REFERENCE COORDINATE SYSTEMS: AN UPDATE

Ivan I. Mueller
Department of Geodetic Science & Surveying

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The Ohio State University
Research Foundation
1314 Kinnear Road
Columbus, Ohio 43212
PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The Science Advisor is Dr. David E. Smith, Code 921, Geodynamics Branch, and the Technical Officer is Dr. Robert J. Coates, Code 601, Crustal Dynamics Project, Space and Earth Sciences Directorate, both at NASA/Goddard Space Flight Center, Greenbelt, Maryland 20771. The work is carried out under NASA Grant NSG 5265, OSU Research Foundation Project No. 711055.

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ABSTRACT

A common requirement for all geodetic investigations is a well-defined coordinate system attached to the earth in some prescribed way, as well as a well-defined inertial coordinate system in which the motions of the terrestrial frame can be monitored. This paper deals with the problems encountered when establishing such coordinate systems and the transformations between them. In addition, problems related to the modeling of the deformable earth are discussed.

This paper is an updated version of the earlier work ‘Reference Coordinate Systems for Earth Dynamics: A Preview’, by the author published in the Proceedings of IAU Colloquium 56 on Reference Coordinate Systems for Earth Dynamics, Sept. 8-12, 1980, Warsaw, Poland, E.M. Gaposchkin and B. Kolaczek, eds., D. Reidel, 1981.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ii</td>
</tr>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>vi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. CONVENTIONAL INERTIAL SYSTEMS (CIS) OF REFERENCE</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Basic Considerations</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Inertial Systems in Practice</td>
<td>8</td>
</tr>
<tr>
<td>2.2.1 Extragalactic Radio Source System</td>
<td>8</td>
</tr>
<tr>
<td>2.2.2 Stellar System</td>
<td>11</td>
</tr>
<tr>
<td>2.2.3 Dynamical Systems</td>
<td>13</td>
</tr>
<tr>
<td>2.3 Conclusions</td>
<td>18</td>
</tr>
<tr>
<td>3. CONVENTIONAL TERRESTRIAL SYSTEMS (CTS) OF REFERENCE</td>
<td>21</td>
</tr>
<tr>
<td>3.1 Brief History of the Past Decade</td>
<td>23</td>
</tr>
<tr>
<td>3.2 The New CTS</td>
<td>24</td>
</tr>
<tr>
<td>3.3 Reference Frame Ties</td>
<td>27</td>
</tr>
<tr>
<td>3.3.1 Ties Between the CIS Frames</td>
<td>27</td>
</tr>
<tr>
<td>3.3.2 Ties Between the CTS Frames</td>
<td>29</td>
</tr>
<tr>
<td>4. MODELING THE DEFORMABLE EARTH</td>
<td>33</td>
</tr>
<tr>
<td>4.1 Precession</td>
<td>33</td>
</tr>
<tr>
<td>4.2 Nutation</td>
<td>33</td>
</tr>
<tr>
<td>4.3 Earth Rotation</td>
<td>38</td>
</tr>
<tr>
<td>4.4 Deformations</td>
<td>39</td>
</tr>
<tr>
<td>4.4.1 Tidal Deformations</td>
<td>39</td>
</tr>
<tr>
<td>4.4.2 Plate Tectonic Mass Transfer</td>
<td>40</td>
</tr>
<tr>
<td>4.4.3 Other Deformations</td>
<td>41</td>
</tr>
<tr>
<td>4.4.4 Current (1988) Practice</td>
<td>41</td>
</tr>
<tr>
<td>4.5 Recent Developments</td>
<td>42</td>
</tr>
<tr>
<td>4.5.1 Expected Changes in the Adopted Series of Nutation</td>
<td>42</td>
</tr>
<tr>
<td>4.5.2 Expected Change in the Constant of Precession</td>
<td>43</td>
</tr>
<tr>
<td>4.5.3 Intermediate Reference Frame Issues</td>
<td>43</td>
</tr>
<tr>
<td>5. THE INTERNATIONAL EARTH ROTATION SERVICE</td>
<td>45</td>
</tr>
<tr>
<td>5.1 The MERIT-COTES Programs</td>
<td>45</td>
</tr>
<tr>
<td>5.2 The International Earth Rotation Service</td>
<td>47</td>
</tr>
<tr>
<td>References</td>
<td>51</td>
</tr>
<tr>
<td>Appendix 1 Principal Recommendations of the MERIT and COTES Working Groups</td>
<td>55</td>
</tr>
<tr>
<td>Appendix 2 Resolution of International Astronomical Union (1985)</td>
<td>56</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1. Construction of conventional reference systems .......................................................... 2
2. Conventional terrestrial and quasi-inertial systems of reference with some possible connections ........................................................................................................ 7
3. Reference frame connections ......................................................................................... 28

LIST OF TABLES

1. VLBI Recording Systems ............................................................................................... 9
2. VLBI Antennas Used for Astrometry ............................................................................ 10
3. J2000.0 Catalogues of Extragalactic Compact Sources ............................................. 11
4. Data in Modern Lunar and Planetary Ephemerides .................................................. 15
5. Transformation Parameters Between Different CTS Frames .................................. 30
6. Transformation Parameters from the Individual 1984.0 CTS Systems to the ‘Global’ CTS ................................................................. 32
7. Corrections to the Long-Period Terms of the IAU 1980 Nutation Series .................... 43
1. INTRODUCTION

Geodynamics has become the subject of intensive international research during the last decades, involving plate tectonics, both on the intraplate and interplate scale, i.e., the study of crustal movements, and the study of earth rotation and of other dynamic phenomena such as the tides. Interrelated are efforts improving our knowledge of the gravity and magnetic fields of the earth. A common requirement for all these investigations is the necessity of a well-defined coordinate system (or systems) to which all relevant observations can be referred and in which theories or models for the dynamic behavior of the earth can be formulated. In view of the unprecedented progress in the ability of geodetic observational systems to measure crustal movements and the rotation of the earth, as well as in the theory and model development, there is a great need for the definition, practical realization, and international acceptance of suitable coordinate system(s) to facilitate such work. Manifestation of this interest has been the numerous specialized symposia organized during the past decade or so, such as those held in Stresa (Markowitz and Guinot, 1968), Morioka (Melchior and Yumi, 1972; Yumi, 1971), Torun (Kolaczek and Weiffenbach, 1974), Columbus (Mueller, 1975b, 1978, 1985), Kiev (Fedorov, Smith and Bender, 1980), San Fernando (McCarthy and Pilkington, 1979), Warsaw (Gaposchkin and Kolaczek, 1981), and Coolfont (Wilkins and Babcock, 1987). There seems to be general agreement that only two basic coordinate systems are needed: a Conventional Inertial System (CIS), which in some 'prescribed way' is attached to extragalactic celestial radio sources, to serve as a reference for the motion of a Conventional Terrestrial System (CTS), which moves and rotates in some average sense with the earth and is also attached in some 'prescribed way' to a number of dedicated observatories operating on the earth's surface. In the latter, the geometry and dynamic behavior of the earth would be described in the relative sense, while in the former the movements of our planetary system (including the earth) and our galaxy could be monitored in the absolute sense. There also seems to be a need for certain interim systems to facilitate theoretical calculations in geodesy, astronomy, and geophysics as well as to aid the possible traditional decomposition of the transformations between the frames of the two basic systems. This scheme is shown in Fig. 1.

The Earth Model block represents the current best knowledge of the geometry and dynamic behavior of the earth, partially deduced from the measurements made at the Dedicated Observatories. This model is continuously improving as more data of increasing accuracy
becomes available, and it includes both local (L) and global (G) phenomena which have theoretical foundations based on physical reality and are mathematically describable. In the final and ideal situation, which may be achieved only after several iterations over an extended period of time, the global part of the model should be identical to the connection between the CIS and CTS frames. Departures (v) from the model (L') observed at the observatories (j) or at other station (i) are of course most important since they represent new information based on which the model can be improved, after observational random and systematic errors have been taken into proper consideration. The model could eventually include the solid earth as well as the oceans and the atmosphere.

As we will see later, there is an understanding on how the two basic reference systems should be established; operational details are part of a recent international agreement. There are still, however, a number of open questions which have to be discussed further. These include the type of interim systems needed and their connections to be CIS and CTS, the type(s) of observatories, their number and distribution, whether all instruments need to be permanently located there or only installed at suitable regular intervals to repeat the measurements; how far the model development
should go so as not to become impractical and unmanageable; and how independent observations should be referenced to the CTS, i.e., what kind of services need to be established for the user of the systems.

In order to clarify some of the conceptual aspects of various reference systems and frames, we propose to use specific terms suggested in (Kovalevsky and Mueller, 1981) that have been used somewhat inconsistently in the past:

The purpose of a reference frame is to provide the means to materialize a reference system so that it can be used for the quantitative description of positions and motions on the earth (terrestrial frames), or of celestial bodies, including the earth, in space (celestial frames). In both cases the definition is based on a general statement giving the rationale for an ideal case, i.e., for an ideal reference system. For example, one would have the concept of an ideal terrestrial system, through the statement that with respect to such a system the crust should have only deformations (i.e., no rotations or translations). The ideal concept for a celestial system is that of an inertial system so defined that in it the differential equations of motion may be written without including any rotational term. In both cases the term ‘ideal’ indicates the conceptual definition only, and no means are proposed to actually construct the system.

The actual construction implies the choice of a physical structure whose motions in the ideal reference system can be described by physical theories. This implies that the environment that acts upon the structure is modeled by a chosen set of parameters. Such a choice is not unique: there are many ways to model the motions or the deformations of the earth; there are also many celestial bodies that may be the basis of a dynamical definition of an inertial system (moon, planets, or artificial satellites). Even if the choice is based on sound scientific principles, there remains some degree of imperfection or arbitrariness. This is one of the reasons why it is suggested to use the term ‘conventional’ to characterize this choice. The other reason is related to the means, usually conventional, by which the reference frames are defined in practice.

At this stage, there are still two steps that are necessary to achieve the final materialization of the reference system so that one can refer coordinates of objects to them. First, one has to define in detail the model that is used in the relationship between the configuration of the basic structure and its coordinates. At this point, the coordinates are fully defined, but not necessarily accessible. Such a model is called a conventional reference system. The term ‘system’ thus includes the description of the physical environment as well as the theories used in the definition of the coordinates. For example, the FK4 (conventional) reference system is defined by the ecliptic as given by Newcomb’s theory of the sun, the values of precession and obliquity, also given by
Newcomb, and the Woolard theory of nutation. Once a reference system is chosen, it is still necessary to make it available to the users. The system usually is materialized for this purpose by a number of points, objects or coordinates to be used for referencing any other point, object or coordinate. Thus, in addition to the conventional choice of a system, it is necessary to construct a set of conventionally chosen (or arrived at) parameters (e.g., star positions or pole coordinates). The set of such parameters, materializing the system, define a *conventional reference frame*. For example, the FK4 catalogue of over 1500 star coordinates defines the FK4 frame, materializing the FK4 system.

Another example is the CTS for the deformable earth defined through the time varying coordinates of a number of terrestrial observatories whose positions are periodically reobserved by some international service. The frame of this CTS could then be derived from the changing coordinates through transformations containing rotational (and possibly translational) parameters. The service, as part of the system definition, would have to make the assumption that the progressive changes of the reference coordinates of the observatories, defining the frame, do not represent rotations (and translations) in a statistically significant sense.
2. CONVENTIONAL INERTIAL SYSTEMS (CIS) OF REFERENCE

2.1 Basic Considerations

The first law of Newton is as follows: 'Every body persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state of forces impressed on it' (Newton, 1686). It should be obvious that the above law of inertia cannot hold in any arbitrary reference frame so that only certain specific reference frames are acceptable. In classical mechanics, reference frames in which the above law is valid are called inertial frames. Such 'privileged' frames move through space with a constant translational velocity but without rotational motion. Another privileged frame in classical mechanics is the quasi-inertial, which also moves without rotational motion, but its origin may have acceleration. Such a frame would be, for example, a nonrotating geocentric Cartesian coordinate system whose origin due to the earth orbit around the sun would move with a nonconstant velocity vector. Inertial reference frames thus are either at rest or are in a state of uniform rectilinear motion with respect to absolute space, a concept also mentioned by Newton and visualized as being observationally defined by the stars of invariable positions, a dogma in his time.

The refinement of classical mechanics through the theory of relativity requires changes in the above concepts. The theory of special relativity allows for privileged systems, such as the inertial frame but in the space-time continuum instead of the absolute space (Moritz, 1967). Transformation between inertial frames in the theory of special relativity are through the so-called Lorentz transformations, which leave all physical equations, including Newton’s laws of motion, and the speed of light invariant. The special theory of relativity holds only in the absence of a gravitational field.

In the theory of general relativity, Einstein defined the inertial frames as 'freely falling coordinate systems' in accordance with the local gravitational field which arises from all matter of the universe. Thus the inertial frames lose their privileged status. Concerning the existence of inertial frames in the extended portions of the space-time continuum, Einstein (1956) states that 'there are finite regions, where, with respect to a suitably chosen space of reference, material
particles move freely without acceleration, and in which the laws of special relativity hold with remarkable accuracy.' In other words, one can state (Weinberg, 1972) that

'At every space-time point in an arbitrary gravitational field, it is possible to choose a locally inertial coordinate system such that, within sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate system in the absence of gravitation.'

(i.e., as in the theory of special relativity). Our sphere of interest, the area of the solar system, where the center of mass of the earth-moon system is 'falling' in an elliptic orbit around the sun, in a relatively weak gravitational field, seems to qualify as such a 'small region.' Thus we may assume that inertial or quasi-inertial frames of reference exist, and any violation of principles when using classical mechanics can be taken into account with small corrections appropriately applied to the observations and by an appropriate 'coordinate' time reference. The effects of special relativity for a system moving with the earth around the sun are of the order of $10^{-8}$, while those of general relatively are $10^{-9}$ (Moritz, 1979). Since $10^{-8}$ on the earth's surface corresponds to about 6 cm, corrections at least for special relativity effects are needed when striving for such accuracies. Other than this, the problem, in the conceptual sense, need not be considered further.

Since the definition of the CIS may be based on dynamical properties of the solar system as well as on the kinematics of extragalactic sources, we are led to distinguish between two kinds of quasi-inertial systems (Fig. 2) (Kovalevsky and Mueller, 1981):

(a) **Conventional kinematical systems**, based on the assumption that the proper motions of some celestial bodies have known statistical properties. In the case of extragalactic sources, it is postulated that remote galaxies have no rotational component in their motions.

(b) **Conventional dynamical systems**, based on the theory of the motion of some bodies in the solar system (including artificial earth satellites) constructed in such a way that there remains no rotational term in the equations of motion.

If in the framework of Newtonian mechanics, both definitions are equivalent, this is not true in the theory of general relativity. A dynamical system of coordinates is a local reference that is locally tangent to the general space-time manifold. In contrast, the kinematical frame defined by the apparent directions of remote objects is a coordinate system that is subject to relativistic effects such as the geodesic precession. Even if this is being suitably corrected for, there remains a basic difference between the concepts, and this is another good reason to use the terminology 'quasi-inertial' to characterize both kinematical and dynamical systems.
Fig. 2  Conventional terrestrial and quasi-inertial systems of reference with some possible connections.
It is now well agreed that the best future CIS will be based on the position of extragalactic radio sources. But even if such a system is due to play a major role among conventional quasi-inertial systems, there may be great advantages, in some cases, to use a dynamical system. This is the case, for instance, when artificial satellites are used to monitor the earth rotation. This is why a certain hierarchy among these systems has been proposed in which the CIS, based on extragalactic radio sources is designated as a primary system, a role which used to be played by the FK4 System. Other systems, and in particular all the conventional dynamical systems, will have to be connected to the primary system in order to give consistent results (see later).

As mentioned, the actual availability of the systems is obtained through their realization in the form of reference frames. This materialization can be done in two different ways so that one can distinguish between two kinds of reference frames (Kovalevsky and Mueller, 1981):

(a) Conventional kinematic frames. The fiducial points are presently stars or extragalactic radio sources. In case of the latter, it is necessary to provide connection to stellar catalogues, so that the celestial system can be made available to optical instrumentation.

(b) Conventional dynamic frames. In such frames, one or several moving objects are used as the materialization of the system. The theory supporting the corresponding reference system provides the apparent ephemeris of the objects (satellites or planets) as a function of time and the observed successive positions are the fiducial points needed to refer the observations to the system.

It is to be noted that there is not necessarily a bi-univocal correspondence between the two types of frames and quasi-inertial systems. For instance, the FK4 or FK5 stellar systems are dynamical (due to the method of determination of the equinox), while their frame is stellar.

2.2 Inertial Systems in Practice

2.21 Extragalactic Radio Source System. This system is attached to radio sources which generally either are quasi-stellar objects (quasars) or galactic nuclei. Very long baseline interferometers rotating with the earth determine the declinations of these sources with respect to the instantaneous rotation axis of the earth (see Section 4.2), as well as their right ascension differences with respect to a selected source (3C273, NRAO 140, Persei (Algol), etc.) In addition, the observations also determine changes in the earth rotation vector with respect to a selected initial state, the baseline itself, and certain instrumental (clock) corrections. The frame of the Radio Source-CIS can be defined by the adopted true or mean coordinates of appropriately selected sources referred to some standard epoch. The mean coordinates naturally will depend on the model of the transformation from the true frame of date to the adopted mean standard. If,
however, the reduction procedure is correct (see more on this later), there are no known reasons for nonradial relative motions of the sources, i.e., for the rotation of the frame. Thus such a frame could be considered inertial or at least quasi-inertial. The equatorial system of coordinates may be retained for convenience, but the frame could be attached to the sources in any other arbitrary way should this be necessary.

As far as the accuracy of the Radio Source-CIS is concerned, the question has meaning only in the sense of the formal precisions of the source positions in the catalogue. At the Torun meeting, this number was $0.1'$ (Moran, 1974); at Warsaw it was $0.01'$ (Purcell et al., 1980). Now the precision is of the order of $0.001'$ (Ma, 1989). The problem on this level is that the densification of such a catalogue is very difficult, since only a relatively few well-defined point-like radio sources have been observed. Others have structures such that identification of the center of the radiation with such accuracy may not be possible. This situation may change when the astrometric satellites (see below) are launched, or other new technological developments take place.

VLBI instrumentation has undergone considerable development since the initial efforts in the early 1960's. Table 1 describes the primary recording systems.

<table>
<thead>
<tr>
<th>System</th>
<th>In Use</th>
<th>Basic Design</th>
<th>Sample Rate</th>
<th>Tape Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark I</td>
<td>1967-78</td>
<td>Digital recording on computer tape</td>
<td>0.72</td>
<td>3</td>
</tr>
<tr>
<td>Mark II</td>
<td>1971-</td>
<td>Digital recording on various TV recorders</td>
<td>4</td>
<td>64-246</td>
</tr>
<tr>
<td>Mark III</td>
<td>1977-</td>
<td>Digital recording</td>
<td>112</td>
<td>13</td>
</tr>
<tr>
<td>Mark IIIA</td>
<td>1984-</td>
<td>Digital recording</td>
<td>112</td>
<td>164</td>
</tr>
</tbody>
</table>

VLBI networks, since they are composed of independent elements, vary with time and availability. Table 2 shows the stations which have contributed significantly to the current astrometric data base.

Two connected element interferometer (CEI) instruments are also regularly used for astrometric measurements. The National Radio Astronomy Observatory interferometer in Green Bank, West Virginia, has a 35-km baseline and operates continuously as part of a program to monitor UT1. The Very Large Array (VLA) near Socorro, New Mexico, while primarily a mapping instrument is also used for differential and absolute astrometry. It consists of 27 25-m antennas laid out in a Y pattern with the longest arm 21 km.
Table 2 VLBI Antennas Used for Astrometry (Ma, 1989)

<table>
<thead>
<tr>
<th>Location</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gilmore Creek, Alaska, USA</td>
<td>26 m</td>
</tr>
<tr>
<td>Goldstone Deep Space Station, California, USA</td>
<td>64</td>
</tr>
<tr>
<td>Hartbeesthoek Radio Observatory, So. Africa</td>
<td>26</td>
</tr>
<tr>
<td>Hat Creek Radio Observatory, California, USA</td>
<td>26</td>
</tr>
<tr>
<td>Harvard Radio Astronomy Station, Texas, USA</td>
<td>26</td>
</tr>
<tr>
<td>Haystack Observatory, Massachusetts, USA</td>
<td>37</td>
</tr>
<tr>
<td>Kashima Space Research Center, Japan</td>
<td>26</td>
</tr>
<tr>
<td>Kokee Tracking Station, Hawaii, USA</td>
<td>9</td>
</tr>
<tr>
<td>Madrid Deep Space Station, Spain</td>
<td>64</td>
</tr>
<tr>
<td>Maryland Point, Maryland, USA</td>
<td>26</td>
</tr>
<tr>
<td>Mojave Base Station, California, USA</td>
<td>12</td>
</tr>
<tr>
<td>National Radio Astronomy Observatory, West Virginia, USA</td>
<td>43</td>
</tr>
<tr>
<td>Onsala Space Observatory, Sweden</td>
<td>20</td>
</tr>
<tr>
<td>Owens Valley Radio Observatory, California, USA</td>
<td>40</td>
</tr>
<tr>
<td>Richmond, Florida, USA</td>
<td>18</td>
</tr>
<tr>
<td>Tidbinbilla Deep Space Station, Australia</td>
<td>64</td>
</tr>
<tr>
<td>Westford, Massachusetts, USA</td>
<td>18</td>
</tr>
<tr>
<td>Wettzell, Fed. Repub. Germany</td>
<td>20</td>
</tr>
</tbody>
</table>

There are at present several catalogs of extragalactic radio sources in the J2000.0 system. They vary considerably in number of sources, distribution of sources, and precision. See Table 3 for a summary (Ma, 1989).

An extragalactic reference frame which will serve as the initial system of the International Earth Rotation Service (IERS) was compiled on the basis of four individual catalogues from the NASA Goddard Space Flight Center, the Jet Propulsion Laboratory (JPL) and the U.S. National Geodetic Survey (NGS). The compilation was carried out at the IERS (Arias et al., 1988a) and includes 228 extragalactic, compact sources divided into primary, secondary and complementary sources depending upon geometrical and physical considerations as well as observational histories. Unfortunately, this reference frame contains no sources south of -45°, and of the 23 primary sources which define the directions of the axes, eight are in the Southern Hemisphere between the equator and -29°. This points up the fact that the distribution of well-observed radio sources and radio interferometry baselines is far from ideal for the purposes of a global reference frame.

Ma (1983) intercompared the catalogues of JPL and NASA, based on 45 overlapping sources and found an RMS difference of about 0.005 in both right ascension and declination. A recent study by Arias et al. (1988b) intercompared JPL, NASA and NGS 1984-1986 catalogues based on 19-128 overlapping sources and found the directions of the axes of their respective reference frames consistent within 0.003. This is considered a remarkable agreement on account of the diversity of observing strategies and data analysis.
### Table 3  J2000.0 Catalogues of Extragalactic Compact Sources

<table>
<thead>
<tr>
<th>Organization</th>
<th>Instrument</th>
<th>Baseline Length (km)</th>
<th>No. of Sources</th>
<th>Uncertainties mas</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRAO</td>
<td>CEI</td>
<td>35</td>
<td>36</td>
<td>20-40</td>
<td>Wade &amp; Johnston, 1977</td>
</tr>
<tr>
<td>NRAO</td>
<td>CEI</td>
<td>35</td>
<td>16</td>
<td>10</td>
<td>Kaplan et al., 1982</td>
</tr>
<tr>
<td>JPL</td>
<td>Mark II</td>
<td>8000-11000</td>
<td>836</td>
<td>300</td>
<td>Morabito et al., 1982-86</td>
</tr>
<tr>
<td>NSF</td>
<td>VLA</td>
<td>&lt;27</td>
<td>700</td>
<td>20-100</td>
<td>Perley, 1982</td>
</tr>
<tr>
<td>JPL</td>
<td>Mark II</td>
<td>8000-11000</td>
<td>117</td>
<td>1-5</td>
<td>Fanselow et al., 1984</td>
</tr>
<tr>
<td>NASA</td>
<td>Mark III</td>
<td>800-6000</td>
<td>85</td>
<td>0.3-13</td>
<td>Ma et al., 1986</td>
</tr>
<tr>
<td>NGS</td>
<td>Mark III</td>
<td>800-6000</td>
<td>26</td>
<td>0.5</td>
<td>Robertson et al., 1986</td>
</tr>
<tr>
<td>NASA</td>
<td>Mark III</td>
<td>800-11000</td>
<td>101</td>
<td>0.2-9</td>
<td>Ma, 1988</td>
</tr>
<tr>
<td>JPL</td>
<td>Mark III</td>
<td>8000-11000</td>
<td>128</td>
<td>0.5-7</td>
<td>Sovers et al., in press</td>
</tr>
</tbody>
</table>

The premier instrument for future radio astrometry will be the Very Long Baseline Array, currently under construction. It will consist of ten 25-m antennas spaced from Hawaii to Puerto Rico, each equipped with ten receivers from .33 GHz to 43 GHz.

Until the VLBA becomes fully operational in the mid-1990's, there are several ongoing programs which will continue to expand and refine the extragalactic catalogue. The NASA Crustal Dynamics Project has a VLBI survey program to expand its catalogue of unresolved sources to take advantage of improvements in sensitivity. The US Naval Observatory is starting an astrometric program using North American VLBI stations to densify the grid of optical/radio sources in the Northern Hemisphere. The JPL survey work will be further refined to support planetary spacecraft navigation using differential VLBI.

### 2.22 Stellar System

This system is attached to stars in the FK5 catalogue, i.e., the adopted right ascensions and declinations of the FK5 define the equator and the equinox and thus the frame of the Stellar-CIS. The FK5 is the fifth fundamental catalogue in a series which began with the FC in 1879 (Fricke and Gliese, 1978). In the fundamental catalogues the equator is determined from zenith distance (or distance difference) observations of the stars themselves, but the equinox determination also necessitates measurements of the sun or other members of the planetary system. It was always tacitly assumed that coordinate systems attached to the fundamental catalogues were quasi-inertial. However, as more and more observations became available for proper motions and on the various members of the planetary systems, certain small rotations were discovered, which required changes in the positions of the fundamental equator and equinox, in the proper motions and in the precessional constant (all intricately interwoven) when one fundamental catalogue replaces the other. This slow and painstaking process should lead to a quasi-inertial system. We hope that the FK5 is such a system.
When the FK4 was compiled, a small definitive correction to the declination of FK3 was applied, but there seemed to be no need to change the position of the equinox or the precessional constant (Fricke, 1974). The FK5 is a considerably different and improved catalogue. The main changes with respect to the FK4, regarding the issue of the coordinate systems, are as follows (Fricke, 1979a): 1) New value of general precession in longitude adopted by the IAU in 1976 was used (more on this later). 2) The centennial proper motions in right ascension were increased by \( 0'085/\text{century} \) to eliminate the motion of the FK4 equinox with respect to the dynamical equinox (the FK4 right ascensions are decreasing with time due to an error in the FK4 proper motions, see below). 3) Rotation of the FK4 equinox at 1950 by the amount of \( 0'035 \) so that the FK5 and the dynamic equinoxes will be identical (the FK4 right ascensions at 1950 are too small). 4) Elimination of inhomogeneities of the FK4 system by means of absolute and quasi-absolute observations. 5) Determination of individual correction to positions and proper motions of FK4 stars. 6) Addition of 3130 fundamental stars to extend the visual magnitude to about 9.5, to be published in the \textit{FK5 Extension}.

It should be mentioned that the above improvements were possible because of the availability and/or reanalysis of observations of the sun (1900-1970), of lunar occultations (1820-1977), of Mars (1941-1971), of minor planets (1850-1977), and the JPL DE-108 Ephemeris based on optical or radar observations of the sun, planets and some space probes (Mariner 9, Viking). All in all the number of these observations exceeds 350,000. In addition, more than 150 catalogues of star observations have become available since the completion of the FK4 (Fricke, 1979b).

One should also take note here of the work in progress at the Astronomisches Rechen-Institut Heidelberg providing FK5 coordinates of a few extragalactic radio sources with radio and optical positions and thus the connections between the Stellar-CIS and the Radio Source-CIS, though with somewhat limited accuracy (~\( 0'1 \)). Improvement of this particular problem is expected from the Space Telescope (Van Altena, 1978) which could increase the number of radio stars, observable by VLBI, in the FK5 to about 50. Such missions (e.g., Hipparcos) could also contribute to the determination of the fundamental equator and equinox with increased accuracies, by observations of the minor planets. This, of course, would mean improved ties with the planetary-CIS (discussed below) which nowadays is based on the observations mentioned in connection with the establishment of the FK5 equator and equinox. The astrometric satellite Hipparcos is described to be able to measure relative positions of some 100,000 stars to a precision of \( 0'0015 \) and annual proper motions to \( 0'002 \) over a lifetime of 2.5 years (Barbieri and Bernacca, 1979). A second mission ten years later could improve this figure by a factor of 5. This compares well indeed with
the precision of ground-based observations of 0'.04 at best, requiring something like 50 years to obtain proper motions of comparable precision (0'.002).

As far as the accuracy of the FK5-CIS is concerned, the question again is meaningful only in the sense of how precise the star positions in the FK5 are. The 1535 'Basic FK5' stars have a mean precision of 0'.02 and 0.7 mas per year in proper motion. The 980 stars in the bright (magnitudes 5.5-7.0) 'FK5 Extension' are precise to about 0'.03, while the 2150 additional faint (magnitudes 6.5-9.5) stars to about 0'.06.

An important extension of the FK5 is the International Reference Star (IRS) catalogue which is almost completed and will include about one star per square degree. It will include the AGK3R stars in the northern hemisphere, the SRS (Southern Reference Stars) catalogue in the Southern Hemisphere (Zverev et al., 1986) and some additional stars to insure the homogeneity of the distribution on the celestial sphere (Smith, 1986). A special effort was made to obtain a homogeneous system of proper motions (Corbin, 1978).

Further extensions should be based on the IRS itself or on future larger and more homogeneous catalogues like the Hipparcos catalogue mentioned above (Froeschle and Kovalevsky, 1982).

2.23 Dynamical Systems. The dynamics expressed in the equations of motion define a number of nonrotating planes which could be the basis of reference frames. Considering the observable planes that could be the basis of such a Dynamic-CIS, there are the planetary (including the earth-moon barycenter) orbital planes, the equator, the lunar orbital plane, and the orbital planes of certain high flying, thus only slightly perturbed, artificial earth satellites (e.g., Lageos or GPS). Since all of these planes have relative rotations, it is possible to derive a mean plane for a given epoch from an observable apparent plane, or a nonobservable invariant plane could be adopted (Duncombe et al., 1974). At this point, the definition of the origin of the system becomes important also, because relativistic effects necessitate the distinction between proper and coordinate times. In the radio-source or stellar quasi-inertial systems, the question of origin can be settled through appropriate corrections for aberration and parallax, etc., but here it is also necessary that a uniform and unambiguous time scale referenced to a nonrotating frame of specified origin be established (coordinate time). The practical implications of a global coordinate time scale is not treated here, but the problem should not be ignored (cf. Ashby and Allan, 1978; Guinot, 1989). In more practical (observational) terms one can distinguish between Planetary, Lunar and (artificial) Satellite CIS's, each frame defined, in theory, by two of the above-mentioned planes, and in practice by the available ephemerides (see Fig. 2).
In the case of the *planetary systems*, the defining planes are the equator and the ecliptic, their intersection being the line of the equinoxes. In practical terms the frame of the Planetary-CIS is defined by the ephemerides of the centers of masses of the planets, including the barycenter of the earth-moon system. The ephemerides, such as the JPL DE-200, are based on observations of the sun, the planets, and space probes (see Table 4). Since most modern ephemerides are computed through the numerical integration of the orbital equations of motion, the degree of satisfaction that can be obtained depends only on the completeness of the modeling, including the astronomical constants, the determination of the starting conditions and, of course, on the type, accuracy and distribution of the observed data. In this sense each planetary ephemeris defines its own reference frame. These should agree with each other within the observational accuracies. Connection between the Planetary-CIS’s and the Stellar-CIS’s is through the determination of the equinox and the equator, as explained earlier (Fig. 2).

In the case of the *lunar system*, the main references are the orbital plane of the moon and the equator of the earth. In practice the Lunar-CIS frame is again defined by the lunar ephemeris, which nowadays is most accurately determined from lunar laser observations made from the surface of the earth to reflectors deposited on the lunar surface. For this reason, the adequacy of the definition also depends on how well the lunar rotation (librations) can be computed. Since the most frequently used lunar ephemerides are generally calculated through numerical integration, the above dependence on modeling (especially on the effect of tidal dissipation in the earth), and on initial conditions, apply here also. The identity of the coordinate frame, such defined, may be compared to the other frames to certain accuracies (Fig. 2). Lunar occultation of stars, or the earlier Markowitz moon-camera photography, provide a connection to the Stellar-CIS; differential VLBI observations between radio sources deposited on the moon and the extragalactic ones would tie to the Radio Source-CIS. The connection to the Planetary-CIS is through solar eclipse observations, and also through the planetary ephemeris used when calculating the lunar ephemeris. There are also some other looser connections stemming from the orientation of the earth when its nonspherical gravitational effects on the lunar motions are taken into consideration. Present observations reveal a residual rotation (or accelerations) of the order of a few seconds of arc per century squared. This seems to be the present stability (i.e., the accuracy) of this quasi-inertial frame. It is unlikely that without stronger connections to a frame of better stability, this rotation can be eliminated. As it is, the accuracy of this CIS should compare favorably with that defined by the FK5 but only over a period of, say, a decade (Kovalevsky, 1979).

Data types to which modern planetary and lunar ephemerides are adjusted are listed in Table 4. The post-fit rms residuals indicate the accuracy of the data. The values listed without brackets are
the units of the original observations; those within brackets give the comparable values for comparison purposes.

Table 4 Data in Modern Lunar and Planetary Ephemerides (Williams and Standish, 1989)

<table>
<thead>
<tr>
<th>Type of Observation</th>
<th>Time Span</th>
<th>Post-Fit Rms (km)</th>
<th>Residuals</th>
<th>No. of Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar Ranging</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>1966-</td>
<td>1.5</td>
<td>[0.0002]</td>
<td>500</td>
</tr>
<tr>
<td>Venus</td>
<td>1965-</td>
<td>1.5</td>
<td>[0.0002]</td>
<td>1000</td>
</tr>
<tr>
<td>Mars</td>
<td>1967-</td>
<td>2.2</td>
<td>[0.0003]</td>
<td>40000</td>
</tr>
<tr>
<td>Mars Closure</td>
<td>1969-82</td>
<td>0.15</td>
<td>[0.0002]</td>
<td>200</td>
</tr>
<tr>
<td>Spacecraft Ranging</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ma9 Orbiter (Mars)</td>
<td>1972-73</td>
<td>0.040</td>
<td>[0.0002]</td>
<td>600</td>
</tr>
<tr>
<td>Viking Lander (Mars)</td>
<td>1976-80</td>
<td>0.007</td>
<td>[0.000003]</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>1980-82</td>
<td>0.012</td>
<td>[0.000006]</td>
<td>400</td>
</tr>
<tr>
<td>Spacecraft Tracking (Range, Doppler)</td>
<td>1973-80</td>
<td>[200, 400]</td>
<td>[0.05]</td>
<td>20000</td>
</tr>
<tr>
<td>Pion&amp;Voy (Jup,Sat)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lunar Laser Ranging</td>
<td>1969-70</td>
<td>0.00100</td>
<td>[0.0005]</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>1970-75</td>
<td>0.00030</td>
<td>[0.00016]</td>
<td>1700</td>
</tr>
<tr>
<td></td>
<td>1976-85</td>
<td>0.00015</td>
<td>[0.00008]</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>1985-</td>
<td>0.00006</td>
<td>[0.00003]</td>
<td>600</td>
</tr>
<tr>
<td>Radio Astrometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jupiter, ..., Neptune</td>
<td>1983-</td>
<td>[100, ..., 600]</td>
<td>0.03</td>
<td>10</td>
</tr>
<tr>
<td>Ring Occultation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uranus</td>
<td>1978-</td>
<td>[1500]</td>
<td>0.1</td>
<td>14</td>
</tr>
<tr>
<td>Optical Transits (Manual)</td>
<td>1911-</td>
<td>[700]</td>
<td>1.0</td>
<td>37000</td>
</tr>
<tr>
<td>Sun, Mercury, Venus</td>
<td></td>
<td>[150, ..., 10000]</td>
<td>0.5</td>
<td>18000</td>
</tr>
<tr>
<td>Mars, ..., Neptune</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optical Transits (Photoelectric)</td>
<td>1982-</td>
<td>[100, ..., 6000]</td>
<td>0.3</td>
<td>1000</td>
</tr>
<tr>
<td>Mars, ..., Neptune</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Astrolabe</td>
<td>1961-</td>
<td>[100, ..., 4000]</td>
<td>0.3</td>
<td>1500</td>
</tr>
<tr>
<td>Mars, ..., Uranus</td>
<td>1961-</td>
<td>[100, ..., 4000]</td>
<td>0.3</td>
<td>1500</td>
</tr>
<tr>
<td>Astrometry</td>
<td>1914-</td>
<td>[15000]</td>
<td>0.5</td>
<td>1600</td>
</tr>
</tbody>
</table>

Earlier ephemerides of the moon and planets, based upon optical observations, have inherited errors directly from the catalogues upon which they have been based. These errors amount to a number of tenths of an arcsecond in angular position and a number of tenths of an arcsecond per century in angular motion; i.e., errors comparable to those that are known to exist in the FK4 fundamental
reference system. Modern ephemerides based upon ranging observations show at least an order of magnitude improvement over their optically based predecessors. Williams and Standish (1989) selected the most important data types and calculated how sensitive these data are to changes in certain ephemeris elements. The sensitivities, in turn, indicate how well each of these elements may be determined through the data fitting, keeping in mind that the statistics of the actual determinations are improved due to the large number of observations but also that there are correlations among the various parameters. The summary of their findings is quoted below:

'The lunar laser ranging data is sensitive to a change in the lunar mean anomaly and its rate at levels of 0:0006 and 0:02/cy respectively. The data is also sensitive to the rate of the lunar longitude with respect to inertial space at a level of 0:04/cy. This rate error is dominated by the uncertainties in the precessional rates of the lunar perigee; the precessional rates themselves are due to the perturbations which depend on the orbital elements and gravitational harmonics of the earth and moon. At times away from the data span, the uncertainty (1:0/cy²) in the tidally induced acceleration in longitude becomes predominant.

'For the planets, the most important data are the ranges to the Viking landers on Mars. We show that these ranges have a remarkable sensitivity to a number of differential angles: the difference in heliocentric longitudes between earth and Mars at a level of 0:00001, each longitude with respect to the perihelion of Mars at a level of 0:00004 and each longitude with respect to the perihelion of the earth at a level of 0:0002. Further, the corresponding level for the inclination of Mars' orbit upon the ecliptic is about 0:0002.

'Radar ranging to Mercury and Venus determines the longitudes of these planets with respect to the longitude of the earth (and therefore to Mars). These sensitivities are on the order of 0:005 and 0:003 respectively, since the data are accurate to the level of 1.5 km. The sensitivities to the inclinations upon the ecliptic are two orders of magnitude worse than that for Mars.

'Solar perturbations upon the lunar orbit provide sensitivity to both the differential longitude between the heliocentric earth and the geocentric moon and to the inclination of the lunar orbit to the ecliptic; 0:001 and 0:007 respectively.

'Since the lunar ranges are taken from the spinning earth, sensitivities to the earth's orientation, coupled with the terrestrial coordinates of the observing station, allow determinations of

1) the mutual inclinations of the equator, the ecliptic and the lunar orbital plane (0:002);
2) the longitude of the earth and moon with respect to the dynamical equinox (0:005);
and
3) a tie between the ephemeris frame and the terrestrial reference system (0:001 in longitude, comparable to 0.0001 seconds in UT0).

'Finally, the fact that the lunar retroreflectors and the Viking landers are situated on the surfaces of the bodies, the ranges are sensitive to the physical orientations of the bodies themselves. The lunar librations affect the LLR data; the spin rate, obliquity and equinox of Mars influence the Viking ranges.
'The analytical sensitivity analyses have been substantiated by numerical examples though the correspondence is not exact because of differences in numbers of observations, correlations, additional data and other perturbing forces. However, even when all of these factors are considered, it is seen that the dynamical reference system may be determined better than 0.01 in position with respect to the dynamical equinox. Further, the mean motions of earth and Mars with respect to inertial space may be determined as well as 0.003/cy during the times of the highly accurate ranging data; the uncertainty for Mars will grow to about 0.015/cy over the course of many decades away from the present data.'

In the case of satellite systems, the problem is compounded by additional modeling problems related to the force field in which the satellite moves and by the fact that nowadays there are no direct connections to other frames of reference. Modern satellite tracking techniques (laser, GPS, etc.) all basically observe ranges or range differences and contain no direct directional information. The main reference planes, the orbital plane of the satellite and the equator, intersect along the line of nodes, the initial orientation of which therefore must be defined more or less arbitrarily. In the ‘old days’ of satellite geodesy, when satellites were observed photographically in the background of stars, this direction could be determined with respect to the stellar frame, though not much better than a few tenths of a second of arc. The accumulation of errors in describing the motion of the node with respect to a selected zero point, even for the most suitable high flying and small heavy spherical satellites (Lageos), may prevent a Satellite-CIS from being accurate over a long period of time, say beyond several months. In any case, in observational terms such a frame would be defined by the satellite ephemeris made available to the users by organizations which provide for the continuous tracking of the satellite in question. A current example would be the Precise Ephemeris of the U.S. Navy Navigational Satellite (Transit) System. As far as the connections to other systems are concerned, the only accurate possibility seems to be indirectly through the tracking stations. If two observational systems occupy the same station, one observing the satellite, the other, say, the radio sources, either simultaneously or after a short time interval (during which the movement of the station can be modeled), the connection between the satellite and radio source frames can be established (see Fig. 2). In fact, the now classical disparity between the JPL and SAO frames came to light through just such an arrangement, when the SAO longitudes determined from satellite camera tracking (thus in the FK4 frame) differed by those determined by JPL space probe tracking (in the planetary frame) by an amount (about 0'7 in the early 1970’s) consistent with the FK4 equinox motion with respect to the dynamical equinox, mentioned earlier. Only through such continuously maintained connections can the lifetime of a Satellite-CIS be extended, thus its accuracy increased.
2.3 Conclusions

From the above discussion, the following conclusions can be drawn:

1. The most accurate, long-term CIS is the one attached to extragalactic radio sources. It is accessible through VLBI observations. Other systems can be accurately connected to it by station collocation, space astrometry, VLA and differential VLBI observations (Fig. 2). The number of primary radio sources must be increased, especially in the Southern Hemisphere, to achieve isotropy.

2. The CIS attached to the FK5 is somewhat less accurate. Direct access to it is through optical star observations, which by nature are generally less accurate than VLBI observations. Its main value is in defining the fundamental mean system of coordinates and thereby providing a direction (the FK5 equinox) for the time (UT1) definition, and for the possible orientation of the Radio Sources-CIS. The latter function, however, stems from more of a traditional requirement and not from theoretical needs.

3. Of the Dynamical-CIS's, the accuracy of the planetary (including lunar) system is better than the FK5, with a rotational stability of 3 mas per century. The satellite systems by themselves are suitable for medium-term to short-term work only. The rotational stability can be extended by connections to the Radio Source-CIS through accurate and continuous observations at collocated stations or differential VLBI. Lunar laser ranging provides the best connection between the lunar and planetary systems.

4. If a dynamical system is based on the motion of planets, the ecliptic plays a privileged role and, naturally, the ecliptic is used in the definition of coordinates. Since equatorial coordinates are preferred to ecliptic ones for obvious instrumental reasons, the intersection of the ecliptic and the equator, the vernal equinox, becomes the natural origin of right ascensions. When the dynamical system is geocentric, the natural reference plane is the Laplace plane whose position depends upon the relative magnitude of the perturbations. For the moon, the solar effects are dominant and, practically, the Laplace plane is the ecliptic and, again, the equinox is the natural origin of equatorial coordinates. In the case of artificial satellites the perturbations due to the earth flattening are predominant so that the Laplace plane is the equator. The equator is, therefore, the natural fundamental plane, but the origin may be arbitrary.

Similarly, the choice of the equinox in the stellar systems is justified by the fact that they are partially dynamical systems based upon planetary theories. However, in the construction of the
corresponding stellar frame, the difficulty of maintaining the theoretical origin is so serious that one is led to distinguish between the dynamical equinox which defines the origin of the system and the catalogue equinox which is the origin of the frame. In practice, the actual origins of the stellar reference frames are purely conventional and are not the dynamical equinox.

5. The situation will become even more conspicuous for frames derived from conventional kinematic systems. Even if, for the sake of continuity, the origin and the fundamental plane of such a system should be close to the equinox and the equator, they should be conventional points defined only by the realization of the corresponding frame. Otherwise, it would be necessary to introduce a complex dynamical model to define the origin at the expense of introducing inaccuracies in the system and an uncertainty in its realization by the frame. In practice, the solution might be analogous to the present situation for the terrestrial reference frame (see Section 3.2). One would establish an international organization that would provide the coordinates of radio sources in the conventional kinematic frame, taking into account eventual changes in the number and position of the reference sources, due, for instance, to the disappearance or motion of quasars or better measurements, in such a way that the changes should not introduce a rotation (or translation) of the system in the average statistical sense. It is an almost unavoidable conclusion that for geodetic and geodynamic applications the most useful CIS is just such a system (Kovalevsky and Mueller, 1981; Guinot, 1986).
3. CONVENTIONAL TERRESTRIAL SYSTEMS (CTS) OF REFERENCE

As mentioned in the Introduction, the CTS is in some 'prescribed way' attached to observatories located on the surface of the earth. The connection between the CTS and CIS frames of common origin by tradition (to be preserved) is through the rotations (Mueller, 1969)

\[
[\mathbf{C}^T_\mathbf{S}] = \mathbf{SNP} \ [\mathbf{C}^T_\mathbf{S}]
\]

where \( \mathbf{P} \) is the matrix of rotation for precession, \( \mathbf{N} \) for nutation (to be discussed in Section 4), and \( \mathbf{S} \) for earth rotation (including polar motion). Polar motion thus is defined as the angular separation of the third (Z) axis of the CTS and the axis of the earth for which the nutation (N) is computed (e.g., instantaneous rotation axis, Celestial Ephemeris Pole, Tisserand mean axis of the mantle (see Section 4)).

Geodynamic requirements for CTS may be discussed in terms of global or regional problems. The former are required for monitoring the earth's rotation, while the latter are mainly associated with crustal motion studies in which one is predominantly interested in strain or strain rate, quantities which are directly related to stress and rheology. Thus for these studies, global reference frames are not particularly important although it is desirable to relate regional studies to a global frame.

For the rotation studies one is interested in the variations of the earth's rotation rate and in the motions of the rotation axis both with respect to space (CIS) and to the crust (CTS). The problem therefore is threefold:

(i) To establish a geometric description of the crust, either through the coordinates of a number of points fixed to the crust, or through polyhedron(s) connecting these points whose side lengths and angles are directly estimable from observations using the new space techniques (laser ranging or VLBI). The latter is preferred because of its geometric clarity.

(ii) To establish the time-dependent behavior of the polyhedron due to, for example, crustal motion, surface loading or tides.

(iii) To relate the polyhedron to both the CIS and the CTS. For the global tectonic problems only the first two points are relevant although these may also be resolved through point (iii).
In the absence of deformation, the definition of the CTS is arbitrary. Its only requirement is that it rotate with the rigid earth, but common sense suggests that the third axis should be close to the mean position of the rotation axis and the first axis be near the origin of longitudes.

In the presence of deformations, particularly long-periodic or secular ones, the definition is more problematic, because of the inability to separate rotational (and translational) crustal motions of the crust from those of the CTS.

One geophysical requirement of the reference system is that other geophysical measurements can be related to it. One example is the gravity field. The reference frame generally used when giving values of the spherical-harmonic coefficients is tied to the mean axis of figure of the earth. This frame should be simply related with sufficient accuracy to the CTS as well as to the CIS in which, for example, satellite orbits are calculated. Another example is height measurements with respect to the geoid.

The vertical motions may require some special attention, because absolute motions with respect to the center of mass have an immediate geophysical interest and are realizable. Again, if the center of mass has significant motions with respect to the crust, such a motion will be absorbed in the future CTS, if defined as suggested above. At present there is no compelling evidence that the center of mass is displaced significantly, at least at the decade time scale.

Apart from the geometrical considerations, the configuration of observatories should be such that (i), there are stations on most of the major tectonic plates in sufficient number to provide the necessary statistical strength, and (ii) the stations lie on relatively stable parts of the plate so as to reduce the possibility that tectonic shifts in some stations will not overly influence, at least initially, the parameters defining the CTS frame.

Finally one should realize that the problem of the geometric origin of the CTS is linked to that of a geocentric ephemeris frame. The center of mass of the earth is directly accessible to dynamical methods and is the natural origin of a geocentric satellite-based dynamical system. But, as such, it is model dependent, and, unless the terrestrial reference frame is also constructed from the same satellites (as is the case in various earth models such as GEM, SAO, GRIM), there may be inconsistencies between the assumed origin of a kinematically obtained terrestrial system and the center of mass. A time-dependent error in the position of the center of mass, considered as the origin of a terrestrial frame, may introduce spurious apparent shifts in the position of stations that may then be interpreted as erroneous plate motions. To avoid this problem the parameters defining the CTS frame should include translational terms as well.
3.1 Brief History of the Past Decade

Until 1984 the internationally adopted Woolard series of nutation, based on a rigid earth model, was computed for the instantaneous rotation axis of the rigid earth, and the Z axis of the CTS was the Conventional International Origin (CIO), defined by the adopted astronomic latitudes of the five International Latitude Service (ILS) stations, located approximately on the 39°08' parallel. These were assumed to be motionless relative to each other, and without variations in their respective verticals (plumblines) relative to the earth. Thus, conceptually, polar motion was to be determined from latitude observations only at these ILS stations. This had been done for over 80 years, and the results are the best available long-term polar motions, properly, but not very accurately, determined. The first axis of the CTS was defined by the assigned astronomic longitudes of time observatories (around 50) participating in the work of the Bureau International de l'Heure (BIH).

Due to the fact that in most geodetic and astronomical applications accurate shorter-term variations of polar motion were needed, which were not available with sufficient accuracy from the ILS observations, and especially not fast enough, polar motion was also determined from latitude and/or time observations at a larger number of observatories participating in the work of the International Polar Motion Service (IPMS), as well as of the BIH. In the resulting calculations, the earlier definition of the CIO could not be maintained. The common denominator being the Woolard series of nutation, observationally the Z axis of the CTS was thus defined by the coordinates of the pole as published by the IPMS or by the BIH. Thus it was legitimate to speak of IPMS and BIH poles of the CTS (in addition to the CIO). The situation had become even more complicated because Doppler and laser satellite tracking, VLBI observations, and lunar laser ranging also can determine variations in the earth rotation vector (including polar motion), some of which were incorporated in the BIH computations. Further confusion arose due to the fact that the BIH had two systems: the BIH 1968 and the BIH 1979, the latter due to the incorporation of certain annual and semiannual variations of polar motion determined from the comparisons of astronomical (optical) results with those from Doppler and lunar laser observations (Feissel, 1980).

Though naturally every effort was made to keep the IPMS and BIH poles of the CTS as close as possible to the CIO, the situation was not satisfactory from the point of view of the geodynamic accuracy requirement of a few parts in 10\(^9\). The accuracy of the pole position was estimated to be 0.01', and that of the UT1, 1 ms (\(-5 \times 10^{-8}\)) for five-day averages (Guinot, 1978). These figures, of course, did not include biases from the definition problems mentioned.
From 1984 onward, the IAU 1980 (Wahr, 1981) series of nutation for the nonrigid earth gives the space position of the Celestial Ephemeris Pole (CEP) (see later). The pole of the CTS officially remained the same as before.

3.2 The New CTS

There seemed to be general agreement that the new CTS frame conceptually be defined similarly to the CIO-BIH system (Bender and Goad, 1979; Guinot, 1979; Kovalevsky, 1979; Mueller, 1975a), i.e., it should be attached to observatories located on the surface of the earth. The main difference in concept was that these can no longer be assumed motionless with respect to each other. Also they must be equipped with advanced geodetic instrumentation like VLBI or lasers, which are no longer referenced to the local plumblines. Thus the new transformation formula, again assuming a common origin, may have the form

\[ [\vec{O} \vec{B} \vec{S}]_j = \vec{L}_j' + [\vec{C} \vec{T} \vec{S}]_j = \vec{L}_j' + \text{SNP} [\vec{C} \vec{T} \vec{S}]_j \]

(1)

where \( \vec{L}_j' \) is the vector of the 'j' observatory's movement on the deformable earth with respect to the CTS, computed from suitable models (see Fig. 1 and Section 4); \( \text{NP} \), the nutation and precession matrices computed with the new 1976 IAU constants and the 1980 IAU series of nutation (see Section 4); and \( S \), the rotation matrix between the CTS and the true frame for which the nutation is computed (see eqs. (7)-(9) in Section 4).

In the above equation the coordinates of the observatory 'j' \([\vec{C} \vec{T} \vec{S}]_j \), in the Conventional Terrestrial Reference Frame, are related to the coordinates determined by the technique 'o' \([\vec{O} \vec{B} \vec{S}]_j^o \), in its own reference frame, through the well-known transformation equation:

\[ [\vec{C} \vec{T} \vec{S}]_j + \vec{S}_o + R_1(\vec{\beta}_1^o) R_2(\vec{\beta}_2^o) R_3(\vec{\beta}_3^o) [\vec{C} \vec{T} \vec{S}]_j + c[\vec{C} \vec{T} \vec{S}]_j + \vec{L}_j' = [\vec{O} \vec{B} \vec{S}]_j^o + \vec{v}_j \]

(2)

where \( \vec{S}_o \) contains the three translation components between the CTS frame and that inherent in the technique 'o', \( \vec{\beta}_o \) are the three (usually very small) rotations, and \( c \) a differential scale factor. \( \vec{L}_j' \) is the vector of deformation, not containing global rotations nor translations, and \( \vec{v}_j \) is the residual vector.

Another set of equations derived in (Zhu and Mueller, 1983) relate other parameters in eq. (1), specifically the earth rotation parameters (ERP) in the matrix \( S \) determined by the technique 'o' in its own frame of reference, to those referring to the CTS frame:
\[ x_p - \beta_2^o + \alpha_1^o \sin \theta + \alpha_2^o \cos \theta = x_p^o + v_x p \]
\[ y_p - \beta_1^o - \alpha_1^o \cos \theta + \alpha_2^o \sin \theta = y_p^o + v_y p \]
\[ \omega_d UT1 + \beta_3^o - \alpha_3^o = \omega_d UT1^o + v_{UT1} \]  \hspace{1cm} (3)

where \( x_p^o, y_p^o \) and \( UT1^o \) are the observed ERP's; \( x_p, y_p \) and \( UT1 \) are those referenced to the CTS; \( \omega_d \) is the conversion factor, \( \vec{v} \) is the residual vector of the observed ERP's and \( \theta \) the sidereal time. Finally, \( \alpha_1^o \) are the small rotations between the Conventional Inertial Reference Frame and the Inertial Frame inherent in the technique 'o', i.e.,

\[ [\vec{C}_I S]^o = R_1(\alpha_1) R_2(\alpha_2) R_3(\alpha_3) [\vec{C}_I S] \]  \hspace{1cm} (4)

For each ERP series 'k' of 1-1.2 years length (or longer), generated by the technique 'o', one can fit the following type of circular model:

\[ a_{10}^o k + a_{20}^o k \cos A + a_{30}^o k \sin C + a_{40}^o k \cos C = x_k^o + v_x k \]
\[ a_{50}^o k - a_{20}^o k \sin A + a_{30}^o k \cos A - a_{40}^o k \sin C + a_{50}^o k \cos C = y_k^o + v_y k \]  \hspace{1cm} (5)

where \( A \) is the annual frequency and \( C \) the Chandler frequency, \( x_k^o, y_k^o \) are the observed ERP's in the series \( k \), and \( v \) the residuals.

The coefficients \( a_{10}^o k \) allow the computation of the amplitude of the annual motion

\[ \sqrt{(a_{20}^o k)^2 + (a_{30}^o k)^2} \],

that of the Chandler motion

\[ \sqrt{(a_{40}^o k)^2 + (a_{50}^o k)^2} \],

as well as the coordinates of the center of the polhode \( a_{10}^o k \) and \( a_{50}^o k \).

If two techniques, \( 01 \) and \( 02 \), are collocated or tied together by local surveys at the station 'j', the following additional relationship holds

\[ [\vec{O}_B S]^o_j - [\vec{O}_B S]^o_{j2} = \vec{\Delta}_j + \vec{v}_{\Delta j} \]  \hspace{1cm} (6)

where \( \vec{\Delta}_j \) is the coordinate difference vector from the local survey and \( \vec{v}_{\Delta j} \) the residual vector.

Equations (2) - (6) can be used as observation equations by an international service to determine the parameters defining and maintaining the CTS and providing the relationship versus
the terrestrial frame of each technique ($\vec{\delta}^0$, $\vec{\beta}^0$, c), versus the CIS ($x_p$, $y_p$, UT1), and the latter's relationship to the technique inertial frame ($\vec{\alpha}^0$):

\[
[C'I'S_\j] \text{ and } \vec{L}_\j
\]

for each observatory 'j', to define the CTS

\[
\delta^0, \beta^0, c, \alpha^0 \text{ and } \vec{\alpha}^0 \j
\]

for each technique 'o' and ERP series 'k'

\[
x_p, y_p, \text{UT1}
\]

for the service to provide the S matrix in eqs. (1) and (9)

As far as the origin of the CTS is concerned, it could be centered at the center of mass of the earth, and its motion with respect to the stations can be monitored either through observations to satellites or the moon, or, probably more sensitively, from continuous global gravity observations at properly selected observatories (Mather et al., 1977). For the former method, the condition

\[
\sum_D w_D \delta_D^0 = 0
\]

could be imposed on the above adjustment. The summation would be extended to all the above dynamic techniques D with given relative weights $w_D$. A similar condition could also be imposed on the scale extended to techniques defining the best scales (probably VLBI):

\[
\sum_S w_S cS^0 = 0
\]

Other conditions between two independent techniques 01 and 02 may include the following obvious relations

\[
a_1^{01} - a_1^{02} = \beta_2^{01} - \beta_2^{02}
\]

\[
a_0^{01} - a_0^{02} = \beta_1^{01} - \beta_1^{02}
\]

The above method of determining ERP or some variation thereof needs to be initialized in a way to provide continuity. This could be done through the IPMS or BIH poles, and the BIH zero meridian, at the selected initial epoch (or averaged over a well-defined time interval, say 1 to 1.2 years), uncertainties in their definition mentioned elsewhere being mercifully ignored, i.e.,

\[
\sum_{1-1.2 \text{ yrs}} (x_p - x_p^{\text{BIH}}) = 0
\]

\[
\sum_{1-1.2 \text{ yrs}} (y_p - y_p^{\text{BIH}}) = 0
\]

etc.
It is probably not useless to point out that in the system described above, the most important information for the users of the service are the ERP’s and the transformation parameters, but for scientists new knowledge about the behavior of the earth will come from the analysis of the residuals after the adjustment.

The IAU and IUGG recently made practical recommendations on the establishment of such a (or very similar) Conventional Terrestrial System, including the necessary plans for supporting observatories and services by establishing the International Earth Rotation Service, effective 1 January 1988 (Wilkins and Mueller, 1986). The goal of the service is the determination of the total transformation between the CTS and CIS. Thus the service will publish not only ERP determined from the repeated comparisons (the past situation), but also the models and parameters described above in eqs. (1) - (6), i.e., the parameters defining the whole system. (See Section 5.)

3.3 Reference Frame Ties

3.31 Ties Between the CIS Frames. ‘Measurements are inherently simpler to make and generally more accurate in their “natural” frame and hence should always be reported as such. However, to benefit from the complementarity of the various techniques, knowledge of the frame interconnections (both the rotation and the time-variable offset) is essential’ (Dickey, 1989). These are summarized in Figs. 2 and 3.

Recent activity in this area is indicated by the number of boxes and lines in Fig. 3, entitled Connections 1986 (the accuracy cutoff here is 0.05); a similar figure in an earlier paper (Williams et al., 1983) had fewer boxes and connecting lines. For example, ten lines instead of fifteen connected the targets with the techniques, and radio stars were listed as prospects for the future. The lunar planetary system, integrated in a joint ephemeris, is by its nature unified by the dynamics (Williams and Standish, 1989). The radio frame is tied to the ephemeris frame in several ways; one is via differential VLBI measurements of planet-orbiting spacecraft and angularly nearby quasars (Newhall et al., 1986). Another is the determination of a pulsar’s position in the ephemeris frame (via timing measurements) and the radio frame (via radio interferometry, see Backer et al., 1985). Very Large Array (VLA) observations of the outer planets (Jupiter, Saturn, Uranus and Neptune) or their satellite provide an additional tie between these two frames (Muhleman et al., 1985).

As for an optical-radio frame tie, a preliminary link has been established between the FK5 optical frame and the JPL radio reference frame via the differential VLBI measurement of optically bright radio stars and angularly nearby quasars coupled with comparisons of their optical positions (see Lestrade et al., 1988), and also by the use of the optical positions of quasars (Purcell, 1979).
The optical and ephemeris frames are tied by optical observations of the planets. Dickey (1989) also treats a few of the frame ties in greater detail; for example, for the connection between the radio and the ephemeris frames. For the other ties, the highlights are given with reference to a more detailed account.

Dickey (1989) also outlines the future with ongoing and planned efforts in several areas: Improved ephemeris-radio frame ties can be accomplished by VLBI observations of pulsars, additional VLA observations of the outer planets and satellites, and future differential VLBI experiments (such as that with orbiting spacecraft around Jupiter and Saturn). The millisecond pulsar PSR1937+214, having a period of 1.6 ms, has exceptionally low timing noise. Its position in the ephemeris frame can be measured to ~1 mas. This will allow a radio-planetary frame tie, limited only by the accuracy of an interferometric position measurement. Roughly, a factor of five
improvement (down to 0.01) is expected here with the full implementation of VLBI observations. An initial experiment of this type has been executed by R. Linfield and C. Gwinn.

As already mentioned, for optical astrometry, Hipparcos will measure a network of stars over the entire sky with accuracies of ~2 mas (Kovalevsky, 1980), while the Space Telescope will measure small fields with similar differential accuracy. However, the Space Telescope can observe much fainter objects (Jeffreys, 1980) and could observe the optical counterparts of extragalactic radio sources, all but possibly one of which are too faint for Hipparcos. A joint program would produce an accurate stellar network linked to the quasar radio frame by the Space Telescope. The occultations of stars by planets and planetary rings can provide an additional link between the optical and ephemeris frames. Also, optical interferometry offers exciting possibilities with the potential resolution being two or three orders of magnitude finer than that of VLBI (Reasenberg, 1986). More details are given in (Dickey, 1989).

3.32 Ties Between the CTS Frames. Boucher and Altamimi (1987) established relationships between a number of Conventional Terrestrial Reference Frames based on colocated observation stations and eq. (2). The selected sets of station coordinates defining each CTS are as follows:

(i) CTS (VLBI). Three sets of station coordinates have been selected:

CTS(NGS) 87 R01. The coordinate data are derived from a composite set of Mark III VLBI observations collected under the aegis of project MERIT, POLARIS, and IRIS and conducted between September, 1980, and January, 1987. Westford coordinates were fixed to their initial values. The IRIS terrestrial frame is made more nearly geocentric by applying the BTS 1985 translations (Carter et al., 1987).

CTS(GSFC) 87 R01. The data acquired since 1976 by the NASA Crustal Dynamics Project and since 1980 by the NGS POLARIS/IRIS programs. The terrestrial frame is defined by the position of the Haystack 37-M antenna and the BIH Circular D values for 1980 October 17 (Ma et al., 1987).

CTS(JPL) 83 R05. The coordinate data are from the JPL Time and Earth Motion Precision Observations (TEMPO) project, using the DSN radio telescopes. The reference frame solution is tied to the BIH on 20 December 1979 (Eubanks et al., 1984).

(ii) CTS (Lunar Laser Ranging). The coordinate data are from the JPL solution: SSC(JPL) 87 M01 containing four stations, two at Fort Davis, one at Haleakala (Maui), and one at Grasse. The
nominal planetary and lunar ephemeris DE121/LE65 was used in the reduction. The ephemeris uses the equator and equinox of B1950.0. It is on the dynamical equinox and has a zero point consistent with the FK5 catalogue (Newhall et al., 1987).

(iii) CTS (Satellite Laser Ranging). Two sets of station coordinates have been selected:

CTS(CSR) 86 L01. The solution is based on Lageos ephemeris from May, 1976, to September, 1986, using the model Lageos Long Arc 8511. The force model, referred to as the CSR 8511 system, adheres closely to the MERIT standards. The tectonic plate motion model AM1-2 of Minster and Jordan (1978) was used and the epoch of the derived station coordinates is 1983 January 1. The GM value is 398600.4404 km$^3$/s$^2$ (Schutz et al., 1987).

CTS(DGFI) 87 L01. The solution is computed from Lageos observations covering the period 1980 to end 1984 and based on five yearly solutions. By the rates of change of the yearly solutions, the station coordinates then were related to the same reference epoch 1984.0. The reference frame was defined by the three coordinates (longitude, latitude of Yaragadee (7090) and latitude of Wettzell (7834)) which were held fixed in the five solutions. The GM value is 3.98600448 E$^{+14}$ m$^3$s$^{-2}$, initial ERP series were from homogeneous BIH series and other constants from MERIT Standards (Reigber et al., 1987).

(iv) CTS (Doppler). Station coordinates are from DMA Doppler project SSC(DMA) 77 D01 solution, and other Doppler campaigns containing more than 100 station positions. They are determined in the NSWC9Z2 datum by point positioning using Precise Ephemerides.

Three comparisons have been performed to get an idea about the consistency of different solutions and relations between these solutions related to a same technique. Table 5 summarizes these different comparisons.

| Table 5 Transformation Parameters Between Different CTS Frames (Boucher and Altamimi, 1987) (the uncertainties are given in the second line) |
|---|---|---|---|---|---|---|---|---|
| CTS | $\delta_1$ m | $\delta_2$ m | $\delta_3$ m | $(c-I)10^{-6}$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | CS$^*/$RMS |
| NGS 87R01 - | 1.697 | -0.998 | 0.339 | 0'+003 | -0'+001 | -0'+001 | -0'+003 | 12 |
| GSFC87R01 | 0.006 | 0.006 | 0.007 | 0.001 | 0.000 | 0.000 | 0.000 | 1 cm |
| CSR 86L01 - | -0.007 | -0.026 | 0.074 | 0.015 | 0.013 | -0.009 | 0.125 | 37 |
| DGFI87L01 | 0.023 | 0.023 | 0.023 | 0.003 | 0.001 | 0.001 | 0.001 | 12 cm |
| CSR 86L01 - | -0.080 | 0.040 | 0.080 | 0.015 | 0.004 | -0.003 | 0.009 | 35 |
| CSR 85L07 | 0.024 | 0.023 | 0.022 | 0.003 | 0.001 | 0.001 | 0.001 | 11 cm |

*Number of collocated stations.
The first comparison is between two VLBI solutions CTS(NGS) 87 R01 and CTS(GSFC) 87 R01 containing 12 colocated stations. Note the 1 cm of rms issued from this comparison. The origin difference between the two solutions is due to the arbitrary choice of the VLBI origin in the definition of the terrestrial frame.

The second comparison is between two SLR solutions CTS(CSR) 86 L01 and CTS(DGFI) 87 L01 containing 37 colocated sites. In this case the rms is about 12 cm. Note also a rotation of 125 mas about the Z axis between the two solutions.

The last comparison is between the two last SLR solutions of CSR of 85 and 86 giving an RMS of about 11 cm. Note here that the scale factor has been decreased of about $1.5 \times 10^{-8}$ from 85 to 86 solution.

The slightly larger scatter (10 cm level) of SLR data is mainly explained by the mixture of good third generation stations (4 cm level) with some older ones (20 to 50 cm).

A combination of all above data has also been performed incorporating 51 colocated sites and making use of the plate tectonic absolute motion model AMO-2 derived from the global RM-2 model (Minster and Jordan, 1978).

Table 6 lists the transformation parameters of the individual systems with respect to a global one whose origin is constrained to that of JPL 87M01 (LLR) and CSR 86L01 (SLR), the scale to CSR 86L01 (SLR), and the orientation to NGS 87R01 (VLBI). Some conclusions about the origin, scale and orientation of the individual CTS's with respect to the global one: Knowing that the origin of the adjusted system is from SLR and LLR, the origin of all VLBI solutions remains arbitrary. The level of consistency of the scale factor is $10^{-8}$ for the different solutions. Some variations for VLBI and LLR solutions are due to a relativistic bias in the definition of the terrestrial system (Hellings, 1986; Boucher, 1986). The orientation of the individual terrestrial systems is usually realized through BIH values. The differences in orientation of the different solutions are arbitrary and of some mas level.

Tables similar to Table 6 are published in the Annual Reports of the BIH (or IERS) giving the transformation parameters for all CTS techniques participating in the work of the Service, as per Section 3.2.
Table 6 Transformation Parameters from the Individual 1984.0 CTS Systems to the ‘Global’ CTS
(Boucher and Altamimi, 1987) (the uncertainties are given on the second line)

<table>
<thead>
<tr>
<th>CTS</th>
<th>$\delta_1$ m</th>
<th>$\delta_2$ m</th>
<th>$\delta_3$ m</th>
<th>$(c-1)10^{-6}$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
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<tr>
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<td>0.002</td>
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<tr>
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<td>0.006</td>
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</table>

32
4. MODELING THE DEFORMABLE EARTH

In this section we will try to highlight the modeling problems associated with the components of transformation between the CIS and CTS mentioned in Section 3.

4.1 Precession (P)

At the XVIth General Assembly in Grenoble in 1976, the IAU adopted a new speed of general precession in longitude of 5029.0966 per Julian century at the epoch J2000.0 (JED 2451545.0) effective 1984. This value when referred to the beginning of the Besselian year B1900.0 is 5026.767 per tropical century, which may be compared to the previously adopted value of 5025.64 per tropical century at B1900.0. The change was calculated by Fricke (1977) from proper motions of stars in the systems GC, FK3, N30 and FK4. From the results, a correction to Newcomb’s lunisolar precession in longitude was recommended. This value combined with a correction to Newcomb’s planetary precession, due to the improved 1976 IAU values of planetary masses, resulted in the above new precessional constant. Expressions to compute the effect of precession from one epoch to another were developed by Lieske et al. (1977); and the usual equatorial parameters, \( z, \theta, \zeta_0 \), to be used in the precession matrix (Mueller, 1969),

\[
P = R_3(-z) R_2(\theta) R_3(-\zeta_0)
\]

(7)

to and from the epoch J2000.0 were computed by Lieske (1979). This transformation is the currently adopted one between the CIS (say, the FK5 at J2000.0) and an interim ‘Mean Equator and Equinox Frame’ of some date (see Section 4.52).

4.2 Nutation (N)

The nutation story is much more complex. First of all, the nutation matrix is (Mueller, 1969)

\[
N = R_1(-\epsilon - \Delta \epsilon) R_3(-\Delta \psi) R_1(\epsilon)
\]

(8)

where \( \epsilon \) is the obliquity of the ecliptic, \( \Delta \epsilon \) is the nutation in obliquity, and \( \Delta \psi \) the nutation in longitude, computed from a certain theory of nutation. This matrix allows transformation from the aforementioned interim mean frame of date to the (also) interim true frame of the same date. This
part is clear and without controversy. The complexities are in the agreement reached on the theory of nutation when computing the above parameters. Kinoshita et al. (1979) give an historical review:

‘In astronomical ephemerides, nutation has been computed until now by the formulae which were given by Woolard (1953). The coefficients of the formulae are calculated assuming that the Earth is rigid. However, it has been found in recent analyses of observations ... that some coefficients of actual nutations are in better agreement with values calculated by the nonrigid Earth theory.

‘Moreover, Woolard (1953) gave the nutation of the axis of rotation. Therefore, a small and nearly diurnal variation appears in the latitude and time observations, which is the so-called dynamical variation of latitude and time, or Oppolzer terms. In the global reduction of latitude and time observations, such as polar motion or time services, the Oppolzer terms have been until now removed from the data at each station (cf. BIH Rapport Annuel 1977, p. A-3) or counted out as a part of the nonpolar common z and τ-terms (IPMS Annual Report 1974, p. 11). On the other hand, Atkinson (1973) pointed out that if the (forced) nutation of the axis of figure is calculated instead of rotation axis, such a complicated treatment becomes unnecessary.

‘Considering these situations, the IAU investigated the treatment of nutations, together with the system of astronomical constants which should be used in new ephemerides, and set up the “Working Group of IAU Commission 4, on Precession, Planetary Ephemeris, Units, and Time-Scales.” The results by the Working Group are given in the report of Joint Meeting of Commissions 4, 8, and 31, in Grenoble, 1976 (Duncombe et al., 1976). In the report, the proposal by Atkinson is adopted, and the formula for computing the (forced) nutation of figure axis is shown clearly and in detail, by using the equation-numbers given by Woolard (1953). However, the amendments of coefficients taking account of the nonrigidity of the Earth have not been adopted. In regard to this problem, it was noted that there should be a possibility of making further amendments in Kiev Symposium ....

‘At the IAU Symposium No. 78 in Kiev in 1977, the problem with the nonrigid values of nutation was discussed, and a series of new values were recommended which seemed to be based on Molodenskij’s nonrigid theory. In the Symposium, however, it was recommended that the axis for which the nutation should be computed was the axis of rotation. This recommendation reversed the resolution given at Grenoble.

‘In accordance with the resolution at the Kiev Symposium, an “IAU Working Group on Nutation under Commission 4” was set up and is investigating these two problems, in order to prepare a fully documented proposal for the next IAU General Assembly in Montreal in 1979. In the second draft of the Working group circulated on Nov. 16, 1978, the following conclusions are reported: (1) as for the axis to be referred, the Grenoble resolution is still valid, and (2) as for the coefficients of nutation series, the value in which the nonrigidity of the Earth is taken into account should be adopted as a working standard of astronomical observations. In the draft, a table of nutation series is given, and the numerical values in the table are based on the rigid theory by Kinoshita (1977), with use of IAU (1976) System of Astronomical Constants, and are modified by Molodenskij’s nonrigid theory (Molodenskij, 1961).’
As we understand it, the Kinoshita theory above is for the nutation of the axis of maximum moment of inertia of the 'mean shape of the elastic mantle' (briefly, 'mean axis of figure of the mantle'). To add to the history, after the above-quoted Working Group Report was circulated, a new proposal was made by J.M. Wahr and M.L. Smith of CIRES that it would be preferable to adapt the nonrigid earth results of Wahr (1979) for the earth model 1066A developed by Gilbert and Dziewonski (1975). This model is a rotating, elliptically stratified linearly elastic and oceanless earth with a fluid outer core and a solid inner core. The nutations are computed for the 'Tisserand mean figure axis of the surface', which is also a mean mantle fixed axis (Wahr, 1979). The IAU in Montreal in 1979 considered both proposals and opted for the Kinoshita et al. (1979) series. A few months later in December, 1979, the IUGG in Canberra, in Resolution No. 9 addressed to the IAU, requested reconsideration in favor of the Wahr model. The IAU subsequently adopted Wahr’s series as the IAU 1980 Theory of Nutation.

It should be pointed out that regardless of the fact that in geodetic or geodynamic applications we are only concerned with the total transformation SNP, it is of scientific importance to understand clearly the definition of the interim true equator and equinox frame of date, more specifically, the exact definition and the desirability (from the observability point of view) of the axis for which the nutation is computed.

In order to simplify the discussion, let us start with the rigid model. The motion of each of the axes, i.e., the axis of figure (F) (maximum moment of inertia), of the angular momentum (H), and the instantaneous rotation axis (I) are described by differential equations. If we want to refer to one of these axes we have to consider the complete solution of the differential equations, i.e., the free solution and the forced solution components. Confusion can arise if one refers to only one solution component (forced or free), but still calls it axis of figure, instantaneous rotation axis, etc. It is mandatory to point out which solution component one refers to. Neglecting to do so has been the reason for the by now classical confusing controversy about the Atkinson papers, though Atkinson (1975, p. 381) clearly states: ‘Accordingly, when we speak of computing the nutations for either axis, we mean here computing the forced motion only, excluding the appropriate fraction of the noncomputable Chandlerian wobble.’ Unfortunately, he, and others as well, then continue to use the term 'axis of figure' sometimes in the sense of the axis of maximum moment of inertia and at other times in the sense of the forced motion of the axis of figure.

A remark concerning the ‘Eulerian pole of rotation’ (E₀) as given by Woolard seems in order also. Quoting once again Atkinson (1976):

‘The wording of the resolution on nutation, and the notes on it, which have been circulated by the Working Group, avoid all explicit mention of the axis of figure, even
though they specify that the coefficients which Woolard gives for that axis shall be inserted, and they refer to the "Eulerian pole of rotation" although this cannot ever, in principle, coincide with the celestial pole and really has no more direct connection with the observations than is shown for it in [his] Fig. 2, i.e., none at all.'

The difference between the Eulerian pole of rotation (E₀) and the pole which Atkinson talks about is due to a homogeneous solution component. (E₀) is obtained from the complete solution of (I) by subtracting the periodic diurnal body-fixed motions of (I).

Consequently, the point E₀ has no periodic motion with respect to the crust, but it does have such a motion in space which is exactly the free nutation. Although this spatial motion is conceptually insignificant considering the observation technique (fundamental observations at both culminations), one gets another point, which is called the (true) Celestial Pole (C) in (Leick and Mueller, 1979), by subtracting the forced body-fixed motions of (H) from the complete nutation set of (H). The thus obtained axis (C) has no periodic diurnal spatial motion because the homogeneous solution of the angular momentum (H) is constant (zero). Equivalently, one can say that the nutations of (C) correspond to the forced solution of the axis of figure (rigid case, of course). This is the pole which Atkinson talks about and which is called (mistakenly) the 'mean axis of figure'. There is no doubt that this is the point to which the astronomical observations as well as lunar laser ranging refer, and the nutation should be adopted for this point. As for terminology, the IAU in 1979 named this (C) pole appropriately the Celestial Ephemeris Pole because its motion characteristics, i.e., no periodic diurnal motion relative to crust or space, have always been associated with the concept of the celestial pole. It would be preferred that the word 'figure' be dropped entirely for several reasons. First, one intuitively associates the axis of figure with the one for which the moment of inertia is maximum. This is true for the (C) only if the free solution (Chandler) is zero. But this is, generally, not the case. Second, the conceptual definition of (C) can easily be extended to elastic models or models with liquid core (the IAU 1980 case). Moreover, in order to emphasize that the observations take place on the earth surface, it would be useful to denote the actual pole accessible to the fundamental observation techniques by another designation, e.g., (CO), similarly to UT0. The 'O' would indicate that the nutations of this pole can in principle be determined only from observations because of the lack of a perfect earth model. Any nutation set based on a model is only an approximation to the nutations of the (CO). In this sense the rigid earth nutations of (I), (H) or (F') are all equivalent. Each of these nutations defines its own pole which has a diurnal motion around the (CO). The purpose of the measuring efforts is to find the corrections to the adopted set of nutations in order to get those of the (CO), the only pole which is observable.
Some have suggested the term ‘zero excitation figure axis’ for what is called above the (CO). The term ‘zero excitation’ would not reduce the confusion. The spatial motion of this axis is computed by adding Atkinson’s terms to Woolard’s series, but this is equivalent to the forced motion of the axis of figure (rigid case). The observed motion of the (CO) relative to the crust only appears as a motion of zero excitation (free motion) at the first sight. Since the conceptual observation time of one position determination is one day, the observed position of the (CO) will always include effects due to oceans, atmospheric mass redistribution, etc., i.e., the geophysical nutations. These motions are better known as the annual polar motion and the subharmonics. Therefore, the zero-excitation pole is not directly observable. On the other hand, the concept of the (CO) can still be used in this case since it is by definition the pole which has no periodic diurnal motions relative to the crust or to space.

There is also the common offset of both the rotation axis and the (CO) caused by the tidal deformation (McClure, 1973). This is an offset of (I) and (CO) relative to (H) for the perfectly elastic model as compared with the rigid model. We have to remember, again, that the observations refer to the (CO). Therefore, any nutation correction which is derived from observations (based on an adopted set of nutations) will automatically give the corrections to the (CO). Consequently, there is no need for a special consideration of this possible separation, at least not for those harmonic motions whose amplitudes are derived from observations. In fact, the analysis of the observed fortnightly term seems to contradict somewhat the predicted amplitude for the perfectly elastic model.

From the above discussion, it also seems clear that ideas advocating the adoption of nutations for the axis of angular momentum violate the concept of observability. It is true that the direction of (H) in space is the same for the rigid, elastic, or any other reasonable earth model. But this property is not of much interest to the astronomer or geodesist who tries to determine the orientation of the earth. It is conceptually simpler to refer to an axis which is observable. More on this in Section 4.4.

Returning now to the problem of the IAU 1979/1980 adopted sets of nutations, from the geodetic point of view there seemed to be little difference whether the Kinoshita series was retained or the Wahr set was adopted. Using more and more realistic earth models is certainly appealing. On the other hand, severely model-dependent developments are liable to change as models improve (see Section 4.5I). Improving the nutation series from geodetic observations leads to earth model improvements, one of the main goals in geophysics. This is one of the dichotomies between geodesy and geophysics.
4.3 Earth Rotation (S)

The two components of the S matrix (Mueller, 1969),

\[
S = R_2(-x_p) R_1(-y_p) R_3(GAST)
\]  

are the rotational angle of the first (X) axis of the CTS with respect to the first axis of the interim true equator and equinox frame of date, measured in the equator of the Celestial Ephemeris Pole (or whatever is defined in the N matrix), also known as the Greenwich Apparent Sidereal Time (GAST), and the polar motion coordinates (x_p, y_p) referred to the same pole and the Z axis of the CTS.

In this connection it should be mentioned that some authors prefer a different 'true' frame, which would have 'no rotation' about the Z axis (Guinot, 1979; Murray, 1979; Kinoshita et al., 1979). It is in such an interim frame where, for example, a nutational theory can be conveniently developed, or satellite orbits calculated (Kozai, 1974). Such a frame can be obtained from the CIS by a modified NP transformation, where

\[
N = R_1 (-\Delta \epsilon \cos M + \Delta \psi \sin \epsilon \sin M) R_2 (\Delta \psi \sin \epsilon \cos M + \Delta \epsilon \sin M)
\]

and

\[
P = R_3 (-z + M) R_2(\theta) R_3 (-\xi_0)
\]

where M is the precession in right ascension.

In this case the rotation of CTS about the Z axis (\(\phi\)) is the Apparent Sidereal Time from which the general precession and nutation in right ascension are removed. What is left, thus, is the rotational angle of the X axis of the CTS directly with respect to that of the CIS. Such a definition of the sidereal angle would, of course, necessitate the redefinition of UT1, a possibility for controversy. It should be noted also that the above transformation is independent of the ecliptic, a preference of many astronomers. More on this in Section 4.5.

Here there is not very much modeling that can be considered really useful. Of course, the rotation rate of the earth could be modeled as constant and possibly in the UTC scale. This would then mean that observed departures could immediately be referenced to that scale, a current practice. If one really wanted to go overboard, polar motion could also be modeled with the Chandlerian cycle of, say, 428 days and a circular movement of radius 0'15, centered at the Z axis of the CTS. More complex models may be developed (e.g., Markowitz, 1976, 1979), but since there are no valid physical concepts yet for the excitation of the amplitude of the Chandler motion, such modeling would not serve much purpose.
Reviewing Sections 4.1–4.3, one could conclude that if the phenomena of precession, nutation and polar motion as well as the concepts of the ecliptic and the vernal equinox can be disconnected from the realization of a reference frame, and be regarded as simply describing various aspects of the Earth's complicated motions, then a great simplification will have been achieved. Of course all of the above phenomena and concepts are basic, and a knowledge of them is absolutely necessary. This knowledge will continue to be supplied by the classical, dynamical observations, radio astrometry and pulsar observations. However, it is now possible to consider these items in their proper context and to define a reference frame which is independent of them. Such independence will benefit not only the reference frame, but also aid in the study of the very phenomena from which the concept of a reference frame will have been freed. Essentially, observations will have been decoupled from the observing platform. As a result of this, the accuracy of the reference frame will become primarily dependent upon the precision and accuracy of the underlying measurements, and will have a minimal, noncritical dependence upon any companion theories. More on this in Section 4.53.

4.4 Deformations (L′)

The deformations which reasonably can be modeled at the present state of the art are those due to the tidal phenomena and to tectonic plate movements.

4.41 Tidal Deformations. Tides are generated by the same forces which cause nutation; thus models developed for the latter should be useful for the former. One would think that for earth tides it may not be necessary to use the theories based on the very sophisticated earth models: the amplitude of the phenomena being only around 30 cm, an accuracy of 3% should be adequate for centimeter work. This should be compared, for example, with the accuracy of the Wahr nutation model claimed to be at the 0.3% level. However, the tides and nutations differ in one important respect. The nutations hardly depend upon the elasticity and are affected only slightly by the liquid core (this is one reason why modern theories such as those of Wahr and Kinoshita give only slightly different results). Thus, except perhaps for the largest terms, one can depend upon theory when dealing with nutation. The tides, on the other hand, depend intimately upon the internal properties of the earth, and one must use tidal theories with caution (Newton, 1974). Additional problems are handling the transformation of the potential into physical displacements and on the calculations of regional (ocean loading) or local tidal deformations.

As far as the transformation of the tidal potential into displacement is concerned, the traditional way to do this is through the Love numbers for the solid effect and through 'load' numbers for
ocean loading. These numbers, however, are spherical approximations which, for the purely elastic earth, are global constants. For more sophistication, elliptic terms can be added, but they will change the results by 1-2% only. A liquid core model produces resonance effects, which will result in a frequency dependency. The actual numbers representative for a given location can be determined only through in situ observations, such as gravity, tilt, deflections, which are all sensitive to certain Love number combinations and frequencies. Difficulties in this regard include the frequency dependence of the Love number. For example, the Love number h for radial (vertical) displacement can be determined locally from combined gravity and tilt meter observations by the analysis of the O1 tidal component, but the real radial motion of geodetic interest is influenced by the M2 and other semidiurnal tidal components.

Tidal loading effects have been successfully computed by Goad (1979) using the 1° square Schwiderski (1978) M2 ocean tide model. Global results show agreement with gravimetrically observed deformation on the 0.5 μgal (5×10^{-10}) level. From this it would seem that with good quality ocean tide models and with proper attention to the frequency dependence, this problem is manageable.

Suitable equations for displacement, gravity change, deflection change, tilt and strain calculations due to tides may be found in (Melchior, 1978; Vanicek, 1980) and in (Wahr, 1979) for the elliptic case.

As a conclusion one can reasonably state that the global and regional station movements due to tides can be estimated today within centimeters. Local effects, however, can be sizable and unpredictable, and therefore they are best determined from in situ observations. Thus most of the tidal effect in fact can and should be removed from the observations.

**4.42 Plate Tectonic Mass Transfer.** The concept that the earth lithosphere is made up of a relatively small number of plates which are in motion with respect to each other is the central theme of global plate tectonics. The theory implies the transfer of masses as the plates move with velocities determined from geologic evidence (see, e.g., Solomon and Sleep, 1974; Kaula, 1975; or Minster and Jordan, 1978). Material rises from the asthenosphere and cools to generate new oceanic lithosphere, and the lithospheric slabs descend to displace asthenospheric material (see, e.g., Chapple and Tullis, 1977). A good example of how such a theory can be used to estimate the vertical motions of observatories located on the lithosphere (in terms of changes in geoid undulations) is given in (Larden, 1980), based on specific models constructed in (Mather and Larden, 1978). The results indicate that changes in the geoid can reach 150 mm/century.
Horizontal displacements can be estimated from the plate velocity models mentioned directly with certain possible amendments (Bender, 1974).

### 4.43 Other Deformations

If one wants to carry the modeling further, it is possible to estimate seasonal deformations due to variations in air mass and groundwater storage, for which global datasets are available (Van Hylckama, 1956; Stolz and Larden, 1979; Larden, 1980). A more esoteric effect would be the expansion of the earth (e.g., Dicke, 1969; Newton, 1968). The rate of possible expansion is estimated to be 10 - 100 mm/century.

One could continue with other modeling possibilities, but there is a real question on the usefulness of modeling phenomena of this level of magnitudes and uncertainties. As a general philosophy, one could accept the criteria that modeling should be attempted only if reliable and global data is available related to the phenomena is question, and if the magnitudes reach the centimeter per year level or so.

One last item which should be brought up is the fact that the issue of referencing observations and/or geodynamic phenomena is not exhausted by the establishment of reference frames of the Cartesian types discussed in this paper. An outstanding issue is still the geoid as a reference surface. Though it is true that three-dimensional advanced geodetic observational techniques do not need the geoid as a reference, there are still others, such as spirit leveling, which are used in the determination of crustal deformations in the local scale. In addition, the geoid is needed to reference gravity observations on a global scale (one should remember that a 1 cm error in the geoid corresponds to a 3 µgal error in the gravity reduction, which is (or soon will be) the accuracy of modern gravimeters). Further, in connection with the use of satellite altimetry for the determination of the departures of sea surface topography from the equipotential geoid (a topic of great oceanographic interest), there is a requirement for a geoid of at least 10 cm accuracy. The determination of such a geoid globally, or even over large areas, is a very difficult problem, which, however, is not the subject of the present paper.

### 4.44 Current (1988) Practice

Some of the above effects can be modelled with good accuracy. A review of current models can be found in MERIT Standards (Melbourne, 1983). Two models are of particular interest for terrestrial frames (Boucher, 1987):

(i) The solid earth tide correction for ground station positions. Especially important is the vertical component

\[ \Delta h = -0.121 \left( \frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) \text{ meters}, \]
the permanent tidal deformation, where $\phi$ is the latitude of the station.

(ii) Tectonic plate motion correction for the horizontal components of ground station position: The usual ones, such as the series of Minster-Jordan models, are defined through a set of angular velocity vectors $\Omega_p$, one for each plate, and expressed in the terrestrial system, so that the velocity of a point of coordinate $\mathbf{X}$ is

$$\dot{\mathbf{X}} = \Omega_p \wedge \mathbf{X}$$

Two absolute motion models are usually used: AMO-2, derived from the RM-2 model by applying a 'no global rotation' condition; AM1-2, which minimizes the motion of a set of hot spots, also derived from RM-2 (Minster and Jordan, 1978).

AMO-2 depends only on the adopted plate boundaries, whereas AM1-2 depends on the selection of the hot spots which are more subject to uncertainties. On the other hand, AMO-2 corresponds to the type of law of evolution one wants to give to terrestrial frames and has been consequently adopted by MERIT Standards (Update 1, December 1985). Nevertheless, AM1-2 leads to a system linked to the mantle which is needed to express a geopotential model without secular variations due to a residual rotation of the system. It is therefore favoured by groups which perform dynamical analysis of satellite tracking data.

4.5 Recent Developments

4.51 Expected Changes in the Adopted Series of Nutation. Recent analysis of modern highly accurate observations (e.g., VLBI) indicates significant departures from the IAU 1980 nutation series. None of the existing theories based on various Earth models can adequately explain these departures from Wahr's model. Apparently more efforts are required both in theory and in observations to arrive at a resolution. In the interim, the corrections in Table 7, based on (Herring et al., 1986), and further analysis are being recommended until such time when adequate theoretical coefficients can be determined. See also (Carter, 1988; Sovers and Edwards, 1988; and Kinoshita, 1988).

Assuming that the CTS is to be maintained unchanged, corrections to the nutation terms in longitude ($\Delta \psi$) and obliquity ($\Delta e$) would theoretically change the polar motion components and GAST, utilized in the transformation equation (1), i.e., in the matrix $S$, as follows (Zhu and Mueller, 1983):

42
\[ \Delta x_p = \delta \Delta \varepsilon \sin \theta + \delta \Delta \psi \sin \varepsilon \cos \theta \]
\[ \Delta y_p = -\delta \Delta \varepsilon \cos \theta + \delta \Delta \psi \sin \varepsilon \sin \theta \]
\[ \Delta \text{(GAST)} = \delta \Delta \psi \cos \varepsilon \]  

(12)

where \( \theta \) is the sidereal time. As it is seen, the theoretical effects on polar motion are diurnal terms (\( \delta \Delta \psi \) and \( \delta \Delta \varepsilon \) being long periodic).

Table 7  Corrections to the Long-Period Terms of the IAU 1980 Nutation Series

<table>
<thead>
<tr>
<th>Period (years)</th>
<th>( \Delta \varepsilon ) In Phase (mas)</th>
<th>( \Delta \varepsilon ) Out of Phase (mas)</th>
<th>( \Delta \psi ) In Phase (mas)</th>
<th>( \Delta \psi ) Out of Phase (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6</td>
<td>2.15</td>
<td>1.81</td>
<td>-5.55</td>
<td>3.37</td>
</tr>
<tr>
<td>9.3</td>
<td>-0.24</td>
<td>0.0</td>
<td>1.20</td>
<td>0.0</td>
</tr>
<tr>
<td>1.0</td>
<td>2.08</td>
<td>-0.24</td>
<td>5.23</td>
<td>-0.61</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.41</td>
<td>-0.47</td>
<td>1.02</td>
<td>-1.18</td>
</tr>
<tr>
<td>0.037</td>
<td>0.32</td>
<td>0.0</td>
<td>-0.81</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\( \Delta \) precession constant -2.7 mas/yr

4.52 Expected Change in the Constant of Precession. Modern, LLR and VLBI, observations also indicate a possible correction of \(-0.2\) to \(-0.3\)/Julian century to the IAU 1976 constant of precession. This correction is uncertain due to the relatively short time span of available observations.

Williams and Melbourne (1982) and Zhu and Mueller (1983) investigated the effects of such a change. The effect on polar motion is a diurnal periodic term with an amplitude increasing linearly in time; on the GAST it is a linear term.

4.53 Intermediate Reference Frame Issues. The complete transformation from the CIS to the terrestrial frame CTS is given by eq. (1). In geodetic applications generally only the complete transformation SNP is needed. Changes in the ‘intermediate’ reference frame defined by the NP transformation must either by ‘absorbed’ in the S matrix by changing appropriately \( x_p, y_p \) and GAST (UT1), or the CTS must change its orientation. There are seven options to choose from, and they are a matter of preference (Zhu and Mueller, 1983). One of these which would neither change the CTS orientation nor the UT1 is probably preferred by geodesists. It would however change the definition of the Greenwich Mean Sidereal Time by referring it to a point on the equator insensitive to precession. As mentioned in Section 4.3, a similar option has been advocated by Guinot (1979) during the past decade but for different reasons. A recent proposal by
Capitaine and Guinot (1988) is based on the observation that the classical definition of GAST representing the rotation of the Earth (i.e., CTS) is not satisfactory mainly for two reasons:

(i) It is referred to the true equinox of date which is an inadequate and unnecessary intermediate reference point because modern observations of the CTS’s orientation in space (especially VLBI) are practically insensitive to the orientation of the ecliptic and consequently to the position of the equinox.

(ii) The presently adopted expression converting GAST to UT1 (Aoki et al., 1982) neglects some cross-terms between precession and nutation which are of the order of 0.001 and should now be considered.

The definition advocated would thus be better adapted to the new methods of observation and would provide an accuracy of the order of 0.0001. It would also result in a new definition of Universal-Time which would remain valid even if the adopted model for the NP transformation is revised (see also (Capitaine et al., 1986)). The proposal is not without its critics. See (Aoki and Kinoshita, 1983; Aoki, 1988).

Related to the above issue is the definition of the third axis of the intermediate frame as defined by the transformation model NP, specifically, by the adopted theory of nutation (see Section 4.2). This pole, the Celestial Ephemeris Pole (CEP), conceptually has no diurnal motion with respect to an Earth-fixed or a space-fixed reference frame. Some of the modern observational techniques, however, are not very sensitive to this axis and, in fact, on the level of 0.001 accuracy, define a variety of technique dependent conventional poles. Capitaine et al. (1985) and Capitaine (1986) point out that clarification of this issue is necessary in order to intercompare and interpret polar motion coordinates determined at the level of 0.001 accuracy, by means of a variety of techniques ranging from VLBI to superconducting gravimetry.
5. THE INTERNATIONAL EARTH ROTATION SERVICE

5.1 The MERIT-COTES Programs

The acronyms MERIT and COTES refer to two international programs that were started independently, but which developed together. MERIT refers to an international program to monitor the earth's rotation and intercompare the techniques of observation and analysis with a view to making recommendations about the form of a new international service. On the other hand, the objective of the COTES program was to provide a basis for recommendations on the establishment and maintenance of a new conventional terrestrial reference system for the specification of positions on or near the earth's surface. The two programs were linked when it became clear that the observational campaign planned for MERIT and the new earth rotation service would provide results that could be used for COTES. In particular, in order to determine the earth rotation parameters to high accuracy, it is necessary to establish the positions of the observing sites (or 'stations') in a worldwide network that provides a suitable basis for a new terrestrial reference system. The observational data and results that have been obtained in the course of these programs have been collected together for further analysis and for use in current and future scientific studies and practical applications.

Project MERIT was conceived in 1978 at IAU Symposium No. 82 on 'Time and the Earth's Rotation.' The Symposium recommended the appointment of a 'working group to promote a comparative evaluation of the techniques for the determination of the rotation of the earth and to make recommendations for a new international program of observation and analysis in order to provide high quality data for practical applications and fundamental geophysical studies.' Two years later, in 1980, the participants in IAU Colloquium No. 56 on 'Reference Coordinate Systems for Earth Dynamics' recommended the setting up of a working group 'to prepare a proposal for the establishment and maintenance of a Conventional Terrestrial Reference System.' Information discussions at the First MERIT Workshop in 1981 were followed eventually by the merging of the two groups and the production of a Joint Summary Report (Wilkins and Mueller, 1986). This report describes briefly the development of the programs of observation and analysis and gives recommendations for new terrestrial and celestial reference systems and for the setting up of a new International Earth Rotation Service (IERS); this report also includes references to earlier reports.
that describe the techniques used, the organizational arrangements and the programs of the activities, and that give the principal results and references to relevant papers.

The MERIT and COTES programs have been very successful in stimulating the use and development of new techniques of observations using laser ranging and radio interferometry; they also led to improvements in the results from optical astrometry and the Doppler (radio) tracking of satellites, which were in regular use before 1978. Coordinators were appointed for each technique and for certain associated activities, such as the operation of a Coordinating Center for the combination and dissemination of results, the preparation of MERIT Standards, and the collocation of equipment of different techniques.

The quantities measured by each of the techniques that were used in the programs are as follows:

Doppler tracking of satellites: The Doppler shifts (range-rates) in the radio transmissions from Transit navigation satellites.

Satellite laser ranging: The time for pulses of laser light to travel to and from geodetic satellites carrying retroreflectors.

Lunar laser ranging: Time of flight for pulses of laser light to travel to and from retroreflectors on the surface of the moon.

Optical astrometry: Directions to stars measured with respect to local reference frames.

Connected-element radio interferometry, and

Very long baseline radio interferometry: Differences between the travel times of the radio emission from quasars to two or more radio telescopes.

Organizational arrangements for the regular transmission and processing of data already existed for optical astrometry and Doppler tracking, but for the other techniques it was necessary to set up both operational centers and analysis centers. The operational centers coordinated the observations, collected the observational data, computed earth rotation parameters on a rapid-service basis from 'quick-look data', and distributed the observational data (perhaps after some processing) to the analysis centers, which determined both earth rotation parameters and station coordinates from all the available data.

There were several designated periods when all stations were requested to make observations and send them as quickly as possible to the operational centers. The first was the MERIT Short Campaign from 1 August to 31 October 1980. This was primarily a test of the technical and
organizational arrangements, but it also produced much valuable data and showed clearly the potential of the new techniques. The MERIT Main Campaign covered the 14-month period from 1 September 1983 to 31 October 1984 and included the first COTES Intensive Campaign, which ran from 1 April until 30 June 1984. The data were analyzed independently at two or more analysis centers for each technique, and many excellent series of earth rotation parameters and sets of station coordinates were obtained. These data are still being studied to determine, for example, the systematic differences between the reference systems of the various techniques. The results have established beyond doubt the very close correlation between the short-period variations in the length of day and in the angular momentum of the atmosphere. The pole of rotation has been shown to move much more smoothly than had earlier been thought, but there is still controversy about the sources of excitation of the 14-month term in the motion.

5.2 The International Earth Rotation Service

By the end of the MERIT Main Campaign it had become clear that laser ranging and radio interferometry were able to provide more precise estimates of polar motion, universal time and length of day than could optical astrometry and the Doppler tracking of satellites, which were the prime contributors to the international services in 1978. This conclusion has since been substantiated by the more detailed analyses of the data that have been reported at the MERIT Workshop and Conference held at Columbus, Ohio, on 29 July - 2 August 1985 (Mueller, ed., 1985). The accuracy of the regular determination of the coordinates of the poles by SLR and VLBI is about 5 cm, compared with 30 cm by optical astrometry and Doppler tracking, while for UT and excess length of day the accuracy is about 0.2 ms and 0.06 ms, compared with 1 ms and 0.2 ms.

It must be realized, however that other factors besides precision had to be taken into account before recommendations about the future international services could be formulated. Perhaps the most important factor was whether it is reasonable to expect that the organizations concerned are likely to continue to make and process observations at an appropriate level and to make the results available to the international community without restriction. The MERIT Main Campaign was a period of special activity, and it cannot be assumed that any technique would provide results of the same high quality (as judged by the combination of precision, accuracy, frequency, reliability and promptness) on a long-term basis.

The International Latitude Service was initially set up a set of five dedicated stations, but it was eventually replaced by the International Polar Motion Service which relied on receiving data from a much larger number of instruments which provided local services and data for other scientific purposes as their prime justification. It is to be expected that any new International Earth
Rotation Service will also have to depend largely on the use of observations and results that are obtained for other national and international programs.

In particular it must be recognized that an important application of the Service will be the establishment and maintenance of the new conventional terrestrial reference system. The permanent stations used for monitoring earth rotation will comprise a primary geodetic network of large scale and high precision that will be densified, partly by the use of mobile systems using the same techniques, but mainly by the use of other geodetic techniques, such as the use in radio interferometric mode of signals for navigation satellites.

The choice of the techniques to be used in the new service depends on the subjective evaluation of many factors and not merely on a comparison of the potential quality of the determination of each rotation parameters. Although it is conceivable that a single VLBI network could provide an adequate international earth rotation service, the general conclusions of the discussions in the MERIT and COTES working groups is that the new service should be based on both laser ranging and VLBI and should also utilize any other appropriate data that are made available to it.

The three recommendations given in Appendix 1 were adopted at a joint meeting of the MERIT Steering Committee and the COTES Working Group that was held at Columbus, Ohio, on 3 August 1985. Earlier drafts had been subject to critical review at the MERIT Workshop on 30 July and by interested participants in the Conference on Earth Rotation and Reference Systems held 31 July to 2 August. The joint meeting also adopted a draft resolution for consideration by a Joint Meeting of the IAU Commissions 19 and 31 on 22 November 1985 during the XIXth General Assembly of the IAU at New Delhi. Amended versions of this resolution were adopted by the Joint Meeting and subsequently by the Union on 28 November 1985. A further recommendation concerning the assignment of responsibility within the IAU for matters relating to the celestial and terrestrial reference systems was adopted by the MERIT/COTES meeting on 3 August and served to stimulate a discussion within the IAU, but no decision was announced.

The final version of the IAU resolution on the MERIT/COTES program and recommendations is given in Appendix 2. In effect the resolution endorsed this report and the principal recommendations on concepts, organization and interim arrangements. As a consequence the MERIT and COTES Working Groups were replaced by a Provisional Directing Board for the new International Earth Rotation Service which was to come into operation on 1 January 1988. The IAU resolution was endorsed by the Executive Committee of the International Association of Geodesy in March, 1986 (Mueller and Wilkins, 1986). The recommendations of the Provisional
Directing Board were considered and adopted by the IUGG during its XIXth General Assembly in 
Vancouver, B.C., in August, 1987 (Appendix 3).

With this last action, after ten years of preparation the new International Earth Rotation Service 
became a reality.

Organization of the Service. For each technique of observation (VLBI, SLR and LLR), 
prospective host organizations were invited to submit proposals for participation in one or more of 
the following ways:

- as a coordinating center,
- as an observing station or a network of stations,
- as a data collection (and distribution) center for quick-look and/or full-rate observational data. 
  Such a center could, if appropriate, also process the data to form normal point data for use in 
  analyses, or the task could be carried out by separate centers,
- as a quick-look operational center that would provide rapid service results,
- as a full-rate analysis center that would determine ERP's, station coordinates and other 
  parameters to a regular schedule.

Several of these activities might be carried out by one center, and the actual organization would 
differ according to the number of observing stations and networks and to the nature of the 
processing required. There will be no need for associate analysis centers in the formal structure, 
although it is expected that many groups will wish to analyze data provided by the Service. Offers 
of the deployment of mobile systems for use in improving the terrestrial reference system would be 
welcomed.

The principal tasks of the Central Bureau are specified in Recommendation B in Appendix 1, 
and some of them would be carried out by sub-bureaus. There is a need also for separate centers 
for relevant data from other fields, such as data on atmospheric angular momentum (AAM) and 
appropriate geodetic data (e.g., GPS results). The former might prove to be useful in predicting the 
variations in the rate of rotation of the earth, while the latter would be useful in the establishment 
and maintenance of the terrestrial reference system.

Kovalevsky and Mueller in their 1980 review of the Warsaw Conference listed a number of 
actions required to assure that the reference system issue be resolved "early and that the uniformity 
is assured by means of international agreements." There were the following:
Re CTS:

1. Selection of observatories whose catalogue will define the CTS.
2. Initiation of measurements at these observatories.
3. Recommendation on the observational and computational maintenance of the CTS (e.g., permanent versus temporary and repeated station occupations, constraints to be used).
4. Decision on how far and which way the earth deformation should be modeled initially.
5. Plans and recommendations for the establishment of new international service(s) to provide users with the appropriate information regarding the use of the CTS frame.

Re CIS:

6. Selection of extragalactic radio sources whose catalogue will define the CIS.
7. Improvement of the positions of these sources to a few milliseconds (arc).
8. Final decision on the IAU series of nutation and to assure that it describes the motion of the Celestial Ephemeris Pole.
9. Early completion of the FK5 and revision of astronomical equations due to the changed equinox (e.g., transformation between sidereal and Universal times).
10. Extension of the stellar catalogues (FK5 and later Hipparcos) to higher magnitudes.
11. Connection of the FK5, and later Hipparcos, reference frames to the CIS frame.

Eight years later it is gratifying to note that significant progress has been made on all items.
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APPENDIX 1
PRINCIPAL RECOMMENDATIONS OF THE MERIT AND COTES WORKING GROUPS

A. Technical Recommendation on Concepts

The IAU/IUGG MERIT and COTES Joint Working Groups recommend that the following concepts be incorporated in the operation of an international earth orientation service:

(1) The Conventional Terrestrial Reference System (CTRS) be defined by a set of designated reference stations, theories and constants chosen so that there is no net rotation or translation between the reference frame and the surface of the earth. The frame is to be realized by a set of positions and motions for the designated reference stations.

(2) The Conventional Celestial Reference System (CCRS) be defined by a set of designated extragalactic radio sources, theories and constants chosen so that there is no net rotation between the reference frame and the set of radio sources. The frame is to be defined by the positions and motions of the designated radio sources. The origin of the frame is to be the barycenter of the solar system.

(3) This international service should provide the information necessary to define the Conventional Terrestrial Reference System and the Conventional Celestial Reference System and relate them as well as their frames to each other and to other reference systems used in the determination of the earth rotation parameters. The information should include, but not be limited to, pole positions, universal time, precession, nutation, dynamical equinox, positions of the designated reference stations and radio sources, and crustal deformation parameters.

B. Recommendation for the Organization of a New International Earth Rotation Service

The IAU/IUGG MERIT and Cotes Joint Working Groups recommend that IAU and IUGG establish a new international service within FAGS for monitoring the rotation of the earth and for the maintenance of the Conventional Terrestrial Reference System to replace both the International Polar Motion Service (IPMS) and the Bureau International de l'Heure (BIH) as from 1 January 1988.

The new service will be known as the International Earth Rotation Service (IERS) and will consist of a Directing Board, a Central Bureau, coordinating centers and observatories. The Central Bureau, the centers and the observatories will be hosted by national organizations.

The Directing Board will exercise organizational, scientific and technical control over the activities and functions of the Service including such modifications to the organizational structure and participation in the Service as are appropriate to maintain an efficient and reliable service while taking full advantage of advances in technology and theory. The voting membership of the Directing Board will consist of one representative each of the IAU, the IUGG, the Central Bureau, and each of the coordinating centers. Additional nonvoting members may be appointed to advise the Board on complex technical and scientific issues.
The Central Bureau will combine the various types of data collected by the Service to derive and disseminate to the user community the earth rotation parameters in appropriate forms, such as predictions, quick-look and refined solutions, and other information relating to the rotation of the earth and the associated reference systems. The Central Bureau will conduct research and analysis to develop improved methods of processing and interpreting the data submitted. The Central Bureau may include sub-bureaus that carry out some of the specific tasks of the Central Bureau.

Coordinating centers will be designated for each of the primary techniques of observation to be utilized by the Service as well as for other major activities which the Directing Board may deem appropriate. Initially, there will be three centers for (1) very long baseline interferometry (VLBI), (2) satellite laser ranging (SLR), and (3) lunar laser ranging (LLR). Additional coordinating centers may be designated for the improvement of the determination of the earth rotation parameters and the maintenance of the conventional reference system by other techniques and to ensure that relevant data on the atmosphere, oceans and seismic events are available.

The coordinating centers will be on the same level as the Central Bureau in the organizational structure of the Service and will be responsible for developing and organizing the activities by each technique to meet the objectives of the Service. Associated with the coordinating centers there may be network centers for subsets of observatories that may, for reasons of geometry or system compatibility, work more efficiently as autonomous units. There may also be associated analysis centers to process the observational data regularly or for special applications and studies. These centers may submit their results directly to the Central Bureau.

National Committees for the International Unions for Astronomy and for Geodesy and Geophysics will be invited to propose before 1 January 1987 national organizations and observatories that will be willing to host the Central Bureau or one of the centers and/or to provide observational data for use by the Service.

It is essential that the new service have redundancy throughout the organizational structure to insure the uninterrupted timely production of consistent, accurate, properly documented earth orientation and reference frame parameters, even in the event that one of the host national organizations should terminate its participation. A widespread distribution of observatories that regularly make high precision observations by one, or preferably more, modern space techniques by fixed and/or mobile equipment will be needed for this purpose, and national organizations are urged to provide appropriate resources.

APPENDIX 2
RESOLUTION OF INTERNATIONAL ASTRONOMICAL UNION (1985)

The following resolution was adopted by the XIXth General Assembly of the International Astronomical Union at New Delhi on 28 November 1985.

The International Astronomical Union

recognizing the highly significant improvement in the determination of the orientation of the earth in space as a consequence of the MERIT/COTES program of observation and analysis, and

recognizing the importance for scientific research and operational purposes of regular earth orientation monitoring and of the establishment and maintenance of a new Conventional Terrestrial Reference Frame,
thanks all the organizations and individuals who have contributed to the development and implementation of the MERIT and COTES programs and to the operations of the International Polar Motion Service and the Bureau International de l’Heure,

endorses the final report and recommendations of the MERIT and COTES Joint Working Groups;

decides

(1) to establish in consultation with IUGG a new International Earth Rotation Service within the Federation of Astronomical and Geophysical Services (FAGS) for monitoring earth orientation and for the maintenance of the Conventional Terrestrial Reference Frame; the new Service is to replace both the IPMS and the BIH as from 1 January 1988,

(2) to extend the MERIT/COTES program of observation, analysis, intercomparison and distribution of results until the new service is in operation,

(3) to recommend that an optical astrometric network be maintained for the rapid determination of UT1 for so long as this is recognized to be useful,

(4) to set up a Provisional Directing Board to submit recommendations on the terms of reference, structure and composition of the new service, and to serve as the Steering Committee for the extended MERIT/COTES program,

invites National Committees for the International Unions for Astronomy and for Geodesy and Geophysics to submit proposals for the hosting of individual components of the new service by national organizations and observatories, and

urges the participants in Project MERIT to continue to determine high precision data on earth rotation and reference systems and to make the results available to the BIH until the new service is in operation.

APPENDIX 3

RESOLUTION 1 OF THE INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS, XIX GENERAL ASSEMBLY
VANCOUVER, 21 AUGUST 1987

The International Union of Geodesy and Geophysics

Noting that the improved determination of the Earth’s orientation parameters resulting from the MERIT and COTES programmes of observation and analysis is highly significant,

considering the importance for scientific research and operational purposes of regularly monitoring the Earth’s orientation and of establishing and maintaining a new conventional terrestrial frame of reference,

approving the replacement of the International Polar Motion Service (IPMS) and of the Bureau International de l’Heure (BIH) by the International Earth Rotation Service (IERS) which will be responsible both for earth rotation and for the associated conventional frames of reference, and
recognizing that organizations in many countries have indicated their willingness to participate in such a new service,

endorses the recommendations of its Provisional Directing Board on the terms of reference, structure and composition of the new service,

decides to establish, in cooperation with the International Astronomical Union, the International Earth Rotation Service within the Federation of Astronomical and Geophysical Data Analysis Services (FAGS) a from 1 January 1988,

thanks all organizations and individuals who have helped to develop and implement the MERIT and COTES programmes, all who have operated IPMS and BIH in the past and all who have indicated their willingness to participate in the new Service.