ON A USEFUL FUNCTIONAL REPRESENTATION OF CONTROL SYSTEM STRUCTURE

by

Harvey L. Malchow

December 1988

The Charles Stark Draper Laboratory, Inc.

555 Technology Square
Cambridge, Massachusetts 02139
ON A USEFUL FUNCTIONAL REPRESENTATION OF CONTROL SYSTEM STRUCTURE

Harvey L. Malchow

Charles Stark Draper Laboratory

An alternative functional structure for control systems is proposed. The structure is represented by a three element block diagram and three functional definitions. It is argued that the three functional elements form a canonical set. The set includes the functions prescription, estimation and control. General overlay of the structure on parallel state and nested-state control systems is discussed. Breakdown of two real nested-state control systems into the proposed functional format is displayed. Application of the process to the mapping of complex control systems R&D efforts is explained with the Mars Rover Sample and Return mission as an example. A previous application of this basic functional structure to Space Station performance requirements organization is discussed.

Introduction

Many introductory diagrams of feedback control systems have the following format(1,2,3)

While this graphic description is economical and nominally correct, it does not explicitly display all the necessary functions* generally represented in a controller. For example, one problem with Figure 1 from a functional standpoint is that it does not explicitly display the estimation function. The feedback loop implicitly returns to the control error junction an estimate of the feedback signal. That is, the plant feedback signal is operated on by the wire (or by the uncertainty principle in the

* We draw on the definition given by Pulliam and Price4 for necessary function namely: "... a function is a thing or event that is needed to achieve the mission."
most taxing rhetorical case) and so the pure feedback signal originating at the plant always arrives corrupted at the summing junction, and is therefore always just an estimate whether it is the product of a simple transducer output or the more complex product of a filtering process. Gelb\(^5\) defines estimation as the "process of extracting information from data ...". Thus in this broad sense, a measurement on the plant becomes a state estimate when it is used to compute a state error signal. The use of the measurement with whatever mapping or filtering is applied, is an implicit conversion of measurement data to information, and is therefore, by Gelb's definition, an estimation process. We argue, therefore, that the act of providing a feedback estimate is a fundamental and necessary function in control systems\(^*\), and that this function is always present whether implicitly or explicitly so that estimation should be granted canonical functional status.

We also argue that the act of providing an input to a control system ought to have the status of a canonical function. Within the definition of necessary function it is therefore argued that no controller can function without an input. Control systems are always driven by an input. For regulating systems the input is constant, and for Tracking systems it is time dependent. For some systems, for example with missile guidance systems and with computer driven machinery, the function of prescribing an input state sequence is a complex discipline with sufficient status to stand alone as a technical specialty. Takahashi\(^8\) has argued for inclusion of the input function as part of the control system for tracking systems: "If the command signal is deterministic in a tracking control problem, a "generator" of the signal is considered part of the system." We propose to include inputs to regulating control systems as "generators" in the interest of generalization.

Another problem with Figure 1 is that the signal driving the control box is restricted to the difference between the input and the feedback signals. In general, the controller operates on a function of the input and feedback signals which is not necessarily a simple difference. For example, in the Space Shuttle orbital maneuvering system\(^7\) the control actuating signal is driven by a vector cross-product (which looks like a differencing driver only for small angles).

\(^*\) It may be argued that open loop controllers do not require estimation. However, one can counter-argue that the designer always "peeks" to see how the system is doing and thereby closes the loop as a living feedback estimator.
This paper suggests a revised basic functional block diagram for feedback control loops based upon the above arguments. The purpose of the revised diagram is to create a mental image of feedback control loops that contains the essential functions and therefore facilitates structured thinking about control problems. It is shown how various examples of working control loops from a variety of applications fit within the proposed elemental functional structure. It is then shown how this structure can be used to lay out work plans and to organize performance requirements matrices for controllers for complex systems. Such systems may have many state vectors and modes of operation such as occurs with multi-staged space vehicles.

The Proposed Functional Structure

We begin by reconstructing Figure 1. If we recognize that in general the controller acts upon a function of the comparison between the input and feedback states (not necessarily a simple difference), and if we allocate to the controller the task of forming that comparison, then the comparison process can be placed inside the controller box with the resulting diagram shown in Figure 2.

![Control Loop With Comparison Junction Inside Controller](image)

This form is exhibited for example in Hsu and Cruz. Ogata explicitly displays the comparison action inside the controller box.

The next step is to make the implicit estimation function explicit, and install it in the feedback loop. The result is Figure 3.
Many authors use this diagram although some, including Ogata\textsuperscript{10}, use the term "measuring element" or "measurement" instead of estimator.

We now take the important step of elevating the input action to function status. However before showing the result a few comments are in order. First, in the diagrams we have used a personalized noun form (controller and estimator) for the functions of control and estimation. The natural implication for the input function is therefore that it be termed the "inputer". In the interest of assigning an actual word in current usage we substitute the term "prescriber". Usage of this term is not without precedent (see for example Kwakernaak\textsuperscript{11} and Adams\textsuperscript{12}). Also, the root word prescribe has an advantage over other possible choices in that it admits both the gerund form prescribing and the action noun form prescription. Secondly, the form in which the feedback estimate and the input or prescribed state are entered into the control law is mathematically symmetric, i.e. as a difference, cross product, or some other symmetric form. To emphasize this symmetry we construct the block diagram with the prescriber in symmetric form opposite the controller from the estimator. The blocks have equal size to represent the equality of status of the two inputs from the standpoint of the controller. The result is Figure 4 which we term the "canonical functional form".
Fig. 4 Canonical Functional Feedback Control System.

The functional blocks represented in Figure 4 are all present in the general linear dynamical equation for an undisturbed control system with time dependent input:

\[ \dot{q}(t) = L(t)q(t) + K_r(t)r(t) - K_f(t)y(t) \]

Here \( q \) is the system state, and the left two terms represent the plant dynamics, \( r(t) \) is the prescribed state, \( K_r \) is a controller gain, and \( K_f \) times \( y \) is the estimated state scaled by a controller gain (i.e., \( K_f \) implicitly includes both estimator and controller functions). A similar construct is displayed by Franklin and Powell for discrete MIMO systems, namely:

\[ x(k + 1) = \Phi x(k) + \Gamma N r(k) - \Gamma K \hat{x}(k) \]

with \( r(k) \) the prescribed state. Since Figure 4 represents all the elements of the above equations, we declare the figure to be a necessary and sufficient structure.

Definitions of the Functions

It is appropriate at this point to define the elemental functions. We choose the point of view of "state control" to facilitate this process. All control systems control certain states of the system. The canonical functions must therefore bear some association with the states either as inputs to or outputs from the functions.
Function Definitions

**PRESCRIBER:** The PRESCRIBER sets forth desired values of those plant states that are controlled by the control system.

**ESTIMATOR:** The ESTIMATOR estimates those states that are controlled by the control system.

**CONTROLLER:** The CONTROLLER compares linear functions of the estimated and prescribed states and sends appropriate actuation signals to the plant.

The prescriber defines the purpose of the control system which is to drive the plant state to a desired set or sequence of sets of state values. The estimator operates on measurements of the plant that are related to the state $x$ through the relationship $y = H(x)$. This equation is inverted to produce a feedback proportional to the estimated state. Note that since the closed loop transfer function is dimensionless one must have dimensional agreement between the prescribed and estimated inputs to the controller. In many systems the controller performs a straight state differencing between prescribed and estimated states. Gavrilov\(^\text{15}\) has suggested that control systems involve three internal actions including the control action, the monitoring action, and the input action, and that structure resembles the one proposed here. However, Gavrilov includes disturbances and measurement noise in the input category, and later qualifies the input category of function to include monitoring, which is a definite departure from our structure.

**Nested Loops, General Format**

For independent states of a system the overall control system is represented simply by an independent set diagrams of the type shown in Figure 4. As stated by Meerov\(^\text{16}\): "... when there are $n$ controlled variables present, the whole system will consist of at least $n$ control loops interconnected in one way or another.". For interdependent states as well as time derivatives of the same state one can recast Figure 4 in a nested structure. The structure for nested loops depends upon the
interaction between the controller of one loop and the prescriber of the next. If the following prescriber changes its prescribed state as a result of the preceding control action (as it might do in Artificial Intelligence systems for example) then the structure is applied as drawn in Figure 5a.

![Fig.5a General Nested Loop Structure.](image)

If the prescribed state is not a function of the preceding control, then the preceding control acts as an internal prescriber, and the internal and external prescribed states can be added inside the following controller as illustrated in Figure 5b.

![Fig.5b Nested Loop for Simple External Prescribers.](image)

Many real control systems such as those shown in the following examples are structured as in Figure 5b.
Examples of Nested Loops

We display two examples, one involving two nested state vectors, and one involving four. The first is a proposed U.S. Space Station flight control system structure\textsuperscript{17}. The flight control system is responsible for control of (at least) the following two state vectors: 1) Space Station attitude, and 2) control moment gyro (CMG) stored momentum. The controller for CMG momentum alters the Space Station attitude so that gravity gradient torques can unload stored momentum. Meanwhile the station attitude controller attempts to maintain alignment with the local vertical coordinate frame. In the structure shown in Figure 6, the momentum controller output is expected to be the lone dynamic driver of attitude, and the external prescribed attitude is constant at some average torque equilibrium value. The CMG prescriber declares a desired target CMG momentum which may be biased in anticipation of disturbances, and the controller generates a desired attitude offset which becomes in essence a prescribed attitude for the attitude controller.

Some schemes\textsuperscript{18} weight the CMG momentum controller output, and combine that output with a weighted prescribed attitude from the attitude control loop. In that case the revised block structure of Figure 6 resembles that of Figure 5a.

In Figure 7 we display a four-state nested control system\textsuperscript{19} for controlling the Space Shuttle Orbiter when it is changing its orbital velocity. The highest level state vector, representing the outer loop, is the VGO or velocity-to-go vector.
VGO is set externally by the VGO prescriber which is part of the Shuttle PEG7 (Powered Explicit Guidance, mode 7) guidance algorithm. VGO is estimated by the User Parameter Processor logic which reads IMU data. The estimated VGO is compared to the prescribed value in the PEG7 guidance logic which issues an appropriate thrust direction command (which we label THR) as long as the VGO estimate is sufficiently different than the prescribed VGO. The THR prescriber merely prescribes a thrust direction in body coordinates that is through the center of mass. The THR estimator feeds back accelerometer data, and the THR controller, using "cross product steering" converts the thrust direction error into a body rate command. The body attitude rate prescriber prescribes a nominal rate of zero in all axes. The rate error is mapped by the rate controller into an Orbital Maneuvering System (OMS) engine gimbal deflection command. The OMS deflection prescriber presets deflections according to known Shuttle mass distribution. Of the four prescriber functions in this example, only the guidance VGO prescriber and the OMS deflection prescriber are nontrivial. The others, which are often implicit in descriptions of this particular control system, are included in Figure 7 in the interest of generality.

The proposed functional structure allows one to resolve some arguments about the meaning of other functional terms used in connection with control systems. For example, while "navigation" is clearly an estimation function, the roll of "guidance" in flight control systems has been defined by a range of functions. For example Blakelock says:
"The Guidance system performs all the functions of a navigation system plus generating the required correction signal to be sent to the control system."

whereas Wolverton\textsuperscript{22} is more inclusive:

"Guidance may be defined as the processes of measurement, data extraction and smoothing computation and control which are required to assure that a space vehicle reaches a desired destination from a given launch point."

and Beck\textsuperscript{23} is more exclusive:

"Guidance’s purpose, then is to determine where we want to be and how best to get there."

The first of these definitions has guidance performing an estimation function on position (navigation) in addition to calculating an error signal (on an unidentified state vector). The second definition seems to include estimation and control functions, and is notable for its absences of prescriptive function. The third is a purely prescriptive function. In the Space Shuttle OMS system, the guidance function performs both prescriptive and control tasks according to our definitions. It prescribes a VGO, then exercising a control function, tests the prescribed VGO against the VGO estimate and issues a control signal to the spacecraft. For Shuttle first stage operations however, guidance is a purely prescriptive function\textsuperscript{24}, and in that case, an attitude sequence is prescribed by extraction from a tabular reference. So while "guidance" can be uniquely defined as a prescriptive process, in practice it sometimes incorporates the other basic functions.

**Mapping Control Systems**

The stated function structure is useful for systematically identifying control system design problem areas. If the canonical functional structure is complete, then for each controlled state the basic function set must necessarily be invoked. For a complex system then, the overall control problem can be mapped out in a $3 \times N$ array, with $i = 3$ representing the three canonical functions, and $N$ representing the controlled states. If a system uses different controllers, estimators, or prescribers
for different modes of operation, the space of problem areas is expanded to 3 by m by N, where m is the number of distinct modes. This structure is illustrated by example in Figure 8, where two states are controlled in MODE 1, and three states in MODE 2.

MODE 1
STATE 1 STATE 2
prescribe prescribe
control control
estimate estimate

MODE 2
STATE 1 STATE 2 STATE 3
prescribe prescribe prescribe
control control control
estimate estimate estimate

Fig. 8 General Control System Mapping Structure.

Example of a System Structure Map

As a specific application consider the proposed Mars Rover Sample and Return Mission\(^2^5\). We begin breaking the problem down by listing flight modes and their associated controlled states (those normally associated with flight control - other states such as thermal and power control can of course be dealt with in the same manner). Table 1 is a hypothetical partial listing which ends at Mars orbit stage separation to save space.
Table 1. Controlled States for Various Flight Modes.

<table>
<thead>
<tr>
<th>Flight Mode</th>
<th>Earth Orbit States</th>
<th>Trans-Mars Boost States</th>
<th>Trans-Mars Coast States</th>
<th>Mars Deboost States</th>
<th>Mars Orbit States</th>
<th>Stage Separation States</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>earth relative attitude</td>
<td>inertial attitude</td>
<td>trans-Mars trajectory</td>
<td>inertial attitude</td>
<td>Mars Orbit</td>
<td>relative position</td>
</tr>
<tr>
<td></td>
<td>earth orbit</td>
<td>inertial thrust vector</td>
<td>trans-Mars coast</td>
<td>inertial thrust vector</td>
<td>Mars Orbit</td>
<td>relative position</td>
</tr>
<tr>
<td></td>
<td>antenna pointing</td>
<td>antenna pointing</td>
<td>mars orbit</td>
<td>antenna pointing</td>
<td>Mars Orbit</td>
<td>relative position</td>
</tr>
<tr>
<td></td>
<td>sol.array pointing</td>
<td>sol.array pointing</td>
<td>Mars orbit</td>
<td>sol.array pointing</td>
<td>Mars Orbit</td>
<td>relative position</td>
</tr>
<tr>
<td></td>
<td>CMG momentum</td>
<td>CMG momentum</td>
<td>Mars orbit</td>
<td>CMG momentum</td>
<td>Mars Orbit</td>
<td>relative position</td>
</tr>
</tbody>
</table>

We break down the problem areas for column 1 of Table 1 (the Earth orbit flight mode) into the task array shown in Figure 9.

Some of the problem areas may turn out to be trivial, for example prescribing a solar array pointing sequence may just mean stating that the array shall point at the sun. However if night-side feathering or differential drag pointing are considered this prescriptive function becomes nontrivial. The aim of the general process is to be all inclusive at the occasional expense of overkill.
Organizing Performance Requirements

Performance requirements for complex systems are often organized within an arbitrarily defined structure. Subsystem designers are asked to submit their requirements to the system level designers, and the collection is interleaved in a sequence with loose associations. Reference 26 is a case in point. Although the report is exhaustive and functionally complete, the flight control system related performance requirements are displayed on thirty-odd scattered pages, under a dozen different headings, and in some cases, the information is repetitive.

Using the structures of Figures 8 and 9, and Table 1, it is possible to construct organized sets of performance requirements. As demonstrated previously, the concepts of control system structure, controlled states, and distinctive operational modes lead to the layout of a complete set of place holders into which performance requirements may be entered. Table 2 is an example of such a layout that has been previously composed for the US Space Station program.

Table 2. Partial Space Station Performance Requirements Layout.

<table>
<thead>
<tr>
<th>FLIGHT CONTROL (incl. G, N&amp;C) PERFORMANCE REQUIREMENTS SUMMARY (1 of 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>attitude (deg)</strong></td>
</tr>
<tr>
<td>Pres: &lt; 3.99</td>
</tr>
<tr>
<td>Est.: ±0.013 sigma</td>
</tr>
<tr>
<td>Cont: ±1.0</td>
</tr>
<tr>
<td><strong>REBOOST</strong></td>
</tr>
<tr>
<td>Est.: TBD</td>
</tr>
<tr>
<td>Cont: TBD</td>
</tr>
<tr>
<td><strong>EVA</strong></td>
</tr>
<tr>
<td>Pres: TBD</td>
</tr>
<tr>
<td>Est.: TBD</td>
</tr>
<tr>
<td>Cont: TBD</td>
</tr>
</tbody>
</table>
Conclusions

Control systems, including those that involve many state vectors, can be broken down into a three function structure. Identification of this canonical functional structure is of practical as well as pedantic value. It allows for a systematic mapping out of control system design problem areas, and it provides for construction of a complete array of performance requirement place holders.

Acknowledgements

The author appreciates critical readings by colleagues Neil Adams, Phil Hattis and Les Sackett. This work was funded in part under NASA Contract NAS-9-17560.
References

13. Ref.10, pp. 128.
26"Space Station Program Definition and Requirements Section 3: Space
27Malchow, H., and Croopnick, S., "Organizing Performance Requirements for
Complex Dynamical Systems", IEEE Transactions on Engineering
28Malchow, H., "Space Station Requirements Review Charts", Memo SS-88-04,
C.S. Draper Lab., Dept. 10c, May 1988.