SOI calculations.
\[
\text{GAMAM} = -\text{GAMAM}
\]
Appendix G. Subroutine GAMACALC

The subroutine GAMACALC receives the parameters orbital perigee radius (RPX), orbital velocity at LP (VV), orbital radius at LP (RRX), and the gravitational parameter of the body being orbited (CMUX). It calculates and returns time of flight (TIMX), the cosine of the flight path angle (COSGAMX), velocity at periapses (VPX), circular velocity at periapses (VCIRX), true anomaly (THETAX), and an indicator describing whether the orbit is elliptical or hyperbolic (TRAJ$).

1. Initialize indicators to presume an elliptical orbit. (IHYPE, TRAJ$).
2. Calculate the vis-viva parameter
   \[ QQX = \frac{RRX \times VELX^2}{CMUX}. \]
3. If the orbit is just barely hyperbolic (QQX is within one-millionth of 2), force the calculation to consider it elliptical (reduce QQX to just under 2).
4. If the orbit is still hyperbolic, reset the indicators to show this. Print a message on the screen announcing a hyperbolic orbit.
5. Calculate the semi-major axis of the orbit
   \[ AAX = \frac{RRX}{2-QQX}. \]
6. If the semi-major axis is very large (greater than 10^{12}) or very small (less than -10^{12}), the orbit is trapped near a parabolic trajectory. Make it hyperbolic:
   \[ AAX = -10^{12}. \]
7. Calculate the orbit eccentricity
   \[ EEX = 1 - \frac{RPX}{AAX}. \]
8. Calculate the semi-latus rectum
   \[ PPX = AAX \times (1 - EEX). \]
9. Calculate the flight path angle
   a. The angular momentum of the orbit at perigee is
      \[ HP = RPX \times VELPERIGEE \times \cos(GAMAX). \]
      But at perigee, GAMAX is zero, so
      \[ HP = RPX \times VELPERIGEE. \]
   b. At LP, angular momentum is
      \[ HX = RRX \times VELX \times \cos(GAMAX). \]
   c. Angular momentum is constant along a given orbit, so
      \[ HP = HX \]
      \[ RPX \times VELPERIGEE = RRX \times VELX \times \cos(GAMAX) \]
      \[ GAMAX = \arccos((RPX \times VELPERIGEE) / (RRX \times VELX)) \]
d. The velocity at perigee is

\[ \text{VELPERIGEE} = \sqrt{\left(\frac{\text{CMUX} \times \text{RAPOGEE}}{\text{AAX} \times \text{RPX}}\right)} \]

Since \( \frac{\text{RAPOGEE}}{\text{AAX}} = (1 + \text{EEX}) \), then

\[ \text{VELPERIGEE} = \sqrt{\left(\frac{\text{CMUX} \times (1 + \text{EEX})}{\text{RPX}}\right)} \]

or

\[ \text{RPX} \times \text{VELPERIGEE} = \sqrt{\left(\text{RPX} \times (1 + \text{EEX}) \times \text{CMUX}\right)} \]

e. Substituting (d) into (c) above yields

\[ \text{GAMAX} = \arccos\left(\frac{\sqrt{\left(\text{RPX} \times (1 + \text{EEX}) \times \text{CMUX}\right)}}{\text{RRX} \times \text{VELX}}\right) \]

e. Substituting from (2) above yields

\[ \text{GAMAX} = \arccos\left(\frac{\sqrt{(\text{FU\text{?X} \times (1 + \text{EEX})/\text{RRX} \times \text{QQX}}))}}{\text{RRX} \times \text{QQX))}\right). \]

10. Calculate the perigee velocity

\[ \text{VPX} = \sqrt{\left(\text{CMUX} \times (1 - \text{EEX}) \times \text{RPX}\right)} \]

11. Calculate the circular velocity at perigee

\[ \text{VCIRX} = \sqrt{\left(\frac{\text{CMUX}}{\text{RPX}}\right)} \]

12. Calculate the true anomaly

\[ \text{THETAX} = \arccos\left(\frac{(\text{PPX} / \text{RRX}) - 1}{\text{EEX}}\right) \]

13. Calculate the eccentric anomaly

\[ \text{AEX} = \arccos\left(\frac{\text{EEX} + \cos(\text{THETAX})}{1 + \text{EEX} \times \cos(\text{THETAX})}\right) \]

14. Compare the orbital radius at LP (\( \text{RRX} \)) to the apogee radius (\( \text{AAX} \times (1 + \text{EEX}) \)). If the orbital radius at LP is greater than the apogee radius, print a message on the screen indicating the difference in nautical miles. Also display the following:

a. LP latitude (\( \text{ALAT} \))

b. LP longitude (\( \text{ALON} \))

c. vis-viva parameter (\( \text{QQX} \))

d. semi-major axis (\( \text{AAX} \))

e. eccentricity (\( \text{EEX} \))

f. flight path angle (\( \text{GAMAX} \))

g. perigee velocity (\( \text{VPX} \)).

15. Calculate the time of flight.

a. If the orbit is elliptical:

\[ \text{TIMX} = \sqrt{\left(\frac{\text{AAX}^3}{\text{CMUX}} * \text{AEX} - \text{EEX} \times \sin(\text{AEX})\right)} \]

b. If the orbit is hyperbolic:

\[ \text{TIMX} = \sqrt{\left(-\text{AAX}^3/\text{CMUX})\times(\text{EEX}\times\sinh(\text{EEX}) - \log(cosh(\text{EEX}) + \sinh(\text{EEX}))\right)} \]
Appendix H. Subroutine POSVELMO

The subroutine POSVELMO receives the parameter TIM (number of Julian centuries from the year 2000) and returns the moon’s position and velocity at that time. Specifically, it returns the moon’s distance from the Earth’s center, in Earth radii (RRM); velocity in the x-direction (along the Earth-Moon line), in feet per second (XDLO); and velocity in the y-direction (direction of the moon’s orbit), in feet per second (YDLO).

1. Calculate a fraction of time that represents half a second.
   \[ \text{DELT} = \frac{0.5 \text{ seconds}}{\left(36525 \text{ days/century} \times 24 \text{ hrs/day} \times 3600 \text{ seconds/hr}\right)} = 1.58440\text{E-10 centuries.} \]

2. Call the subroutine MOON with the parameter (TIM minus DELTA) to determine the moon’s distance in Earth radii at half a second before TIM. This distance is RM1.

3. Call the subroutine MOON with the parameter (TIM plus DELTA) to determine the moon’s distance in Earth radii at half a second after TIM. This distance is RM2.

4. Calculate the velocity of the moon in the -x (radial) direction.
   \[ \text{XDLO} = \frac{\left(\text{RM}_2 \text{ Earth radii} - \text{RM}_1 \text{ Earth radii}\right)}{(1 \text{ sec})} \times 20,925,741 \text{ ft/radii.} \]

5. Calculate the average radius of the Lunar orbit during the one second centered on TIM.
   \[ \text{RRM} = \frac{(\text{RM}_1 + \text{RM}_2)}{2}. \]

6. Determine the radius of the Lunar orbit from the Earth-Moon barycenter.
   \[ \text{RRMB} = \text{RRM} - 0.7412789 \text{ Earth radii.} \]

7. Calculate the moon’s velocity in the direction of its orbit.
   a. Moon’s apogee (Apo) = 62.83308 Earth radii.
   b. Moon’s perigee (Per) = 55.68264 Earth radii.
   c. Eccentricity (e) = (Apo - Per) / (Apo + Per) = 0.06033.
   d. Semi-latus rectum (p) = Apo(1 - e^2) = 62.60439 Earth radii 
      \[ = 1,310,038,967 \text{ feet.} \]
   e. Earth’s gravitational parameter (mu) = 1.407646822E+16 ft^3/sec^2.
   f. Angular momentum (h) = \(\text{sqr(mu} \times p)\)
      \[= 4.29427E+12 \text{ ft}^2/\text{sec} \times (1 \text{ earth radii} / 20,925,672.57 \text{ ft}) \]
      \[= 205,215.4 \text{ ft*Earth radii/second.} \]
   g. Y-velocity (YDLO) = h / RRMB = 205,215.4/RRMB.
Appendix I. Subroutine MOON

The subroutine MOON receives the parameter T (number of Julian centuries from the year 2000) and returns the approximate location of the moon in geocentric coordinates at that time. Specifically, it returns the right ascension of the moon (RAM), declination of the moon (DECM), and distance to the moon in Earth radii (RM). The formulae are from The Astronomical Almanac of the Year 1984, page D46. All degrees are converted to radians with the conversion factor $C = \pi/180$.

1. Calculate the ecliptic coordinates of the moon.
   a. Moon’s ecliptic longitude
      \[
      \text{LAM} = 218^{\circ}.32 + 481267^{\circ}.833T \\
      + 6^{\circ}.29 \times \sin(134^{\circ}.9 + 477198^{\circ}.85T) \\
      - 1^{\circ}.27 \times \sin(259^{\circ}.2 - 413335^{\circ}.38T) \\
      + 0^{\circ}.66 \times \sin(235^{\circ}.7 + 890534^{\circ}.23T) \\
      + 0^{\circ}.21 \times \sin(269^{\circ}.9 + 954397^{\circ}.70T) \\
      - 0^{\circ}.19 \times \sin(357^{\circ}.5 + 35999^{\circ}.05T) \\
      - 0^{\circ}.11 \times \sin(186^{\circ}.6 + 966404^{\circ}.05T)
      \]
   b. Moon’s ecliptic latitude
      \[
      \text{BETA} = 5^{\circ}.13 \times \sin(93^{\circ}.3 + 483202^{\circ}.03T) \\
      + 0^{\circ}.28 \times \sin(228^{\circ}.2 + 960400^{\circ}.87T) \\
      - 0^{\circ}.28 \times \sin(318^{\circ}.3 + 6003^{\circ}.18T) \\
      - 0^{\circ}.17 \times \sin(217^{\circ}.6 + 407332^{\circ}.20T)
      \]
   c. Horizontal parallax
      \[
      \text{PIE} = 0^{\circ}.9508 \\
      + 0^{\circ}.0518 \times \cos(134^{\circ}.9 + 477198^{\circ}.85T) \\
      + 0^{\circ}.0095 \times \cos(259^{\circ}.2 - 413335^{\circ}.38T) \\
      + 0^{\circ}.0078 \times \cos(235^{\circ}.7 + 890534^{\circ}.23T) \\
      + 0^{\circ}.0028 \times \cos(269^{\circ}.9 + 954397^{\circ}.70T)
      \]
   d. Semi-diame of moon’s orbit
      \[
      \text{SD} = 0.2725 \times \text{PIE}
      \]
   e. Distance to the moon in Earth radii
      \[
      \text{RM} = 1 / \sin(\text{PIE})
      \]

2. Form the geocentric direction cosines to rotate into geocentric coordinates.
   a. $l = \cos(\text{BETA}) \cos(\text{LAM})$
   b. $m = 0.9175 * \cos(\text{BETA}) \sin(\text{LAM}) - 0.3978 * \sin(\text{BETA})$
   c. $n = 0.3978 * \cos(\text{BETA}) \sin(\text{LAM}) + 0.9175 * \sin(\text{BETA})$
      where $l = \cos(\text{DECM}) \cos(\text{RAM})$, $m = \cos(\text{DECM}) \sin(\text{RAM})$, $n = \sin(\text{DECM})$. 
3. Then:

a. \( \text{RAM} = \arctan(m/l) \) [right ascension]

b. \( \text{DECM} = \arcsin(n) \) [declination]

The errors will rarely exceed 0.2 Earth radii in distance (RM), 0.3° in right ascension (RAM), and 0.2° in declination.
Appendix J. Subroutine VELTRANS

The subroutine VELTRANS converts an orbital velocity vector into rectangular coordinates (see figure K-5). Parameters received by the subroutine are velocity (VEL), flight path angle (GAMA), azimuth (AZM), latitude above the Earth-Moon plane (ALAT), and longitude from the Earth-Moon line (ALON). A set of intermediate calculations is performed to express the velocity vector in terms of a radial component, a latitudinal component, and a longitudinal component. Each of these three components is further resolved into x-, y-, and z-components. Finally, all three x-components, all three y-components, and all three z-components are summed to provide the total x-, y-, and z-components of velocity.

1. Conversion of velocity vector into spherical coordinates.

From the geometry, the radial component of velocity, R, is calculated to be

\[ \text{VEL} \times \sin(\text{GAMA}). \]  
(See figure K-6).

The component along the orbital path, \( R_\text{ orbital} \), is

\[ \text{VEL} \times \cos(\text{GAMA}). \]

This orbital path component of velocity is further resolved into a latitude component, LAT, and a longitude component, LON (see figure K-7). Again, from the geometry,

\[ \text{LON} = R \times \sin(\text{AZM}) \]  
and
\[ \text{LAT} = R \times \cos(\text{AZM}). \]

2. Conversion of spherical coordinates into rectangular coordinates.

a. Conversion of radial component into rectangular coordinates.

Refer to figure K-8. The projection of R onto the x-y plane is

\[ R \times \cos(\text{LAT}). \]

The negative x-component of this is

\[ R \times \cos(\text{LAT}) \times \cos(\text{LON}) \]

so the x-component, \( X \), is

\[ -R \times \cos(\text{LAT}) \times \cos(\text{LON}). \]

The negative y-component of R is

\[ R \times \cos(\text{LAT}) \times \sin(\text{LON}) \]

so the y-component, \( Y \), is

\[ -R \times \cos(\text{LAT}) \times \sin(\text{LON}). \]

The z-component, \( Z \), is

\[ R \times \sin(\text{LAT}). \]

b. Conversion of latitude component into rectangular coordinates.

Refer to figures K-9 and K-10. LAT is perpendicular to the radial vector, R. A line in the z-direction that meets the tip of LAT and intersects the radius vector produces the angles \( a \) and \( b \), where

\[ b = 90 - \text{LAT} \]

\[ a + b + 90 = 180. \]  
Therefore,
\[ a = \text{LAT}. \]

From the geometry, the z-component of LAT, ZLAT, is

\[ \text{LAT} \times \cos(\text{LAT}). \]
The projection of LAT onto the x-y plane is 
LAT * sin(LAT). (see figures K-11).
The x-component of this, XLAT, is 
LAT * sin(LAT) * cos(LON).
The y-component of this, YLAT, is 
LAT * sin(LAT) * sin(LON).

c. Conversion of longitude component into rectangular coordinates.

Refer to figures K-12 and K-13. LON is always parallel to the x-y plane, so the 
z-component of LON, ZLON, is always zero. Using the same proof as in (b) 
above, it can be seen that the angle between LON and the y-component of LON is 
equal to LON. From the geometry, the x-component of LON, XLON, is 
LON * sin(LON).
The negative y-component of LON is 
-LON * cos(LON),
so the y-component, YLON, is 
-LON * cos(LON).

3. Sum of the rectangular coordinates.

All of the x-, y-, and z-components are summed to provide the complete rectangular 
coordinates of the velocity vector. 
VXX = X + XLAT + XLON 
VYX = Y + YLAT + YLON 
VZX = Z + ZLAT + ZLON.
The projection of LAT onto the x-y plane is LAT * sin(LAT). (see figures K-11).

The x-component of this, XLAT, is LAT * sin(LAT) * cos(LON).

The y-component of this, YLAT, is LAT * sin(LAT) * sin(LON).

c. Conversion of longitude component into rectangular coordinates.

Refer to figures K-12 and K-13. LON is always parallel to the x-y plane, so the z-component of LON, ZLON, is always zero. Using the same proof as in (b) above, it can be seen that the angle between LON and the y-component of LON is equal to LON. From the geometry, the x-component of LON, XLON, is LON * sin(LON).

The negative y-component of LON is LON * cos(LON),

so the y-component, YLON, is -LON * cos(LON).

3. Sum of the rectangular coordinates.

All of the x-, y-, and z-components are summed to provide the complete rectangular coordinates of the velocity vector.

\[ VXX = X + XLAT + XLON \]
\[ VYX = Y + YLAT + YLON \]
\[ VZX = Z + ZLAT + ZLON. \]
Appendix K. Figures
Figure K-10