Common Source/Multiple Load vs. Separate Source/Individual Load Photovoltaic System

Joseph Appelbaum
Lewis Research Center
Cleveland, Ohio

Prepared for the
4th International Photovoltaic Science and Engineering Conference
cosponsored by The Institute of Radio and Electronics Engineers Australia
and The University of New South Wales
Sydney, Australia, February 14–17, 1989
TO: Distribution
FROM: 1900/Report Control
       Lewis Research Center
SUBJECT: TM101465

Attached find corrected copies of NASA TM101465, Appelbaum. Please DESTROY copies received earlier.

Report Control
COMMON SOURCE/MULTIPLE LOAD VS SEPARATE SOURCE/INDIVIDUAL
LOAD PHOTOVOLTAIC SYSTEM

Joseph Appelbaum*
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135 U.S.A.

Summary

A comparison of system performance is made for two possible system setups:
(1) individual loads powered by separate solar cell sources, (2) multiple
loads powered by a common solar cell source. A proof for resistive loads is
given that shows the advantage of a common source over a separate source photo-
voltaic system for a large range of loads. For identical loads, both systems
perform the same.

1. INTRODUCTION

In designing a multiple load photovoltaic (PV) system, the designer may wish
to compare the performance of two possible setups: a common PV array powering
all loads, or separate PV arrays powering individual loads [1]. A criterion for
comparing the load performance may be the "energy utilization" defined by

\[ \eta^e = \frac{\int T P(T) dT}{\int T P_m(T) dT} \]  

where the numerator is the input energy to the loads by the PV array, and the
denominator is the maximum available energy that the PV array can supply; \( P \)
is the PV array output power, and \( P_m \) is its maximum output power; both are
functions of the solar insolation, \( T \) is time.

In a common source system there is an interaction between the loads such that
it is possible that the operation of one load may be improved at the expense of
another load; or the total performance of all the loads powered by a common
source may also be improved. In a common source system, there exists an addi-
tional option where in certain cases, such as at low insolation, it might be
advantageous to disconnect a load from the system in order to improve the oper-
ation of another load. A proof is given in the next section, for resistive
loads, that shows the advantage of a common source system over a wide range of
load resistance.

2. COMPARISON OF SYSTEM PERFORMANCES WITH COMMON AND SEPARATE SOLAR CELL
SOURCES

The I-V characteristics of a solar cell array is shown in Fig. 1. Broadly
speaking, the I-V characteristics may be divided into two ranges; range I
where the solar cell behaves more likely as a current source, and range II
where the solar cell behaves more likely as a voltage source.

The approximate I-V characteristics equation of the solar cell array source,
neglecting the shunt resistance, is given by:

*This work was done while the author was a National Research Council - NASA
Research Associate; on sabbatical leave from Tel Aviv University.
\[ I = I_{ph} - I_0 \left\{ \exp \left[ \Lambda (V + IR_s) \right] - 1 \right\} \]  

(2)

where \( I_{ph} \) is the photocurrent, \( I_0 \) is the reverse saturation current, \( R_s \) is the series resistance, \( \Lambda = q/\text{AKT} \), and \( V = IR \) for resistive loads. The source terminal current and voltage are \( I \) and \( V \), respectively.

The I-V equation of \( N_p \) such sources connected in parallel and \( N_s \) such sources connected in series, which form a larger array, is given by:

\[ I_c = N_p I_{ph} - N_p I_0 \left\{ \frac{\exp \left[ \Lambda V_c + I_c \frac{N_s}{N_p} R_s \right]}{N_s} - 1 \right\} \]  

(3)

where \( I_c \) and \( V_c \) are the larger array (or common) terminal current and voltage, respectively. A resistive load of value \( R_C \), which is connected to a common solar cell array is seen by an individual source in the common array with a resistance value of:

\[ R = R_C \frac{N_p}{N_s} \]  

(4)

For the case where \( N_p = N_s = 1 \), we have \( I_c = I \), \( V_c = V \), \( R_C = R \) and Eq. (2) applies.

Resistive loads \( R_1 \) and \( R_2 \) connected to identical separate solar cell array sources are shown in Fig. 2. The load currents are, respectively:

\[ I_1 = I_{ph} - I_0 \left\{ \exp \left[ \Lambda (R_1 + R_s) \right] - 1 \right\} \]  

(5)

\[ I_2 = I_{ph} - I_0 \left\{ \exp \left[ \Lambda (R_2 + R_s) \right] - 1 \right\} \]  

(6)

Now let these two loads be connected in parallel to a common solar cell array source (\( N_s = 1, N_p = 2 \)) formed by two identical solar cell sources, as shown in Fig. 3. The equivalent load resistance is \( R_C = R_1 R_2/(R_1 + R_2) \) and the common load current \( I_c \) is:

\[ I_c = 2I_{ph} - 2I_0 \left\{ \exp \left[ \Lambda I_c (R_c + R_s/2) \right] - 1 \right\} \]  

(7)

For a particular case of identical loads where \( R_1 = R_2 = R \), the load current of a separate source system is:

\[ I = I_{ph} - I_0 \left\{ \exp \left[ \Lambda (R + R_s) \right] - 1 \right\} \]  

(8)

and the load power is:

\[ P = I^2 R \]  

(9)
The total power of two such systems is:

\[ P_s = 2P = 2I^2R \]  

(10)

where "s" stands for separate.

Now, for the common source system we have:

\[ I_c = 2I = 2I_{ph} - 2I_o \left\{ \exp \left[ A2I(R/2 + R_s/2) \right] - 1 \right\} \]

or

\[ I_c = 2 \left\{ I_{ph} - I_o \left\{ \exp \left[ AI(R + R_s) \right] - 1 \right\} \right\} \]  

(11)

and

\[ P_c = I_c^2 R_c = 2I^2R = 2P \]  

(12)

comparing with Eq. (10) we get:

\[ P_s = P_c \]  

(13)

The result shows that for identical resistive loads, the load powers are the same for both the separate and common source systems. (Following the same procedure one can show that this result is valid for any number of identical loads connected to a common source of appropriate size.)

The "energy utilization" is defined in eq. (1). For a given array size and insolation profile the \( \int P_m \, dT = \text{constant independently whether the cells are utilized or not.} \) The energy utilization of the two separate source system (Eq. (10)) is:

\[ n_s^e = \int P_s \, dT / 2 \int P_m \, dT = \int P \, dT / \int P_m \, dT \]  

(14)

and the energy utilization of the common source system (Eq. (12)) is:

\[ n_c^e = \int P_c \, dT / 2 \int P_m \, dT = \int P \, dT / \int P_m \, dT \]  

(15)

where \( P_m \) is the peak power of an individual solar cell array source.

The results show that identical resistive loads which are connected to separate solar cell sources perform the same as if they are connected in parallel to a common solar cell source of the appropriate size.

We shall now analyze the performance of nonidentical resistive loads, for example, \( R_m \) and \( R_1 \) in range I and \( R_m \) and \( R_2 \) in range II, where \( R_m \) is the load resistance corresponding to the maximum power point \( P_m \) of the solar cell array source, Fig. 1. We examine first an extreme case in range I.
(current source range) for \( R_1 = 0 \) and \( R_m \). For the separate source system we have \( P_1 = 0 \), \( P_m = I_m^2 R_m \) and \( P_S = P_1 + P_m = P_m \), therefore, the energy utilization of both separate systems is:

\[
\eta_S^e = \frac{\int P_S \, dT}{2 \int P_m \, dT} = 50 \text{ percent} \tag{16}
\]

For the common source system, \( P_c = 0 \) since \( R_1 \) short circuits \( R_m \), i.e., \( \eta_C^e = 0 \), therefore \( \eta_S^e > \eta_C^e \).

Another extreme case is in range II (voltage source range) for \( R_2 = \infty \) and \( R_m \). For the separate source system we have \( P_m = I_m^2 R_m \) and \( P_2 = 0 \), therefore:

\[
\eta_S^e = \frac{\int P_S \, dT}{2 \int P_m \, dT} = 50 \text{ percent} \tag{17}
\]

and for the common source system, \( P_2 = 0 \) and \( R_m \) is now powered by a double size source, therefore \( P_c > P_m \) and,

\[
\eta_C^e = \frac{\int P_c \, dT}{2 \int P_m \, dT} > \eta_S^e \tag{18}
\]

It can be shown that not only for \( R_2 = \infty \) is the \( \eta_C^e > \eta_S^e \), but also for large \( R_2 \). We have mentioned three distinct cases:

1. \( R_1 = R_2 = R_m \) (identical loads whether optimal or nonoptimal), where both the separate and the common solar cell source systems have the same energy utilization;

2. \( R_1 = 0 \) and \( R_m \), where the separate source systems have a higher energy utilization than the common source system; and

3. For \( R_m \) and large \( R_2 \), where the common source system has a higher energy utilization than the separate source systems.

These three cases are shown in Fig. 4 by the cross marks. The solid line describes the energy utilization of the common source system \( \eta_C^e \), and the dashed line the separate source systems \( \eta_S^e \). Two possible trajectories (connecting the cross marks) of \( \eta_S^e \) are shown in Fig. 4 in range I. In one case, the energy utilization of separate source systems is always higher than the common source system; in the second case, up to point "a" the separate source systems have a higher energy utilization, but between points "a" and "b" the common source system has a higher energy utilization. In range II, the common source system is always superior (\( \eta_C^e > \eta_S^e \)).
An actual comparison of performances of a common source/multiple load versus separate source/individual load photovoltaic systems is shown in Fig. 5. The individual solar cell array is 1400 Wp, 176 V open-circuit voltage and 13.613 A short-circuit current. The optimal resistive load for this array is 13.5 $\Omega$ calculated by Eq. (1). For both types of systems, the optimal load was kept constant and the second load was varied between 0 and 30 $\Omega$. The energy utilizations $\eta_C$ and $\eta_S$ were calculated (Eq. (1)), and plotted in Fig. 5. The figure clearly shows that for a wide range of load resistances (greater than 4.85 $\Omega$, and including the vicinity of the optimal load for a good system design) the common source system is superior.

3. CONCLUSIONS

In designing a multiple load photovoltaic system, the designer may wish to compare the performances of separate and common solar cell array source systems. A mathematical proof is given in this paper that shows the advantage of a common source system for a wide range of load resistances. Other load types than resistive loads may exhibit similar behavior. Identical loads, either optimal or nonoptimal, of any type perform the same when powered either by separate sources or by a common source.

4. REFERENCE

FIGURE 3. - COMMON SOLAR CELL SOURCE SYSTEM.

FIGURE 4. - ENERGY UTILIZATION OF SEPARATE AND COMMON SOURCE SYSTEM IN THREE EXTREME CASES $R_1 = R_2 = R_m^2$, $R_1 = 0$ AND $R_m^2$, $R_m$ AND LARGE $R_2$.

FIGURE 5. - ENERGY UTILIZATION OF SEPARATE AND COMMON SOURCE SYSTEMS FOR RESISTIVE LOADS.
A comparison of system performance is made for two possible system setups: (1) individual loads powered by separate solar cell sources, (2) multiple loads powered by a common solar cell source. A proof for resistive loads is given that shows the advantage of a common source over a separate source photovoltaic system for a large range of loads. For identical loads, both systems perform the same.