Wear Consideration in Gear Design for Space Applications

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Prepared for the
Fifth International Power Transmission and Gearing Conference
sponsored by the American Society of Mechanical Engineers
WEAR CONSIDERATION IN GEAR DESIGN FOR SPACE APPLICATIONS

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ABSTRACT

A procedure is described that was developed for evaluating the wear in a set of gears in mesh under high load and low rotational speed. The method can be used for any low-speed gear application, with nearly negligible oil film thickness, and is especially useful in space stepping mechanism applications where determination of pointing error due to wear is important, such as in long life sensor antenna drives.

A method is developed for total wear depth at the ends of the line of action using a very simple formula with the slide to roll ratio Vs/Vr. A method is also developed that uses the wear results to calculate the transmission error also known as pointing error of a gear mesh.

INTRODUCTION

Gears used in space applications are usually thought of as "instrument gears" with negligible load that must maintain position accuracy throughout their operational life or as deployment/actuator gears that may have inertial loads but a very low number of duty cycles.

More and more frequently the combined instrument/power drives are emerging on the space mechanisms scheme such as drives for weather satellites where pointing accuracy (as in instruments) as well as substantial inertial load are imposed due to momentum compensated antennas. This type of drive may also have a continuous duty cycle for up to 10 years. Such drives cannot be corrected by use of anti-backlash devices because this contributes to more wear as a result of the additional anti-backlash device loading springs. Also "position accuracy" is not corrected by such devices that are subject to severe wear.

A gear arrangement must be devised that will minimize wear effects due to gear ratio and provide addendum modification that will minimize the tooth wear rate. This paper describes a method that has been developed for analyzing the total gear tooth wear and its effect on pointing accuracy. The results can also be used to analyze the resulting effect on gear dynamic loading.

APPROACH TO SOLUTION

Before describing a detailed analytical approach, it is necessary to understand the background assumptions and analysis conditions assumed. First of all there are three lubrication regimes (Peterson, 1980) in geared drives: (1) Boundary (all asperity contact - no film), (2) mixed, and (3) full film (no asperity contact). This paper will deal with wear in the boundary lubrication regime only. Wear in the full film lubrication regime is negligible.

Second it is presumed that people designing gears for space applications will have a gear design computer program from which parameters can be pulled for use in the analysis presented herein. This saves considerable time and allows use of the simplified method presented at the end of this paper.

The conventional approach to wear (Dudley, 1962 and Akin, 1973) is to consider that wear can consist of one or more of the following modes (1) low speed lubricated wear, (2) pitting initiated wear, (3) abrasive wear, and (4) scuffing or scoring that may eventually lead to seizure. Only lubricated low speed adhesive wear is considered in this paper. The sections to follow provide a quantitative analysis of low speed adhesive wear in gears. Although spur gears are used here as the example case, the method presented would be applicable to other types of gears.

DEVELOPMENT OF THE THEORY

The boundary lubrication regime is defined quantitatively as gears operating at a speed and lubricant viscosity such that the lambda ratio $\lambda$ is less than 0.4 ($0 < \lambda < 0.4$) where:

$$\lambda = \frac{\text{Lubricant EHD film thickness}}{\text{Composite Asperity RMS height}}$$

This allows the assumption that substantial asperity interaction exists as is the case.

Realizing that the substance of wear in gear action is due to sliding along the line of action, as shown in Fig. 1, the slide-to-roll ratio (also known as specific
sliding) and the compressive stress must be calculated. The necessary formulas may be found in Gay (1970) and other references, to determine the sliding distance per pass at any point on the tooth profile. The Archard wear equation (Rabinowicz, 1965) will be used to calculate depth wear (h) as follows

\[ h = \frac{K W_N X}{3pA_a} \]  

(2)

where
- \( K \) wear coefficient (dimensionless)
- \( W_N \) load normal to the tooth (along line of action)
- \( X \) total sliding distance (for total life)
- \( p \) flow pressure of substrate (hardness)
- \( A_a \) apparent area of contact (Hertzian area)

The wear coefficient (K) is determined experimentally but usually falls within the range of 3x10^-8 to 6.7x10^-5 for lubricated gears and as high as 3.7x10^-3 for clean unlubricated surfaces (Peterson, 1980) in an earth environment.

The normal load \( W_N \) is usually calculated from the applied torque (T) as follows

\[ W_N = \frac{T}{R_b} = \frac{T}{R \cos \phi} \]  

(3)

where
- \( R_b \) is the involute base circle radius
- \( R \) pitch radius
- \( \phi \) pressure angle (transverse)

Determination of the total sliding distance (x) past a specific point of contact is the most important calculation to be made. It can be determined as follows

\[ x = V_{ss} b_c Y \]  

(4)

where
- \( V_{ss} \) slide-to-roll ratio = \( V_s/V_r \) determined at the end of action where it is a maximum
- \( V_s \) \( V_{r1} - V_{r2} \) = sliding velocity
- \( V_r \) rolling velocity
- \( \rho \) \( R \sin \phi_i \) = radius of curvature at point i
- \( \omega \) rotational velocity, rad/sec
- \( \phi_i \) pressure angle at instantaneous point of contact i
- \( b \) width of Hertzian band of contact
- \( C_y \) total number of contact cycles (life)

The slide-to-roll ratio \( V_{ss} \) may be determined from equations in Khiralla (1978) or from a Gear Design Computer program.

The Hertzian band of contact (b) may be determined by the method of Dudley (1954 or 1984):

\[ b = \left[ \frac{16W_N P}{(F \epsilon') \sqrt{1 - \nu^2}} \right]^{1/2} \]  

(5)

where
- \( P \) = \( P_p + P_g \) = relative radius of curvature
- \( F \) = face width gear
- \( E' = E_1E_2 \) and when \( E_1 = E_2 \): \( E' = E/2 \)
- \( \nu \) = Poisson's ratio

The contact compressive stress can be used to calculate \( b \) as follows:

\[ b = \frac{3.63 S_c \rho}{E} \]  

(for steel gears only)  

(6)

where
- \( S_c = \left[ \frac{0.35 W_N E'}{F_p} \right]^{1/2} \) = compressive stress (AGMA, 1982)

The flow pressure or hardness can be approximated by:

\[ p = 1500 BH(psi) \]

where
- \( BH \) = Brinell hardness number

Then the apparent area of contact can be calculated from

\[ A_a = b F = \text{Hertzian contact area} \]  

(7)

where: \( F \) = gear face width. In combining Eqs. (2), (4), and (7), the wear formula can be presented as:

\[ h = \frac{K W_N V_{ss} b_c Y}{3pB} \]  

so that the b's can be cancelled resulting in the formula:

\[ h = \frac{K W N V_{ss} C_y}{3pB} \]  

(8)

providing a very simple formula with only one variable \( (V_{ss}) \) as a function of position on the line of action. In practice a gear computer program may be used to calculate the \( V_{ss} \) values at the extreme positions on the line of action as shown in Table I. The normal
wear on the pinion must be added to the wear on the
gear to obtain the total wear at a specific point on
the line of action.

Thus, if we observe in Eq. (8) that the grouped
parameters

\[ \frac{K W N}{3 P F} \]

form a constant for a given gear set and that
the total wear can be formulated as:

\[ \frac{h_{pl}}{h_{g1}} = \frac{K W N}{3 P F} \left( V_{ss1} C_{y1} + V_{ss2} C_{y2} \right) \]  

(8)

Further, the contact cycles for the pinion can be
calculated from:

\[ C_{y1} = m_{g} C_{y2} \]

where

\[ m_{g} \] gear ratio. Thus:

\[ \frac{h_{pl}}{h_{g1}} = \frac{K W N}{3 P F} \left( m_{g} V_{ss1} + V_{ss1} \right) \] at OD of gear, and

\[ \frac{h_{p2}}{h_{g2}} = \frac{K W N}{3 P F} \left( m_{g} V_{ss2} + V_{ss2} \right) \] at OD of pinion.

The task now is to balance the wear at the extreme
ends of the line of action (see Fig. 2) so that
\[ \frac{h_{p1}}{h_{g1}} = \frac{h_{p2}}{h_{g2}} \]. This can be accomplished by adjusting the
addendum of the pinion and gear on standard centers.

Thus \( \Delta a_{p} = -\Delta a_{g} \) is adjusted until:

\[ EV_{1} = \left( m_{g} V_{ss1} + V_{ss1} \right) = \left( m_{g} V_{ss2} + V_{ss2} \right) \] for the opposite ends of the line of action where

\[ \Delta a_{p} \] addendum modification on pinion = \( \Delta N_{p} / 2P_{d} \)

\[ \Delta a_{g} \] addendum modification on gear = \( \Delta N_{g} / 2P_{d} \)

\[ \Delta N_{p, g} \] virtual change in number of teeth

\[ P_{d} \] diametral pitch

Therefore, the only special data needed for inputs
would be \( \Delta N_{p} \) and \( \Delta N_{g} \) and the outside diameters \( d_{o} \)
and \( D_{o} \) calculated from

\[ d_{o} = \frac{N_{p} + 2 + \Delta N_{p}}{P_{d}} \]

and

\[ D_{o} = \frac{N_{g} + 2 - \Delta N_{g}}{P_{d}} \] both on standard centers.

Finally, as an end result the designer would want

\[ e_{1} = \frac{180 C_{y1}}{R_{b}} \] degrees, and

\[ e_{5} = \frac{180 C_{y5}}{R_{b}} \] degrees, so that by design

\[ e_{1} = e_{5} \]

If Eq. (10) is not true, the addendum modifications
\( \Delta a_{p} = -\Delta a_{g} \) are iterated until Eq. (10) is true within
an acceptable tolerance (say 1 percent).

A SPECIFIC EXAMPLE

Let

Diametral Pitch \( P_{d} = 48 \)

Number of teeth in pinion \( N_{p} = 24 \)

Pressure angle \( \phi_{g} = 20\degree \)

Number of teeth in gear \( N_{g} = 120 \)

Gear base radius \( R_{b} = 1.1746 \)

Gear ratio \( = 5 \)

Addendum modification \( \Delta a_{p} = -\Delta a_{g} = 0 \)

Outside diameter of pinion \( d_{o} = 0.54166 \)

Outside diameter of gear \( D_{o} = 2.54166 \)

The results of these inputs to a gear computer program
provided the output shown in Table 1. The underlined
output values \( V_{ss1} \) and \( V_{ss2} \) are substituted into
Eq. (10) as follows

\[ EV_{1} = \left( 5 \cdot 2.47011 + 0.7118 \right) \times \left( 5 \cdot 4.3864 \right) + \left( -0.7814 \right) = 13.0623 \times 2.9744 \]

\( a \) considerable mismatch results here. The pointing
errors due to wear that result here would be from
Eqs. (12) and (13):

\[ e_{1} = \frac{KW_{N} v C_{y1}}{3P_{d}} \] \( EV_{1} \)

\[ e_{5} = \frac{KW_{N} v C_{y5}}{3P_{d}} \] \( EV_{5} \]

or

\[ e_{1} = e_{5} \]

or

\[ e_{5} = 0.005671/1.1746 = 0.0048 \text{ rad, or } 0.28\degree \]
also the top land of the pinion would have been reduced from 0.0144 to 0.0109 and worse the pinion root (LPC) would have been reduced by wear 0.0236 in. (66 percent) of the tooth thickness. After several iterations the final design resulted. Let:

\[
N_p = 24, \quad N_g = 120, \quad \Delta N_p = 0.96, \quad \Delta N_g = -0.96
\]

Outside diameter of pinion \(d_0 = 0.561666\)

Outside diameter of gear \(D_0 = 2.521666\)

The computer output is shown in Table II. The underlined output values are substituted into Eq. (10) as follows

\[
EV_1 = (5)(-0.6725 + 0.4021) = 5(5.5332 + 1.1422)2 = LV_5 = 3.7646 \cong 3.8082 \text{ within } 1\%.
\]

A good match is provided here. The pointing errors that would result are:

\[
e_1 = 0.00718/1.1746 = 0.00611 \text{ rad, or } 0.350^\circ
\]

\[
e_5 = 0.00726/1.1746 = 0.0618 \text{ rad, or } 0.354^\circ
\]

This is a substantial reduction in pointing error \(e_1\) caused by wear from the original standard gear design.

It should be noted here that the above errors \(e_1\) and \(e_5\) are due to wear only and must be added to the original basic error due to backlash and tooth-to-tooth composite errors to get a total pointing error at the end of life after \(C_y_1\) and \(C_y_2\) cycles of the pinion and gear respectively.

The gear wear life can be the critical failure criteria. As an example, consider an instrument that is considered inoperable if its pointing error exceeds 0.2°. We calculate its life as follows. Let's say the desired life is 15 000 000 cycles \((C_y)\) over three years and we determine that allowable backlash is

\[
e_a = 0.2^\circ \times \pi/180 \times R_b
\]

\[
e_a = 0.00349 \times 1.1746 = 0.0041 \text{ in. at wear out.}
\]

Subtract 0.0018 in. for initial backlash plus tooth-to-tooth composite error (when new) to get 0.0023 in. allowable for wear. Using formula (9) we determine that calculated wear depth \(h\) over 3 years is 0.049 in. Thus the predicted wear life will be 0.0023/0.0049 \times 3 years = 1.4 years or 16.9 months. Since this is less than is desired, a design modification would be necessary.

It should also be noted that the choice of gear arrangement can also be very important in minimizing gear drive pointing error, but this is beyond the scope of this paper.

CLOSURE

The basic tools have been provided in this paper for the minimization of pointing error due to wear through modification of standard gear tooth geometry, also an example has been provided to clearly illustrate the use of the method. This method is easily adapted to other types of gears using the same principles. This is a first attempt at quantization of gear wear and especially its effect on gear life in terms of an allowable pointing inaccuracy.

REFERENCES

AGMA, 1982, "Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth," Standard 218.01, American Gear Manufacturers Association, Alexandria, VA.


TABLE I. - COMPUTER PRINTOUT FOR SLIDING AND HERTZ STRESS AT 5 POINTS ON THE TOOTH PROFILE

<table>
<thead>
<tr>
<th>Location on tooth profile (a)</th>
<th>Roll angle, $^\circ$, deg</th>
<th>Rolling velocity, in./s</th>
<th>Sliding velocity, in./s</th>
<th>Hertzian stress, $q$, lb/in.²</th>
<th>Specific sliding ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V_r1</td>
<td>V_r2</td>
<td>V_s1</td>
<td>V_s2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6.82</td>
<td>1.22</td>
<td>4.23</td>
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<td>3.01</td>
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<td>5.88</td>
<td>3.30</td>
<td>2.58</td>
<td>-2.58</td>
</tr>
</tbody>
</table>

*aCode for locations on tooth profile:
1 = Start of Active Profile (SAP)
2 = Lowest Point of Single Tooth Contact (LPSTC)
3 = Pitch point (P)
4 = Highest Point of Single Tooth Contact (HPSTC)
5 = End of Active Profile (EAP)

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<td>V_r2</td>
<td>V_s1</td>
<td>V_s2</td>
<td></td>
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<td>0.00</td>
</tr>
<tr>
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<td>3.46</td>
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<td>-1.61</td>
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<td>37.53</td>
<td>6.71</td>
<td>3.13</td>
<td>3.58</td>
<td>-3.58</td>
</tr>
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FIGURE 1. - SLIDE DISTANCE ON LINE OF ACTION.

FIGURE 2. - WEAR DEPTH ON LINE OF ACTION.
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