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FOR STAGNATION POINT TURBULENT FLOWS

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ABSTRACT

The problem of plane stagnation point flow with freestream turbulence is examined from a basic theoretical standpoint. It is argued that the singularity which arises from the standard $K - \epsilon$ model is not due to a defect in the model but results from the use of an inconsistent freestream boundary condition. The inconsistency lies in the implementation of a production equals dissipation equilibrium hypothesis in conjunction with a freestream mean velocity field that corresponds to homogeneous plane strain - a turbulent flow which does not reach such a simple equilibrium. Consequently, the adjustment that has been made in the constants of the ϵ -transport equation to eliminate this singularity is not self-consistent since it is tantamount to artificially imposing an equilibrium structure on a turbulent flow which is known not to have one.

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1. INTRODUCTION

The calculation of stagnation point turbulent flows has a variety of important engineering applications in boiler tubes, gas turbines, and ramjet combustors. Most of the earlier work [1,2] on this subject was based on the use of algebraic eddy viscosity models that do not allow for the detailed calculation of the turbulence statistics which can play an important role in determining wall friction and heat transfer coefficients. Consequently, the most recent work on the subject has been based on the use of more sophisticated two-equation turbulence models of the $K - \epsilon$ type which have the advantage of allowing for the direct calculation of the turbulent kinetic energy [3,4]. Unfortunately, a problem with a singularity in the turbulent kinetic energy has arisen when the traditional dissipation rate transport equation of the $K - \epsilon$ model is applied to plane stagnation point turbulent flow. Strahle and his co-workers [3,4] introduced an ad hoc modification of the constants in the ϵ -transport equation which eliminated this singularity. However, this readjustment of constants is somewhat unsettling since it yields an ϵ -transport equation which is incapable of collapsing most of the homogeneous turbulence data that is commonly used to benchmark turbulence models.

In this paper, it will be shown that the singularity in the turbulent kinetic energy that occurs when the standard $K - \epsilon$ model is applied to plane stagnation point flows arises from the use of an inconsistent freestream boundary condition and not from a defect in the model. To be specific, this commonly used formulation of turbulent stagnation point flow is ill-posed since a production equals dissipation equilibrium hypothesis is used in the freestream in conjunction with a mean velocity field that corresponds to homogeneous plane strain - a turbulent flow which does not possess such an equilibrium solution. In homogeneous plane strain turbulence, the kinetic energy and dissipation rate grow unbounded with time. Hence, it will be shown that the ad hoc adjustment of the constants of the ϵ -transport equation which has been used to eliminate this singularity [3,4] is not self-consistent since it results in the imposition of an equilibrium structure (with a bounded turbulent kinetic energy and dissipation rate) on a freestream turbulence whose mean velocity field does not permit such a solution. The mathematical origins and implications of this inconsistency will be discussed in detail along with possible alternative formulations that are well posed.

2. THE $K - \epsilon$ MODEL AND STAGNATION POINT TURBULENT FLOWS

The problem to be considered is that of plane stagnation point flow with freestream turbulence as shown in Figure 1. Outside of a boundary layer of thickness δ , the mean flow is assumed to be irrotational and incompressible with a background turbulence superimposed on it. This outer flow is taken to be of the form [3,4]

$$\bar{u}_\infty = x, \quad \bar{v}_\infty = -y \quad (1)$$

where $\mathbf{V}_\infty = \bar{u}_\infty \mathbf{i} + \bar{v}_\infty \mathbf{j}$ is the outer mean velocity which is non-dimensionalized. The mean velocity near the wall is a solution of the mean continuity and Reynolds equations which take the dimensionless form

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \quad (2)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = -\frac{\partial \bar{P}}{\partial x} + \frac{1}{Re} \nabla^2 \bar{u} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \quad (3)$$

$$\bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} = -\frac{\partial \bar{P}}{\partial y} + \frac{1}{Re} \nabla^2 \bar{v} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \quad (4)$$

where \bar{P} is the mean pressure, Re is the Reynolds number, and τ_{ij} is the Reynolds stress tensor. For simplicity, since it will not alter the critical conclusions to be arrived at in this paper, we will consider the inner flow to be incompressible. The system of equations (2)-(4) for the inner mean flow \bar{u}, \bar{v} are not closed and must be supplemented with a turbulence model. In the $K - \epsilon$ model, the Reynolds stress tensor is given by

$$\tau_{ij} = -\frac{2}{3} K \delta_{ij} + C_\mu \frac{K^2}{\epsilon} \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) \quad (5)$$

where K is the turbulent kinetic energy, ϵ is the turbulent dissipation rate, and C_μ is a dimensionless constant which is usually taken to be 0.09. The turbulence quantities K and ϵ are determined from modeled versions of their transport equations which, for the Lam-Bremhorst model [5] that was considered by Strahle and his co-workers [3,4], takes the form

$$\bar{u} \frac{\partial K}{\partial x} + \bar{v} \frac{\partial K}{\partial y} = \frac{1}{Re} \nabla^2 K + \frac{\partial}{\partial x} \left(C_\mu \frac{K^2}{\epsilon} \frac{\partial K}{\partial x} \right) + \frac{\partial}{\partial y} \left(C_\mu \frac{K^2}{\epsilon} \frac{\partial K}{\partial y} \right) + P - \epsilon \quad (6)$$

$$\bar{u} \frac{\partial \epsilon}{\partial x} + \bar{v} \frac{\partial \epsilon}{\partial y} = \frac{1}{Re} \nabla^2 \epsilon + \frac{\partial}{\partial x} \left(\frac{C_\mu K^2}{\sigma_\epsilon \epsilon} \frac{\partial \epsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{C_\mu K^2}{\sigma_\epsilon \epsilon} \frac{\partial \epsilon}{\partial y} \right) \quad (7)$$

$$+ C_{\epsilon 1} f_1 \frac{\epsilon}{K} P - C_{\epsilon 2} f_2 \frac{\epsilon^2}{K}$$

where

$$\mathcal{P} = \tau_{ij} \frac{\partial \bar{v}_i}{\partial x_j} \quad (8)$$

is the turbulence production; σ_ϵ , $C_{\epsilon 1}$ and $C_{\epsilon 2}$ are dimensionless constants which typically assume the values of 1.3, 1.45, and 1.90, respectively; and f_1 and f_2 are wall damping functions which vanish at the wall and approach unity far away from the wall [5]. Sufficiently far from boundaries, at high Reynolds numbers where the molecular viscosity can be neglected, the modeled transport equations (6) - (7) reduce to those of the more commonly used $K - \epsilon$ model of Hanjalic and Launder [6].

The equations of motion (2)-(7) for stagnation point turbulent flows are solved subject to the boundary conditions

$$\bar{u} = \bar{v} = K = \frac{\partial K}{\partial y} = 0, \quad \epsilon = \frac{1}{Re} \frac{\partial^2 K}{\partial y^2} \quad (9)$$

at the wall $y = 0$, along with the freestream boundary conditions (for $y \rightarrow \infty$)

$$\bar{u} = \bar{u}_\infty, \quad \bar{v} = \bar{v}_\infty \quad (10)$$

$$K_\infty = \frac{1}{2\sqrt{C_\mu}} \epsilon_\infty. \quad (11)$$

All of the boundary conditions except for (11) can be obtained as a rigorous consequence of the Navier-Stokes equations assuming that \bar{u}, \bar{v} and K are Taylor expandable near the wall. Boundary condition (11) is obtained by a production equals dissipation hypothesis, i.e., by *assuming* that

$$\mathcal{P} = \epsilon \quad (12)$$

in the outer flow which is based purely on experimental observations for similar (although not identical) stagnating turbulent flows.

It will now be shown that the outer flow boundary condition (11) is fundamentally inconsistent with the mean velocity field (1). This outer mean velocity has the following nonzero gradients

$$\frac{\partial \bar{u}_\infty}{\partial x} = 1, \quad \frac{\partial \bar{v}_\infty}{\partial y} = -1 \quad (13)$$

and, hence, corresponds to the case of homogeneous plane strain turbulence (see Tucker and Reynolds [7] and Rogallo [8]). It is now well established that homogeneous plane strain turbulence does *not* reach an equilibrium state where production is balanced by dissipation. In fact, the turbulent kinetic energy, dissipation rate, and length scales *grow* monotonically with time in homogeneous plane strain turbulence.[†] As an illustration, the time evolution

[†]It is precisely this unbounded growth of the length scales in numerical simulations of homogeneous plane strain turbulence that force a termination of such computations after relatively short elapsed times [8].

of the turbulent kinetic energy (non-dimensionalized by its initial value) taken from the direct numerical simulations of homogeneous plane strain conducted by Lee and Reynolds [9] is shown in Figure 2. These results are suggestive of a long-time exponential growth of the turbulent kinetic energy which has been postulated based on alternative arguments [8] (while the precise functional dependence has not been rigorously established, both experimental and computational evidence indicate conclusively that there is a monotonic, unbounded growth in time of the turbulence level). The commonly used $K-\epsilon$ model (where $C_{\epsilon 1} = 1.45$ and $C_{\epsilon 2} = 1.90$) properly mimics this behavior as can be seen in Figure 3. These computations, which were conducted using a Runge-Kutta-Fehlberg numerical integration scheme, indicate that after an early decay (the turbulence was initially undergoing an isotropic decay), the turbulent kinetic energy grows monotonically and becomes unbounded in the limit as $t \rightarrow \infty$. It can be shown analytically that the long-time growth of kinetic energy predicted by the traditional $K - \epsilon$ model is exponential.

For a general homogeneous plane strain turbulence, with mean velocity gradients

$$\frac{\partial \bar{v}_i}{\partial x_j} = \begin{pmatrix} S/2 & 0 \\ 0 & -S/2 \end{pmatrix}, \quad (14)$$

the $K - \epsilon$ model yields transport equations for K and ϵ which are given by

$$\frac{dK}{dt} = C_\mu \frac{K^2}{\epsilon} S^2 - \epsilon \quad (15)$$

$$\frac{d\epsilon}{dt} = C_{\epsilon 1} C_\mu K S^2 - C_{\epsilon 2} \frac{\epsilon^2}{K}. \quad (16)$$

These equations can be manipulated into the alternative dimensionless form

$$\frac{dK^*}{dt^*} = \left(C_\mu \frac{SK}{\epsilon} - \frac{\epsilon}{SK} \right) K^* \quad (17)$$

$$\frac{d}{dt^*} \left(\frac{\epsilon}{SK} \right) = C_\mu (C_{\epsilon 1} - 1) - (C_{\epsilon 2} - 1) \left(\frac{\epsilon}{SK} \right)^2 \quad (18)$$

where $t^* = St$, $K^* = K/K_0$, and

$$\epsilon^* \equiv \frac{\epsilon}{\epsilon_0} = \left(\frac{\epsilon}{SK} \right) \left(\frac{SK_0}{\epsilon_0} \right) K^* \quad (19)$$

given that $(\cdot)_0$ denotes the initial value. Equation (18) has an equilibrium solution of the form

$$\left(\frac{SK}{\epsilon} \right)_\infty = \left(\frac{\alpha}{C_\mu} \right)^{\frac{1}{2}} \quad (20)$$

(in the limit as $t^* \rightarrow \infty$) where

$$\alpha = \frac{C_{\epsilon 2} - 1}{C_{\epsilon 1} - 1} \quad (21)$$

which is approximately 2 for the standard $K - \epsilon$ model.[†] Then, from equations (17) and (19), it follows that for $t^* \gg 1$ we have

$$K^* \sim \exp \left[\sqrt{\frac{C_\mu}{\alpha}} (\sqrt{\alpha} - 1) t^* \right] \quad (22)$$

$$\epsilon^* \sim \exp \left[\sqrt{\frac{C_\mu}{\alpha}} (\sqrt{\alpha} - 1) t^* \right]. \quad (23)$$

It is therefore clear that the traditional $K - \epsilon$ model predicts an exponential growth in time of the turbulent kinetic energy and dissipation rate where a structural equilibrium is reached with respect to their ratio – the turbulent time scale K/ϵ (in fact, SK/ϵ has a universal equilibrium in the limit as $t \rightarrow \infty$ which is completely independent of the strain rate S and the initial conditions K_0 and ϵ_0).

As shown above, the traditional $K - \epsilon$ model (where $\alpha > 1$) predicts an exponential growth in the turbulent kinetic energy and dissipation rate for plane strain which is consistent with physical and numerical experiments. On the other hand, if we take $C_{\epsilon 1} = C_{\epsilon 2}$ as suggested by Strahle, et al. [3,4], the $K - \epsilon$ model erroneously predicts a production equal dissipation equilibrium wherein both K and ϵ approach a *finite asymptote* in the limit as $t \rightarrow \infty$. This solution is of the form

$$K_\infty = \frac{1}{S\sqrt{C_\mu}} \epsilon_\infty \quad (24)$$

where ϵ_∞ is bounded and is determined by the initial conditions and the strain rate (it can be shown that $\epsilon_\infty/\epsilon_0 = (\sqrt{C_\mu}SK_0/\epsilon_0)^\beta$ where $\beta = C_{\epsilon 1}/(C_{\epsilon 1} - 1)$). It is clear that if Eq. (24) is nondimensionalized it becomes identical to Eq. (11).

Now, we will return to the problem of stagnation point flow. By a Galilean transformation

$$y = U_0 t \quad (25)$$

(where U_0 is the characteristic mean velocity), the temporally evolving version of homogeneous plane strain turbulence can be converted to a spatially evolving problem (in the coordinate y) governed by the equations

$$U_0 \frac{dK}{dy} = C_\mu \frac{K^2 S^2}{\epsilon} - \epsilon \quad (26)$$

$$U_0 \frac{d\epsilon}{dy} = C_{\epsilon 1} C_\mu K S^2 - C_{\epsilon 2} \frac{\epsilon^2}{K}. \quad (27)$$

[†]It is a simple matter to show that α is the equilibrium value of the ratio of production to dissipation.

This spatially evolving version of the problem (which is actually the way that the physical experiments are conducted [7]) has the *same* solution as the temporally evolving version if we set

$$t^* = \frac{yS}{U_0}. \quad (28)$$

As before, the standard $K - \epsilon$ model predicts an exponential growth of K and ϵ in y which properly mimics the experiments; the modified $K - \epsilon$ model where $C_{\epsilon 1} = C_{\epsilon 2}$ erroneously predicts a production equals dissipation equilibrium where K and ϵ approach *finite* asymptotes as $y \rightarrow \infty$ (see Figure 4). These results have a direct bearing on the stagnation point flow problem. The boundary conditions (10) - (11) must be matched in the limit as $y \rightarrow \infty$. This is usually accomplished by marching in the y -direction from the wall starting at $y = 0$ (see Figure 1). However, as can be seen from the previous analogy, if we march in the y direction from the wall with velocity U_0 , then the standard $K - \epsilon$ model *properly predicts* an exponential growth in the turbulent kinetic energy and dissipation rate

$$K^* \sim \exp \left[\sqrt{\frac{C_\mu}{\alpha}} (\sqrt{\alpha} - 1) \frac{yS}{U_0} \right] \quad (29)$$

$$\epsilon^* \sim \exp \left[\sqrt{\frac{C_\mu}{\alpha}} (\sqrt{\alpha} - 1) \frac{yS}{U_0} \right] \quad (30)$$

for $yS/U_0 \gg 1$. If a free stream boundary condition is used where K_∞ and ϵ_∞ are bounded, an ill-posed problem results; this arises from the inconsistency discussed above and *not* from a defect in the standard $K - \epsilon$ model.

Although the singularity in the plane stagnation point problem can be eliminated by setting $C_{\epsilon 1} = C_{\epsilon 2}$, it is highly undesirable to do so since this results in a miscalibration of the $K - \epsilon$ model for homogeneous turbulent flows to the point where qualitatively incorrect results can be predicted. Consequently, the specific quantitative results obtained from this alternatively calibrated $K - \epsilon$ model for stagnation point flows are likely to be dubious. Rather than rendering the problem well posed by an inconsistent recalibration of the model, it would appear to be preferable to consider an alternative formulation of the problem which is not intrinsically ill-posed. Such an alternative formulation would require an outer flow with a mean velocity field that is compatible with a bounded turbulent kinetic energy and dissipation rate which are statistically steady. One such example would be stagnation point flow that arises about a semi-infinite Rankine solid (see Figure 5). For this problem, the outer mean velocity is obtained by the superposition of a uniform stream with a source located at point P (a velocity field that can be written in closed form). It is a simple matter to show that in the limit as $r \rightarrow \infty$

$$\bar{u}_\infty = U_\infty, \bar{v}_\infty = 0 \quad (31)$$

for this flow. The mean velocity (31) has no spatial gradients, and hence no source for turbulence production; consequently any background turbulence will decay yielding equilibrium values of

$$K_\infty = 0, \varepsilon_\infty = 0. \quad (32)$$

in the limit as $r \rightarrow \infty$. In addition, in the outer region of the turbulent boundary layer there will be a region where production is approximately balanced by dissipation so that (11) would be an appropriate boundary condition. No problems with singularities would arise with this alternative formulation of stagnation point flow.

3. CONCLUDING REMARKS

It has been demonstrated in this paper that the problem of plane turbulent stagnation point flow, as it is usually formulated, constitutes an ill-posed problem. This ill-posed formulation arises since the outer mean flow corresponds to a homogeneous plane strain turbulence which has no simple equilibrium structure; the turbulent kinetic energy and dissipation rate grow exponentially with time. The standard $K - \epsilon$ model was shown to properly mimic this exponential growth in time of the turbulence intensity which is the source of the singularity in the stagnation point flow problem. Hence, this singularity is in no way due to a defect in the model but rather results from the use of an inappropriate boundary condition.

There is no doubt that the singularity in the $K - \epsilon$ model for stagnation point flow can be eliminated by setting $C_{\epsilon 1} = C_{\epsilon 2}$ in the dissipation rate transport equation as suggested by Strahle and his co-workers [3,4]. However, in the opinion of the author, it is highly undesirable to do this since the recalibrated model yields completely erroneous results for most homogeneous turbulent flows (e.g., its prediction of a production equals dissipation equilibrium for plane strain and plane shear turbulence which is in contradiction to the results of physical and numerical experiments). An alternative formulation of the stagnation point flow problem based on a semi-infinite Rankine solid was discussed which is well posed (i.e., no singularities would arise from the implementation of boundary conditions). Of course, other alternative formulations of turbulent stagnation point flows exist which are also well posed (for example, flow past a circular cylinder or the three-layer model of Traci and Wilcox [10] for plane stagnation point flow). It is true that these alternative formulations are not quite as easy to compute since a similarity solution may not exist. Nonetheless, if we are to gain a better understanding of the physics of turbulent stagnation point flows, we must avoid making ad hoc adjustments in the constants of turbulence models which render incorrect predictions for simpler flows that are analogous.

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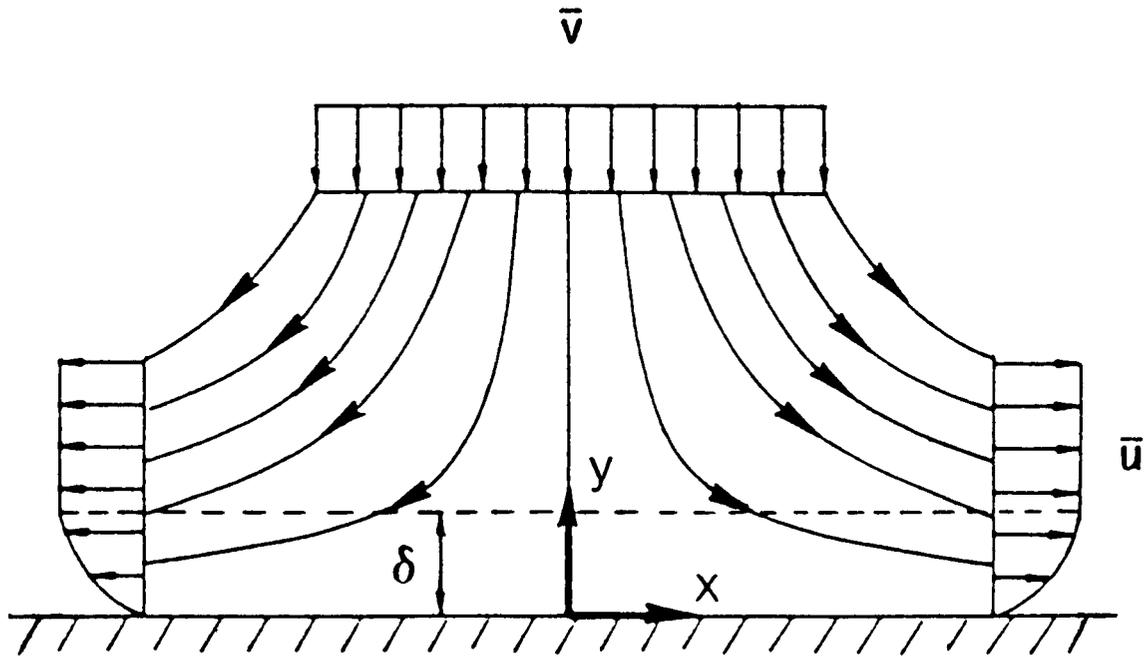


Figure 1. Plane stagnation point flow.

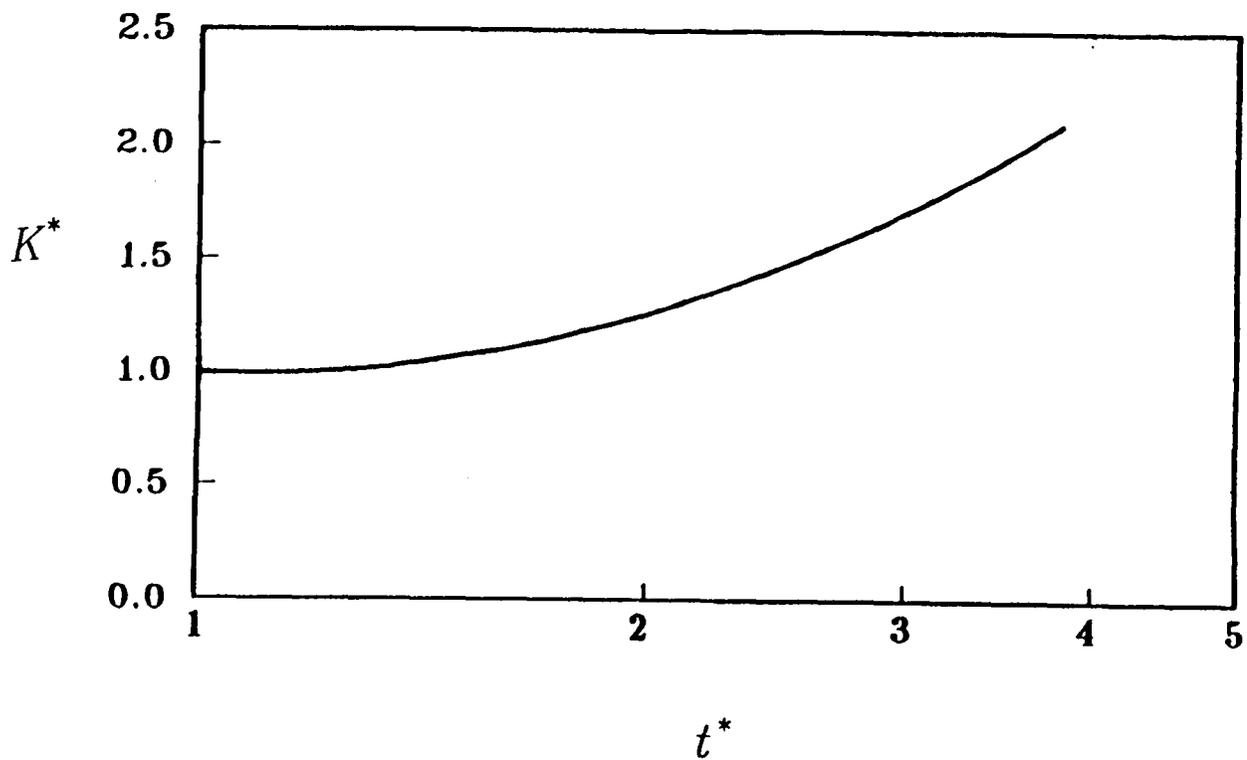


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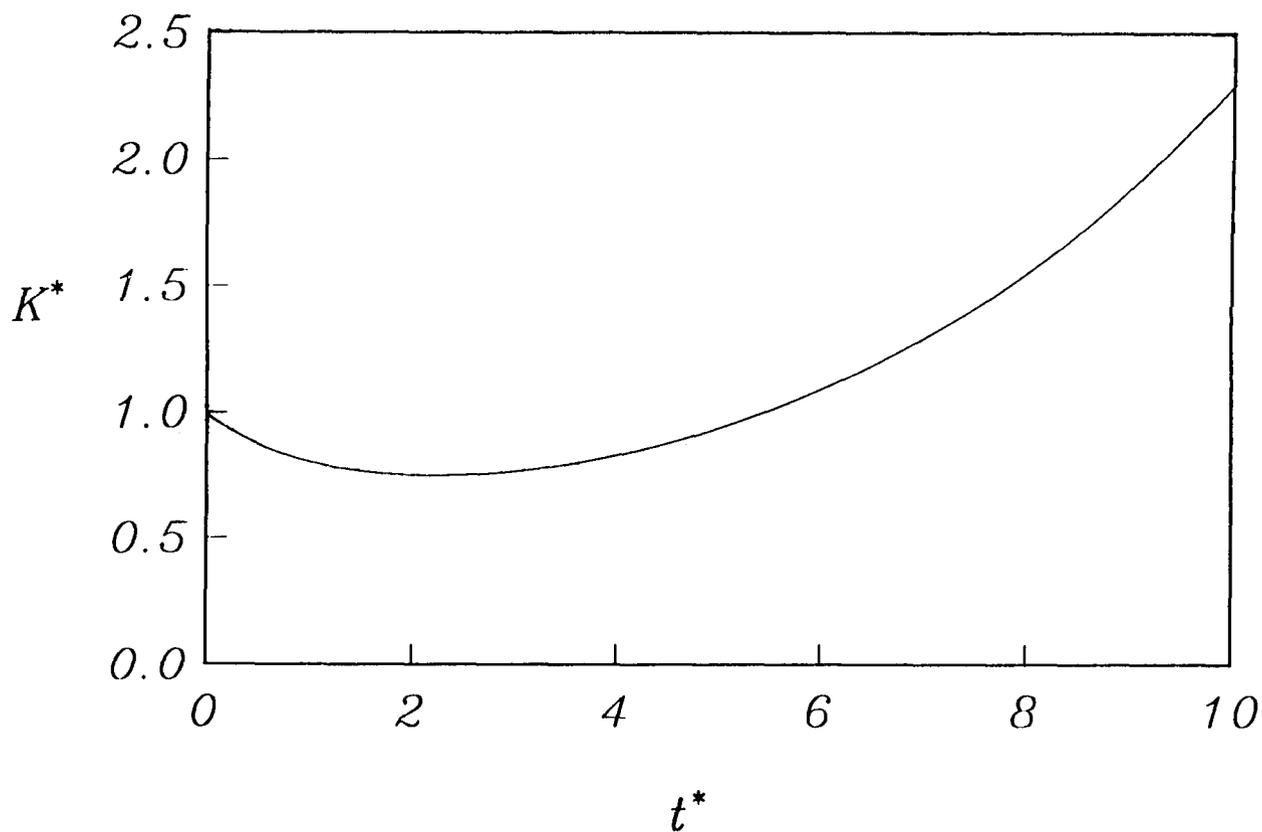


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(a)

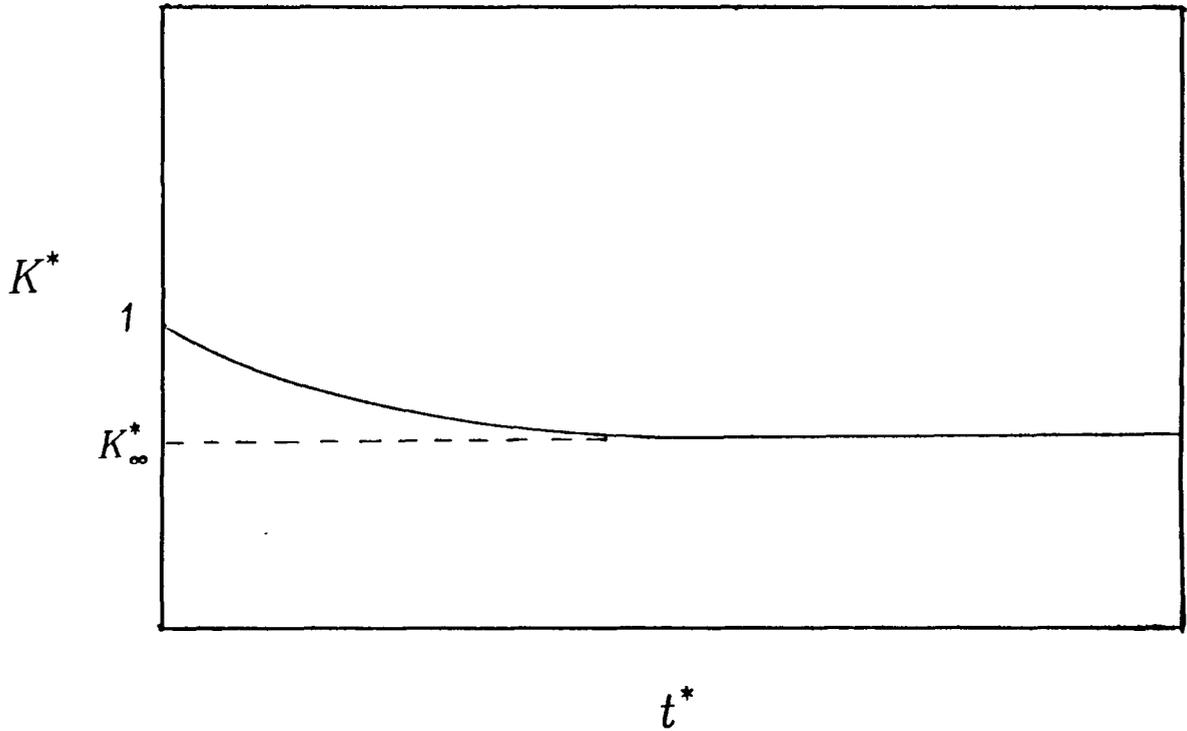


Figure 4. Time evolution of the turbulent kinetic energy predicted by the modified $K - \epsilon$ model (where $C_{\epsilon 1} = C_{\epsilon 2}$) for homogeneous plane strain:

- (a) $SK_0/\epsilon_0 < 1/\sqrt{C_\mu}$
- (b) $SK_0/\epsilon_0 = 1/\sqrt{C_\mu}$
- (c) $SK_0/\epsilon_0 > 1/\sqrt{C_\mu}$.

(b)

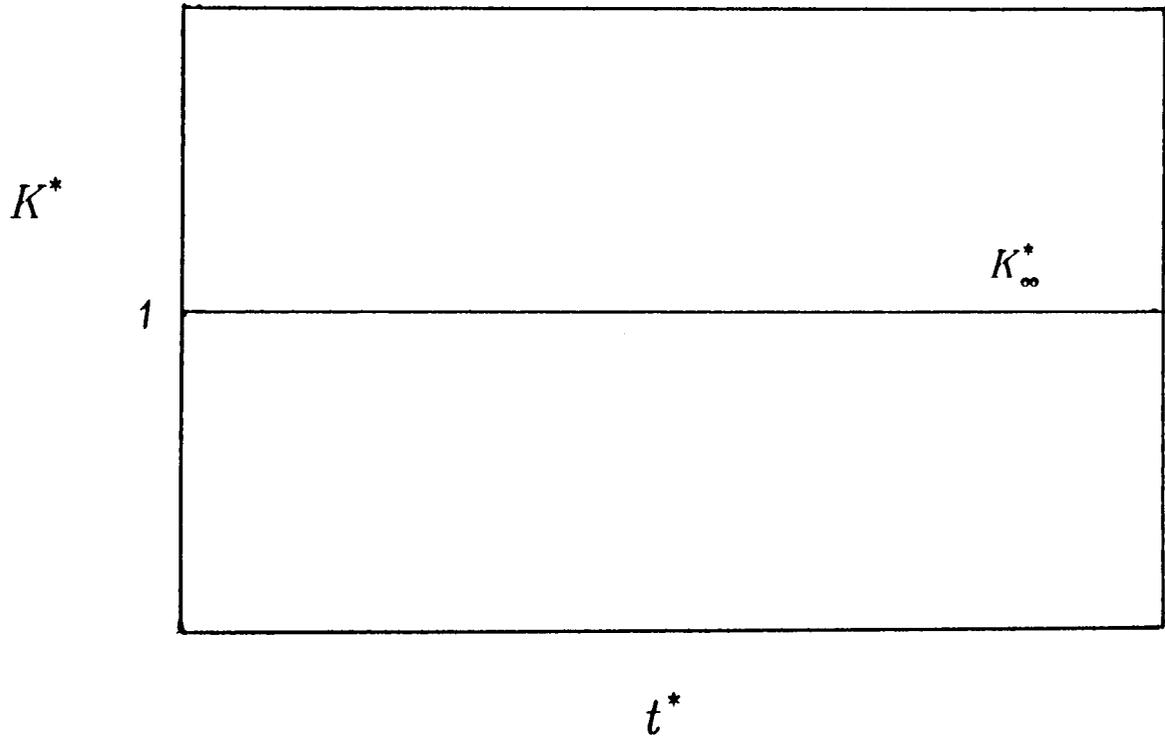


Figure 4. Time evolution of the turbulent kinetic energy predicted by the modified $K - \epsilon$ model (where $C_{\epsilon 1} = C_{\epsilon 2}$) for homogeneous plane strain:

- (a) $SK_0/\epsilon_0 < 1/\sqrt{C_{\mu}}$
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- (c) $SK_0/\epsilon_0 > 1/\sqrt{C_{\mu}}$.

(c)

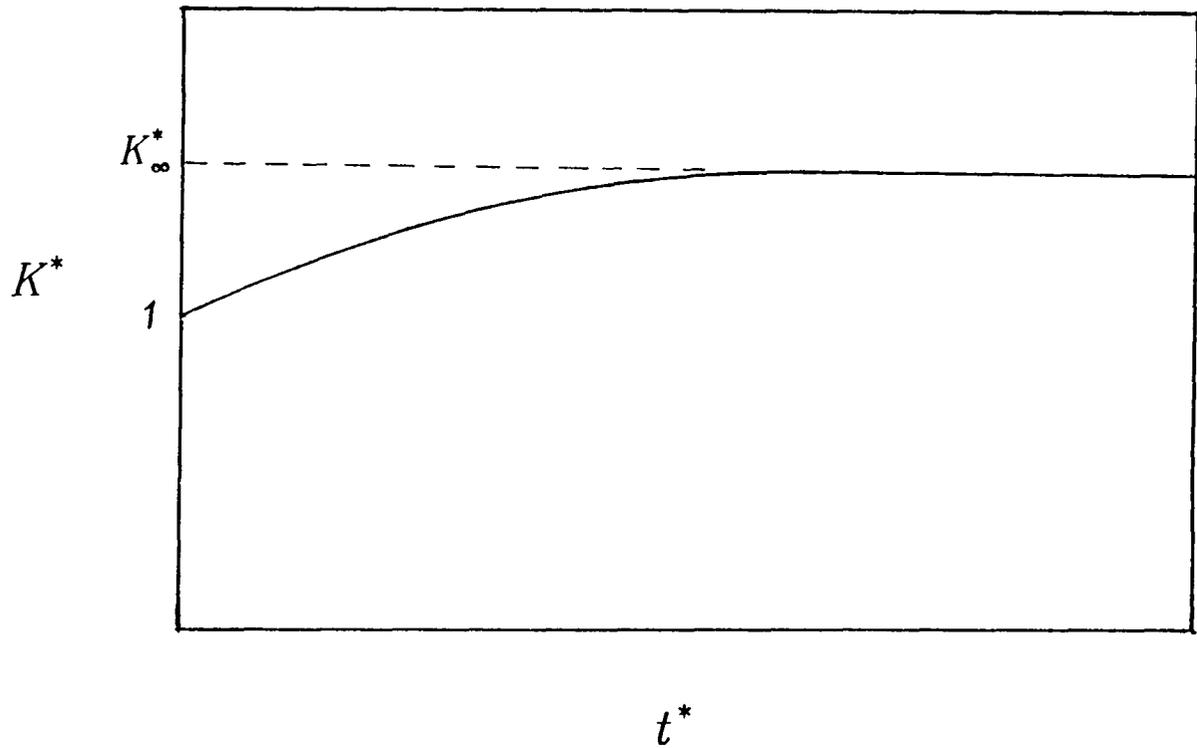


Figure 4. Time evolution of the turbulent kinetic energy predicted by the modified $K - \epsilon$ model (where $C_{\epsilon 1} = C_{\epsilon 2}$) for homogeneous plane strain:
(a) $SK_0/\epsilon_0 < 1/\sqrt{C_\mu}$
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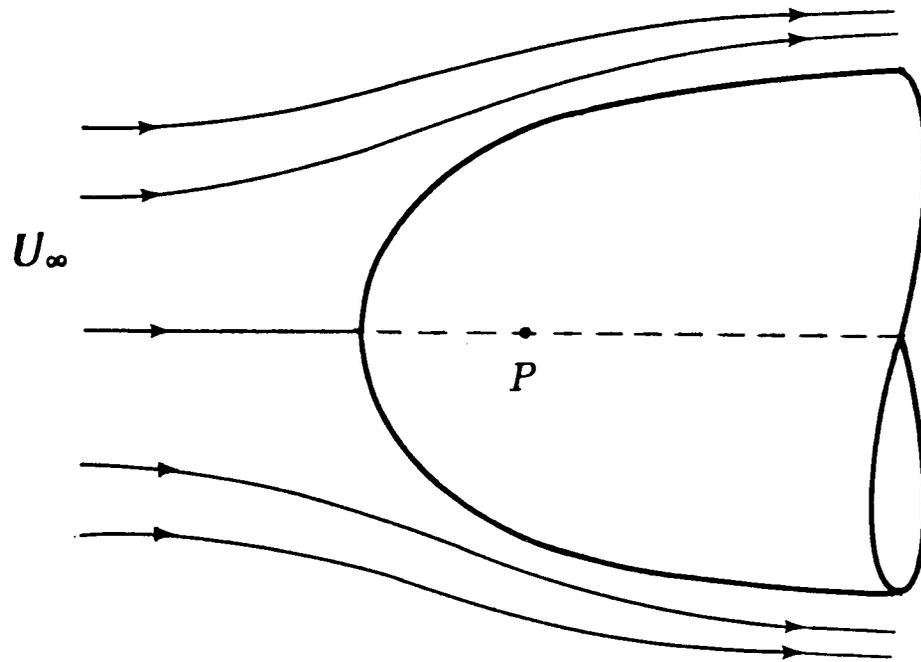


Figure 5. Stagnation point flow for a semi-infinite Rankine solid.



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