FINAL REPORT FOR THE UNMANNED MULTIPLE EXPLORATORY PROBE SYSTEM (MEPS)
FOR MARS OBSERVATION

Volume II. Calculations and Derivations

A design project by students in the Department of Aerospace Engineering at Auburn University, Auburn, Alabama, under the sponsorship of NASA/USRA Advanced Design Program.

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June, 1988
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ABSTRACT

This volume of the final report on the unmanned Multiple Exploratory Probe System (MEPS) details all calculations, derivations, analyses, and computer programs that support the information presented in the first volume.
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INTRODUCTION

In Volume One of the report on the Multiple Exploratory Probe System (MEPS) the final design is presented. However, in most cases the reasoning or rationale behind many of the design decisions are not given to the reader. Volume Two alleviates this problem by presenting calculations, derivations, computer programs, and additional arguments for the final design.

Several areas are discussed in this volume. First, the structural mass calculation and the structural analysis are presented. The calculation of the propulsive burns for the Earth-Mars transfer are shown, as well as the burns required for the aerobraking and the satellite. The secondary and main propulsion systems are studied to obtain the mass of propellant (oxidizer and fuel) required for the entire trip and the size of the propellant tanks; engine analysis is also presented in more detail in this volume. The necessary equations for the aerobraking program are derived, and the lander system is analyzed for determination of mass and stagnation temperature. In addition the recovery system is optimized. Appendices present various programs and supplemental plots.
STRUCTURAL MASS CALCULATION

The total structural mass was calculated by summing the mass of each component of the structural system. The structural mass will be divided into stringers, bulkheads, the cylindrical shells, the caps on the ends of the modules, and the connectors. Aluminum will be used for the structural material (specific weight = 173.4 lb/ft³).

Stringers

There are 36 longitudinal stringers arranged circumferentially along the length of each module. Each stringer is assumed to have a cross-sectional area of 1 in² = 0.006944 ft². Therefore, the total stringer mass is

\[ 36 \times (\text{module length}) \times (0.006944 \text{ ft}^2) \times (173.4 \text{ lbm/ft}^3) \]

mass (lbm) = 43.365 \times \text{length}

Bulkheads

The bulkheads are I-beams located along the interior circumference. The cross-sectional area of this beam is 1.25 in² = 0.008681 ft². For the mass of each bulkhead,

\[ (2\pi) \times (12.5 \text{ ft}) \times (0.008681 \text{ ft}^2) \times (173.4 \text{ lbm/ft}^3) \]

mass = 118.261 lbm (per bulkhead)

The \(2\pi\times12.5 \text{ ft}\) term is the circumference of the bulkhead, where 12.5 ft is the radius of the module.
Cylindrical Shells

The structural mass of the cylinder obviously depends on the length of the module. Using a thickness of 1/4 in (.020833 ft),

\[(2 \cdot \pi \cdot 12.5 \text{ ft}) \times (0.020833 \text{ ft}) \times (173.4 \text{ lbm/ft}^3) \times \text{length} \]

\[\text{mass (lbm)} = 283.83 \times \text{length} \]

End Caps on Cylinders

The end cap is a plate modeled on the end of the module, and two caps are designed for each module. Using the same thickness as the cylindrical shells (.020833 ft),

\[2(\pi \cdot (12.5 \text{ ft})^2 \times (0.020833 \text{ ft}) \times (173.4 \text{ lbm/ft}^3)) \]

\[\text{mass} = 3547.98 \text{ lbm} \]

Connectors

Each connector is one inch thick and are five feet in length (with the exception of the aerobrake connector, which is 10 feet long). The design of the MEPS vehicle requires a total of four 5-foot long and one 10-foot long connectors; only two 5-foot long connectors and the 10-foot long connector will remain on the vehicle during aerobraking. For the total mass of the connectors,

\[(2 \cdot \pi \cdot 12.5 \text{ ft}) \times (0.08333 \text{ ft}) \times (\text{length}) \times (173.4 \text{ lbm/ft}^3) \]

\[\text{mass for Earth configuration (lbm)} = 1135 \times 30 \text{ ft} = 34050 \]

\[\text{mass for Mars configuration (lbm)} = 1135 \times 20 \text{ ft} = 22700 \]
**Total Vehicle Mass**

Using the individual calculations given above, the total mass of the MEPS vehicle can be determined:

**Equatorial Lander:**

- **Stringers** (43.37 lbm/ft * 30 ft) = 1305 lbm
- **Bulkhead** (4 * 118.26 lbm) = 475 lbm
- **Cylinder** (283.83 lbm/ft * 30 ft) = 8515 lbm
- **Caps** = 3550 lbm
- **Total Mass** = 13845 lbm

**Satellite/CIC:**

- **Stringers** (43.37 lbm/ft * 25 ft) = 1085 lbm
- **Bulkhead** (3 * 118.26 lbm) = 355 lbm
- **Cylinder** (283.83 lbm/ft * 25 ft) = 7100 lbm
- **Caps** = 3550 lbm
- **Total Mass** = 12100 lbm

**Secondary Propulsion:**

- **Stringers** (43.37 lbm/ft * 10 ft) = 435 lbm
- **Bulkhead** (2 * 118.26 lbm) = 240 lbm
- **Cylinder** (283.83 lbm/ft * 10 ft) = 2840 lbm
- **Caps** = 3550 lbm
- **Total Mass** = 7065 lbm

**Main Propulsion:**

- **Stringers** (43.37 lbm/ft * 70 ft) = 3040 lbm
- **Bulkhead** (5 * 118.26 lbm) = 595 lbm
- **Cylinder** (283.83 lbm/ft * 70 ft) = 19870 lbm
- **Caps** = 3550 lbm
- **Total Mass** = 27055 lbm

**Polar Lander:**

- **Stringers** (43.37 lbm/ft * 50 ft) = 2170 lbm
- **Bulkhead** (6 * 118.26 lbm) = 710 lbm
- **Cylinder** (283.83 lbm/ft * 50 ft) = 14195 lbm
- **Caps** = 3550 lbm
- **Total Mass** = 20625 lbm

**Total Vehicle (Structural Mass):**

- **Earth departure** = 111,500 lbm
- **Mars arrival** = 67,100 lbm
PROPOSED STRENGTH ANALYSIS

The following analysis approach will be used to find (a) the proper material for the stringers, module skin, and bulkheads; (b) the correct material and thickness of the connectors; and (c) the material composition and overall number of pins for module connections.

A complete finite element model for the MEPS vehicle is being placed on MSC/pal. This model includes the proper lengths of the modules and connectors (using aluminum as the initial material for all components) and the payload mass inside each module. The force cases for dynamic analysis of the system are being obtained from the propulsion and orbital insertion analyses. These cases will include thrust from the initial departure burn, acceleration and thrust from the deceleration burn to insert into Martian orbit, and the acceleration and drag from the maximum-force aerobraking pass. The model and forces will be translated into a NASTRAN input file using a program available on MSC/pal2.

NASTRAN will be executed using the input cases outlined above and the output will be evaluated; to utilize this evaluation, the following procedure should be applied. If the evaluation shows that the structural integrity of the connector is in doubt, two cases should be run. First, use aluminum for the module and a metal matrix composite (MMC) for the connector;
second, use aluminum for both components but increase the thickness of the connector. If the evaluation shows question in the module's strength, the following runs will be conducted sequentially until a proper solution is reached: (1) increase the stiffener and bulkhead sizes (areas); (2) change the material of the module to MMC and use the original stiffener and bulkhead sizes alone; (3) use aluminum for all components but increase the thickness of the module; and (4) increase the sizes of all components while still using aluminum.

Once the proper sizes for a minimum stress on the entire vehicle have been determined, an analysis technique outlined by R. E. Peterson in *Stress Concentration Design Factors* (Wiley Press, 1974) will be used to determine the stress in a pin hole of a connector. From this information, strength or failure of the connection can be determined. If the pin shows failure, then the material of the pins must be changed; however, if the pin hole indicates failure, then the thickness around the hole will be increased. Also from this stress information, and some assistance from Dr. W. A. Foster, Jr. of Auburn University, the minimum amount of pin connections can be determined.
ANALYSIS OF EARTH-MARS TRAJECTORY

After MEPS has been moved to the ecliptic plane, a propulsive burn will be conducted to start the vehicle on the journey to Mars. A Hohmann (minimum energy) transfer will be employed to save fuel, although the time of flight will be extended. This section will introduce the analysis of the Earth-Mars trajectory, including the magnitudes of the required propulsive burns and the determination of the time of flight.

Earth Departure

The semi-major axis of the transfer is defined as
\[ a_r = \frac{r_{\oplus}}{2} (r_{\oplus} + r_{\oplus}) = \frac{r_{\oplus}}{2} (4.908 \text{ E11} + 7.477 \text{ E11}) \text{ ft} \]
\[ a_r = 6.193 \text{ E11 ft} \]

To determine the propulsive burns for the start of the transfer, the approach to the problem must be considered. The transfer between Earth and Mars will require determination of relative velocities, as the situation is not a simple transfer between two orbits about the same body. Now two bodies must be taken into account—the Earth and the sun.

The velocity of the vehicle relative to the sun can be expressed in terms of velocities about earth:
\[ V_{\oplus} = V_{\oplus} + V_{\oplus} = V_{\oplus} + V_{\oplus} \]
where \( V_{\oplus} \) is the hyperbolic excess speed. This speed can be expressed as
\[ V_{\oplus} = V_{\oplus} - V_{\oplus} \]
A fundamental equation used in astrodynamics is the Vis-Viva equation. This equation allows the calculation of a velocity at a point in an orbit if the parameters of the orbit are known:

\[ v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a}\right)} \]

where \( \mu \) is the gravitational parameter of the body (planet) which influences the vehicle, \( r \) is the distance from the body where the propulsive burn is applied, and \( a \) is the semi-major axis of the transfer ellipse.

Applying the Vis-Viva equation to find the velocity of the vehicle relative to the sun, the required inputs are

\[ \mu_\odot = 4.687 \text{E}21 \text{ ft}^3/\text{sec}^2 \]
\[ r_\odot = 4.908 \text{E}11 \text{ ft} \]
\[ a = 6.193 \text{E}11 \text{ ft} \]

The resulting velocity is

\[ v_{\odot/\odot} = 107,383.46 \text{ ft/sec} \]

The velocity of the Earth is calculated by assuming the Earth is in a circular orbit about the sun. Using the equation for velocity in a circular orbit,

\[ v_\odot = \sqrt{\frac{\mu_\odot}{r_\odot}} \]

and the appropriate values given above,

\[ v_\odot = 97,722.64 \text{ ft/sec} \]

The hyperbolic excess speed can now be determined:

\[ v_\infty = v_{\odot/\odot} - v_\odot \]
\[ v_\infty = (107,383.46 - 97,722.64) \text{ ft/sec} \]
\[ v_\infty = 9660.82 \text{ ft/sec} \]
The burnout velocity is expressed as

\[ V_{\infty} = \sqrt{V_{\infty}^2 + \frac{2\mu g}{r}} \]

At an radius of 22,567,193.75 ft (3714.153 n mi) from the center of the Earth, with

\[ \mu = 1.408 \text{ E}16 \text{ ft}^3/\text{sec}^2 \]

the burnout velocity has the value of

\[ V_{\infty} = 36621.86 \text{ ft/sec} \]

The velocity of the vehicle relative to the Earth is given by the expression for circular velocity, using the gravitational parameter for the Earth and the radius of 22,567,193.75 ft:

\[ V_{\infty}/\mu = 24,978.28 \text{ ft/sec} \]

With this value and the value for the burnout velocity the required propulsive burn can be calculated:

\[ \Delta V = V_{\infty} - V_{\infty}/\mu \]

\[ \Delta V = 11643.58 \text{ ft/sec} \]

**Mars Capture**

The equations for analyzing the propulsive burn to allow capture by Mars are similar to those used for the Earth departure analysis.

The hyperbolic excess speed is expressed as

\[ |V_{\infty}| = |V_{\infty}/\mu + V_{\delta}| \]

The velocity of the spacecraft relative to the sun is determined by the Vis-Viva equation, with the following inputs:

\[ \mu = 4.687 \text{ E}21 \text{ ft}^3/\text{sec}^2 \]

\[ r_{\odot} = 7.477 \text{ E}11 \text{ ft} \]

\[ r_{\odot} = 6.193 \text{ E}11 \text{ ft} \]
Substitution into the Vis-Viva equation yields the value of this velocity:

\[ V_{\infty} = 70,489.39 \text{ ft/sec} \]

The velocity of Mars about the sun is calculated under the assumption that Mars is moving in a circular orbit about the sun:

\[ V_0 = \sqrt{\frac{\mu}{r_0}} \]
\[ V_0 = 79174.22 \text{ ft/sec} \]

Thus the hyperbolic excess speed has the value of

\[ V_* = 8684.83 \text{ ft/sec} \]

The magnitude of the propulsive burn required for Mars capture is expressed as

\[ \Delta V = V_{\infty} - V_0 \]

where \( V_{\infty} \) is the velocity of the spacecraft at the point of closest approach to Mars. Because the vehicle is on a hyperbolic approach to Mars the velocity of the vehicle relative to Mars is expressed with the same equation as the burnout velocity used for the Earth departure:

\[ V_{\infty} = \sqrt{V_0^2 + \frac{2\mu_{\delta}}{r_p}} \]
\[ \mu_{\delta} = 1.5066 \times 10^{15} \text{ ft}^3/\text{sec}^2 \]
\[ r_p = 12,774,573.5 \text{ ft (2102.46 n mi)} \]
\[ V_{\infty} = 17,643.74 \text{ ft/sec} \]

The elliptic orbit of the vehicle after Mars capture is defined as 1,640,500 ft (270 n mi) x 108,151,603 ft (17800 n mi).

Given the radius of Mars,

\[ r_0 = 11,134,073.5 \text{ ft} \]
the semi-major axis of the ellipse can be calculated using the following:

\[ a = \frac{1}{2}(1,640,500 \text{ ft} + r) + (108,151,603 \text{ ft} + r) \]

\[ a = 66,030,125 \text{ ft} (10,867.37 \text{ n mi}) \]

At periapsis the velocity of the vehicle is determined (using the Vis-Viva equation) to have the following value:

\[ V_{p} = 14,596.52 \text{ ft/sec} \]

Using the expression for the propulsive burn required for Mars capture,

\[ \Delta V = V_{p} - V_{\infty} \]

\[ \Delta V = 3047.22 \text{ ft/sec} \]

**Time of Flight**

The time required for the transfer from Earth to Mars can be calculated from the period of the elliptical orbit. The orbit period is defined as

\[ \tau = \sqrt{\frac{2\pi}{\mu_\odot}} \cdot a^{3/2} \]

Substitution of the appropriate values yields

\[ \mu_\odot = 4.687 \text{ E21 ft}^3/\text{sec}^2 \]

\[ a = 6.193 \text{ E11 ft} \]

\[ \tau = 44,728,466.79 \text{ seconds} = 517.7 \text{ days} \]

Since this period is the time of flight for an entire elliptic orbit, the required time to reach Mars is one-half the period, or

\[ \tau_{m} = 258.85 \text{ days} \]
PROPELLANT ANALYSIS FOR THE SECONDARY PROPULSION SYSTEM

The secondary propulsion system will be employed upon approach to Mars. The purpose of this engine system is to slow down MEPS to obtain an elliptic orbit about Mars and to help in the final stages of orbit circularization. The analysis presented in this section concerns the calculation of the propellant mass (oxidizer and fuel) for each ΔV burn and the required volume of the fuel tanks.

The masses of each MEPS module which will be placed into orbit about Mars are presented below. These values do not include propellant.

<table>
<thead>
<tr>
<th>Module</th>
<th>Mass (lbm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerobrake</td>
<td>12,000</td>
</tr>
<tr>
<td>Equatorial Lander/Rover</td>
<td>65,000</td>
</tr>
<tr>
<td>Satellite System</td>
<td>3,500</td>
</tr>
<tr>
<td>CIC</td>
<td>3,000</td>
</tr>
<tr>
<td>Structure</td>
<td>67,100</td>
</tr>
</tbody>
</table>

The total mass of MEPS excluding propellant is 140,000 lbm.

Four ΔV burns will be required during the circularization process at Mars. The first burn will place the MEPS vehicle system into a highly elliptic orbit about Mars. During the appropriate orbit a second ΔV burn will be performed at the orbit apoastron to lower the periapsis into the Martian atmosphere. Following aerobraking, a third burn moves the periapsis out of
the atmosphere, and the fourth ΔV burn will provide final orbit circularization by adjusting the apoapsis. The calculated values of these burns will now be presented.

Burn 1 (orbit capture) 3047.90 ft/sec
Burn 2 (lower periapsis) 75.1214 ft/sec
Burn 3 (raise periapsis) 301.6568 ft/sec
Burn 4 (adjust apoapsis) 12.1129 ft/sec

The secondary propulsion system will use three engines similar to the Space Shuttle Orbiting Maneuvering System (OMS). These engines have a specific thrust of 280 seconds. With the help of an equation relating the burn to the specific thrust and initial and final masses, the propellant mass (initial mass before burn) required for each burn can be calculated.

\[ \Delta V = \text{Isp} \cdot g_\ast \cdot \ln(M_i/M_f) \]

\[ M_i = M_f \cdot e^{(\Delta V/\text{Isp} \cdot g_\ast)} \]

Use of this equation will begin the propellant analysis required for the secondary propulsion system; for each burn the mass of the propellant (oxidizer and fuel) must be determined. The final vehicle mass is 140,000 lbm. Substitution of this mass and the value for the ΔV burn (12.1129 ft/sec) results in the total vehicle mass prior to apoapsis adjustment (or, following periapsis raising):

\[ M_{i_4} = M_{i_3} = (140,000 \text{ lbm}) \cdot e^{(12.1129/(280 \cdot 32.174))} \]

\[ M_{i_3} = 140,188.36 \text{ lbm} \]
The propellant mass required for apoapsis adjustment is determined simply by subtracting the total mass following adjustment from the mass prior to the maneuver:

\[ M_{p,\Delta} = (140,188.36 - 140,000) \text{ lbm} \]

\[ M_{p,\Delta} = 188.36 \text{ lbm} \]

Following this procedure, the propellant mass breakdown is given in the accompanying table.

<table>
<thead>
<tr>
<th>Burn Number</th>
<th>( \Delta V ) (ft/sec)</th>
<th>Total Vehicle Mass (lbm)</th>
<th>Propellant Mass (lbm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3047.90</td>
<td>205,026.35</td>
<td>58,850.44</td>
</tr>
<tr>
<td>2</td>
<td>75.12</td>
<td>146,175.91</td>
<td>1213.86</td>
</tr>
<tr>
<td>3</td>
<td>301.66</td>
<td>144,962.05</td>
<td>4773.69</td>
</tr>
<tr>
<td>4</td>
<td>12.11</td>
<td>140,188.36</td>
<td>188.36</td>
</tr>
</tbody>
</table>

Note that the total vehicle mass on approach to Mars is determined to be 205,026.35 lbm. The total mass of the propellant used during the \( \Delta V \) burns is 65,026.35 lbm.

To calculate the mass of oxidizer (\( N_2O_4 \)) and fuel (MMH) the oxidizer/fuel ratio (1.65) will be used. Every 1.65 parts of oxidizer is accompanied by 1 part of fuel, for a total of 2.65 parts of propellant. From the ratio,

\[ \text{mass of } N_2O_4 = (1.65/2.65) \cdot M_p \]

\[ \text{mass of MMH} = (1.0/2.65) \cdot M_p \]
For the total mass of propellant given above (65,030 lbm),

- mass of $\text{N}_2\text{O}_4 = 40,491$ lbm
- mass of MMH = 24,540 lbm

The volumes of the nitrogen tetroxide and mono-methyl hydrazine are calculated with the equation for density:

$$\text{Volume} = \frac{\text{mass}}{\text{density}}$$

where

- $\rho_{\text{N}_2\text{O}_4} = 85.50$ lbm/ft$^3$
- $\rho_{\text{MMH}} = 53.83$ lbm/ft$^3$

Using the oxidizer and fuel masses given above, the respective volumes are

- $V_{\text{N}_2\text{O}_4} = 473.57$ ft$^3$
- $V_{\text{MMH}} = 455.88$ ft$^3$

The shape of the tanks can now be determined. If cylindrical tanks (25 ft. diameter) are used, the length may be calculated using

$$l = \frac{V}{\pi R^2}$$

For the nitrogen tetroxide,

$$l_{\text{N}_2\text{O}_4} = 0.965 \text{ foot}$$

and for the mono-methyl hydrazine,

$$l_{\text{MMH}} = 0.929 \text{ foot}$$

Note that the required length is only one foot, which is very impractical.

Spherical tanks will now be considered. For the radius of the tank,

$$R = \left( \frac{3 \cdot V}{4 \cdot \pi} \right)^{\frac{1}{3}}$$
Calculation of the sphere radii for the oxidizer and fuel tanks yields, respectively,
\[ R_{N_2O_4} = 4.835 \text{ feet} \]
\[ R_{MM} = 4.774 \text{ feet} \]
From this analysis the spherical tank is the optimum design. The tanks can be contained side-by-side within the cylindrical compartment of the MEPS vehicle; the radii of the spheres may be increased to five feet for ease of construction, thus providing for a compartment 10 feet in length with sufficient room for the 3 engines.
ANALYSIS OF THE MAIN PROPULSION SYSTEM

The following section contains the analysis of the engines considered for MEPS. Four engines were compared on the basis of thrust, specific thrust, weight, and burntime; the Space Transportation Main Engine (STME) was chosen for the main propulsion system. The necessary data (propellant mass and volume, module length/tank size) is presented for the STME, and a staging analysis is shown.

Engine Comparisons

Four engines were compared in the analysis of the main propulsion system—the J-2, RL-10-A-1, Space Shuttle Main Engine (SSME), and the Space Transportation Main Engine (STME). The J-2 was used for the third stage of the Saturn rockets. The original RL-10 engine was used for the early Saturn rockets, and has seen use on the Titan; the RL-10-A-1 is more of an engine design as this engine has not been produced. The SSME is currently in operation on the Space Shuttle orbiters, while the STME is a second generation SSME-based engine which also has not gone into production.

By using Newton’s Second Law, and assuming the initial mass (engine and propellant) to be equal, the four engine candidates can be compared:

\[ \Sigma F = T = m \cdot a \]

\[ T = \frac{M \cdot dV}{dt} \]
Rewriting the latter equation as an expression for time,

$$dt = \frac{M \cdot dV}{T}$$

$$\Delta t = \frac{M \cdot \Delta V}{T}$$

This final equation is used for calculation of the burn time of each engine. Note that the comparisons were made on the basis of thrust levels of approximately equal magnitudes; for approximately 450,000 lb of thrust the appropriate number of engines must be considered.

<table>
<thead>
<tr>
<th>Thrust (lbf)</th>
<th>Engines</th>
<th>Isp (sec)</th>
<th>Mass (lbm)</th>
<th>Burn Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-2</td>
<td>200,000</td>
<td>418</td>
<td>3480 (6960)</td>
<td>635.53</td>
</tr>
<tr>
<td>RL-10-A-1</td>
<td>15,000</td>
<td>433</td>
<td>298 (8642)</td>
<td>584.40</td>
</tr>
<tr>
<td>SSME</td>
<td>470,000</td>
<td>433</td>
<td>6700</td>
<td>540.90</td>
</tr>
<tr>
<td>STME</td>
<td>435,000</td>
<td>449</td>
<td>7455</td>
<td>584.40</td>
</tr>
</tbody>
</table>

**RL-10-A-1**

$$t = \frac{(702588.2 \text{ lbf}) \cdot (11641.26 \text{ ft/sec})}{2 \cdot (200000 \text{ lbf}) \cdot (32.174 \text{ lbm-ft/lbf-sec}^2)}$$

Total mass leaving orbit = 702588.2 lbm
Delta-V burn required = 11641.26 ft/sec

A short duration burn time is desirable because of the decreased risk of course deviation during the burn (ref 3). The shortest burn time is achieved by the engine with the greatest
thrust; by the above data this engine is the SSME. The next
lowest burn time occurs with the RL-10-A-1 engines and the STHE.
The RL-10-A-1 was found to be unfeasible since 29 units are
required to obtain a comparable thrust level. Although the STHE
has a longer burn time than the SSME, the STHE is designed to be
more reliable and less expensive than the SSME (ref 2); thus the
Space Transportation Main Engine is selected over the Space
Shuttle Main Engine.

Comparison of the STHE to the J-2 engine is based on thrust,
weight, burn time, and design. The two J-2 engines produce
400,000 lbf of thrust and weigh 6960 lbm; the STHE weighs
slightly more but produces greater thrust (435,000 lbf). The
burn time of the STHE is considerably less than that of the J-2.
In addition, the STHE is being designed specifically for reusabi-
ility and space applications (one design of the STHE nozzle ex-
pands the flow at the exit to the optimum pressure for operation
in the vacuum of space).

STHE Engine Information

The engine data required for the analysis of the MEPS
mission will now be presented. Some of the engine particulars
have been previously stated.

Isp = 449 seconds
Thrust = 435,000 lbf
Mass = 7455 lbm
Oxidizer/Fuel Ratio = 6.0
Area Ratio = 55/141
The initial mass of the MEPS vehicle prior to leaving Earth may be calculated using the following analysis:

\[
\Delta V = \text{Isp} \cdot g \cdot \ln(M_i / M_f) \\
M_i = M_f \cdot e^{(\Delta V / (\text{Isp} \cdot g))}
\]

The vehicle mass before the engines and the polar lander system are released is 381,505 pounds (see previous section). For a required delta-V burn of 11641.26 ft/sec to begin the Earth-Mars transfer,

\[
M_i = (381,505 \text{ lb}) \cdot e^{(11641.26/((449) \cdot 32.174))} \\
M_i = 854,020 \text{ lb}
\]

The mass of the propellant is the difference between the initial mass (vehicle plus propellant) and the final mass (vehicle only). For the given conditions,

\[
M_p = M_i - M_f \\
M_p = 854,019.4 \text{ lb} - 381,501.4 \text{ lb} \\
M_p = 472,518 \text{ lb}
\]

The oxidizer/fuel ratio for the engine is given as 6.0. Every six parts of liquid oxygen must be accompanied by one part of liquid hydrogen; thus a total of seven parts of oxidizer and fuel will be available. Using this development, the masses of the liquid oxygen and liquid hydrogen can be determined.

\[
M_{O_2} = 6/7 \cdot M_p = 6/7 \cdot 472,518 \text{ lb} = 405,015.43 \text{ lb} \\
M_H_2 = 1/7 \cdot M_p = 67,502.57 \text{ lb}
\]

Using the densities of the oxidizer and fuel, and the relationship between density and volume, the volumes of the liquid
oxygen and liquid hydrogen can be calculated. Knowing these volumes will allow the sizing of the fuel and oxidizer tanks.

\[ \rho_{\text{oxygen}} = 71.07 \text{ lb/ft}^3 \text{ at } -297^\circ \text{ F} \]
\[ \rho_{\text{hydrogen}} = 4.42 \text{ lb/ft}^3 \text{ at } -423^\circ \text{ F} \]

The density is defined as the mass per unit volume. Therefore,

\[ V_{\text{oxygen}} = 405,015 \text{ lb} \cdot (1 \text{ ft}^3/71.07 \text{ lb}) = 5699 \text{ ft}^3 \]
\[ \text{Volume} = 42,727.7 \text{ gallons} \]

\[ V_{\text{hydrogen}} = 67,503 \text{ lb} \cdot (1 \text{ ft}^3/4.42 \text{ lb}) = 15,271.95 \text{ ft}^3 \]
\[ \text{Volume} = 114,499.8 \text{ gallons} \]

A pressure vessel is normally spherical, or cylindrical with hemispherical ends. The diameter of the MEPS vehicle must be considered to determine which type tank will hold the liquid oxygen and hydrogen. For the diameter of 25 feet, the volume of a spherical tank is

\[ V = 4/3 \pi R^3 \]
\[ = 4/3 \pi (12.5 \text{ ft})^3 = 8181.23 \text{ ft}^3 \]

This volume falls between the required volumes for the oxidizer and fuel. Thus, a spherical tank will be employed for the liquid oxygen, and the cylindrical/hemispherical tank will be used for the liquid hydrogen.

For the calculated volume of liquid oxygen the corresponding tank size is determined to be

\[ \text{volume} = 5699 \text{ ft}^3 = 4/3 \pi R^3 \]
\[ R = 11.08 \text{ ft} \]

If boil-off of the liquid oxygen (12.0 lb/hr) is considered, the actual size of the tank must be increased to account for the
expendable oxidizer. Based on a thirty day transport and construction period, the volume of the LO₂ lost to boil-off is
\[
\text{mass} = (720 \text{ hrs}) \times (12.0 \text{ lb/hr}) = 8640 \text{ lb}
\]
\[
\text{volume} = (8640 \text{ lb}) \times (1 \text{ ft}^3 / 71.07 \text{ lb}) = 121.57 \text{ ft}^3
\]
This additional volume yields an increase in the diameter of the spherical tank to 22.5 feet.

The volume of liquid hydrogen is much larger than that of the liquid oxygen and, as mentioned previously, a cylindrical tank with hemispherical endcaps will be required. For the tank to fit snugly inside the MEPS vehicle (diameter of 25 feet), the length of the tank can be calculated:
\[
\text{volume} = \frac{4}{3} \pi (12.5)^3 + \pi (12.5)^2 h = 15,271.95 \text{ ft}^3
\]
\[
\text{length} = h = 14.4 \text{ ft}
\]
The tank size will increase under consideration of boil-off. The rate for LH₂ is 18.0 lb/hr, and for the same thirty day period used earlier,
\[
\text{mass} = (720 \text{ hrs}) \times (18.0 \text{ lb/hr}) = 12,960 \text{ lb}
\]
\[
\text{volume} = (12,960 \text{ lb}) \times (1 \text{ ft}^3 / 4.42 \text{ lb}) = 2932.13 \text{ ft}^3
\]
The change in the length of the tank is now determined:
\[
\text{length} = h = (15,271.95 + 2932.12 - 8181.23) / 490.87
\]
\[
h = 20.45 \text{ ft}
\]
Since the endcaps have a radius of 12.5 feet, the total length of the LH₂ tank is
\[
25 \text{ ft} + 20.45 \text{ ft} = 45.45 \text{ ft}
\]
If the two tanks are mounted bulkhead to bulkhead, the total length of the main propulsion module is

$$45.45\text{ ft} + 22.5\text{ ft} = 67.95\text{ ft}$$

**Staging Analysis**

In order to determine the optimum number of stages for Earth departure, the following equation is used:

$$\lambda = \left[ e^{-\frac{c}{\alpha \beta}} - \xi \right]^n$$

The results of the analysis performed on the STME are plotted below. From this plot, one stage is shown to be the optimum configuration for the engines.
AEROBRAKE ANALYSIS

The aerobrake will be used for orbit circularization about Mars. This section will present the determination of the weight of the aerobrake and the calculation of the propulsive burns required for the circularization analysis.

Weight Determination

Using information provided by Bill Willcockson, OTV Program Manager at Martin Marietta Aerospace (Denver), the mass of an aerobrake can be sized with the aerobrake area. From values presented in reference 7, 

\[
\text{Area} = 142 \text{ ft}^2 \\
\text{mass of rigid surface insulation (RSI)} = 401 \text{ lbm} \\
\text{mass of flexible surface insulation (FSI)} = 8890 \text{ lbm} \\
\text{structure weight} = 11032 \text{ lbm}
\]

The weight of the RSI will remain 401 lbm since a diameter of 25 feet is used for the Martin Marietta brake as well as the proposed brake. The weight of the FSI will require a calculation. For the Martin aerobrake,

\[
A = \frac{\pi}{4} \left( (142 \text{ ft})^2 - (25 \text{ ft})^2 \right) = 15345.895 \text{ ft}^2
\]

Obtaining a weight to area ratio,

\[
\frac{\text{weight}}{\text{area}} = \frac{8890 \text{ lbm}}{15345.895 \text{ ft}^2} = .57931 \text{ lbm/ft}^2
\]

For the MEPS aerobrake,

\[
A = \frac{\pi}{4} \left( (95 \text{ ft})^2 - (25 \text{ ft})^2 \right) = 6597.345 \text{ ft}^2
\]
The weight of the FSI is calculated using the weight/area ratio previously determined:

\[ W_{\text{FSI}} = (0.57931 \text{ lbm/ft}^2)(6597.345 \text{ ft}^2) \]

\[ W_{\text{FSI}} = 3821.895 \text{ lbm} \]

To determine the weight of the aerobrake structure, use the weights using a weight/area ratio:

\[ \frac{W_{\text{structure}}}{\% \cdot (95 \text{ ft})^2} = \frac{11032 \text{ lbm}}{\% \cdot (142 \text{ ft})^2} \]

\[ W_{\text{structure}} = 4937.701 \text{ lbm} \]

**Propulsive Burns Used For Mars Orbit Circularization**

Although aerobraking will be applied during the MEPS mission, complete orbit circularization will require propulsive burns using the secondary propulsion system. These burns must be considered for three different maneuvers: lowering the periapsis prior to aerobraking, and raising the periapsis and adjusting the apoapsis after braking. Calculation of each \( V \) will be made by applying the Vis-Viva equation:

\[ V = \sqrt{\mu g \cdot (2/r - 1/a)} \]

where \( r \) is the length of either the periapsis or apoapsis, measured from the center of Mars, and \( a \) is the semi-major axis of the elliptic orbit.

To decrease the periapsis, the burn will be applied at the apoapsis of the initial elliptic orbit about Mars. The desired periapsis altitude has been determined to be 314,976 ft (51.84 nautical miles). The lengths of the initial periapsis and apoapsis are 120,598,076.5 ft (19848.27 n mi) and 12,774,573.5 ft (2102.46 n mi), respectively, measured from the center of Mars.
The lengths of the semi-major axes of the two different elliptical orbits (same initial apoapsis, two different periapses) are determined:

\[ r_1 = 12,774,573.5 \text{ ft} (2102.46 \text{ n mi}) \]

\[ a_{v_1} = \frac{1}{2}(r_1 + r_s) = 66,686,325 \text{ ft} (10,975.37 \text{ n mi}) \]

\[ r_2 = 11,449,049.5 \text{ ft} (1884.31 \text{ n mi}) \]

\[ a_{v_2} = 66,023,563 \text{ ft} (10,866.29 \text{ n mi}) \]

Applying the Vis-Viva equation, the velocities at the apoapsis for each elliptical orbit are calculated:

\[ \sqrt{\frac{2}{r_1} - \frac{1}{a_{v_1}}} \]

\[ V_1 = 1546.98 \text{ ft/sec} \]

\[ \sqrt{\frac{2}{r_2} - \frac{1}{a_{v_2}}} \]

\[ V_2 = 1471.85 \text{ ft/sec} \]

The propulsive burn required to lower the periapsis is determined by taking the difference between the apoapsis velocities given above:

\[ \Delta V = V_2 - V_1 = -75.1214 \text{ ft/sec} \]

The minus sign indicates the burn will be applied in the direction opposite that of the MEPS vehicle (retrofire).

The same analysis is performed for the burns to raise the periapsis and apoapsis. The necessary inputs and output are presented.
to raise the periapsis to 1,640,500 ft (270 n mi):

\[
\begin{align*}
r_* &= 12,717,736.7 \text{ ft} \ (2093.11 \text{ n mi}) \quad \text{-- from program} \\
r_* &= 11,449,049.5 \text{ ft} \ (1884.31 \text{ n mi}) \\
a_e &= 12,083,391.5 \text{ ft} \ (1988.71 \text{ n mi}) \\
r_* &= 12,774,573.5 \text{ ft} \ (2102.46 \text{ n mi}) \\
a_e &= 12,746,155.2 \text{ ft} \ (2097.79 \text{ n mi})
\end{align*}
\]

\[\Delta V = 301.6568 \text{ ft/sec}\]

to raise the apoapsis to 12,774,573.5 ft (2102.46 n mi) from the center of Mars:

\[
\begin{align*}
r_* &= 12,774,573.5 \text{ ft} \ (2102.46 \text{ n mi}) \\
r_* &= 12,717,736.7 \text{ ft} \ (2093.11 \text{ n mi}) \\
a_e &= 12,746,155.2 \text{ ft} \ (2097.79 \text{ n mi}) \\
r_* &= 12,774,573.5 \text{ ft} \ (2102.46 \text{ n mi}) \\
a_e &= 12,774,573.5 \text{ ft} \ (2102.46 \text{ n mi})
\end{align*}
\]

\[\Delta V = 12.1129 \text{ ft/sec}\]
DERIVATIONS FOR THE AEROBRAKING PROGRAM

The program included in Appendix A is used to execute the iterations for the aerobraking process. With inputs concerning the orbit of a vehicle about Mars, and parameters of the aerobrake, the complete aerobraking passage can be analyzed. The output presents the time for aerobraking, the drag forces that act on the aerobrake, and the parameters of the final orbit.

The program requires several derivations--the location of the intersection of a circle (Mars atmosphere) and an ellipse (vehicle orbit); the length of segment between the intersection points (total distance travelled within the atmosphere); and the drag coefficient of the aerobrake.

Intersection Points

The equations of an ellipse and a circle are given, respectively, as

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

\[(x - ae)^2 + y^2 = r^2\]

where \(a\) is the semi-major axis and \(e\) is the eccentricity of the orbit; and \(a\cdot e\) is the location of the center of the circle representing the Martian atmosphere (i.e., the center of Mars). In addition, the trajectory equation, which gives the location of any point on the ellipse, is defined as

\[ r = \frac{p}{1 + e \cdot \cos \nu} \]
In the above equation all of the variables (semi-latus rectum, eccentricity, and argument of periapsis) are given parameters of an elliptic orbit.

Solving for $y^2$ in the two equations,

$$y^2 = b^2 - \frac{b^2x^2}{a^2}$$
$$y^2 = r^2 - (x - ae)^2$$

Equating the $y^2$ terms,

$$b^2 - \frac{b^2x^2}{a^2} = r^2 - (x - ae)^2$$

$$b^2 - \frac{b^2}{a^2} \cdot x^2 = r^2 - x^2 + 2aex - a^2e^2$$

$$(1 - \frac{b^2}{a^2}) \cdot x^2 - 2aex + (b^2 - r^2 + a^2e^2) = 0$$

Using the quadratic equation to solve for $x$,

$$x = \frac{2ae \pm \sqrt{4a^2e^2 - 4\cdot(b^2 - r^2 + a^2e^2)\cdot(1 - b^2/a^2)}}{2\cdot(1 - b^2/a^2)}$$

Choosing only the negative value of the square root (due to the geometry of the problem), the $x$-location of the intersection points is known. Substitution of $x$ back into an expression for $y$ will yield the complete location of the points.

**Segment Length**

The values of $x$ and $y$ obtained as the intersection points will be used in this derivation. The angle created between radii from the center of Mars is denoted as $\theta$, $s$ is the segment length, $R$ is the radius, $c$ is the chord of the arc, and $d$ is the distance from the center of Mars to the $x$-position of the intersection points.
Using the geometry of an arc,

\[ c = 2y \]
\[ d = (x - \text{ae}) \]
\[ R = (d^2 + y^2)^{1/2} \]
\[ \theta = 2 \cdot \tan^{-1} \left( \frac{c}{2d} \right) = 2 \cdot \tan^{-1} \left( \frac{c/2}{d} \right) = 2 \cdot \tan^{-1} \left( \frac{y}{x - \text{ae}} \right) \]

Finally,

\[ s = R\theta \]

Thus the segment is known, and this value can be used to determine the change in velocity due to drag forces during aero-braking.

**Determination of the Drag Coefficient**

From hypersonic equations for a cone, the drag coefficient is given as

\[ C_s = 2 \cdot \sin^2 \theta_v + (1 - 3 \cdot \sin^2 \theta_v) \cdot \sin^2 \alpha \]

where

\[ \theta_v = \text{cone half-angle} \]
\[ \alpha = \text{angle of attack} \]

For the MEPS mission the design for zero angle of attack (using momentum wheels and the cone's inherent stability) allows cancellation of the second term. Therefore (1:681),

\[ C_s = 2 \cdot \sin^2 \theta_v \]
ANALYSIS FOR THE OBSERVATION SATELLITE

This section will contain the calculations of the propulsive burns required for orbital transfer of the satellite. In addition, the determination of the solar array panel (applicable to the CIC as well) will also be presented.

Propulsive Burns

A transfer between a 1,640,500 ft (270 n mi) orbit and a 2,313,105 ft (327.64 n mi) orbit will be required to put the satellite into the observation orbit. For these calculations a Hohmann (minimum energy) transfer will be assumed.

Calculation of the altitudes is the first step:

\[ r_0 = 11,134,073.5 \text{ ft (1832.47 n mi)} \]
\[ r_1 = (11,134,073.5 + 1,640,500) \text{ ft} \]
\[ = 12,774,573.5 \text{ ft (2102.46 n mi)} \]
\[ r_f = (11,134,073.5 + 2,313,105) \text{ ft} \]
\[ = 13,447,178.5 \text{ ft (2005.25 n mi)} \]

The gravitational parameter of Mars is given as

\[ \mu_\text{M} = 1.5066 \text{ E15 ft}^3/\text{sec}^2 \]

For a Hohmann transfer, first calculate the circular velocities of the two orbits:

\[ V_c = \sqrt{\frac{\mu_\text{M}}{r}} \]
\[ V_1 = \sqrt{\frac{1.5066 \text{ E15 ft}^3/\text{sec}^2}{12774573.5 \text{ ft}}} = 10,859.7819 \text{ ft/sec} \]
\[ V_f = \sqrt{\frac{1.5066 \text{ E15 ft}^3/\text{sec}^2}{13447178.5 \text{ ft}}} = 10,584.8341 \text{ ft/sec} \]
Now determine the semi-major axis of the transfer ellipse:

\[ a_r = \frac{r_i + r_p}{2} = 13,110,876 \text{ ft (2157.82 n mi)} \]

To obtain the propulsive burn required to leave the initial orbit, the Vis-Viva equation of astrodynamics (see trajectory analysis section) will be used. The result is

\[ V_{r_i} = 10,998.2401 \text{ ft/sec} \]

The burn is found by subtracting the circular velocity from the velocity at the periapsis:

\[ \Delta V_i = V_{r_i} - V_i = 138.4 \text{ ft/sec} \]

The speed at the apoapsis and the propulsive burn required to achieve the final circular orbit are calculated in a similar manner:

\[ V_{r_2} = 10,448.0164 \text{ ft/sec} \]

\[ \Delta V_2 = 136.64 \text{ ft/sec} \]

Using these burns the mass of propellant required for the transfer can be calculated (the proper equation may be found in the section on the secondary propulsion system). First obtain the mass ratio for each \( \Delta V \):

\[ \left( \frac{M_i}{M_{r_i}} \right) = 1.0155 \quad \left( \frac{M_i}{M_{r_2}} \right) = 1.0153 \]

The final mass of the satellite in the observation is approximately 3500 lbm. Backing out the mass required by the second \( \Delta V \),

\[ M_i = M_r + M_s \]

\[ 1.0153 \cdot M_r = M_r + M_s \]

\[ 0.0153 \cdot (3500 \text{ lbm}) = M_s \]

\[ M_s = 53.55 \text{ lbm} \]
Obtaining the propellant mass used in the first $\Delta V$ with the same calculations,

$$1.0155 \cdot M_r = M_r + M_p$$

$$0.0155 \cdot M_r = M_p$$

$$0.0155 \cdot (3500 \text{ lbm} + 53.55 \text{ lbm}) = M_p$$

$$M_p = 55.08 \text{ lbm}$$

The total mass of propellant is

$$M_{rT} = M_r + M_p$$

$$M_{rT} = 108.93 \text{ lbm}$$

**Area for the Solar Array**

The area of a solar array panel is calculated using

$$A_a = \frac{P_a}{S \cdot \eta \cdot F \cdot \cos \Gamma} \quad \text{(ref 5)}$$

where

- $P_a = \text{available power (1500 watts)}$
- $S = \text{solar intensity at Mars (54.14 mW/ft}^2\text{)} \quad \text{(5:410)}$
- $\eta = \frac{\text{power output of array}}{\text{power input of sun}} \quad \text{(9.6\%)}$
- $F = \text{sum-total of array design and degradation factors}$
  - misc. assembly and degradation $\quad 0.95$
  - radiation (for silicon cells) $\quad 0.74$
  - configuration (flat plate array) $\quad 1.00$
- $F = 2.69 \quad \text{(5:123-125)}$
- $\Gamma = \text{angle between sun's rays and the normal to the panel}$
  - $\Gamma = 0$
  - $\cos \Gamma = 1$

Calculation of the area yields

$$A_a = \frac{1500 \text{ W}}{(54.14 \text{ W/ft}^2) \cdot 0.096 \cdot 2.69 \cdot 1} = 108 \text{ ft}^2$$
CALCULATION OF APPROXIMATE MASS OF MARS LANDER

To calculate the approximate mass of a Mars lander the masses of the major components of the lander must be estimated. These major components are (1) the Sample Return Vehicle (SRV); (2) the rovers; (3) the automated laboratory; (4) an upper and lower aeroshell; (5) a platform for the SRV to sit upon; (6) landing gear; and (7) a recovery system consisting of a solid rocket motor and three parachutes.

Mass of the Sample Return Vehicle (SRV)

The mass of the solid rocket booster that will propel the SRV can be obtained from a program written to simulate the launch of a solid rocket booster; booster specifications include the fuel, payload, and planet of launch. This program, titled "Stages", can be used to study the effect of changing propellant mass on the final altitude and velocity achieved by the rocket.

To use "Stages" (listed in Appendix A), the following parameters for a launch must be known or assumed:

1. the combustion temperature of the propellant (°R)
2. the density of the propellant (lbm/in³)
3. the propellant cross-sectional diameter (feet)
4. the propellant burn rate (in/sec)
5. the specific heat ratio and perfect gas constant for the burning propellant
6. the radius of the planet (n. mi.), the latitude of launch site (degrees), the angular velocity of the planet surface, and the gravitational acceleration on the planet surface

7. the desired altitude after launch

8. the mass of the final payload to be put into orbit.

The first seven parameters were designated for a launch from a pole of Mars of a rocket propelled by the propellant DB/AP-HMX/AL (Double Base/Aluminum Perchlorate-Cyclotetramethylene Tetranitramine/Aluminum), selected for its high combustion temperature (6700 degrees Rankine) and burn rate (.55 in/sec). The rocket booster was designed to have a propellant cross-sectional area of 2.91667 feet and a deadweight ratio (ratio of booster non-propellant mass to total booster mass) of 0.12. The desired orbit was specified to be circular at an altitude of 270.0 nautical miles.

The eighth parameter (payload mass) was designed to be a lightweight vessel that would carry up to 100 lbm of Martian soil and air samples in a refrigerated chamber; on board the ship would be a small reaction control system and an aeroshell. The mass of this vehicle is estimated to be 1000.0 lbm (200 lbm for the refrigeration chamber, 500 lbm for the reaction control system, 100 lbm of samples, 50 lbm for the aeroshell, and 150 lbm for onboard guidance and control computers).

Once these parameters for the SRV launch have been specified, a "target burn time" (which is equal to the mass of propellant divided by the mass consumption rate) is entered into the "Stages" subroutine named "Launch". This subroutine is a numeri-
cal integration of the equation of motion for a single-stage rocket being launched in a gravity field, and will fire for the entire "target burn time" unless the propellant is completely consumed or the target altitude is reached. As the launch proceeds, the spacecraft is rotated through a "pitch program", arbitrarily selected to vary the direction of the rocket's weight vector as its altitude increases.

To optimize the propellant mass, and thus the initial mass of the SRV, five plots of data from "Stages" are constructed (see Appendix B). These plots show:

1. variation of final altitude with "target burn time" (Figure B.1)
2. variation of final velocity with "target burn time" (Figure B.2)
3. variation of payload ratio (payload mass/initial mass) with "target burn time" (Figure B.3)
4. variation of "excess mass" (mass excluding payload mass after launch) with "target burn time" (Figure B.4)
5. variation of final acceleration with "target burn time" (Figure B.5)

The first plot is used to determine a minimum value for "target burn time" (TBT) by observing that below a particular value for TBT the desired altitude is not reached due to insufficient propellant mass. The second plot is then used to find the range of values of TBT above the minimum altitude value for which the final velocity is at least sufficient to achieve a circular orbit at the design altitude.
The third, fourth, and fifth plots are used to find the optimum value of TBT from the range of TBT values determined with the first two plots. An optimum payload ratio can be selected from the third plot; an optimum "excess mass" can be selected from the fourth plot; and an optimum final acceleration can be selected from the fifth plot.

For the SRV the payload ratio was optimized because the ratio will yield a shorter and less massive booster than the booster for which the final acceleration is minimum. From Figure B.1 the desired altitude is reached for TBT greater than or equal to 350 seconds. From the second plot the required orbital velocity is achieved only for TBTs ranging from 100 to 460 seconds; therefore the optimum range of TBTs is between 350 and 460 seconds. From the third plot the TBT for the highest payload ratio is found to be 350 seconds. From the fourth plot excess mass is seen to be a minimum for the optimum range at TBT of 350 seconds. From Figure B.5 final acceleration is seen to be a maximum for the optimum range at TBT equal to 350 seconds.

The value used for the payload ratio of the SRV, based upon optimization by use of "Stages", is 0.068120. This ratio yields an initial SRV mass of approximately 14,700 lbm.

Mass of the Upper and Lower Aeroshell

The masses of the upper and lower aeroshell can be estimated by determining the approximate geometry of the aeroshells and selecting a material with which the aeroshells will be made. The material that is selected for the shell must be strong enough to
withstand large aerodynamic forces and aerothermodynamic heating incurred upon descent through the Martian atmosphere. The maximum temperature that will occur on the lander during descent will be located at the stagnation point of the vessel, which is located at the center of the lower aeroshell. The stagnation point temperature that the lander encounters at an altitude of 100 nautical miles (calculated as 1630 degrees Rankine in Appendix C) is used to determine the type of material to be used for the upper and lower aeroshell (as a first approximation).

An "outer blanket" of carbon-carbon heat-transfer resistant tiles, or a one-piece carbon-carbon sheet, will cover the bottom of the lower aeroshell. The inner part of the lower aeroshell and the upper aeroshell will be composed of CLAD 2014 aluminum alloy (density of .101 lbm/in³). Modeling the upper aeroshell as a conic frustum 23 feet high, with a base diameter of 25 feet, a top diameter of seven feet, and a thickness of .30 inches, an approximate upper aeroshell volume of 233,989.7 cubic inches and a mass of approximately 23,633.0 lbm are determined. Modeling the lower aeroshell as a segment of a sphere with a base diameter of 25 feet, a segment height of five feet, and a thickness of .30 inches yields an approximate lower aeroshell volume of 20,722 cubic inches and a mass of approximately 2093.0 lbm.

**Mass of Platform and Landing Gear**

A metal disk twenty-five feet in diameter and one-half inch thickness is used to model the platform which the SRV, rovers, and autonomous laboratory sit upon; this platform has a volume
of 35,342 cubic inches. A strong material that can withstand the effects of the exhaust plume of the launching SRV is needed to comprise the platform; AM-355 stainless steel is chosen for its favorable resistance to high temperature and corrosion. AM-355 stainless steel has a density of .282 lbm/in³, so the mass of the platform is approximately 9966 lbm.

Each strut of the landing gear was modeled as a quarter-inch thick AM-355 stainless steel pipe, one foot in outer diameter and five feet long, fastened to a square AM-355 stainless steel pipe with sides four feet in length and a thickness of one-half inch. The total volume of each strut is 1857 cubic inches and the total mass of each strut is approximately 525 lbm. The landing gear system will consist of four struts so the total landing gear mass is 2100 lbm.

Mass of the Rovers and Automated Laboratory

Each rover is to be no more massive than 2500 lbm (5000 lbm for the two rovers on each lander). The mass of the automated laboratory will not exceed 1000 lbm.

Mass of the Lander Recovery System

The mass of the recovery system for the lander was determined by use of a program written by D. Bell. The program calculates the optimum recovery system mass, consisting of one to six parachutes and a solid fuel retrorocket, for a generic ve-
hicle of specifiable mass (minus recovery system mass) that is landing on the surface of Earth or Mars. The program inputs are:

1. the mass of the vehicle without recovery system
2. the desired terminal velocity for the main parachutes
3. the number of parachutes desired
4. the specific impulse, thrust, and mass fraction of the solid rocket motor used for descent
5. the required velocity upon impact with the planet surface
6. the desired height above the ground at which a constant-velocity descent of the vehicle begins (the rocket is fired such that the thrust equals the weight of the vehicle -- "constant velocity falling height")

A set of plots can be obtained by making a series of runs of this program (see Figures D.1, D.2, D.3). These plots are used to optimize the main chute terminal velocity, impact velocity, constant velocity falling height, the mass of the parachute system, and the mass of the solid rocket motor required to land the vehicle.

The total (approximate) mass of a lander is found to be 58,500 lbm, excluding the mass of the recovery system. A set of runs of the optimizing program were made using this value of the vehicle mass; the results can be seen in the figures of Appendix D. The optimum main chute terminal velocity for this vehicle is determined to be 75 feet per second. The optimum impact velocity for the vehicle is 10 feet per second (assuming that the terminal velocity under consideration for the vehicle is the optimum value). The optimum constant velocity falling height is...
five feet (assuming that the terminal velocity and impact velocity are optimum values. The optimum recovery system mass for the lander (using the aforementioned variables) is determined to be as follows:

- parachute system mass = 1610 lbm
- solid rocket motor mass = 2639 lbm
- total recovery system mass = 4249 lbm

A more detailed breakdown of the mass of the recovery system is shown in Appendix D. This breakdown is the final output of the optimizing program.

**Statement of Approximate Total Mass of Mars Lander**

As stated at the beginning of this section, the approximate total mass of the Mars lander is the sum of the masses of its major components:

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>MASS (lbm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRV</td>
<td>14,700</td>
</tr>
<tr>
<td>Upper Aeroshell</td>
<td>23,633</td>
</tr>
<tr>
<td>Lower Aeroshell</td>
<td>2,093</td>
</tr>
<tr>
<td>Platform</td>
<td>9,966</td>
</tr>
<tr>
<td>Landing Gear</td>
<td>2,100</td>
</tr>
<tr>
<td>Rover Systems</td>
<td>5,000</td>
</tr>
<tr>
<td>Laboratory</td>
<td>1,000</td>
</tr>
<tr>
<td>Parachute System</td>
<td>1,610</td>
</tr>
<tr>
<td>Solid Rocket Motor</td>
<td>2,639</td>
</tr>
</tbody>
</table>

**TOTAL APPROXIMATE LANDER MASS** 62,749
LIST OF REFERENCES


APPENDIX A

Computer Program for the Aerobraking Analysis
THIS PROGRAM RUNS THROUGH AN AEROBRAKE ANALYSIS OF MARS
INPUTS ARE MADE IN SI UNITS, AND OUTPUT IS WRITTEN IN
ENGLISH UNITS

REAL MU, MASS, MASSE
OPEN(UNIT=7, FILE='AEROBRKE.DAT', STATUS='OLD')

PRINT*, 'INPUT THE PERIAPSIS ALTITUDE IN km'
READ(6, * ) PERIAP

DEFINE THE PERIAPSIS FROM THE CENTER OF MARS
PERIAP = PERIAP + 3393.5
PRINT*, 'INPUT THE INITIAL SEMI-MAJOR AXIS IN km'
READ(6, * ) AINIT
PRINT*, 'INPUT THE DESIRED APOAPSIS DISTANCE IN km'
READ(6, * ) APOAPF

CALCULATE THE TIME FOR THE FIRST 1.5 ORBITS BEFORE MAKING
PERIAPSIS CHANGE (AT APOAPSIS) FOR AEROBRAKING PROCESS.
THE ELLIPTIC ORBIT FOR THIS PERIOD IS (500 km X 33363 km)
SMAJ1 = .5*(500. +3393.5)*(33363. +3393.5)

THE GRAVITATIONAL PARAMETER MU (kg^3/sec^2)
MU = 42656.
PI = 3.141592654
PERD1 = 2*PI*SQRT(SMAJ1**3/MU)
TIME1 = 1.5*PERD1

CALCULATE THE PARAMETERS OF THE ELLIPTIC ORBIT ABOUT MARS
USING THE PERIAPSIS FROM INPUT. THIS ORBIT IS ACTUALLY ONLY
HALF AN ORBIT, MAKING THE JOURNEY FROM APOAPSIS TO AEROBRAKING
PERIAPSIS
PERIOD = 0.0
APOAP = 0.0
CALL PARAMS(PERIAP, APOAP, AINIT, PERIOD)
PERDHR = PERIOD/3600.

DETERMINE THE TIME FOR THE HALF-ORBIT FROM THE APOAPSIS TO
THE PERIAPSIS OF AEROBRAKING
TIME2 = .5*PERIOD

OBTAIN THE INPUTS FOR THE AEROBRAKING PROCESS
PRINT*, 'INPUT THE ATMOSPHERIC DENSITY FOR THE ALTITUDE (kg/m^3)'
READ(6, * ) RHO
PRINT*, 'INPUT THE MASS OF THE SPACE VEHICLE (kg)'
READ(6, * ) MASS

CONVERT THE MASS TO ENGLISH UNITS
MASSE = MASS*32.174/14.57
PRINT*, 'INPUT THE HALF-ANGLE OF THE CONICAL AEROBRAKE (deg)'
READ(6, * ) THETA
PRINT*, 'INPUT THE DIAMETER OF THE AEROBRAKE (m)'
READ(6, * ) DIAM
DIAM = DIAM/1000.

DETERMINE THE AREA OF THE CONICAL AEROBRAKE
THETAR = THETA*PI/180.
PART = 1./((TAN(THETAR)**2)
AREA = PI*(DIAM/2.)**2*SQRT(1.+PART)

DETERMINE THE DRAG COEFFICIENT OF THE AEROBRAKE
BASED ON NEWTONIAN METHODS
CD = 2.*SIN(THETAR)**2

CALCULATE THE ENGLISH-UNIT COUNTERPARTS OF THE ABOVE VALUES
PERAPE = PERIAP+3280.839895
AINITE = AINIT+3280.839895
DIANE = DIAM+3280.839895
AREA = AREA+10763910.42
\[ \rho_e = \rho \times 0.0019435035 \]

The periapsis, apoapsis, semi-major axis, and aerobrake diameter are converted from km to ft. The area is changed from \( \text{km}^2 \) to \( \text{ft}^2 \) and the density from \( \text{kg/m}^3 \) to \( \text{slug/ft}^3 \).

\[ \text{TINTL} = \text{TIM1} + \text{TIM2} \]
\[ \text{TINTLH} = \text{TINTL} / 3600. \]

WRITE(7,*) ' AEROBRAKE ANALYSIS'
WRITE(7,*) ' HALF-ANGLE FOR CONICAL AEROBRAKE (deg): ', THETA
WRITE(7,*) ' DIAMETER OF THE AEROBRAKE (ft): ', DIAGE
WRITE(7,*) ' SURFACE AREA OF THE AEROBRAKE (ft^2): ', AREAE
WRITE(7,*) ' MASS OF SPACE VEHICLE (lb): ', MASSE
WRITE(7,*) ' ATMOSPHERIC CONDITIONS:'
WRITE(7,*) ' PERIAPSIS FROM CENTER OF MARS (ft): ', PERAPE
WRITE(7,*) ' DENSITY (slug/ft^3): ', RHOE
WRITE(7,*) ' INITIAL ORBITAL PARAMETERS:'
WRITE(7,130)
WRITE(7,140) PERAPE, APOAPE, AINIT, PERDHR
WRITE(7,*) ' APPROX TIME (hr) PRIOR TO AEROBRACING: ', TINTLH
WRITE(7,*) ' AEROBRAKE PROCEDURE:'
WRITE(7,100)
WRITE(7,110)
WRITE(7,110)

SET INITIAL CONDITIONS FOR VARIABLES PRIOR TO DO-LOOP

SMAJ = AINIT
X = 0.0
Y = 0.0
SEG = 0.0
PHI = 0.0
ASECTR = 0.0
TINTT = 0.0

DO 50 I = 1,500
ECCNTY = (APOAPE - PERAPE) / (APOAPE + PERAPE)
SMIN = SQRT(SMAJ**2 * (1. - ECCNTY**2))

CALL NTSEC(SMAJ, SMIN, ECCNTY, X, Y)

CALL SEGMENT(X, Y, SMAJ, ECCNTY, SEG, PHI, ASECTR)

DETERMINE THE VELOCITY OF THE SPACECRAFT AT PERIAPSIS
VELCTY = SQRT(MU * (2. / PERAPE - 1. / SMAJ))
THE SEMI-MAJOR AXIS IS THE "OLD" SEMI-MAJOR AXIS.

CALCULATE THE DRAG ON THE VEHICLE DURING THE AEROBRAKING PROCESS.

UNITS ARE (KG*KM/SEC^2) AND (LB)

\[
\text{DRAG} = 0.5 \times CD \times (\text{RHO} \times 1.9) \times \text{VELOCITY} \times \text{AREA}
\]

\[
\text{DRAG} = 0.5 \times \text{CD} \times \text{RHO} \times \text{VELOCITY} \times \text{AREA}
\]

DETERMINE THE TIME (IN MINUTES) OF THE AEROBRAKE PASSAGE

\[
\text{TIME} = 2 \times \text{ASECTR} \times \text{SQRT} (\text{MAJ}/\text{MU}) / \text{SHIN}
\]

\[
\text{TIME} = \text{TIME}/60.
\]

DETERMINE THE NEW SEMI-MAJOR AXIS

\[
\text{ENERGY} = \text{MU}/(2 \times \text{MAJ})
\]

\[
\text{MAJ} = \text{MU}/(-2 \times \text{DRAG} \times \text{SEG}/\text{MASS} + 2 \times \text{ENERGY})
\]

DETERMINE THE PARAMETERS OF THE NEW ELLIPTIC ORBIT.

CALL PARMS(\text{PERIAP}, \text{APOAP}, \text{MAJ}, \text{PERIOD})

C PERIOD OF THE ORBIT IS IN HOURS

\[
\text{PERDHR} = \text{PERIOD}/3600.
\]

CONVERT SI UNITS TO ENGLISH UNITS

\[
\text{MAJ}_{\text{E}} = \text{MAJ} \times 3280.839895
\]

\[
\text{APOAPE} = \text{APOAP} \times 3280.839895
\]

CHECK IF THE APOAPSIS IS LESS THAN THE RADIUS OF THE MARTIAN ATMOSPHERE.

\[
\text{IF}(\text{APOAP} \leq 3643.5) \text{ GO TO 80}
\]

45 WRITE(7,120) I, PERIAP, APOAPE, SMAJE, PERDHR, DRAGE, TIME

\[
\text{IF}(\text{APOAP} \leq \text{APOAPF}) \text{ GO TO 60}
\]

\[
\text{TINTTL} = \text{TINTTL} + \text{PERIOD}
\]

50 CONTINUE

\[
\text{IF}(\text{APOAP} \geq \text{APOAPF}) \text{ GO TO 85}
\]

DETERMINE THE TIME TO TRAVEL THE HALF ORBIT FROM THE AEROBRAKING PERIAPSIS TO THE APOAPSIS.

60 TIMEPA = 0.5 \times \text{PERIOD}

A DELTA-V BURN WILL BE PERFORMED AT THE APOAPSIS TO RAISE THE PERIAPSIS TO 500 km. DETERMINE THE PERIOD OF THE NEW ORBIT, AND THE TIME TO TRAVEL FROM THE APOAPSIS TO THE PERIAPSIS.

\[
\text{MAJAP} = 0.5 \times ((500 + 3393.5) + \text{APOAP})
\]

\[
\text{PERDAP} = 2 \times \text{PI} \times \text{SQRT} (\text{MAJAP}^{3}/\text{MU})
\]

\[
\text{TIMEAP} = 0.5 \times \text{PERDAP}
\]

\[
\text{IF}(\text{APOAP} \equiv \text{APOAPF}) \text{ THEN}
\]

\[
\text{PERDF} = \text{PERDAP}
\]

\[
\text{ELSE}
\]

\[
\text{GO TO 70}
\]

END IF

GO TO 75

BECAUSE THE FINAL APOAPSIS FROM AEROBRAKING IS LESS THAN THE DESIRED APOAPSIS, A DELTA-V BURN WILL HAVE TO BE APPLIED AT THE PERIAPSIS TO RAISE THE APOAPSIS SO THAT THE FINAL CIRCULAR ORBIT IS OBTAINED. THE PERIOD OF THIS ORBIT, AND THE TIME TO COMPLETE ONE ORBIT (THUS FINALIZING THE CIRCULARIZATION OF THE ORBIT ABOUT MARS) IS DETERMINED.

70 SMAF = 500 + 3393.5

\[
\text{PERDF} = 2 \times \text{PI} \times \text{SQRT} (\text{SMAF}^{3}/\text{MU})
\]

75 TINTL = TINTTL + TIMEPA + TIMEAP + PERDF

\[
\text{TIMHR} = (\text{TINTL} \times \text{TIME1} \times \text{TIME2})/3600.
\]
WRITE(7,*) 'TIME BREAKDOWN (hrs): '
WRITE(7,*) ' TIME TO INITIALIZE ORBIT: ', TIME1/3600.
WRITE(7,*) ' TIME TO TRAVEL FROM APOAP TO PERIAP: ', TIME2/3600.
WRITE(7,*) ' TIME FOR AEROBRACING PASSAGE: ', TIMEP/3600.
WRITE(7,*) ' TIME TO TRAVEL FROM PERIAP TO APOAP: ', TIMEA/3600.
WRITE(7,*) ' TIME FOR 1 ORBIT AFTER CIRCULARIZE: ', PERD/3600.
WRITE(7,*) ' TOTAL TIME FOR AEROBRACING PROCESS: ', TIMHR

PRINT*,I
PRINT*, 'AEROBRACING TIME= ', (TIME1+TIMEP+TIME2)/3600.
PRINT*, 'PER TO APO= ', TIMEP/3600.
PRINT*, 'APO TO PER= ', TIMEA/3600.
PRINT*, 'ORBIT AFTER CIRCULARIZATION= ', PERD/3600.
GO TO 90
80 WRITE(7,*) 'AEROBRACING IS NOT POSSIBLE FOR THIS PERIAPSIS'
GO TO 90
85 WRITE(7,*) 'FINAL APOAPSIS HAS NOT BEEN REACHED'

130 FORMAT(3X, 'PERIAPSIS (ft)', 6X, 'APOAPSIS (ft)', 6X,
1 'SEMI-MAJOR AXIS', 5X, 'PERIOD (hrs)')
140 FORMAT(2X, F15.5, 5X, F15.5, 5X, F15.5, 5X, F11.5)
100 FORMAT(4X, 'PASS', 5X, 'PERIAPSIS', 7X, 'APOAPSIS', 7X,
1 'SEMI-MAJOR', 4X, 'PERIOD', 7X, 'DRAG', 8X, 'PASSAGE')
110 FORMAT(3X, 'NUMBER', 7X, 'PASS', 11X, '(ft)', 11X, 'AXIS (ft)',
1 5X, '(hrs)', 7X, '(lb)', 7X, 'TIME (min)')
90 CLOSE (UNIT = 7)
STOP
END

SUBROUTINE PARAHS (RP, RA, A, PERD)
C THIS SUBROUTINE CALCULATES THE APOAPSIS AND PERIOD OF THE
ELLIPSE ORBIT, USING THE VALUES OF THE PERIAPSIS AND SEMI-
MAJOR AXIS FROM THE MAIN PROGRAM
THE PERIAPSIS, APOAPSIS, AND SEMI-MAJOR AXIS ARE IN km
THE PERIOD IS IN sec, THE GRAVITATIONAL PARAMETER IS (km^3/sec^2)

REAL MU
RA = 2. * A - RP
PI = 3.141592654
MU = 42656.0
PERD = 2. * PI * SQRT(A**3/MU)
RETURN
END

SUBROUTINE NTRSEC (A, B, E, X, Y)
C THIS SUBROUTINE CALCULATES THE POINTS OF INTERSECTION OF THE
SPACE VEHICLE'S ELLIPSE ORBIT AND THE ATMOSPHERE'S CIRCULAR
ORBIT, USING THE SEMI-MAJOR AXIS AND THE ECCENTRICITY FROM THE
MAIN PROGRAM, AND THE SEMI-MINOR AXIS FROM SUBROUTINE PARAMS
RADIUS = 250. * 3393.5
X1 = 2. * A * E
X2A = 4. * A ** 2 * E ** 2
X2B = 4. * (B ** 2 - RADIUS ** 2 + A ** 2 * E ** 2) * (1. - B ** 2 / A ** 2)
X2 = SQRT(X2A - X2B)
X3 = 2 * (1. - B ** 2 / A ** 2)

THE INTERSECTION POINTS OF THE ELLIPTIC ORBIT ARE X AND Y
X = (X1 - X2) / X3
Y = SQRT(RADIUS ** 2 - (X - A * E) ** 2)

RETURN
END

SUBROUTINE SEGMENT(X, Y, A, E, SEG, PHI, AREA)
THIS SUBROUTINE CALCULATES THE LENGTH OF THE SEGMENT (km) OF THE
SPACE VEHICLE'S ELLIPTIC ORBIT BOUNDED BY THE MARTIAN ATMOSPHERE
USING THE INTERSECTION POINTS, SEMI-MAJOR AXIS, AND ECCENTRICITY
FROM THE MAIN PROGRAM

C = 2. * Y
D = X - A * E
R = SQRT(D ** 2 + Y ** 2)

PHI IS THE ANGLE OF THE BOUNDED SEGMENT, AND AREA IS THE AREA
OF THE BOUNDED PORTION OF THE ORBIT
PHI = 2. * ATAN(C / (2. * D))
SEG = R * PHI
AREA = .5 * R * SEG

RETURN
END
AEROBRAKE ANALYSIS

HALF-ANGLE FOR CONICAL AEROBRAKE (deg): 70.00000000
DIAMETER OF THE AEROBRAKE (ft): 95.00000000
SURFACE AREA OF THE AEROBRAKE (ft^2): 7543.12402000
MASS OF SPACE VEHICLE (lb): 200000.00000000

ATMOSPHERIC CONDITIONS:
PERIAPSIS FROM CENTER OF MARS (ft): 1.14484910E+07
DENSITY (slug/ft^3): 2.429379489E-10

INITIAL ORBITAL PARAMETERS:

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APPROX TIME (hr) PRIOR TO AEROBRAKING: 48.79166030
AEROBRAKING COMPLETE

TIME BREAKDOWN (hrs):

TIME TO INITIALIZE ORBIT: 36.73029330
TIME TO TRAVEL FROM APOAP TO PERIAP: 12.06136420
TIME FOR AEROBRAKING PASSAGE: 472.14114400
TIME TO TRAVEL FROM PERIAP TO APOAP: 0.94430298
TIME TO TRAVEL FROM APOAP TO PERIAP: 1.02305233
TIME FOR 1 ORBIT AFTER CIRCULARIZE: 2.05304074

TOTAL TIME FOR AEROBRAKING PROCESS: 524.95318600
APPENDIX B

Optimization of Propellant Mass of a Solid-Propellant Rocket
MIKE LISANO
AUGEN UNIVERSITY AEROSPACE ENGINEERING SENIOR DESIGN PROJECT
UNMANNED MARS MISSION

"STAGES"

PROGRAM TO OPTIMIZE THE PROPELLANT MASS OF A SOLID-PROPELLANT
ROCKET LAUNCHING IN AN ARBITRARY GRAVITY FIELD FOR A GIVEN
DESIRED ORBIT (PITCH PROGRAM INCLUDED)

COMMON/ROCK/TCMBM/DF/DIA/BR/GAMMA/RGAS/GC
COMMON/PLAN/GRAV/R/A/OMEGA/RAD/ALAT
COMMON/LAUN/V(610)/G(610)/H(610)/AM(610)/DELV(610)/ACC(610)

UNIT CONVERSION FACTOR GC (LBM-FT/LBF-SEC**2)
GC=32.174

SPECIFICATIONS OF ROCKET:
(COMBUSTION TEMPERATURE OF PROPELLANT, DEGREES RANKINE)
TCMBM=6700.
(DENSITY OF FUEL, LBM/IN**3)
DF=.065
(PROPELLANT CROSS-SECTION DIAMETER, FT)
FDIA=2.91667
DIA=FDIA*12.0
(PROPELLANT BURN RATE, IN/SEC)
BR=0.55
(SPECIFIC HEAT RATIO OF BURNING PROPELLANT (DEFAULT: AIR))
GAMMA=1.4
(PERFECT GAS CONSTANT OF BURNING PROPELLANT (DEFAULT: AIR),
FT-LBF/LBM-R)
RGAS=53.3
(DEADWEIGHT RATIO OF BOOSTER)
DWRAT=.12
(MASS OF PAYLOAD TO BE CARRIED INTO ORBIT, LBM)
AMF=1000.0

DATA FOR LAUNCH:
(RADIUS OF PLANET, N MI)
RAD=1841.0
(LATITUDE OF LAUNCH SITE, DEGREES)
DLAT=0.0
ALAT=DLAT/3.295773
(ANGULAR VELOCITY OF PLANET, RAD/SEC)
OMEGA=.00067
(DESIRED ALTITUDE, N MI)
ALT=270.0
HF=ALT*6080.
TOTAL HEIGHT, FT
R = (RAD * 6080) + HF
(SEMI-MAJOR AXIS OF DESIRED ORBIT, N MI (DEFAULTS TO A=R))
SMA = RAD + ALT
A = SMA * 6080.
(GRAVITATIONAL ACCELERATION ON THE SURFACE OF THE PLANET, FT/SEC**2)
GSLRF = 12.332
(GRAVITATIONAL PARAMETER OF PLANET IN FT**3/SEC**2)
GRAV = 1.5065E15

WRITE(6, 111)
FORMAT('6X,'OPTIMIZATION OUTPUT:')

SUBROUTINE "ROCKET" CALCULATES THE EXHAUST VELOCITY (C) AND TIME RATE OF CHANGE OF MASS (DMDT) OF THE ROCKET:
CALL ROCKET(C, DMDT)
WRITE(6, 1111) C, DMDT
FORMAT('6X,'EXHAUST VELOCITY OF ROCKET =''F12.3,1X,''FT/SEC',
'SUBROUTINE "SPEEDS" CALCULATES THE SPEED OF THE ROCKET DUE TO PLANETARY SPIN BEFORE TAKEOFF, AND THE SPEED OF THE ROCKET IN THE DESIRED ORBIT:
CALL SPEEDS(VSURF, VORB)
WRITE(6, 1111) VSURF, ALT, VORB
FORMAT('6X,'VELOCITY OF PLANET SURFACE =''F12.3,1X,''FT/SEC',
'SUBROUTINE "LAUNCH" CALCULATES THE TOTAL CHANGE IN SPEED AND CHANGE IN ALTITUDE OF A SINGLE STAGE ROCKET BEING LAUNCHED IN THE GRAVITY FIELD OF A GIVEN PLANET. AERODYNAMIC FORCES ON THE ROCKET HAVE BEEN NEGLECTED. THE INITIAL MASS OF THE ROCKET IS INCREMENTED FROM THE MASS REQUIRED FOR A TEN MINUTE BURN TO THAT REQUIRED TO REACH THE DESIRED ALTITUDE (HF) AND VELOCITY (VORB).
CALL LAUNCH(DWRAT, VSURF, GSURF, AMF, DMDT, C, HF, RAD, VORB)

COMMON/ROCK/TCOMB, DF, DIA, BR, GAMMA, RGAS, GC
A = (3.1415927*(DIA**2))/4.
DMDT = A * DF * ER
C = SQRT((2.*GAMMA*RGAS/(GAMMA-1.))*GC*TCOMB)
SUBROUTINE SPEEDS(VSURF, VORB)
COMMON/PLAN/GRAV, R, A, OMEGA, RAD, ALAT
VSURF = CMega*rad*6080.*cos(ALAT)
VORB = sqrt(grav*((2./r)-(1./a)))
RETURN
END

SUBROUTINE LAUNCH(DWRAT, VSURF, GSURF, AMF, DMT, C, HF, RAD, VORB)
COMMON.LAUN/V(610), G(610), H(610), AM(610), DELV(610), ACC(610)

TARGET BURN TIME (SEC)... DETERMINES INITIAL MASS AM(1)
TBURN = 350.0
TIME INCREMENTS (SEC)
DT = 1.0

T = 0.0
AM(1) = (TBURN*DMT)/(1.0-DWRAT) + AMF
AMFF = (AM(1); AMF)*DWRAT + AMF
V(1) = VSURF
DELV(1) = 0.0
G(1) = GSURF
H(1) = 0.0
DTHET = 0.0
I = 1

I = I + 1
T = T + DT
THET = DTHET/57.29578
AM(I) = AM(I-1) - (DMT*DT)
IF (AM(I)*LT.*AMFF) GOTO 1900
DELV(I) = C*log(AM(I-1)/AM(I)) - (G(I-1)*cos(THET)*DT)
V(I) = V(I-1) + DELV(I)
H(I) = H(I-1) + ((V(I) + V(I-1))/2.)*DT*sin(THET)
G(I) = GSURF + ((RAD*6080.*2)/((RAD*6080. + H(I))**2))
ACC(I) = DELV(I)/(DT*32.174)

PITCH PROGRAM (ARBITRARY, FIVE STEP, INITIAL THETA = 0 DEG.,
FINAL THETA = 90 DEG.)
IF (H(I).GE.HF) GOTO 1900
IF (H(I).GT.(HF*.4)) GOTO 1800
IF (H(I).GT.(HF*.15)) GOTO 1700
IF (H(I).GT.(HF*.05)) GOTO 1600
IF (H(I).GT.(HF*.01)) GOTO 1500

500 DTHET = 45.0
GOTO 100
1600 DTHET = 65.0
GOTO 100
700 DTHET = 80.0
GOTO 100
1800 DTHET = 85.0
GOTO 100
1900 T = T - DT
VEL = V(I-1)
HT = H(I-1) / 6080.
AC = ACC(I-1)
AMF = AM(I-1)
PR = AMF/AM(1)
TPR = AM(I-1)/AM(1)
EXC = AM(I-1) - AMF

WRITE("6,212")TBURN,AM(1),T,VEL,HT,AC,AMM,PR,TPR,EXC

RETURN
END

*******************************************************************************
OPTIMIZATION OUTPUT:

EXHAUST VELOCITY OF ROCKET = 8968.144 FT/SEC

PROPELLANT CONSUMPTION RATE = 34.395610 LBM/SEC

VELOCITY OF PLANET SURFACE = 783.551 FT/SEC

DESIRED ORBIT ALTITUDE = 270.000 N MI

VELOCITY OF ROCKET IN DESIRED ORBIT = 10833.867 FT/SEC

DATA FOR SINGLE-STAGE ROCKET LAUNCH:

THE TARGET TIME OF BURN IS 250.000 SEC

THE INITIAL MASS OF THE ROCKET IS 10771.480 LBM

THE TOTAL TIME OF BURN IS 250.000 SEC

THE FINAL VELOCITY IS 14047.491 FT/SEC

THE FINAL ALTITUDE IS 210.3703 N MI

THE FINAL ACCELERATION IS 4.35 GS

THE FINAL MASS OF THE ROCKET IS 2172.578 LBM

THE DESIGN PAYLOAD RATIO OF THE ROCKET IS 0.092838

ACTUAL FINAL MASS/INITIAL MASS IS 0.201697

THE EXCESS MASS AFTER FIRING IS 1172.578 LBM
OPTIMIZATION OUTPUT:

EXHAUST VELOCITY OF ROCKET = 8968.144 FT/SEC
PROPELLANT CONSUMPTION RATE = 34.395610 LBM/SEC
VELOCITY OF PLANET SURFACE = 783.551 FT/SEC
DESIRED ORBIT ALTITUDE = 270.000 N MI
VELOCITY OF ROCKET IN DESIRED ORBIT = 10833.867 FT/SEC

DATA FOR SINGLE-STAGE ROCKET LAUNCH:
 THE TARGET TIME OF BURN IS 350.000 SEC
 THE INITIAL MASS OF THE ROCKET IS 14680.072 LBM
 THE TOTAL TIME OF BURN IS 336.000 SEC
 THE FINAL VELOCITY IS 13328.603 FT/SEC
 THE FINAL ALTITUDE IS 268.750 N MI
 THE FINAL ACCELERATION IS 3.03 GS
 THE FINAL MASS OF THE ROCKET IS 3123.147 LBM
 THE DESIGN PAYLOAD RATIO OF THE ROCKET IS 0.068120
 ACTUAL FINAL MASS/INITIAL MASS IS 0.212747
 THE EXCESS MASS AFTER FIRING IS 2123.147 LBM
OPTIMIZATION OUTPUT:

EXHAUST VELOCITY OF ROCKET = 8968.144 FT/SEC

PROPELLANT CONSUMPTION RATE = 34.395610 LBM/SEC

VELOCITY OF PLANET SURFACE = 783.551 FT/SEC

DESIRED ORBIT ALTITUDE = 270.000 N MI

VELOCITY OF ROCKET IN DESIRED ORBIT = 10833.867 FT/SEC

DATA FOR SINGLE-STAGE ROCKET LAUNCH:

THE TARGET TIME OF BURN IS 450.000 SEC

THE INITIAL MASS OF THE ROCKET IS 18588.664 LBM

THE TOTAL TIME OF BURN IS 394.000 SEC

THE FINAL VELOCITY IS 10955.684 FT/SEC

THE FINAL ALTITUDE IS 268.4356 N MI

THE FINAL ACCELERATION IS 1.37 GS

THE FINAL MASS OF THE ROCKET IS 5036.794 LBM

THE DESIGN PAYLOAD RATIO OF THE ROCKET IS 0.053796

ACTUAL FINAL MASS/INITIAL MASS IS 0.270961

THE EXCESS MASS AFTER FIRING IS 4036.794 LBM
FIGURE B.1

FINAL ALTITUDE VS. "DESIGN BURN TIME"

ORIGINAL PAGE IS OF POOR QUALITY
FIGURE B.2

FINAL VELOCITY VS. "DESIGN BURN TIME"

INITIAL PROPELLANT MASS/BURN RATE, SEC

FINAL VELOCITY, FT/SEC
FIGURE B.3

PAYLOAD RATIO VS. "DESIGN BURN TIME"
FIGURE B.4

EXCESS MASS VS. "DESIGN BURN TIME"

INITIAL PROPELLANT MASS/BURN RATE, SEC

FINAL EXCESS MASS, LBM

EXCESS MASS
FIGURE B.5

FINAL ACCELERATION VS. "DESIGN BURN TIME"

INITIAL PROPELLANT MASS/BURN RATE, SEC
APPENDIX C

Calculation of Stagnation Temperature on Mars Lander
Apply the method of Nicolai (5;4;6):

From Stefan’s Law,

\[ T_0 = \left( \frac{\dot{q}_{\text{rad}}}{E \sigma_{\text{st}}} \right)^{25} \]

where \( T_0 \) is the temperature at the stagnation point of the body (degrees Rankine)

\( \dot{q}_{\text{rad}} \) is the radiative heat flux from air to body (Btu/ft²/sec)

\( E \) is the emissivity of the fluid

\( \sigma_{\text{st}} \) is the Stefan-Boltzmann constant (.481 E-12 Btu/ft²/sec/°R)

For the atmosphere of Mars the emissivity is assumed to be approximately the same value as the emissivity of the Earth’s atmosphere, or

\[ E = 0.8 \]

For the flow-body system to be in equilibrium, the heat radiated to the body must equal the heat convected from the body to the flow:

\[ \dot{q}_{\text{rad}} = \dot{q}_{\text{conv}} \]

An empirical formula from Nicolai allows the calculation of the convective heat flux:

\[ \dot{q}_{\text{conv}} = 15 \cdot (\frac{\rho_{\infty}}{R}) \cdot (\frac{u}{1000})^3 \cdot \cos \Delta \]

where \( \rho_{\infty} \) = freestream density (slugs/ft³)

\( u \) = freestream velocity (ft/sec)

\( R \) = radius of curvature of the nose (ft)

\( \Delta \) = sweep angle of wing leading edge (zero degrees)
At an altitude of 100 nautical miles the density of the Martian atmosphere is determined:

\[
\rho_{100 \text{ n.m}} = \frac{P_{100 \text{ n.m}}}{R_{CO_2} \cdot T_{100 \text{ n.m}}}
\]

where \( R_{CO_2} = 35.10 \text{ ft} \cdot \text{lb/(lbm} \cdot \text{°R}) \)

\( T_{100 \text{ n.m}} = 324.6 \text{ °R} \)

\( P_{100 \text{ n.m}} = 0.0042837 \text{ lbf/ft}^2 \)

The values of \( T \) and \( P \) were obtained from Viking data.

The velocity at 100 nautical miles was estimated by assuming that the specific total mechanical energy of the lander at 100 nautical miles is the same as the specific total mechanical energy at 270 nautical miles. This assumption implies that no work is done by drag forces, which would decrease the total energy of the lander. Thus the velocity estimate is high:

\( u_{\infty 100} = 11,744 \text{ ft/sec} \)

This estimate leads to a convective heat flux of

\( q_{\text{conv}} = 2.7199 \text{ Btu/ft}^2/\text{sec} = q_{\text{conv}} \)

and a stagnation temperature of

\( T_s = 1630 \text{ °R} \)
APPENDIX D

Optimization of Recovery System Mass
For Mars Landers
RECOVERY SYSTEM SUMMARY

Weight Statement

Parachute System Weight................................. 1610

- Pilot Chute......................... 8
- Drogue Chute................. 224
- Misc. Straps.................... 69
- Main Parachute
  Support Structure............. 76
- Main Parachute............. 391 x 3
- Fittings and Flotation.. 59

SRM Weight........................................... + 2639

- Propellant............ 2243
- Case Weight......... 396

Recovery System Total Weight.............................. 4249

Basic Vehicle Weight...................................... + 58500

Total Vehicle Weight (Reentry)............................. 62749

Performance Characteristics

Parachute System

1 pilot @ 11.7 ft. diameter
1 drogue @ 54.4 ft. diameter
3 mains @ 129.7 ft. diameter
75 fps terminal velocity

SRM

ISP ....................... 260 sec
mass fraction ............. 0.850
thrust .................... 100000 lbf
burn time ................. 6.04 sec
impact velocity ......... 9.92 fps
constant velocity
falling height........... 5.1 ft
total distance fallen.... 210.2 ft
maximum deceleration... 1.6 g's
FIGURE D.1

MASSES VERSUS TERMINAL VELOCITY

- • SRM MASS (LB)
- ▼ PARACHUTE MASS (LB)
- ▽ TOTAL MASS (LB)

MASSES (LB)

MAIN CHUTE TERMINAL VELOCITY (FT/SEC)
FIGURE D.2  MASSES VERSUS IMPACT VELOCITY

- SRM MASS (LB)
- PARACHUTE MASS (LB)
- TOTAL MASS (LB)
FIGURE D.3
MASSES VERSUS FALLING HEIGHT
- SRM MASS (LB)
- PARACHUTE MASS (LB)
- TOTAL MASS (LB)

MASS (LB)

CONST. VEL. FALLING HT. (FT.)

5000 4000 3000 2000 1000

5 6 7 8 9 10