STRESS ANALYSIS METHOD FOR CLEARANCE-FIT JOINTS WITH BEARING-BYPASS LOADS


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INTRODUCTION

Aerospace structures are commonly fabricated using mechanically fastened joints. To better understand the complex behavior of such joints, it is important to develop accurate stress analysis methods. This is especially important for composite structures since they can be seriously weakened by fastener holes and often have rather complex failure modes. Within a multi-fastener structural joint, fastener holes may be subjected to the combined effects of bearing loads and loads that bypass the hole, as illustrated in figure 1. The ratio of bearing load to bypass load depends on the joint stiffness and configuration.

The combined effects of bearing and bypass loads can be simulated by testing and analyzing single-fastener specimens (figure 1). However, very little research has been published on the stress analysis of single-fastener specimens subjected to combined bearing and bypass loading. In 1981, Ramkumar [1] and Soni [2] used a two-dimensional, finite element stress analysis to determine the stress state around the fastener in a single-fastener laminate subjected to bearing-bypass loads. The fastener load was modeled by imposing zero radial displacements on the load-carrying half of the fastener hole. In the same year, Garbo [3] used his BJSFM analysis to obtain the stresses in a single-fastener laminate subjected to bearing-bypass loads. The fastener load was simulated by specifying a radial stress boundary condition varying as a cosine over half the hole.

In almost all practical applications, some clearance exists between the hole and the fastener. The presence of a clearance leads to a contact
region at the bolt-hole interface that varies nonlinearly with the load and influences the stress state around the hole. Although this nonlinearity complicates the analysis, it is important to make an accurate stress analysis to better understand the complex failure modes. Furthermore, for some combinations of compressive bearing-bypass loads the hole tends to close in on the fastener leading to "dual contact". The effects of clearance have been presented by these authors in references 4-6.

The objective of the present paper is to present a simple, direct stress analysis method for a laminate with a clearance-fit fastener subjected to combined bearing and bypass loads in tension or compression, including dual contact, and then to study the effects of bearing-bypass loads on bolt-hole contact and local stresses. The present approach uses a linear-elastic finite element analysis with an inverse formulation like that in Refs. 4-8. Conditions along the bolt-hole interface are specified by constraint equations that limit nodal displacements to a circular arc corresponding to the bolt diameter. These equations describe the contact conditions more realistically than the distributions usually assumed for radial displacement [1,2,7,8] or stress [3]. Furthermore, the present technique does not need an iterative-incremental method of solution which usually involves tedious node tracking along the contact arc [9-12]. The finite element analysis was performed using the MSC/NASTRAN computer code [13]. The material properties used in the analysis represent a quasi-isotropic T300/5208 graphite/epoxy laminate. The bolt was assumed to be rigid and the interface to be frictionless.

Results are presented as curves relating the nominal bearing stress and the bolt-hole contact angle for various combinations of bearing and bypass loads. Also, the effect of bypass load on the bolt-hole contact angle is
presented for constant bearing loads. Hole boundary stresses for a range of bearing-bypass load proportions in both tension and compression are also presented for a typical bearing load level. Finally, a solution array for a range of bearing-bypass loads, typical load-contact variations, and local stresses are presented for dual-contact situations.

NOMENCLATURE

c \quad \text{radial clearance, m}
c_d \quad \text{diametrical clearance, %}
d \quad \text{hole diameter, m}
E_1 \quad \text{modulus in fiber direction, GPa}
E_2 \quad \text{modulus transverse to fiber, GPa}
G_{12} \quad \text{shear modulus, GPa}
P \quad \text{applied load, N}
P_b \quad \text{bearing load, N}
P_{bp} \quad \text{bypass load, N}
r, \theta \quad \text{polar coordinates, m, degrees}
r_b \quad \text{bolt radius, m}
R \quad \text{hole radius, m}
S_b, S_b^* \quad \text{nominal bearing stress, MPa}
S_{np} \quad \text{nominal net-section bypass stress, MPa}
t \quad \text{specimen thickness, m}
u, v \quad \text{x- and y-displacements, m}
w \quad \text{specimen width, m}
x, y \quad \text{Cartesian coordinates, m}
The configuration and loading analyzed in the present study is shown in figure 2 for a typical tension bearing-bypass case. The gross applied load $P$ is reacted partly in bearing $P_b$ and partly as a bypass load $P_{bp}$ at the other end of the model. The nominal bearing stress $S_b$ and net-section bypass stress $S_{np}$ are defined as shown in figure 2. The bolt was assumed to be rigid and the bolt-hole interface to be frictionless. The diametrical clearance $c_d$ between the hole and the bolt was expressed as a percentage of the hole diameter $d$. The material represented in the analysis was a quasi-isotropic T300/5208 graphite/epoxy laminate with the following properties: $E = 59.46$ GPa and $G = 20.4$ GPa. Isoparametric, quadrilateral and triangular elements were used to model the laminate (4,131 nodes were placed at every 0.9375 degrees along the hole boundary to model the contact angles accurately. Loads $P$ and $P_{bp}$ were applied to the ends of the model. The desired bearing-bypass ratio $\beta$ was obtained by an appropriate choice of $P$ and $P_{bp}$.

For a snug-fit joint ($c_d = 0$), the contact angle $\theta_1$ along the bolt-hole interface does not vary with $S_b$ and a simple linear stress analysis can
be used. However, for a clearance-fit joint \((c_d > 0)\), the contact angle \(\theta_1\) increases nonlinearly with \(S_b\) as shown in figure 3. This nonlinear load-contact variation at the bolt-hole interface greatly complicates the stress analysis. Bypass loads also influence the nonlinear load-contact variation further complicating the analysis for combined bearing-bypass loading.

**Inverse Formulation**

The nonlinear load-contact variation shown in figure 3 could be accounted for in an iterative incremental scheme in which the nominal bearing stress \(S_b\) is incremented in small steps and the corresponding contact angle \(\theta_1\) is determined iteratively at each load step. Such a procedure would require special purpose finite element programming and would also involve tedious node tracking along the contact arc [9-12]. Alternatively, an inverse technique [4] can be used in which a contact angle \(\theta_1\) is assumed and the corresponding bearing stress \(S_b\) is computed by a simple procedure which is described later. The inverse technique is simple to use because the boundary conditions for an assumed contact angle \(\theta_1\) are fixed and known at the outset. For an assumed contact angle, the contact problem is linear, and the analysis procedure can be repeated for a range of contact angles to determine a nonlinear load-contact curve like that in figure 3. Therefore, although the contact problem is nonlinear, the inverse technique requires only linear finite element analyses; linear NASTRAN [13] procedures were used to solve this nonlinear problem. As previously mentioned, the conditions along the bolt-hole interface were specified by displacement constraint equations. The formulation of these constraint equations and the solution procedures are described in the following two sections.
Displacement Constraint Equations

Figure 4 shows the analysis model for a typical case of contact at the bolt-hole interface. The bolt radius \( r_b \) is smaller than the hole radius \( R \) by the amount of radial clearance \( c \). To simplify the analysis, the bolt is assumed to be fixed in space and the origin of the reference coordinate system is located at the center of the undeformed hole. Before load is applied, contact occurs only at point A. After the model is loaded on the two ends by \( P \) and \( P_{bp} \), points along the hole boundary which lie within an assumed contact arc AB contact the frictionless surface of the fixed rigid bolt.

Consider a point \( P(x, y) \) on the hole boundary within the assumed contact arc AB (see figure 4). Let \( u \) and \( v \) be the \( x \)- and \( y \)-displacements necessary to move point \( P \) from its original position to a point on the surface of the bolt. The deformed position of \( P \) may be described by the following equation:

\[
[(x - c) + u]^2 + [y + v]^2 = r_b^2
\]

(1)

By expressing \( x \) and \( y \) in polar coordinates, neglecting the higher order \( u \) and \( v \) terms, and noting that \( r_b = R - c \), Eq. (1) may be rewritten as follows:

\[
A u + B v = C
\]

(2)

where

\[
A = R \cos \theta - c
\]

(3)

\[
B = R \sin \theta
\]

(4)

and
Equation (2) is a constraint equation for the \( u \)- and \( v \)-displacements of any point \( P(x,y) \) on the contact arc \( AB \). The quantities \( A \), \( B \), and \( C \) can be computed at the outset since they are functions of the initial geometry. In the finite element analysis, the displacements of each node within the contact region can be specified by applying Eq. (2) as a multi-point constraint [13]. Note that the hole boundary beyond the contact arc is stress free. This fact will be used later in the analysis.

As mentioned earlier, for some compression bearing-bypass loads, the hole tends to close on the bolt leading to dual contact at the bolt-hole interface. For example, point \( D \) on the hole boundary in figure 4 could move in the positive \( x \)-direction and make contact with the bolt surface. This would correspond to the onset of dual contact. Further loading would lead to dual contact as shown in figure 5. The contact angles \( \theta_1 \) and \( \theta_2 \) for dual contact would also, in general, vary nonlinearly with load. However, the inverse technique can also be used for dual contact. Multi-point constraints, as described by Eq. (2), can be used to specify nodal displacements within the two contact regions. The corresponding combination of bearing and bypass loads can then be determined by the solution procedure described later.

**Solution Procedure for Single Contact**

The correct bearing stress \( S_b \) for an assumed contact angle \( \theta_1 \), was established using a simple procedure and the NASTRAN computer code. Point \( A \) (figure 4), which was assumed to be in contact with the rigid bolt, was fixed. Displacements of all nodes within the assumed contact arc were restricted to those allowed by the multi-point constraints given by Eq. (2). Thus all the
boundary conditions, including those along the bolt-hole interface, are defined. However, since the contact angle $\theta_1$ and the nominal bearing stress $S_b$ are nonlinearly related, the correct $S_b$ corresponding to the assumed $\theta_1$ is still unknown.

For a specified $\theta_1$, the problem is linear and thus, the stresses in the plate are linearly related to $S_b$ and $S_{np}$. A linear equation relating $\sigma_{rr}$, $S_b$, and $S_{np}$ can be written as

$$\sigma_{rr}(r, \theta) = F_1(r, \theta) S_b + F_2(r, \theta) S_{np} + F_3(r, \theta) \tag{6}$$

where $F_1$, $F_2$, and $F_3$ are constants for a given $r$ and $\theta$. The first two terms in this equation represent the $\sigma_{rr}$ components due to the applied bearing and bypass loads, respectively. The third term represents the $\sigma_{rr}$ due to imposing the multi-point constraints and is a function of clearance, as indicated by Eqs. (2-5).

For a given bearing-bypass ratio $\beta$, we can write $S_{np}$ as

$$S_{np} = \frac{S_b}{\beta} \tag{7}$$

Substituting Eq. (7) into Eq. (6) gives

$$\sigma_{rr}(r, \theta) = F_4(r, \theta) S_b + F_3(r, \theta) \tag{8}$$

where

$$F_4(r, \theta) = F_1(r, \theta) + F_2(r, \theta)/\beta \tag{9}$$

The hole boundary region beyond the contact angle $\theta_1$ is stress free. Thus,
\[
\sigma_{rr}(R, \theta) = 0 \quad \theta_1 \leq \theta \leq \pi \tag{10}
\]

This stress boundary condition was imposed at the end of the contact arc as

\[
\sigma_{rr}(R, \theta_1) = 0 \tag{11}
\]

If \( S_b^* \) is the correct bearing stress for an assumed contact angle \( \theta_1 \), then from Eqs. (8) and (11), we have the following relation:

\[ F_4(R, \theta_1) S_b^* + F_3(R, \theta_1) = 0 \tag{12} \]

This equation can be rewritten as follows:

\[ S_b^* = -\frac{F_3(R, \theta_1)}{F_4(R, \theta_1)} \tag{13} \]

For a given \( \beta \), the values of \( F_3(R, \theta_1) \) and \( F_4(R, \theta_1) \) were determined by the following procedure. For an assumed \( \theta_1 \), the stress \( S_b \) was selected arbitrarily, and \( \sigma_{rr} \) was calculated at the end of the contact arc using a finite element analysis. These \( S_b \) and \( \sigma_{rr} \) values were then used in Eq. (8) to get one equation for \( F_3(R, \theta_1) \) and \( F_4(R, \theta_1) \). A second \( S_b \) was selected (arbitrarily) and again the corresponding \( \sigma_{rr} \) at the end of the contact was calculated. The second set of \( S_b \) and \( \sigma_{rr} \) values was used in Eq. (8) to get a second equation for \( F_3(R, \theta_1) \) and \( F_4(R, \theta_1) \). The two equations were solved simultaneously to determine \( F_3(R, \theta_1) \) and \( F_4(R, \theta_1) \), which were then used in Eq. (13) to find \( S_b^* \). This procedure was repeated for a series of assumed \( \theta_1 \) values to determine the corresponding \( S_b^* \) values. These pairs of \( \theta_1 \) and \( S_b^* \) values can then be plotted to establish \( S_b \).
versus $\theta_1$ curves for single contact in the clearance-fit joint. The above procedure was introduced and evaluated in reference 4. It can be successfully applied for both tension and compression bearing-bypass loads which lead to single contact. A slightly different procedure is used for dual contact and is described in the next section.

**Solution Procedure for Dual Contact**

As described earlier, compression bearing-bypass loads may cause dual contact at the bolt. The dual contact angles $\theta_1$ and $\theta_2$ (see figure 5) increase nonlinearly with load. However, for specified $\theta_1$ and $\theta_2$, the stresses in the plate are linearly related to $S_b$, $S_{np}$, and the bolt-hole contact stresses associated with the imposed multi-point constraints. A linear equation relating $\sigma_{rr}$, $S_b$, and $S_{np}$ can be written for the end of each contact arc. For point B (see figure 5),

$$\sigma_{rr}(R, \theta_1) = K_1(R, \theta_1) S_b + K_2(R, \theta_1) S_{np} + K_3(R, \theta_1)$$

(14)

and for point E,

$$\sigma_{rr}(R, \pi - \theta_2) = K_4(R, \pi - \theta_2) S_b + K_5(R, \pi - \theta_2) S_{np} + K_6(R, \pi - \theta_2)$$

(15)

The constants $K_1$ through $K_6$ are similar to those in Eq. (6). Again, the first two terms in both Eqs. (14) and (15) represent the $\sigma_{rr}$ components due to the bearing and bypass loads, respectively. The third term represents the $\sigma_{rr}$ associated with the imposed multi-point constraints. Equations (14) and (15) cannot be simplified further using Eq. (7), as was done earlier for the single-contact case (see Eqs. (6) and (8)), because the bearing-bypass ratio $\beta$ that would lead to contact angles $\theta_1$ and $\theta_2$ is not known a priori. The constants $K_1$ through $K_3$ and $K_4$ through $K_6$ can be
determined by calculating $\sigma_{rr}(R, \theta_1)$ and $\sigma_{rr}(R, \pi - \theta_2)$ for three different (arbitrary) combinations of $S_b$ and $S_{np}$. The calculated stresses and the corresponding values of $S_b$ and $S_{np}$ are then substituted into Eqs. (14) and (15) to yield three equations in $K_1$, $K_2$, and $K_3$ and three equations in $K_4$, $K_5$, and $K_6$. These six equations are solved simultaneously to determine $K_1$ through $K_6$. Now the correct values of $S_b$ and $S_{np}$ corresponding to the assumed $\theta_1$ and $\theta_2$ values can be determined by imposing the boundary conditions at the ends of the contact arcs as $\sigma_{rr}(R, \theta_1) = 0$ and $\sigma_{rr}(R, \pi - \theta_2) = 0$. Substituting these boundary conditions into Eqs. (14) and (15) we have

$$K_1(R, \theta_1) S_b + K_2(R, \theta_1) S_{np} + K_3(R, \theta_1) = 0 \quad (16)$$

and

$$K_4(R, \pi - \theta_2) S_b + K_5(R, \pi - \theta_2) S_{np} + K_6(R, \pi - \theta_2) = 0 \quad (17)$$

The only unknowns in the above equations are $S_b$ and $S_{np}$ which can be determined by the simultaneous solution of Eqs. (16) and (17). The above procedure can be repeated for different sets of assumed contact angles $\theta_1$ and $\theta_2$ to determine the corresponding $S_b$ and $S_{np}$ values. These results can be presented as a $S_b$ versus $S_{np}$ plot as discussed in the following section.

RESULTS AND DISCUSSION

All of the results in this paper were obtained for the finite size plate shown in figure 2 with a clearance $c_d$ of 1.2 percent. First, results
are presented for single contact with both tension and compression bearing-bypass loads. Next, the effect of bypass loading on bolt-hole contact is shown for a constant bearing loading. The corresponding hole boundary stresses are also presented. Results are then presented for dual contact. Contact angles are presented on a $S_b$ versus $S_{np}$ plot and local stresses are presented for a dual-contact case.

Single Contact

The effect of bearing-bypass ratio $\beta$ on the bearing stress-contact angle curves is shown in figure 6 for single contact. Note that $\beta = -\infty$ corresponds to a tension bearing case with no bypass load and $\beta = -\infty$ represents the corresponding compression bearing case. The bearing-bypass ratio $\beta$ was found to have a considerable effect on the bearing stress-contact angle behavior. For tension bearing-bypass at a bearing stress level of 400 MPa [6], a typical bearing strength for graphite/epoxy, the contact angle (figure 6) for $\beta = 1$ is about 30 percent larger than that for $\beta = -\infty$. In compression bearing-bypass at $S_b = 400$ MPa, the contact angle for $\beta = -1$ is about 25 percent smaller than that for $\beta = -\infty$.

The variation of contact angle with bypass stress is shown in figure 7 for three $S_b$ levels. Increasing the tensile bypass loading increased $\theta_1$, while increasing the compressive bypass loading had the opposite effect. The small jog in the curve at $S_{np} = 0$ is caused by the small difference between tension-reacted bearing and compression reacted bearing. For the $S_b = 400$ MPa case, dual contact initiated for a compressive bypass stress of about 450 MPa. The secondary contact angle $\theta_2$ increased rather abruptly as the compressive $S_{np}$ exceeded this value. Additionally, the decreasing trend for $\theta_1$ reversed when dual contact developed.
For $S_b = 400$ MPa, the tangential ($\sigma_{\theta\theta}$) and radial ($\sigma_{rr}$) stresses around the hole boundary are shown in figure 8 for three different values of $S_{np}$ for tension bearing-bypass loading. The peak value of the $\sigma_{rr}$ stress is not very sensitive to $S_{np}$. The $\sigma_{rr}$ peak usually occurs at 0 degrees for small values of $S_{np}$. However, the distribution of $\sigma_{rr}$ changes with increased $S_{np}$ values, showing the increased contact angle as in figure 7.

For $S_{np} = 400$ MPa, the $\sigma_{rr}$ peak occurs at around 55 degrees. The increase in the peak $\sigma_{\theta\theta}$ stresses rises nearly proportionally with the increase in $S_{np}$ stress. The location of the peak $\sigma_{\theta\theta}$ is usually a few degrees beyond the contact region. The location of the $\sigma_{\theta\theta}$ peak is important because damage in composite joints often starts at this location [5].

The $\sigma_{\theta\theta}$ and $\sigma_{rr}$ hole boundary stresses for compressive $S_{np}$ stresses are shown in figure 9. For $S_{np} = 0$ ($\beta = -\infty$), $\sigma_{\theta\theta}$ is mostly tensile. For $S_{np} = -133$ MPa, $\sigma_{\theta\theta}$ becomes compressive in the net-section (around 90°) of the joint. For $S_{np} = -400$ MPa, the compression peak of $\sigma_{\theta\theta}$ is larger and more of the hole boundary is in compression. Unlike the tension bearing-bypass cases, the peak value of the $\sigma_{rr}$ stress in compression bearing-bypass increases nearly proportionally with the bypass load and always occurs at $\theta = 0$. For example the peak $\sigma_{rr}$ stress for $S_{np} = 400$ MPa is 40 percent higher than the $S_{np} = 0$ case.

Dual Contact

The $S_b$ and $S_{np}$ values calculated for assumed values of $\theta_1$ and $\theta_2$ are plotted in figure 10. Each solid circular symbol represents a solution corresponding to assumed contact angles $\theta_1$ and $\theta_2$. The dashed curve represents the onset of dual contact. The region to the right of the dashed curve represents the dual-contact region. The double-dashed line
represents the $\beta = -3$ case. Note that this line runs almost parallel to the linear portion of the dual-contact threshold line (dashed line), indicating that the $\beta = -3$ case would never involve dual contact. Thus, dual contact would occur only when $-3 < \beta \leq 0$. The dashed-dot line represents the $\beta = -0.5$ case. It intersects the dual-contact curve (dashed) at $S_{np} = -375$ MPa. Thus dual contact for this case would begin at $S_{np} = -375$ MPa ($S_b = 187.5$ MPa). Note that all the results in figure 10 were obtained for a clearance of 1.2 percent of hole diameter. A smaller clearance would cause dual contact to develop at a lower load level and vice versa.

The effect of dual contact on the $S_b$ versus $\theta_1$ curve is shown in figure 11. The $S_b$ versus $\theta_1$ curve before dual contact was generated using the procedure for single-contact analysis described earlier. The remainder of the curve was constructed from figure 10 by selecting $S_b$, $\theta_1$, and $\theta_2$ values on the $\beta = -0.5$ line. Some interpolation was used. The $S_b$ versus $\theta_1$ relationship changes after the onset of dual contact. The $S_b$ versus $\theta_2$ curve is also nonlinear and $\theta_2$ increases more rapidly than $\theta_1$ with increasing load.

An important consequence of dual contact is that it allows load transfer across the bolt and, therefore, reduces the stress concentration around the fastener hole. Figure 12 illustrates this effect by comparing hole boundary stresses for a case with dual contact to stresses for an open hole case at the same load level. The peak $\sigma_{\theta\theta}$ for the open hole case is 13 percent higher than that for the dual-contact case. This suggests that dual contact is advantageous since it reduces stresses around the hole and increases the joint strength. Smaller bolt-hole clearances promote dual contact and, therefore, should produce higher joint strengths in compression.
CONCLUDING REMARKS

A simple method has been developed for the stress analysis of a laminate with a clearance-fit fastener subjected to combined bearing and bypass loading in tension or compression, including dual-contact situations. This method uses a linear elastic finite element analysis with an inverse formulation. The present method is simple to apply and can be implemented with most general purpose finite element programs. The method was applied to study the effects of bearing-bypass load proportioning on the bolt-hole contact angles and local stresses. The initial clearance between the smooth, rigid bolt and the hole was 1.2 percent of the hole diameter in all analyses. Material properties for the plate represented a quasi-isotropic, graphite/epoxy laminate. The bearing-bypass proportions were expressed in terms of $\beta$, the ratio of bearing stress $S_b$ to bypass stress $S_{np}$.

The bearing-bypass ratio $\beta$ was found to have a considerable effect on the nonlinear contact behavior. Under tension bearing-bypass with $S_b = 400$ MPa, the contact angle was 30 percent larger for $\beta = 1$ than that for $\beta = \infty$. Compressive bypass loading had the opposite effect, the contact angle was 25 percent smaller for $\beta = -1$ than that for $\beta = -\infty$ at $S_b = 400$ MPa. For single-contact situations under tension bearing-bypass, the peak tangential stresses around the hole boundary increased proportionately with the $S_{np}$ stress. The peak radial stress was not very sensitive to $S_{np}$. However, under compressive bearing-bypass loading, both the peak tangential and peak radial stresses were considerably influenced by $S_{np}$. For $\beta$ greater than -3, the hole tended to close on the fastener leading to dual
contact. Dual contact allows load transfer across the fastener and, therefore, reduces the stress concentration around the hole. Dual contact, therefore, has a beneficial effect on the joint strength.

These results illustrate the general importance of accounting for bolt-hole clearance and contact to accurately compute local bolt-hole stresses for combined bearing and bypass loading.
REFERENCES


(a) Multi-fastener joint.

(b) Single-fastener coupon.

Figure 1. - Bearing-bypass loading within a multi-fastener joint.
Figure 2. - Plate configuration for bearing-bypass loading.
Figure 3. - Nonlinear relationship between bearing stress and contact angle.

\[ S_b = \frac{P}{td} \]
\[ c_d = \frac{c}{R} \times 100 \% \]

Figure 4. - Hole clearance notation for single-sided contact.
Figure 5. Contact angle notation for dual contact.
Figure 6. Effect of bearing-bypass ratio on load-contact relationship.
Figure 7. - Variation of contact angle with bypass stress.
Figure 8. - Stresses along hole boundary for tension loading.
Figure 9. S - Stresses along hole boundary for compression loading.
Figure 10. - Solution array for dual contact.
\[ \beta = -0.5 \]
\[ c_d = 1.2\% \]

Figure 11. - Effect of dual contact on \( S_b \) versus \( \theta \) relationship.
Figure 12. - Effect of dual contact on hole boundary stresses.
Within a multi-fastener joint, fastener holes may be subjected to the combined effects of bearing loads and loads that bypass the hole to be reacted elsewhere in the joint. The analysis of a joint subjected to such combined bearing and bypass loads is complicated by the usual clearance between the hole and the fastener. A simple analysis method for such clearance-fit joints subjected to bearing-bypass loading has been developed in the present study. It uses an inverse formulation with a linear elastic finite-element analysis. Conditions along the bolt-hole contact arc are specified by displacement constraint equations. The present method is simple to apply and can be implemented with most general purpose finite-element programs, since it does not use complicated iterative-incremental procedures. The method was used to study the effects of bearing-bypass loading on bolt-hole contact angles and local stresses. In this study, a rigid, frictionless bolt was used with a plate having the properties of a quasi-isotropic graphite/epoxy laminate. Results showed that the contact angle as well as the peak stresses around the hole and their locations were strongly influenced by the ratio of bearing and bypass loads. For single contact, tension and compression bearing-bypass loading had opposite effects on the contact angle. For some compressive bearing-bypass loads, the hole tended to close on the fastener leading to dual contact. It was shown that dual contact reduces the stress concentration at the fastener and would, therefore, increase joint strength in compression. The results illustrate the general importance of accounting for bolt-hole clearance and contact to accurately compute local bolt-hole stresses for combined bearing and bypass loading.