Ground State of High-Density Matter

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Abstract

It is shown that if an upper bound to the false vacuum energy of the electroweak Higgs potential is satisfied, the true ground state of high-density matter is not nuclear matter, or even strange-quark matter, but rather a non-topological soliton where the electroweak symmetry is exact and the fermions are massless. We examine this possibility in the standard SU(3) C ⊗ SU(2) L ⊗ U(1) Y model. The bound to the false vacuum energy is satisfied only for a narrow range of the Higgs boson masses in the minimal electroweak model (within about 10 eV of its minimum allowed value of 6.6 GeV), and a somewhat wider range for electroweak models with a non-minimal Higgs sector.
The idea that restoration of spontaneously broken symmetries occurs at high temperatures is now widely accepted [1], as is the idea that high density also can restore symmetry [2]. In this paper we consider the possibility that high baryon density can restore the electroweak symmetry and the true ground state of high-density baryonic matter may be such a configuration. We will assume the vacuum expectation value of a scalar field is zero in some region of high density, but non-zero in a region of low density. Such a configuration will take the form of a non-topological soliton.

There has recently been a great deal of interest in nontoplogical solitons (hereafter, NTSs) [3]. We will consider NTSs that are a localized region of space containing some number of particles confined to a region of false vacuum. The particles are trapped in the false vacuum region because they have a smaller mass in the false vacuum than in the true vacuum. For a number $N$ of particles greater than some critical value, soliton solutions where some scalar field $\phi$ is at a local maximum of the classical potential will have a lower energy than $N$ free massive particles with $\phi$ equal to the global minimum of the classical potential. Friedberg, Lee, and Sirlin [4] demonstrated that such spherical non-topological scalar field soliton solutions in 3 spatial dimensions existed, and for a large enough $N$ were in fact stable, both classically and quantum mechanically, to arbitrary small perturbations in the fields.

The necessary conditions for having such solutions are: (1) the conservation of an additive quantum number (in our case baryon number) carried by some complex field $\psi$ (in our discussion a fermion field), (2) the presence of a scalar field $\phi$ (here, the electroweak Higgs) that acquires a non-zero vacuum expectation value in the classical ground state, and (3) the mass of the $\psi$ field depends upon the vacuum expectation value of $\phi$. 
Here we address whether such objects exist in the standard theory of the strong and electroweak interactions. Our conclusion is that NTSs may indeed exist in the standard electroweak model. Our analysis compliments and extends the work of Khlebnikov and Shaposhnikov [5]. In their case chiral symmetry was broken in the non-topological soliton, resulting in the presence of nuclear matter in the soliton, whereas in our case chiral symmetry remains unbroken and as we shall see we have quark matter present.

Globally-conserved charges in the standard model are baryon number $B$ and lepton number $L$.\footnote{Even though weak instantons violate $B$ and $L$ separately, conserving only $B - L$, instanton effects are extremely small and will be neglected here.} Quarks will be massless in the region $\phi = 0$ and massive in the true vacuum, hence baryon number is a possibility. Neutrinos carry lepton number, but since they are massless in the standard model (do not couple to the Higgs) there is no mass difference between the false and true vacuum and they will not be trapped. The lepton number of the NTS will be zero. A neutrino star is in principle possible if one includes gravitational interactions, but hard to form because neutrino cooling is very slow in the standard model [6]. Thus, we are led to baryon number as the conserved global charge to satisfy (1). In the minimal\footnote{We differentiate between “standard” and “minimal”. Minimal refers to the Higgs structure, while standard refers to the symmetry. Non-minimal electroweak models contain more than a single Higgs doublet. Non-standard electroweak models might include right-handed $W$'s, etc.} electroweak model the neutral component of the $SU(2)_L$ Higgs doublet receives a vacuum expectation value, satisfying (2). Because quarks are massless in the NTS, but massive outside of it, (3) is satisfied.

Although the standard model has the necessary ingredients for the existence of NTS solutions, we must determine if the NTS has an energy per baryon less than that of the lowest energy free-particle solution.

For a given baryon number there are known to be two possible phases for quarks.
One is the confined, or hadronic, phase where the quarks are bound into nucleons and the minimum energy per unit baryon number is that of bulk nuclear matter. Another mode is the unconfined, or quark-gluon, phase where quarks are free. Ordinary nuclei do not convert to the quark-gluon phase because in the unconfined phase there is a background energy density, which can be thought of as a "bag energy". The bag energy is determined by the bag constant $B_0$ [7]; the currently popular value is $B_0 = (145 \text{ MeV})^4$. The background energy density increases the energy per unit baryon of the quark-gluon plasma above the proton mass. There have been investigations [8] into the possibility that the $u, d, s$ quark-gluon plasma, so-called strange-quark matter, has an energy per baryon lower than nuclear matter due to a lower Fermi energy. (For a given baryon number density, the Fermi energy decreases as the number of fermion species which carry baryon number is increased.) Also, it is known that there is negative pressure, $-B_0$, in the quark-gluon plasma. Thus, the bag constant plays the role of false vacuum energy density in scalar field theory. Additionally, the fermion number density in hand is not much different from that of proton and so it is assumed that the bag constant does not change.

Now we discuss how the above might be modified if there is a large domain where the Higgs field is equal to zero. Quarks, leptons, and the electroweak gauge bosons are massless in this region. Suppose that there is non zero baryon number density and the domain is large enough to neglect surface effects. What is the ground state energy at zero temperature in this phase? Is this phase stable against small fluctuations in the expectation value of $\phi$? Is the energy per baryon less than nuclear matter density?

First, consider the particle content of the minimal $SU(2)_L \otimes U(1)_Y$ model in the unbroken phase. There are four massless vector bosons: $b^i_\mu \ (i = 1, 2, 3)$ of $SU(2)_L$. 
and \(y_\mu\) of \(U(1)_Y\). In the broken phase, the photon field \(A_\mu\) and the transverse components of \(Z^0\) are linear combinations of \(b^3_\mu\) and \(y_\mu\): 
\[
A_\mu = y_\mu \cos \theta_W + b^3_\mu \sin \theta_W,
\]
\[
Z^0_\mu = -y_\mu \sin \theta_W + b^3_\mu \cos \theta_W.
\]
The transverse components of the \(W^\pm\) come from \(b^1_\mu\) and \(b^2_\mu\): 
\[
\sqrt{2}W^\pm_\mu = b^1_\mu \mp i b^2_\mu.
\]

In the minimal model there is one complex Higgs doublet, \(\Phi^T = (\phi^+ \phi^0)\). If the vacuum expectation of the Higgs field is zero, the symmetry is restored and the Higgs field has a negative 'bare' mass squared. However, finite fermion and vector boson density contributes a positive mass squared for the scalar field [2]. As we will argue later, the effective mass of the scalar field will be positive and the zero expectation value of the scalar field will be locally stable, resulting in symmetry restoration.

Assume for now that there is only one generation of fermions. The fermions are massless in the unbroken phase. The left-handed leptons are in an \(SU(2)_L\) doublet, \(L^T \equiv (\nu_e \, e)_L\), and the right-handed electron is an \(SU(2)_L\) singlet. The left handed quarks are also in a doublet, \(Q^T \equiv (u \, d)_L\), and the right-handed quarks are in \(SU(2)_L\) singlets.

The hypercharge of all particles are determined by the Gell-Mann–Nishijima relationship between the charge, hypercharge, and third component of isospin, \(Q = T_3 + Y/2\).

Any particle that is massless outside the NTS will not be trapped inside the NTS, and hence will have vanishing number density and chemical potential. The neutrino is an example: \(\mu(\nu_L) = 0\). Since \(b^3_\mu\) and \(y_\mu\) are components of the massless photon field, they will be able to escape the soliton and they will have zero number density and chemical potential. Since \(b^\pm_\mu\) projects only onto massive \(W^\pm\)'s which decay to massive particles, they will be trapped inside the NTS. Even though the \(SU(2)_L\) nonabelian symmetry is restored, this gauge theory is not asymptotically free and there is no
confinement. Hence they are massless inside the NTS, and their chemical potentials are zero, $\mu(b_\mu^\pm) = 0$.

Chemical equilibrium will establish relations between the chemical potentials of the particles present. Let us first consider chemical equilibrium between the left handed fermions and vector bosons. $u_L$ is connected to $d_L$ and $e_L$ is connected to $\nu_L$ by $b_\mu^\pm$. Since the massless vector bosons must have zero chemical potential,

$$\mu(u_L) = \mu(d_L) + \mu(b_\mu^+) = \mu(d_L)$$
$$\mu(\nu_L) = \mu(e_L) + \mu(b_\mu^+) = \mu(e_L)$$

(1)

Since neutrinos escape and have zero chemical potential, $\mu(e_L) = 0$. Right-handed fermions do not couple to $b_\mu^\pm$ and hence there is no relationship between their chemical potentials and the chemical potential of $b_\mu^\pm$.

Self interactions between vector bosons do not lead to any constraint in chemical potentials other than $\mu(b_\mu^\pm) = 0$. Self interaction between Higgs particles does not lead to any constraint. For a complex scalar field $\phi = (1/\sqrt{2})f \exp(i\omega t)$, the number density $n = f^2\omega$ of Higgs field is zero when the expectation value $f$ of the field is zero because the energy density is $n^2/2f^2$. As it will be shown, the effective chemical potential for Higgs field is positive and large. Hence the dynamical effect of the Higgs boson is that of an off mass-shell particle mediating interactions.

Now consider the conditions for chemical equilibrium arising from the exchange of Higgs particles. Conservation of $T_3$ and $Y$, leads to two further contributions to (1):

$$\mu(u_L) = \mu(u_R) + \mu(e_R)$$
$$\mu(u_L) = \mu(d_R) - \mu(e_R).$$

(2) (3)

Thus we have the conditions for chemical equilibrium as eqs. (1 - 3) between the
chemical potentials. The additional constraints are \( \mu(b^+_{\mu}) = \mu(\nu_L) = \mu(e_L) = 0 \).

The baryon number density is defined as a sum over the net number densities of all quarks \( q \): \( n_B \equiv \sum_q n(q) / 3 \). The lepton number density is defined in a similar manner by summing over all leptons \( l \): \( n_L \equiv \sum_l n(l) \). Here, the net number density of particles means the number density of particles minus that of antiparticles. Using the relations between chemical potentials and the definition of the number density in terms of the chemical potentials, \( n_B \) and \( n_L \) are given by

\[
\begin{align*}
    n_B &= \frac{1}{3} [n(u_L) + n(d_L) + n(u_R) + n(d_R)] \\
        &= \frac{1}{3} [2n(u_L) + n(u_R) + n(d_R)] \\
    n_L &= n(e_L) + n(e_R) + n(\nu_L) \\
        &= n(e_R).
\end{align*}
\]

We require the NTS to be “charge neutral”. For our problem this requires the charge density of \( SU(2)_L \) and \( U(1)_Y \) to vanish. For \( SU(2)_L \) the necessary condition is that the density of the \( T_3 \) generator should be zero. Then, one expects the system to adjust to have the density of \( \bar{T}_3 \) zero. For \( U(1)_Y \), charge neutrality implies \( Y = 0 \). The \( T_3 \) and \( Y \) densities are

\[
\begin{align*}
    T_3 &= \frac{1}{2} [n(\phi^+) - n(\phi^0)] + n(b^+) + \frac{1}{2} [n(u_L) - n(d_L)] \\
        &+ \frac{1}{2} [n(\nu_L) - n(e_L)] = 0 \\
    Y &= \frac{1}{3} [n(u_L) + n(d_L)] + \frac{4}{3} n(u_R) - \frac{2}{3} n(d_R) - 2n(e_R) \\
        &- [n(\nu_e) + n(e_L)] + [n(\phi^+) + n(\phi^0)] \\
        &= \frac{2}{3} n(u_L) + \frac{4}{3} n(u_R) - \frac{2}{3} n(d_R) - 2n(e_R) = 0.
\end{align*}
\]

From \( T_3 = 0 \) and \( n(\phi^i) = 0 \) and the relations in eqs. (1-3) for chemical energy, we find that \( n(b^+) = 0 \).
The above considerations can be easily extended to the case with arbitrary number of generations, say \( \kappa \). Generations will be mixed by the Yukawa interactions, and the composition of fermions will be independent of generation, as they are all massless inside the domain. Thus \( n_B = \frac{1}{3}\kappa [2n(u_L) + n(u_R) + n(d_R)] \). Other generations will have identical compositions.

The fermionic energy density, \( \mathcal{E} \) of a relativistic fermion is

\[
\mathcal{E} = g \frac{\mu^4}{8\pi^2}
\]

for \( g \) degrees of freedom. The number density is given by \( n = g\mu^3/6\pi^2 \). Now each quark of one handedness we have \( g = 3\kappa \) degrees of freedom where the factor of three represents the color, and \( \kappa \) the number of families. We now must evaluate the energy density of the system. We can rewrite eqns. (1 - 3, 4, 6) in terms of their chemical potentials as:

\[
\mu(u_L) = \mu(u_R) + \mu(e_R)
\]

(8)

\[
\mu(u_L) = \mu(d_R) - \mu(e_R)
\]

(9)

\[
\mu^3(u_L) + 2\mu^3(u_R) - \mu^3(d_R) - \mu^3(e_R) = 0 \quad \text{(from } Y = 0)\]

(10)

\[
n_B = \frac{\kappa}{6\pi^2} (2\mu^3(u_L) + \mu^3(u_R) + \mu^3(d_R))
\]

(11)

Defining \( z \equiv \mu(e_R)/\mu(u_L) \), \( x \equiv \mu(u_R)/\mu(u_L) \), \( y \equiv \mu(d_R)/\mu(u_L) \), we find the solution to eqs. (8 - 10): \( z = 0.235 \), \( x = 0.765 \), and \( y = 1.235 \). This can be substituted into eq. (11) to give the number densities of the particles in terms of the fixed \( n_B \) in chemical equilibrium:

\[
\mu(u_L) = 2.39n_B^{1/3} \kappa^{-1/3}
\]

\[
\mu(d_R) = 2.95n_B^{1/3} \kappa^{-1/3}
\]
\[ \mu(u_R) = 1.83n_B^{1/3}\kappa^{-1/3} \]
\[ \mu(e_R) = 0.56n_B^{1/3}\kappa^{-1/3}. \]  

The energy density is a sum of the energy densities of the fermions present, the bag energy density \( B_0 \), and the false vacuum energy density of the Higgs field \( V_0 \).

\[ \mathcal{E} = \sum_{i=\text{fermions}} \mathcal{E}_i + B_0 + V_0 = \sum_{i=\text{fermions}} \frac{g_i\mu_i^4}{8\pi^2} + (B_0 + V_0). \]  

Equilibrium occurs when the positive Fermi pressure of fermions is equal to the negative pressure of the vacuum energy. This happens when the energy per unit baryon number is at local minimum, i.e., \( \delta(\mathcal{E}/n_B) = 0 \). Minimization of the energy per baryon results in the relationship \( \sum_i \mathcal{E}_i = 3(B_0 + V_0) \) or \( (B_0 + V_0) = 1.93n_B^{4/3}\kappa^{-1/3} \). Thus we can rewrite the minimum energy per baryon in terms of \( (B_0 + V_0) \):

\[ \mathcal{E}_{\text{MIN}} = 4(B_0 + V_0) \]  

\[ \mathcal{E}_{\text{MIN}}/n_B = 6.55\kappa^{-1/4}(B_0 + V_0)^{1/4}. \]  

For the case of the true vacuum state, \( V_0 = 0 \), we expect the energy of the gluon-plasma to be larger than the energy per unit baryon of bulk nuclear matter which is about 930 Mev per nucleon, i.e., for the case \( \kappa = 1 \), \( B_0 > (m_{\text{proton}}/6.55)^4 \), which is the case here. However for the NTS to be absolutely stable, the energy per baryon should be less than the energy per baryon of bulk nuclear matter. From eq. (15), this requirement places an upper bound on the total vacuum energy density, \( B_0 + V_0 \), which in turn places an upper bound on the vacuum energy density of the Higgs potential, for \( \kappa = 3 \), \( \kappa = 1 \) does not work

\[ V_0 < (170\text{MeV})^4. \]  

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The question now is whether this bound is acceptable in the standard model. In the standard model with three generations, there are two important parameters, the mass of the Higgs particle and the mass of the top quark. There are theoretical and experimental bounds on these masses, which in turn leads to bounds on $V_0$.

Before we go on, let us ask whether the zero expectation value of Higgs field is locally stable. Suppose that the vacuum expectation value $(\phi^0, \phi^+)$ is $(f, 0)$. Then, the symmetry is broken spontaneously into $U(1)_Q$. One can find the chemical equilibrium of fermions and vector bosons for a given baryonic density in a unitary gauge. The effective mass term for the Higgs field arises because of a finite fermion and vector boson density. The large contribution in our case comes from top quarks and $W^-$ bosons [2]. The effective potential is

$$V_{\text{eff}}(f) = \frac{2}{3} \left[ \frac{g^2}{6\pi^2} \right]^{1/3} n_{T}^{2/3} M_W^2 \phi^2 + n_{W^-} \left( \frac{\phi}{v} \right) M_{W^-}$$

where $M_T, M_{W^-}$ are masses at the true vacuum $\phi = v$. As $n_{W^-}$ is not zero in the $\phi = 0$ limit, the potential forces $\phi$ to be zero. But this limit is singular in the sense that the symmetry is changing discontinuously and so is the chemical composition.

In the standard model the Higgs field is in the $SU(2)$ spinor representation. We write the potential for the one real component which describes the physical Higgs particle. Including one-loop corrections, the potential is [9]

$$V(\phi) = (2A - C)v^2 \phi^2 - A\phi^4 + C\phi^4 \ln \left( \frac{\phi^2}{v^2} \right)$$

where $C$ is given in terms of the masses of the $Z^0$, $W^\pm$, Higgs, and top quark as

$$C = \frac{1}{64\pi^2 v^4} \left( 3M_Z^4 + 6M_W^4 + M_H^4 - 12M_T^4 \right)$$

$$= 1.79 \times 10^{-4} + \left( \frac{M_H}{1232\text{GeV}} \right)^4 - \left( \frac{M_T}{662\text{GeV}} \right)^4.$$  

(19)
We have neglected the contributions of other quarks because their masses are much less than $W$ and $Z$, and we have used $\sin^2 \theta_W = 0.226$.

The potential has a local minimum at $\phi = v$. In the standard model $v = 246$ GeV via the relation $G_F = 1/\sqrt{2}v^2$. There are two conditions that must be placed upon the parameters in the potential:\footnote{There is an additional cosmological bound resulting from the requirement that the transition not be strongly first order. This requires that there is no barrier between the true and false vacua, or $d^2V/d\phi^2 < 0$ at $\phi = 0$ \cite{10}. This will be satisfied if $2A < C$. We will not satisfy this constraint. The no-barrier condition need not be applied if a large neutrino degeneracy prevents symmetry restoration at high density.} (i) The false vacuum energy density at $\phi = 0$ relative to the true vacuum energy density at $\phi = v$ is $V_0 = V(0) - V(v) = (C - A)v^4$. For $\phi = v$ to be the true vacuum, $V_0$ should be larger than zero, or $C \geq A$ \cite{11}. (ii) For the potential to be stable at large $\phi$, $C$ should be larger than zero, or equal to zero with $A < 0$. These two conditions place constraints on the Higgs mass, $M_H^2 = (12C - 8A)v^2$. From condition (i), the Higgs mass must satisfy the inequality $M_H^2 \geq 4Cv^2$. From condition (ii), $M_H^2 \geq (1232 \text{ GeV})^4 \left[ (M_T/662 \text{ GeV})^4 - 1.79 \times 10^{-4} \right]$. The picture of these conditions on the parameter space of Higgs and top quark masses are given in fig. 1.

From eq. (16), the NTS will be stable if $V_0 = (C - A)v^4 = (v^2/8)[M_H^2 - M_H^2(\text{MIN})] \leq (170 \text{ MeV})^4$. Since $v \gg 170$ MeV, $A$ must be very close to $C$ in order for the NTS to be stable. This means that $M_H$ must be very close to its minimum allowed value. Explicit calculation shows that for small top quark masses, the NTS will be stable if the Higgs mass is between its minimum value ($6.6 \text{ GeV}$), and about $10$ eV above this minimum value. This restriction was first realized in \cite{5}. They also pointed out that the degeneracy of minima with $\phi = (0, v)$ naturally emerges in a number of supergravity-induced extensions of the electroweak theory \cite{12}. As the top quark mass increases, $C$ decreases, and the width of the allowed range of Higgs mass in-
creases. The allowed region of $\Delta M \equiv M_H - M_H(MIN)$ for NTS stability is shown in fig. 1.

The present experimental lower bound on the top quark mass is about 40 Gev. The experimental bound on the Higgs mass is more complicated. The bound on very low mass Higgs is rather weak. The energy shift in $\mu$-mesonic atoms due to the additional potential energy from Higgs exchange can be measured in X-ray spectra, and leads to a bound $M_H > 8$ MeV [13]. High-precision, neutron-nuclei scattering experiments leads to a bound $M_H > 13$ Mev [14]. Decays $\pi^+ \rightarrow e^+\nu H$, $K^+ \rightarrow \pi^+ H$, and $\eta' \rightarrow \eta H$ can exclude the mass range $0 - 400$ MeV [15]. Recent analysis of the $Y \rightarrow \gamma\phi$ decay excludes masses in the range $0.5 - 3.0$ GeV [16]. The only region not covered is a small mass interval around the kaon mass. However, one should keep in mind that the lower bound on the Higgs mass is not completely definite. (For a detailed discussion, see [17].)

One can ask what happens when there are two Higgs fields. Then the lowest mass Higgs field corresponds to an angle variable between the vacuum expectation values of the two fields. When one of the Higgs fields is zero, up quark, down quark, or parts of whole generations become massless. Then, we get a similar upper bound on the Higgs particle mass. The parameter space is much larger and so is the possibility.

The NTS configuration may be interesting even if it has higher energy per baryon than nuclear matter. The point is that the NTS may be a metastable configuration; there may be an energy barrier between the NTS minimum and the nuclear matter minimum. Of course such a barrier may also exist in the case where the NTS is the global minimum energy per baryon, suppressing the transition from nuclear matter to NTS.

Finally, if there are such NTSs in the standard model, they may be interesting
astrophysically [5]. One might expect that neutron stars could be converted into NTSs.
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References


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Figure Caption

Fig. 1: The solid line is the minimum Higgs mass as a function of the top quark mass from the two conditions discussed in the text. If the Higgs mass is above this value, but below the value plus $\Delta M(\text{MAX})$ (given by the dashed line) the NTS will be the true ground state of baryonic matter.
Figure 1. Gondolo, Kolb, Lee