PRELIMINARY ORBIT DETERMINATION SYSTEM (PODS) FOR TRACKING AND DATA
RELAY SATELLITE SYSTEM (TDRSS)-TRACKED TARGET SPACECRAFT
USING THE HOMOTOPY CONTINUATION METHOD*

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ABSTRACT

The Preliminary Orbit Determination System (PODS) provides early orbit determination capability in the Trajectory Computation and Orbital Products System (TCOPS) for a Tracking and Data Relay Satellite System (TDRSS)-tracked spacecraft. PODS computes a set of orbit states from an a priori estimate and six tracking measurements, consisting of any combination of TDRSS range and Doppler tracking measurements. PODS uses the homotopy continuation method to solve a set of nonlinear equations, and it is particularly effective for the case when the a priori estimate is not well known. Since range and Doppler measurements produce multiple states in PODS, a screening technique selects the desired state.

PODS is executed in the TCOPS environment and can directly access all operational data sets. At the completion of the preliminary orbit determination, the PODS-generated state, along with additional tracking measurements, can be directly input to the differential correction (DC) process to generate an improved state.

To validate the computational and operational capabilities of PODS, tests were performed using simulated TDRSS tracking measurements for the Cosmic Background Explorer (COBE) satellite and using real TDRSS measurements for the Earth Radiation Budget Satellite (ERBS) and the Solar Mesosphere Explorer (SME) spacecraft. The effects of various measurement combinations, varying arc lengths, and levels of degradation of the a priori state vector on the PODS solutions were considered.

In this paper, it is demonstrated that a poorly known a priori estimate that does not converge in the DC process can be improved through PODS processing, resulting in a solution that is accepted by the DC process. An overview of the system, the test results, and an analysis of these results are presented.

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1. INTRODUCTION

The purpose of preliminary orbit determination methods is to derive an orbit state corresponding to an available set of measurements when, initially, the orbit state is not well known or not known at all. Characteristically, preliminary orbit determination methods use approximate physical models and measurements collected over a limited timespan, usually less than one revolution. These methods are a necessary part of orbit operations procedures. With the expansion at the National Aeronautics and Space Administration (NASA) of spacecraft tracking from the ground-based system [i.e., the Ground Spaceflight Tracking and Data Network (GSTDN)] to a satellite relay system [i.e., the Tracking and Data Relay Satellite System (TDRSS)], it is necessary to have a reliable preliminary orbit determination method available in the Goddard Space Flight Center (GSFC) Flight Dynamics Division (FDD) that functions with TDRSS tracking. This paper reports on the development and provides an evaluation of such a method, called the Preliminary Orbit Determination System (PODS).

The remainder of this section presents background information on preliminary orbit determination, gives requirements for PODS, and lists the topics covered in Sections 2 through 4 of the paper.

1.1 BACKGROUND

Earlier preliminary orbit determination methods in the FDD used angular antenna-pointing observations collected at the ground stations (Reference 1). The capability to process these tracking measurements is a feature of the FDD Goddard Trajectory Determination System (GTDS) Early Orbit Determination (EARLYORB) Program (Reference 2). TDRSS range and Doppler tracking measurements offer a primary source of tracking support for many spacecraft by the FDD. However, the open-loop TDRSS angular antenna-pointing measurements (beam angles azimuth and elevation) are too inaccurate for use even in preliminary orbit determination. Therefore, a preliminary orbit determination method that uses the precise TDRSS range and Doppler tracking exclusively is required. The
problem is basically one of solving a set of nonlinear equations, which specify that the predicted values of the measurements match the observed values. The homotopy continuation method of solving nonlinear systems of equations is particularly well suited for preliminary orbit determination using range and Doppler measurements, especially with inaccurate or even unavailable a priori estimates of the solution (Reference 3 and 4).

1.2 REQUIREMENTS

PODS satisfies the following requirements:

- PODS processes precise TDRSS range and Doppler measurements by one or more Tracking and Data Relay Satellites (TDRSs).
- PODS uses a preliminary orbit determination method with the ability to overcome an inaccurate (or no) a priori value for the target state to be solved for. PODS also resolves multiple solutions that result from TDRSS range and Doppler data. The final solution is accurate enough for subsequent tracking acquisition.
- PODS is operable under the current Flight Dynamics Facility (FDF) operational system, i.e., the Trajectory Computation and Orbital Products System (TCOPS). PODS is flexible in accessing the relay state(s), the a priori target state, and the tracking measurements from different available sources of these data.

1.3 PAPER ORGANIZATION

Section 2 of this paper discusses the theory of the homotopy continuation algorithm and its application to preliminary orbit determination. It also describes the operational use of PODS.

Section 3 of the paper discusses several evaluation studies that were performed to test PODS. These studies include the effects of choosing various a priori
target states, data arc lengths, and data types in obtaining different states. It is demonstrated that the final target state solution selected by PODS is good enough to be successfully used by the GTDS Differential Correction (DC) Program as an a priori target state vector.

Section 4 reviews the results from the evaluation studies, provides a conclusion summary, and lists future enhancements for PODS.

2. THEORY AND APPLICATION OF PRELIMINARY ORBIT DETERMINATION METHODS

The basic equation to be solved to obtain the target state vector, \( \hat{X} \), relates the measured value of the range or Doppler data, \( O_i \), to the modeled value, \( C_i \), as follows:

\[
O_i - C_i(\hat{X}) = 0 \quad (i = 1, 2, \ldots, 6)
\]  

(1)

There are six equations for the six unknown components of \( \hat{X} \), usually the spacecraft position and velocity in Cartesian coordinates at a specified epoch. For simplicity, the modeled values are determined from geometrical distances without atmospheric and measurement corrections.

A procedure for solving Equation (1) was developed using the homotopy continuation method; this procedure is described in Section 2.1. Section 2.1 also contains a discussion of the multiple solutions that arise from TDRSS symmetry in the range and Doppler measurements and presents a method for screening the candidate solutions. Section 2.2 outlines the operational use of PODS under TCOPS.
2.1 HOMOTOPY CONTINUATION METHOD

A general way to solve Equations (1) is the homotopy continuation method. In this method, a continuous mapping parameter, $\lambda$, is first introduced as follows:

$$O^\lambda_i \equiv O^0_i + \lambda (O^1_i - O^0_i) \quad (i = 1, 2, \ldots, 6) \tag{2}$$

where $O^0_i = \text{modeled measurement corresponding to the a priori estimate, } \vec{x}^0$

$O^1_i = \text{real measurement at the unknown solution state } \vec{x}^1$

The quantity $X$ must then be solved for from

$$O^\lambda_i - C_i(\vec{X}) = 0 \quad (i = 1, 2, \ldots, 6) \tag{3}$$

by following the solution curve in the seven-dimensional ($\lambda, \vec{X}$) space, starting at $\lambda = 0$, keeping track of each solution whenever $\lambda = 1$ along the curve.

As an aid in visualizing the solution curve, Figure 1 shows its projection onto the $\lambda$-$z$ plane, where $z$ is the third Cartesian component of position, for a typical orbit (Reference 3). The curve-following begins at the point marked initial state, where $\lambda = 0$, and then passes through four solutions along the line at $\lambda = 1$ before returning to the start. Since this is a smooth curve embedded in seven-dimensional space, the apparently sharp changes and intersections in the figure do not really exist but result from the projection onto the $\lambda$-$z$ plane.

The following is a brief summary of the procedure for following solution curves (Reference 3):

- Given the a priori state at $\lambda = 0$ as the first point, a bootstrap starter is used to develop the second point on the solution curve.
A preliminary value for the next step size change is selected.

The next curve point along the arc is predicted by fitting a polynomial to the previous N backpoints (predictor step) (see Figure 2).

The Newton-Raphson method is used to iteratively refine the predicted state to the corrected state along the hyperplane locally perpendicular to the extrapolating polynomial at the predicted state (corrector step).

The new point is discarded and the step size is corrected, or the new point is accepted and a check is made to see if any candidate solution states have been determined at $\lambda = 1$. 

Figure 1. Projection of Solution Curve Onto $\lambda$-$z$ Plane
Figure 2. Predictor-Corrector Technique for Following a Solution Curve

- If the solution curve has returned to its start, the procedure is terminated.

Usually there are multiple solutions at \( \lambda = 1 \), as seen, for example, in Figure 1. Solutions 1 and 3 (and solutions 2 and 4) are mirror images of each other in the TDRS orbit plane.\(^1\) These multiple solutions are due to the symmetry of range and Doppler data for TDRSS tracking (Reference 3). To determine which of the solutions is correct, a solution screening algorithm is required. Some solution candidates can be rejected because they are not physically correct, for example, when the semimajor axis, eccentricity, or inclination is not within the allowed limits for a particular target's orbit. Usually the

\(^1\)The TDRS orbit plane nearly coincides with the x-y plane. Thus, the mirror image solutions, evident in the projection of the solution curve onto the \( \lambda-z \) plane in Figure 1, are not apparent in similar projections onto the \( \lambda-x \) and \( \lambda-y \) planes.
candidate solutions are not near each other. TDRSS beam angles, which approximately locate the actual orbit, can then be used to reject most candidates, especially one or both of the mirror-image solution pairs.

When no solutions are accepted by the screening process and the candidate solutions are not paired by mirror images, a second solution loop exists that is the mirror image of the first; this solution loop may contain the desired solution. Consequently, each candidate solution, as well as its mirror image, is checked during solution screening.

The homotopy continuation method can be further generalized (Reference 3) to contain up to six continuation parameters ($\lambda_n$) and multiple disconnected loops. For an inaccurate value of the a priori target state that lies on one loop, this generalized method allows jumping from one loop to another at critical points in the search for candidate solutions at $\lambda_n = 1$.

2.2 OPERATIONAL USE OF PODS

To use PODS operationally for an event associated with a particular target, specific steps are followed prior to and immediately after the event. The preliminary steps include setting up sources for observations, relay states, and the a priori target state; generating a generic list of input parameter values; and allocating output files for summary reports and the target solution. Immediately after the event, the operator selects values for the solution epoch, the observations, the a priori target state, the relay state, and the input parameters. (See Reference 5 for detailed requirements specifications for these and a description of the operational steps.)

PODS is then executed operationally within the TCOPS User Interface (UI) environment. Figure 3 shows an overview of the system and its operational environment, including all required input and output interfaces. The foreground and background divisions in this figure indicate the modes of execution of the two separate parts of the system. (A more detailed description of the system is given in Reference 6.)
Figure 3. Operational PODS Under TCOPS
3. EVALUATION STUDIES

PODS was evaluated to determine its strengths and weaknesses in calculating target states under various conditions for different targets. The following four goals formed the basis of the PODS evaluation studies:

1. Determine whether solutions can be found for a priori states of various quality, such as the following:
   a. Nearly Exact—Very close to the actual state
   b. Good—Usually extracted from the TCOPS Vector Hold File
   c. Poor—Usually degraded by long two-body propagation of a good state
   d. Generic—Typical values for the semimajor axis, eccentricity, and inclination of the orbit

2. Ascertain the limitations based on data arc length. Determine whether there is a breakdown for shorter arcs. This question is of major concern, because typical TDRSS tracking for the Landsat-4 and Landsat-5 spacecraft consists of 9- to 24-minute passes, with passes separated by at least one revolution of 99 minutes, and for the Earth Radiation Budget Satellite (ERBS) consists of 9- to 14-minute passes, with passes separated by at least two revolutions of 96.7 minutes each. This tracking schedule required PODS to succeed for short arcs. TDRSS tracking of the Solar Mesosphere Explorer (SME) spacecraft consists of 10- to 65-minute passes for a 95-minute revolution.

3. Determine how successful various combinations of data are (e.g., all range, all Doppler, mixed range and Doppler).
4. Resolve two issues for the final target state calculated in PODS:

a. Demonstrate the effectiveness of PODS by showing that it can determine a solution for the target state from an a priori state for which the GTDS DC Program cannot obtain a solution state. Also show that this PODS solution acts as a successful a priori state in the DC Program.

b. Determine whether screening of candidate solutions is effective by using TDRSS beam angle and physical considerations.

Several PODS executions were made to establish the feasibility of these goals; these executions are summarized in Table 1. Three target spacecraft were analyzed: COBE (using simulated data for December 21, 1987); ERBS (using real

<table>
<thead>
<tr>
<th>TARGET SPACECRAFT</th>
<th>EPOCH</th>
<th>ARC LENGTH (minutes)</th>
<th>DATA TYPE</th>
<th>A PRIORI STATE QUALITY</th>
<th>SOLUTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>COBE</td>
<td>12/21/87, 0°</td>
<td>60</td>
<td>6R</td>
<td>GOOD</td>
<td>2 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>6R</td>
<td>GOOD</td>
<td>1 (NEAR D)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>3R (TDRS-E), 3R (TDRS-W)</td>
<td>GOOD</td>
<td>1 (NEAR D, BUT TOO ECCENTRIC)</td>
</tr>
<tr>
<td>ERBS</td>
<td>04/10/86, 0°</td>
<td>18</td>
<td>6R</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>6R, 2A, 2E</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>3R, 3D</td>
<td>POOR</td>
<td>4 (UNPHYSICAL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>6D</td>
<td>GOOD</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td>ERBS</td>
<td>11/30/87, 0°</td>
<td>11</td>
<td>6R</td>
<td>GENERIC</td>
<td>4 (UNPHYSICAL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>6D</td>
<td>GENERIC</td>
<td>4 (NEAR R)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>3R, 3D</td>
<td>GENERIC</td>
<td>4 (NEAR R)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>4R, 2D</td>
<td>GENERIC</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>2R, 4D</td>
<td>GENERIC</td>
<td>0</td>
</tr>
<tr>
<td>ERBS</td>
<td>12/16/87, 0°</td>
<td>12</td>
<td>6R, 2A, 2E</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>6D</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>4R, 2D, 2A, 2E</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td>SME</td>
<td>01/14/88, 0°</td>
<td>20-50</td>
<td>6D</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>6D</td>
<td>GOOD</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>6R</td>
<td>GOOD</td>
<td>4 (UNPHYSICAL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40-50</td>
<td>6R, 2A, 2E</td>
<td>GOOD</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>6R, 2A, 2E</td>
<td>GENERIC</td>
<td>4 (1 GOOD)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>2R, 4D</td>
<td>GOOD</td>
<td>4 (UNPHYSICAL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>3R, 3D</td>
<td>GOOD</td>
<td>4 (UNPHYSICAL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>4R, 2D</td>
<td>GOOD</td>
<td>4 (UNPHYSICAL)</td>
</tr>
</tbody>
</table>

NOTE: DATA TYPES: A = AZIMUTH; E = ELEVATION; D = DOPPLER; R = RANGE
*PROGRAM UNABLE TO COMPLETE CALCULATIONS
data for April 10, 1986; November 30, 1987; and December 16, 1987); and SME (using real data for January 14, 1988). Data arc lengths ranged from short (11 to 20 minutes) to long (30 to 50 minutes). Data types included range (R), Doppler (D), azimuth (A), and elevation (E), where azimuth and elevation are the TDRSS beam angles used to screen candidate solutions. Relay tracking was by TDRS-East (TDRS-E), except for simulated COBE data, where TDRS-West (TDRS-W) tracking is specifically noted.

The target a priori state quality (generic, poor, good, nearly exact) is also indicated in Table 1. The generic state usually consists of values for the semimajor axis (a), eccentricity (e), and inclination (i) typical of the target, as well as values of zero for the remaining classical or Keplerian elements [i.e., right ascension of the ascending node (Ω), argument of perigee (ω), and mean anomaly (M)].

A good a priori state vector can be extracted from a TCOPS Vector Hold File, where the vector was pregenerated from a GTDS DC solution. A nearly exact a priori state (although not included in the table) leads to a breakdown of the equations in the homotopy continuation method (Reference 3). A poor-quality a priori state can be established by a two-body propagation over a long period, such as 24 hours.

The last column in Table 1, SOLUTIONS, lists all unique solutions for each case in the table. Occasionally, the same solution is repeated while the solution loop is being followed, but this repetition is not indicated in the table. Typical features of unphysical (rejected) solutions noted in this column are unrealistic semimajor axis, eccentricity, inclination, apogee, or perigee.

The remainder of this section discusses the results for ERBS and SME from the perspective of the evaluation goals. The topics covered are as follows: a priori target state (Section 3.1), data arc lengths (Section 3.2), data type combinations (Section 3.3), and final target state (Section 3.4).
3.1 A PRIORI TARGET STATE

The possibility of generating solutions for various values of the a priori target state was studied. The principal example was a long SME data arc starting at 0 hours, 36 minutes, on January 14, 1988 (Figure 4). The good a priori target state vector, extracted from the TCOPS Vector Hold File, was previously generated by executing the GTDS DC Program using a good orbit propagator. Because the data arc was within 2 hours of the a priori state epoch and the final state epoch, there was no appreciable degradation from

Figure 4. SME Orbit as Seen From TDRS-E on January 14, 1988, From 0 to 2 Hours
using a two-body orbit propagator in PODS for the target. Values in Keplerian coordinates for both the good and generic a priori target states are given in Table 2, which summarizes the success status for PODS solutions. Both the good and generic states were successful and give identical solutions for the four Doppler data arcs given in the table. Only the good a priori target state succeeded for longer arcs of range data. Different generic values were tried for the semimajor axis, including a reduction from 8000 to 7500 kilometers, but were unsuccessful.

When an a priori target state has a value close to the solution state, the homotopy continuation algorithm breaks down (Reference 3). This effect was observed when, for a nearly exact a priori value, the correct final state was immediately determined, but the solution curve in the seven-dimensional \((\lambda, \lambda')\) space did not close within specified tolerances.

Table 2. State of PODS Solutions for Various SME a Priori Target States on January 14, 1988, at 0 Hours

| A PRIORI TARGET STATE | SUCCESS STATUS FOR PODS SOLUTIONS\(^a\) |   |   |   |   |   |   |
|-----------------------|----------------------------------------|---|---|---|---|---|
|                       | DOPPLER DATA ARC                      | RANGE DATA ARC | RANGE/DOPPLER DATA ARC |
|                       | (minutes)                | (minutes) | (minutes) |
| GOOD\(^b\)           | Y Y Y Y                                | N Y Y      | N         |
| GENERIC\(^c\)        | Y Y Y Y                                | N N N      | N         |

\(^a\)Y = SUCCESSFUL; N = NOT SUCCESSFUL

\(^b\)GOOD A PRIORI TARGET STATE:
- \(a = 6872\) kilometers; \(e = 0.00079\); \(i = 97.8^\circ\);
- \(\Omega = 20.1^\circ\); \(\omega = 301.7^\circ\); \(M = 157.6^\circ\)

\(^c\)GENERIC A PRIORI TARGET STATE:
- \(a = 8000\) kilometers; \(e = 0.01\); \(i = 100^\circ\);
- \(\Omega = 0^\circ\); \(\omega = 0^\circ\); \(M = 0^\circ\)
3.2 DATA ARC LENGTHS

The continuously tracked SME data arc of 62 minutes duration on January 14, 1988, was used to study the effects of arc lengths from 20 minutes to 50 minutes in 10-minute jumps. Solutions for these data arcs are presented in Table 3 for range and Doppler tracking. The solution using Doppler data at the shortest arc studied, 20 minutes, had dropped by over 50 kilometers in its perigee from the actual value and would have been further degraded for shorter arcs. The range data solution at 40 minutes was poor, as is reflected in its very low perigee of 279 kilometers.

Short-arc studies with ERBS (see Table 1) showed that sometimes good solutions could be obtained (e.g., the 12-minute range and Doppler data arcs on December 16, 1987, and the 18-minute range and Doppler data arcs on April 10, 1986). However, at other times, poor solutions were determined (e.g., the 11-minute range and Doppler data arcs on November 30, 1987, where both solutions were slightly unphysical).

Table 3. SME Solutions for Various Data Arcs on January 14, 1988, at 0 Hours

<table>
<thead>
<tr>
<th>DATA ARC LENGTH (minutes)</th>
<th>DATA TYPE</th>
<th>APOGEE (kilometers)</th>
<th>PERIGEE (kilometers)</th>
<th>a (kilometers)</th>
<th>e</th>
<th>i (degrees)</th>
<th>Ω (degrees)</th>
<th>ω (degrees)</th>
<th>M (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>D</td>
<td>499</td>
<td>422</td>
<td>6839</td>
<td>0.0056</td>
<td>96.9</td>
<td>20.5</td>
<td>85.0</td>
<td>14.1</td>
</tr>
<tr>
<td>30</td>
<td>D</td>
<td>516</td>
<td>460</td>
<td>6866</td>
<td>0.0041</td>
<td>97.5</td>
<td>20.3</td>
<td>78.4</td>
<td>20.5</td>
</tr>
<tr>
<td>40</td>
<td>D</td>
<td>566</td>
<td>519</td>
<td>6921</td>
<td>0.0034</td>
<td>98.2</td>
<td>20.0</td>
<td>8.4</td>
<td>91.6</td>
</tr>
<tr>
<td>40</td>
<td>R</td>
<td>485</td>
<td>279</td>
<td>6760</td>
<td>0.0153</td>
<td>98.6</td>
<td>19.6</td>
<td>150.6</td>
<td>299.9</td>
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<tr>
<td>50</td>
<td>D</td>
<td>569</td>
<td>517</td>
<td>6921</td>
<td>0.0037</td>
<td>98.1</td>
<td>20.0</td>
<td>9.9</td>
<td>90.3</td>
</tr>
<tr>
<td>50</td>
<td>R</td>
<td>558</td>
<td>493</td>
<td>6903</td>
<td>0.0047</td>
<td>99.7</td>
<td>19.0</td>
<td>214.7</td>
<td>240.1</td>
</tr>
</tbody>
</table>

aSELECTED DATA ARE NEARLY UNIFORMLY DISTRIBUTED WITHIN EACH ARC.
bDATA TYPES:
D = DOPPLER
R = RANGE

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Previous studies (Reference 7) of SME showed that solutions became significantly degraded for data arcs of less than one-quarter of a revolution, because the equations that had to be solved became increasingly illconditioned. This limit is approximately 24 minutes for SME and ERBS. The current studies showed that in several cases accurate results were obtained for ERBS for much shorter arcs (see Section 3.4), while SME results were in general agreement with the previous findings.

3.3 DATA TYPE COMBINATIONS

The quality of solution states for various data types (all range, all Doppler, or mixed range and Doppler) varies according to the particular spacecraft conditions. For the three short arcs studied with ERBS, two arcs yielded good states for the separate Doppler and range tracking, but the third gave unphysical states. Unphysical or no solution states were found for mixed range and Doppler tracking in all three arcs. The long arc for COBE was also successful in determining good states for separate range and Doppler tracking.

SME solution states for Doppler-only data were more stable over shorter arcs than for range-only data with the January 14, 1988, arc (see Table 2); however, the reverse was found for the December 9, 1984, arc (Reference 7). In both cases, the mixed range and Doppler solutions were the least satisfactory.

3.4 FINAL TARGET STATE

A test was successfully conducted to demonstrate the primary function of PODS for handling orbit recovery when minimal data are available and the a priori target state vector is not known with certainty or with sufficient accuracy for the DC Program to perform adequately. An 18-minute data arc for ERBS on
April 10, 1986, was selected for the test. The a priori state selected had the generic value, given in Keplerian elements, as follows:

\[
\begin{align*}
    a &= 8000 \text{ kilometers} \\
    e &= 0.01 \\
    i &= 45 \text{ degrees} \\
    \Omega &= \omega = M = 0 \text{ degree}
\end{align*}
\]

Results for all-range data are shown in Figure 5. The DC Program could not generate a solution using the generic state vector and a 4-hour arc consisting of two 18-minute passes. However, after PODS generated a state vector from the generic a priori state and an 18-minute pass, the DC program successfully used the PODS solution as an a priori target state and calculated a final target state for the 4-hour arc using all-range data. This target state, in turn, was successfully used in a differential correction over a 21-hour arc. Similar results were obtained by starting with the generic a priori target state and all-Doppler measurements over the original 18-minute arc, and then using the PODS solution for the a priori target state and 4 hours of mixed range and Doppler data in the DC Program.

A second feature that can be analyzed with this ERBS 18-minute data arc is the multiplicity of solutions and their resolutions by TDRSS beam angle screening. The four candidate solutions generated by PODS from the range data and the generic a priori target state are listed in Table 4. Each candidate solution was used to predict TDRSS beam angles for comparison with the recorded beam angles. A solution was accepted whenever the two sets of values agreed within a specified tolerance. Solutions 1 and 3 are mirror images of each other in the TDRS orbit plane, as are solutions 2 and 4. This symmetry is characteristic of TDRSS range and Doppler measurements in orbit determination (see Section 2.2). Since the TDRS orbit is inclined slightly to the Earth's equatorial
Figure 5. Use of PODS To Aid in the Recovery of the TDRSS-Trackned
ERBS Target on April 10, 1986

Table 4. ERBS Candidate Solutions Using Range Data on April 10, 1986

<table>
<thead>
<tr>
<th>ELEMENTS</th>
<th>A PRIORI TARGET STATE</th>
<th>ERBS CANDIDATE SOLUTIONS¹</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SOLUTION 1</td>
<td>SOLUTION 2</td>
</tr>
<tr>
<td>x (kilometers)</td>
<td>7920</td>
<td>2892</td>
</tr>
<tr>
<td>y (kilometers)</td>
<td>-2718</td>
<td>3269</td>
</tr>
<tr>
<td>z (kilometers)</td>
<td>0</td>
<td>-5715</td>
</tr>
<tr>
<td>ẋ (kilometers/second)</td>
<td>0.0</td>
<td>4.27</td>
</tr>
<tr>
<td>ẏ (kilometers/second)</td>
<td>5.04</td>
<td>6.18</td>
</tr>
<tr>
<td>ż (kilometers/second)</td>
<td>5.04</td>
<td>-0.80</td>
</tr>
<tr>
<td>APOGEE (kilometers)</td>
<td>1702</td>
<td>588</td>
</tr>
<tr>
<td>PERIGEE (kilometers)</td>
<td>1542</td>
<td>554</td>
</tr>
<tr>
<td>a (kilometers)</td>
<td>8000</td>
<td>6949</td>
</tr>
<tr>
<td>e</td>
<td>0.01</td>
<td>0.00248</td>
</tr>
<tr>
<td>i (degrees)</td>
<td>45</td>
<td>55.9</td>
</tr>
<tr>
<td>Ω (degrees)</td>
<td>0.0</td>
<td>59.5</td>
</tr>
<tr>
<td>ω (degrees)</td>
<td>0.0</td>
<td>142.1</td>
</tr>
<tr>
<td>M (degrees)</td>
<td>0.0</td>
<td>120.4</td>
</tr>
</tbody>
</table>

¹Solutions are at 0 hours for the 18-minute data arc (0:31:30 - 0:49:50).
Solution 1 was selected by TDRSS beam angle screening.
plane and is slightly eccentric, the mirror image solution pairs in Cartesian coordinates in Table 4 approximately obey the following:

\[ z \rightarrow -z, \quad \dot{z} \rightarrow -\dot{z} \]

\[ x \rightarrow x, \quad y \rightarrow y, \quad \dot{x} \rightarrow \dot{x}, \quad \dot{y} \rightarrow \dot{y} \]

When solutions 1 and 3 became the a priori states in DC Program runs with TDRSS range tracking, the corresponding DC solutions were also mirror images of each other. To resolve this ambiguity, additional information is needed for selecting the correct solution. The TDRSS beam angle screening in PODS selected solution 1. Solutions 2 and 4, easily rejected by beam angle screening, are also invalid since they are too energetic, with values for the semimajor axis that are too large (reflecting the deliberately chosen too-large a priori value).

4. CONCLUSIONS

This section summarizes the evaluation studies described in this report (Section 4.1) and discusses the conclusions drawn (Section 4.2). In addition, future enhancements to PODS are outlined (Section 4.3).

4.1 EVALUATION SUMMARY

The evaluation studies demonstrated, through various examples, the following points:

- Good and sometimes generic values for the a priori state vector led to the correct PODS solutions.
Shorter data arcs were more unstable, but the cutoff varied on a case-by-case basis.

Range-only and Doppler-only data were more stable than mixed range and Doppler data.

Solution screening by TDRSS beam angles and physical considerations could select a valid solution from multiple candidates.

4.2 CONCLUSIONS

The different kinds of PODS examples given in this study reveal that PODS provides TDRSS tracking capability in preliminary orbit determination as a stand-alone utility under TCOPS. PODS uses the powerful homotopy continuation method with a limited number of measurements and a degraded a priori target state to determine candidate solutions from which the appropriate solution is extracted by solution screening. In addition, PODS is able to generate a solution that can be used to recover an orbit for an event when other systems such as the GTDS DC Program may fail. Some limitations that remain in PODS can be resolved through future enhancements.

4.3 FUTURE PODS ENHANCEMENTS

Future enhancements to PODS that are being developed or considered included the following:

- Improving the solution by taking the selected solution, which was generated using a two-body orbit propagator, and refining it by using a more accurate propagator along with a light-time correction algorithm (Reference 3)

- Generalizing the homotopy continuation algorithm to allow for jumping from an a priori target state loop to a solution loop, when necessary (Reference 3)
Extending TDRSS beam angle screening from range data to Doppler data

Allowing for ground-only and combined ground/relay tracking of the target

REFERENCES

1. Escobal, P. R., Methods of Orbit Determination. New York: John Wiley and Sons, Inc., 1965


5. Computer Sciences Corporation, CSC/SD-87/6737, System Description for the Preliminary Orbit Determination System (PODS), S. Kirschner and E. Nash, November 1987
