LINEAR RESPONSE THEORY FOR ANNEALING OF RADIATION DAMAGE IN SEMICONDUCTOR DEVICES

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SUMMARY REPORT

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INTRODUCTION

This work deals with a theoretical study of the radiation/annealing response of MOS ICs. Although many experiments have been performed in this field, no comprehensive theory dealing with radiation/annealing response has been proposed. Many attempts have been made to apply linear response theory in several works, but no theoretical foundation has been presented.

The linear response theory outlined here is capable of describing a broad area of radiation/annealing response phenomena in MOS ICs, in particular, both simultaneous irradiation and annealing, as well as short- and long-term annealing, including the case when annealing is nearing completion. For the first time, a simple procedure is devised to determine the response function from experimental radiation/annealing data. In addition, this procedure enables us to study the effects of variable temperature and dose rate, effects which are of interest in spaceflight.

In the past, the shift in threshold potential due to radiation/annealing has usually been assumed to depend on one variable: the time lapse between an impulse dose and the time of observation (Eq.(1)). While such a suggestion of uniformity in time is certainly true for a broad range of radiation/annealing phenomena, it may not hold for some ranges of the variables of interest (temperature, dose rate, etc.). We therefore propose here a response function which is dependent on two variables: the time of observation and the time of the impulse dose. This dependence on two variables allows us to extend our theory to the treatment of a variable dose rate.

Finally, the linear theory is generalized to the case in which the response is nonlinear with impulse dose, but is proportional to some impulse function of dose. A method to determine both the impulse and response functions is presented.
I. GENERAL THEORY

We begin with the assumption that the incremental response of a MOS device to radiation/annealing, that is, the shift of the threshold potential, \( \delta V \), is linear with respect to an impulse radiation dose, \( \delta D \), and with respect to some function which depends on the time elapsed between the time of irradiation, \( t \), and the time of observation, \( t' \), (Fig.1):

\[
\delta V(t, t') = R(t-t')\delta D(t'),
\]

Eq. (1) may be rewritten as

\[
\delta V(t, t') = R(t-t')D(t')\delta t',
\]

(1a)

where \( R(t-t') \) is the response function, \( t \) is the time of observation, \( t' \) is the time at which the radiation impulse occurs, \( \delta D \) is the radiation dose impulse and \( D = dD/dt \), \( D \) being the dose rate.

The response function, \( R \), in Eq. (1) is assumed to depend on the time lapse between irradiation and observation, in other words, \( R \) is a function of one variable. This is certainly valid for the case of constant dose rate. We will also assume that this is valid for the case of a slowly varying dose rate. We will refer to this as the "adiabatic" case. Since the case of varying dose rates has not been studied so far, we will defer for the time being an exact definition of what is meant by "slowly varying" with respect to dose rate. For a very rapidly varying dose rate Eq. (1) obviously does not hold true and should be expressed by:

\[
\delta V(t) = R(t, t')\delta D(t').
\]

(1b)

The case of a two-variable response function, \( R(t, t') \), will be treated later.

![Figure 1. Impulse dose.](Image)

To find an expression for the total shift of the threshold
potential, \( \delta V_{\text{tot}} \), we integrate over the total time of irradiation, \( t_r \):

\[
\delta V_{\text{tot}}(t) = \int_0^{t_r} dV(t, t').
\]

(1c)

Substituting Eq. (1) in differential form, we obtain

\[
\delta V_{\text{tot}}(t) = \int_0^{t_r} R(t-t')dD(t'),
\]

(2)

where \( t \) is not necessarily equal to \( t_r \). In particular, for \( t = t_r \), Eq. (2) yields

\[
\delta V_{\text{tot}}(t) = \int_0^{t} R(t-t')dD(t').
\]

(2a)

Derbenwick and Sander\(^1\) proposed an integral for the case of \( t = t_r \), which is only valid for the case of simultaneous irradiation and annealing. The validity of Eq. (2) extends not only to this case, but also the case in which \( t > t_r \), which represents pure annealing after the end of the irradiation. In addition, removing this limitation \( (t=t_r) \) allows us to gain insight into the response function, \( R(t) \), and to find out how it can be determined.

The Response Function

To gain insight into the meaning of the response function, \( R(t) \), we need to study the case in which \( t \gg t_r \). Since, in Eq. (2), \( t' \ll t_r \), then \( t' \ll t \) and \( R(t-t') \approx R(t) \). Eq. (2) then becomes

\[
\delta V_{\text{tot}}(t) \approx \int_0^{t_r} R(t)dD(t') = R(t)D_{\text{tot}},
\]

(3)

where the total integrated dose within the interval from \( t = 0 \) to \( t_r \) (Fig. 2) is

\[
D_{\text{tot}} = D(t_r) - D(0) = D(t_r).
\]

For the case of variable dose rate, \( D_{\text{tot}} \) is

\[
D_{\text{tot}} = \int_0^{t_r} D(t')dt'.
\]
If $D$ is constant, then $D_{\text{tot}} = D_t$.

![Graph showing total dose](image)

Fig. 2. Total dose.

Thus, Eq. (3) gives us the needed insight into the meaning of the response function; namely, that $R(t)$ is the shift of the threshold potential per unit dose when $t \gg t_r$. In other words, to determine $R(t)$ for any time, $t$, all we need is to measure the shift of the threshold potential, $\delta V_{\text{tot}}(t)$. Then,

$$R(t) = \frac{\delta V_{\text{tot}}(t)}{D_{\text{tot}}(t)},$$

provided that $t \gg t_r$.

Thus, the linear response of MOS devices is totally described by Eq. (2). Of course, we have to know an analytical or, at least, a numerical expression for the linear response function $R(t)$. The phenomenological linear response theory of Eq. (1) and (2) does not provide us with the response function; however Eqs. (2) and (4) can serve as a useful tool to plan appropriate experiments and to construct the response function using experimental data. To find the response function, we only need to measure the response to the impulse and substitute it into Eq. (4).

A valuable technique which we shall use below in detail is the treatment of the case of small time, $t$. In this case, the response function may be expanded in a power series of $t$ (see Eq. (7)). Substituting this into the general Eq. (2) and integrating, we obtain the response, $\delta V(t)$, as a power series of $t$. Then, comparing this expansion with experimental data, we can determine the expansion coefficients, i.e., the value of $R(t)$ and its derivatives (first, second, etc.) at $t = 0$.

This will be discussed later. We now need to comment on the definition of small time, $t$. We are dealing here with a kinetic process and it can be characterized by a characteristic time, $t_e$ (or more than one of such parameters). Thus, the condition,
"small time", actually means \( t \ll t_o \).

Plotting \( \delta V(t) \) as a function of \( \ln(t) \) produces a straight line for the long-term annealing, i.e., \( t \gg t_r \), (Fig.3).

![Graph showing \( \delta V_{tot} \) vs. \( \ln(t) \)]

**Fig.3. Shift of threshold potential as function of \( \ln(t) \).**

This means that, for \( t \gg t_r \), (Fig.4),

\[
R(t) = \frac{\delta V(t)}{D_{tot}(t_r)} = A \ln(t) - B, \tag{5a}
\]

where \( t \) is expressed in units appropriate to the experimental data and the constants \( A \) and \( B \) have to be determined from experimental data.

![Graph showing \( R(t) \) vs. \( \ln(t) \)]

**Fig.4. Response function as linear function of \( \ln(t) \).**

As we have already mentioned above, the kinetic process of relaxation should be characterized by some characteristic time, \( t_o \), which makes the argument of the logarithmic function dimensionless:

\[
R(t) = A \ln(t/t_o) - C \tag{5b}
\]

We have introduced a new constant, \( C \), which is related to \( B \) in the following way: By subtracting Eq.(5b) from Eq.(5a), we obtain
\[ C-B = \ln(t) - \ln(t/t_o) = \ln(t_o). \]  \hspace{1cm} (5c)

It is clear from Fig. 4 that the constant \( A \) can be determined easily, since it is the slope and the constant, \( B \), is the \( y \)-intercept. \( B \) is actually arbitrary, since it depends on the units chosen for time measurement. \( t_o \) may be a function of many variables (manufacturing process, temperature, dose rate, etc.) and is therefore difficult to estimate in advance, and the value of \( C \) depends on the value of \( t_o \). Thus, both \( A \) and \( t_o \) remain unknown. We will present below a way to determine the constants \( t_o, C \) and \( A \) from the experimental data.

A close look at Eq. (5b) reveals that it is valid only for large \( t \), i.e., \( t \gg t_o \). Indeed, as \( t \) approaches zero, \( \ln(t) \) approaches infinity, which is a physically meaningless situation. The simplest way to extend Eq. (5b) to small \( t \) is to substitute \( 1 + t/t_o \) for \( t/t_o \) in the argument of the logarithm function:

\[ R(t) = A \ln(1 + t/t_o) - C, \]  \hspace{1cm} (6)

while, for \( t \gg t_o \), we still have

\[ R(t) = A \ln(t/t_o) - C. \]  \hspace{1cm} (5b)

We have so far obtained Eq. (5b) as an expression for \( R(t) \) on the basis of examination of the experimental data. This was based on the condition of long-term annealing \( (t \gg t_e) \). On the other hand, this expression was understood to be valid only for \( t \gg t_e \).

It was then logical to generalize this equation by extending it to the region of small \( t \), i.e., \( t \) comparable to or less than \( t_e \), but still much greater than \( t_e \).

Eq. (6) will be directly compared to the experimental data, i.e., \( \delta V/D \) can be equated to \( R(t) \) (Eq. (6)), under two conditions: 1) the magnitude of \( t \) is comparable to that of \( t_o \) and 2) the data are plotted with respect to \( \ln(1+t) \), rather than to \( \ln(t) \).

Cutoff Factor

Although Eq. (5b) describes pure annealing for large \( t \) in the sense \( t_e < t \) and \( t \gg t_e \), it can be seen that, as \( t \) continues to increase, the logarithm function, and thus \( R(t) \), continues to increase without limit.

The unrealistic physical situation of a continuous increase in annealing without limit can be corrected by introducing a cutoff factor, \( f(t/t_m) \), such that

\[ f(t/t_m) = 1 \text{ for } t < t_m \text{ and } \]
\[ f(t/t_m) \text{ approaches } 0 \text{ for } t > t_m. \]

This new constant, \( t_m \), is defined to be at least an order of magnitude larger than \( t_o \).
There are many possible forms for \( f(t/t_m) \). We offer here the simplest cutoff function:

\[
1/[1+(t/t_m)^2].
\]  

(6a)

Then the modified response function (Fig. 4a) becomes:

\[
R(t) = [A \ln(t/t_o) - C]/[1+(t/t_m)^2].
\]  

(6b)

We require that \( t_m \gg t_o \) such that there is a region such that \( t \approx t_m \) in which Eq. (6b) takes the form of Eq. (6). Thus, for very large \( t \), we have Eq. (6b), which is physically acceptable and in which, for very large \( t \), the response, \( \delta V \approx R(t) \) approaches zero. This satisfies the condition that \( R(t) \) be a reversible process. We shall treat the case of an irreversible process elsewhere.

A way to determine a value for \( t_m \) is to equate the response in Eq. (6) to zero, i.e.,

\[
A \ln(t_m/t_o) - C = 0.
\]

From this we obtain

\[
t_m = t_o e^C/A.
\]  

(6c)

In the next section, we will determine \( C \) and \( A \) from experimental data and will find the values of both \( t_m \) and \( t_o \).

Simultaneous irradiation and annealing, \( t_r = t \).

Substituting Eq. (6) for the response function into Eq. (2a), we obtain the following expression for the shift of the threshold potential:

\[
\delta V(t) = Dt(A(1+t_o/t) \ln(1+t/t_o)-(A+C)).
\]  

(7)

To simplify the study of the time dependence of the response, it is convenient to substitute the shift per unit total dose for the shift in threshold potential:

\[
\delta V(t)/D_{tot} = A(1+t_o/t) \ln(1+t/t_o)-(A+C),
\]  

(8)

where \( D_{tot} = Dt \).
Both Eqs. (7) and (8) can be simplified in the case of large time, \( t \gg t_o \). For the shift per unit dose, we obtain:

\[
\delta V(t)/D_{tot} = A \ln(t/t_o) - (A+C). \tag{8a}
\]

This expression is very convenient to deal with. It is similar to Eq. (5b) for long-term annealing. We must remember, however, that Eq. (5a) gives the total shift in threshold potential, whereas Eq. (8a) gives the shift per unit total dose. For this shift \( t \gg t_o \), we obtain:

\[
\delta V(t) = \int t \ln(t/t_o) - (A+C) dt. \tag{7a}
\]

It is easy to find that \( \delta V(t) \) reaches a minimum at \( t = t_m \), which is the solution of the following equation:

\[
A \ln(t/t_o) - C = 0 \tag{9}
\]

and

\[
t_m = t_o e^{C/A}. \tag{10}
\]

It would be interesting to investigate whether it is possible to reach this minimum experimentally before the process becomes irreversible.

Pure annealing, \( t > t_r \).

We now substitute Eq. (6) into Eq. (2). It is obvious that

\[
\int_{t_r}^{t} R(t-t')dD(t') = \int_{0}^{t} R(t-t')dD(t') - \int_{0}^{t} R(t-t')dD(t').
\]

Making use of this, we obtain:

\[
\delta V(t) = F(t) - F(t-t_r), \tag{11}
\]

where \( F(t) \) is the right-hand side of Eq. (7).

II. CALCULATION OF THE RESPONSE FUNCTION FROM EXPERIMENTAL DATA.

As we have already mentioned above, the parameters \( A, C \) and \( t_o \), which totally define the response function in Eq. (6), should be determined from the experimental data. These parameters are expected to depend on the conditions of irradiation, e.g., the temperature and dose rate. They may also be affected by the process parameters during manufacture, particularly the oxide anneal temperature, but also possibly by dopant species and concentration and other parameters. In short, the quality of MOS devices is "reflected" in these constants.
It is not enough simply to determine and catalog these constants for various MOS devices, but rather to study experimentally the dependence of these parameters on both temperature and dose rate and compare the values obtained with the parameters of the manufacturing process. With this in mind, we come to the realization that there is a strong need for a fast, simple way to determine these constants.

We propose a method based on a study of the linear response for t<<t₀, when R(t) can be expanded in a power series of t:

\[ R(t) = R(0) + R'(0)t + 1/2R''(0)t^2 + 1/3R'''(0)t^3 + \ldots \] (12)

where R'(0) and R''(0) are, respectively, the first and second derivatives of R with respect to t. At the point t=0, R(0) is R(t=0).

In particular, for the ln(t) dependence,

\[ R(t) \approx -C + A(t/t₀) - 1/2A(t/t₀)^2, \] (13)

or, comparing Eq.(13) and Eq.(12),

\[ R(0) = -C, \quad R'(0) = A/t₀, \quad R''(0) = -A/t₀^2 \quad \text{and} \quad R'''(0) = 2A/t₀^3. \] (14)

We next substitute Eq.(12) into Eq.(2) or (2a), depending on whether we are dealing with the case of pure annealing after irradiation or simultaneous irradiation and annealing. After integration, this yields an expression for the shift in threshold potential per unit dose given in the following form:

\[ \delta V(t)/D_{eq} = a₀ + a₁t + a₂t^2 + a₃t^3 + \ldots \] (15)

The coefficients, a₀, a₁, a₂, etc. are determined from the experimental data. From these experimentally determined values, we will determine the constants A, C and t₀.

Experimental Procedure

We used data obtained from experiments performed at the GSFC Radiation Facility and compared these results to our theory. In these experiments, RCA 10⁶ rad-hard CMOS ICs were irradiated with ⁶⁰Co gamma rays with a dose rate equal to approximately 150 rads/min. An IC was irradiated for several hours, taking measurements of the threshold potential (evaluated at a drain current of 300 µA) for one n-channel and one p-channel transistor every ten minutes.

Simultaneous Irradiation and Annealing

Substituting Eq.(12) into Eq.(3), we obtain
\[ \delta V(t) = DR(0) \int_0^t dt' + DR'(0) \int_0^t (t-t') dt' + \]
\[ + \frac{1}{2} DR''(0) \int_0^t (t-t')^2 dt' + \frac{1}{3} DR'''(0) \int_0^t (t-t')^3 dt' + \ldots \]

Integrating, we obtain

\[ \delta V(t) = D_{\text{tot}}[R(0) + \frac{1}{2} R'(0)t - \frac{1}{6} R''(0)t^2] - \frac{1}{2} D_{\text{tot}}R'''(0)t^3, \quad (16) \]

where \( D_{\text{tot}} = D(t) = D_t \).

We obtain the following expression for the shift of threshold potential by substituting Eq. (14) into Eq. (16):

\[ \delta V(t) = D_{\text{tot}}[-C + \frac{1}{2} A(t/t) - \frac{1}{6} A(t/t)^2 - \frac{1}{12} A(t/t)^3], \quad (17) \]

where \( t \ll t_0 \).

It is more convenient to deal with the shift per unit dose in the form of Eq. (15), where the constants are

\[ a_0 = -C, \quad a_1 = A/2t_0, \quad a_2 = -A/6t_0^2, \quad (18) \]

as determined by comparison of Eq. (17) and (15).

For the general case, we have

\[ a_0 = R(0), \quad a_1 = \frac{1}{2} R'(0), \quad a_2 = \frac{-1}{6} R''(0). \quad (19) \]

One can expect a linear dependence of \( \delta V(t)/D_{\text{tot}} \) for small times when

\[ a_2t^2/a_1t = t/2t_0 < 1, \quad (20) \]

or

\[ t < t_0/3. \]

Fig. 5. Radiation-induced shift in threshold potential vs. time.
The first three points showed the linear dependence of $\delta V(t)/D_{tot}$ (30 min.) in both cases, but the fourth point deviated from that dependence (Fig. 5). From this, we could conclude at once that $t_o > 3t = 30$ min. In other words, that $t_o$ was between one and two hours. We used the linear part of the curves (the first 2-3 points) to find $a_0$ and $a_1$, and we then used the next part of the curve to find $a_2$. The constants $a_1$ and $a_2$ allowed us to calculate $t_o$ and $A$:

$$t_o = -1/3 \frac{a_1}{a_2}$$
$$A = -2/3 \frac{a_1^2}{a_2}.$$  

(21)

For the p-channel transistor (Fig. 6), we found $t_o$ to be approximately 110 min. and for the n-channel, $t_o \approx 70$ min.

For the p-channel, the theoretical curve,

$$\delta V(t)/D_{tot} = a_0 + a_1 t + a_2 t^2,$$

deviates from the experimental points only after 70 min., which is 0.64$t_o$, and for the n-channel, the deviation takes place after 45 min., which is also 0.64$t_o$.

![Graph](image-url)

Fig. 6. Shift per unit total dose for p-channel transistor.

For the n-channel (Fig. 7), we then plotted a more precise theoretical curve, adding one more term, $a_3 t^3$:

$$\delta V(t)/D_{tot} = a_0 + a_1 t + a_2 t^2 + a_3 t^3.$$  

The value of $a_3$ was found from Eq. (15) and (15a):

$$a_3 = 3/2 \frac{a_2^2}{a_1} = 0.00196 \approx 0.002.$$  

(22)

We plotted this improved curve to $t = 1$ hr, which gives $t/t_o = 0.86$. There is no point in continuing the curve further, since the condition $t/t_o = 1$ must hold and the expansion (Eq. (17) is
definitely not valid beyond this point. Our purpose here is to analyze the region $t/t_o \leq 1$.

\[ \delta V(t)/D_{tot} = \ln t = 0.7 \frac{mV}{R} \]

**Fig. 7.** Shift per unit total dose for n-channel transistor.

Pure Annealing ($t < t_o$)

The case of pure annealing can be treated in the same way. Instead of Eq. (3), we will use our general equation.

Substituting Eq. (12) in Eq. (2), we obtain

\[
\delta V = D_{tot}[R(0) + R'(0)(t - 1/2t_r) + \\
+ 1/2R''(0)(t^2 - tt_r - 1/3t_r^2) + \\
+ 1/3R''''(0)(t^3 - 3t^2t_r + tt_r^2 - 1/4t_r^3)] \tag{23}
\]

or

\[
\delta V(t)/D_{tot} = R(0) - 1/2R'(0)t_r - 1/6R''(0)t_r^2 - 1/12R''''(0)t_r^3 + \\
[R'(0) - 1/2R''(0)t_r + R''''(0)t_r^2]t + \\
(1/2RR'(0) - 1/2R''''(0)t_r^2) + \\
1/3R''''''(0)t_r^3. \tag{24}
\]

Since we are concerned with the case $t_r/t_o << 1$, Eq. (21) can be simplified:

\[
\delta V(t)/D_{tot} = R(0) + R'(0)t + 1/2R''(0)t^2 + 1/3R''''(0)t^3. \tag{25}
\]

In particular, for the $\ln(t)$ dependence, we substitute Eq. (12) into Eq. (22) and obtain

\[
\delta V(t)/D_{tot} = -C + A(t/t_o) - 1/2A(t/t_o)^2 + 2/3A(t/t_o)^3 \tag{21}
\]

Then, as in the case of simultaneous irradiation and annealing, we can, in principle, determine $a_0$, $a_1$, $a_2$ from experimental data. We can then calculate $t_o$ and $A$:

\[ t_o = -1/2 a_1/a_2 \]
\[ A = -\frac{1}{2} \frac{a_1^2}{a_2}, \]
\[ C = -a_0. \quad (28) \]

Temperature and Dose-Rate Dependence of Response Function.

It is worth noting that our lack of data on the dependence of \( R(t) \) on dose rate and temperature at this point prevents development of the microscopic (quantum) theory of annealing.

If we find that, in some cases, \( R(t) \) does not possess a \( \ln(t) \) dependence (Eq. (1)), our method still allows us to find the value of \( R(t) \) and its derivatives at \( t = 0 \) through Eq. (5). Again, by conducting experiments for different dose rates (or temperatures), one can obtain the dependence of \( R(0), R'(0) \) and \( R''(0) \) on these variables. Since \( R'(0) \) is proportional to \( 1/t_0 \) and \( R''(0) \) is proportional to \( 1/t_0^3 \), we can determine the dependence of \( t_0 \) on temperature or dose rate.

III. LONG-TERM ANNEALING AFTER SEVERAL DISCRETE DOSES.

A study of variable dose rate would gain for us more insight into the problem of annealing. Of course, it must be preceded by a detailed study of temperature and dose rate dependence of \( A, C \) and the characteristic time, \( t_0 \).

We shall treat the specific case of variable dose rate in a stepwise varying dose rate in detail elsewhere. Here we will treat the case of several impulse doses which have different dose rates (Fig. 6). By impulse dose, we mean doses whose duration, \( t ', \) is much less than the annealing time, i.e., \( t' \ll t \). We suggest that the response function has a \( \ln(t) \) dependence, but each dose is described by its own characteristic time, \( t_0 \). We shall assume that \( A \) and \( C \) are independent of dose rate.

Thus, the integral response is the summation of the responses for each dose rate:

\[ \delta V(t) = \sum_{i} R([t-t']/t_0) D_i dt' \quad (29) \]

For the long-term annealing in this case, we obtain

\[ \delta V(t) = \sum_{i} R(t/t_0) D_i, \quad (29a) \]

where \( D_i = D_i t' \) is the individual total dose for a dose rate \( D_i \) and \( t' \) is the duration of the \( i \)-th impulse.
Fig. 8. Three discrete doses with different dose rates.

For the \( \ln(t/t_0) \) dependence, we obtain

\[
\delta V(t) = \sum_{i} [-C + A \ln(t/t_0)] D_i.
\] (30)

It is easy to show that Eq. (30) takes the form of a single impulse with the total dose:

\[
D_{\text{tot}} = \sum D_i,
\] (31)

and with some effective characteristic time, \( t_0 \). To prove this statement and to find this effective characteristic time, it is sufficient to treat the case of two impulse doses. In this case, we have

\[
D_1 \ln(t/t_1) + D_2 \ln(t/t_2) = D_{\text{tot}} \ln(t/t_0).
\] (32)

Then Eq. (31) takes the form:

\[
(D_1/D_{\text{tot}}) \ln(t/t_1) + (D_2/D_{\text{tot}}) \ln(t/t_2) = \ln(t/t_0),
\]

from which

\[
t_0 = (t_1) \frac{D_1}{D_{\text{tot}}} = (t_2) \frac{D_2}{D_{\text{tot}}}.
\] (33)

Eq. (33) can obviously be generalized for any number of impulses:

\[
t_0 = \prod_{i} (t_0) \frac{D_i}{D_{\text{tot}}}. \tag{34}
\]

The principal conclusion to be drawn from Eq. (29) is that the contribution of a single "strong" \( k \)-th impulse, i.e., an impulse whose dose rate is much larger than that of any other \( i \)-th impulse, does not produce much change in \( t_0 \), provided that its contribution to the total dose is small \( (D_k << D_{\text{tot}}) \).
IV. NON-LINEAR EFFECT.

We have assumed that the response is linear with an impulse dose and that the response function depends only on the time lapse between irradiation and measurement. At this point, we will show how existing experimental data should be examined to verify these two assumptions.

We shall now examine nonlinear effects. The output, that is, the shift of the threshold potential, is no longer assumed to be a linear function of an impulse dose, but is proposed to be proportional to an unknown impulse function, $\delta F(D)$:

$$\delta V(t) = R(t-t')\delta F(D).$$  \hspace{1cm} (35)

The total shift is then given by

$$\delta V_{tot} = \int R(t-t')F'(D) dD,$$  \hspace{1cm} (36)

where $F'(D) = dF(D)/dD$. In the limit in which $t \gg t_r$, we obtain

$$V(t, D_{tot}) = R(t)F(D_{tot}).$$  \hspace{1cm} (37)

In contrast to the linear case, we now have not one, but two unknown functions, $R$ and $F$, but only one quantity which can be experimentally determined: $V$. These two functions, however, depend on two different variables, $t$ and $D_{tot}$. This allows us, by fixing one variable, to determine the function which depends on the other variable.

V. Two-Variable Response Function.

Next, we shall treat the case of a two-variable response function, $R(t, t')$. When $t \gg t_r$, we obtain

$$V_{tot} = R(t, 0)D_{tot}.$$  \hspace{1cm} (38)

Eq.(12) gives $R$ only as a function of the first variable. We have devised a method to determine $R$ as a function of the second variable. One needs to provide two impulse doses, $D^{(1)}$ and $D^{(2)}$, separated by an interval $t'$ (Fig.9). We then have the following expression for the shift of the threshold potential:

$$V^{(2)}(t) = R(t, 0)D^{(1)} + R(t, t')D^{(2)},$$  \hspace{1cm} (39)

provided $t \gg t'$.

Substituting the first term on the right-hand side of Eq.(13) by Eq.(12), we obtain

$$R(t, t') = V_{tot}^{(2)}(t) - V_{tot}^{(1)}(t)/D_{tot}^{(2)},$$  \hspace{1cm} (40)
where \( V(1) \) and \( V(2) \) are the shifts of the threshold potentials respectively for the first and second impulses.

Since \( t' \) is arbitrary, provided \( t > t' \), Eq. (40) determines \( R \) as a function of the second variable. Eq. (40), together with Eq. (38), completely determines \( R \) as a function of two variables.

![Diagram](image_url)

**Fig. 9.** Two discrete impulse doses.

**VI. CONCLUSION**

The phenomenological linear response theory developed here is capable of describing the entire range of the reversible kinetic process of irradiation/annealing. Both pure annealing and simultaneous irradiation/annealing have been treated in various time ranges. These include: small time intervals \( (t \ll t_0) \), short-term annealing \( (t \approx t_r) \) and long-term annealing \( (t > t_r, t_0) \), as well as the saturation region \( (\delta V \text{ approaching 0}) \).

A method was also developed to determine the characteristic parameters of the response function. An important advantage of this method is that it is useful to determine the temperature and dose-rate dependence of these parameters.

In dealing with reversible linear kinetic processes, one is naturally interested in the limits of reversibility. To study these limits and to examine the transition are between reversible and irreversible processes, we have presented the case of two variable response functions and have treated a response which is non-linear with respect to an impulse dose.

It was pointed out that in the case of simultaneous irradiation and annealing, especially at the limits of small time, \( t \ll t_0 \), and large time, \( t \gg t_0 \), the response per unit total dose, \( \delta V(t)/D_{\text{tot}} \), is more convenient to deal with than the shift itself.

Finally, we treated the case of several different impulse doses and showed that they act as a single impulse dose with a common...
effective characteristic time. In particular, it was shown that a given impulse dose, if it makes a negligible contribution to the total dose, does not change the effective characteristic time. Such a conclusion is valid, provided that the impulse dose does not exceed the limits of irreversibility for the whole process.

We must realize that our main goal is still to construct a microscopic theory. Only quantum theory can give us an insight into the complicated processes related to radiation damage and annealing and predict temperature and dose-rate dependence.

To build such a theory, however, will be a difficult task, despite the fact that solid-state quantum theory has reached quite a sophisticated state. The main reason is that here we are concerned with kinetic processes, whereas the present quantum theory studies mostly equilibrium or quasi-equilibrium processes. In addition, we are concerned mainly with processes far from equilibrium, and even with the transition to irreversible processes.

To make any progress toward a foundation of this microscopic theory, we need, as a necessary first step, to have a general phenomenological theory. The linear response theory outlined in this paper can serve as an important step toward a quantum theory of radiation damage and annealing.

In conclusion, we have laid the groundwork for analytically characterizing the radiation and annealing response of devices of both variable dose and dose rate, which will greatly aid in the simulation of space radiation effects.

VII. RECOMMENDATIONS

1. We regard as the first priority the study of the temperature dependence of the characteristic time, t*, and the parameters A and C.

2. Additional directions for experimental study are:
   a. Dose-rate dependence
   b. The saturation region, in which annealing approaches completion (δV approaches zero) and
   c. The irradiation conditions beyond which the kinetic processes become irreversible and the extent of these conditions.

3. On the basis of results of these investigations, the theoretical basis of a quantum theory of radiation damage and annealing could be established.
REFERENCES


