Integrated Tools for Control-System Analysis

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Summary

The basic functions embedded within a user-friendly software package (MATRIXx) are used to provide a high-level systems approach to the analysis of linear control systems. Various control-system analysis configurations are assembled automatically to minimize the amount of work by the user. Interactive decision making is incorporated via menu options and at selected points, such as in the plotting sections, by inputting data. One section in this paper describes the frequency and steady-state analysis. There are five evaluations—the singular-value robustness test, the singular-value loop-transfer frequency response, the Bode frequency response, the steady-state covariance analysis, and the closed-loop eigenvalue calculations. Another section describes time-response simulations. These simulations, which are linear except for actuator saturation, may have either a continuous plant or a discrete plant and include a generic controller structure for both feedback and feedforward dynamics. A time response for a random white-noise disturbance is available. This paper describes the configurations and the procedures used for each type of analysis, the restrictions that apply, the type of data required, and a sample problem. One approach to integrating the design and analysis tools is also presented.

1.0. Introduction

Prior to the early 1960's, hand calculations using slide rules and spirules were the major tools for control design; analog computers were the major tools for simulation. During the 1960's and 1970's, as control design and analysis techniques became more complex, use of more powerful digital computers became necessary. Specialized control-system design and analysis programs, usually written in Fortran, were developed for use on digital computers. These programs were significantly more powerful than previously available techniques, but quite often the designer spent a large amount of time debugging software rather than solving the control problem. Typically, the control designer needed a programmer to interface with the computer because of the programming complexity.

Computer-aided control-system design and analysis tools have emerged during the past decade. These programs provide a new generation of user-friendly tools that allow the designer to spend more time on the primary task of control-law design. A summary of 20 computer-aided control-system design software packages is shown in reference 1, and additional packages have been developed since this reference was published. A user-friendly software package described in the software summary MATRIXx (ref. 2) has been purchased by various organizations at Langley Research Center. This paper is not an endorsement of MATRIXx over any other software package. However, since MATRIXx is the software package used by our organization, the control-system analysis tools described in this paper are the fundamental features of this computer program.

The program MATRIXx has the basic functions that allow implementation of a high-level systems approach to control-system analysis. These basic functions include matrix algebra, transformations between frequency and time domain, frequency responses, time simulation, plotting features, and structured programming capability. Also, SYSTEM.BUILD (an optional software module, for use with MATRIXx, that is referred to as BUILD in this paper) provides an environment for building, modifying, and editing simulation models. Version 6.0 of BUILD has a programming capability that allows building blocks to be automatically assembled into various system configurations.

The approach described in this paper is to use the basic functions within MATRIXx to provide a systems approach to the analysis of linear control systems. Using this approach, designers can perform various analyses quickly and accurately. The idea is to minimize the calculations and matrix manipulations by the designer and still provide interactive flexibility. Decisions are made via menu-driven options, which have been programmed into the major parts of the software. At selected points in the software, mainly in the plotting sections, the designer can also interact with the control-system analysis tools by inputting data. For example, the designer may input data to create a vector of variable-density-frequency points for selected frequency-response evaluations.

All the evaluations are developed for discrete control systems. The main input data required are state-space quadruples in discrete form for both a plant and a controller. However, continuous systems can also be evaluated, as is described in this paper. The program BUILD is used to construct the configurations needed by the frequency and time analyses. The system quadruple generated within BUILD is then used in the appropriate evaluation.

Sections 3.0 and 4.0 of this paper describe the control-system analysis capabilities that have been developed. Frequency and steady-state analyses are described in section 3.0. Five evaluations presently available are the singular-value robustness test, the singular-value loop-transfer frequency response, the Bode frequency response, the steady-state covariance analysis to an external disturbance, and the closed-loop eigenvalue calculations. The discussion of the
five evaluations includes a description of menu options, key equations, and control-system analysis diagrams. There are also subsections that describe the plotting of the frequency data and describe an optional user file.

The description of time analyses is in section 4.0 of this paper. Time-response simulations are linear except for actuator saturation, which can be included at the option of the user. The user can also choose between a continuous plant and a discrete plant in the simulation. Subsections describe a generic controller structure that allows feedback and feedforward dynamic controllers, signal-generator options, and control-loop construction. Other subsections describe the time response to external disturbances and the plotting capabilities for the time-response data.

An approach that allows integration of a control-system design program and the analysis tools is described in section 5.0 of this paper. This approach can be used to reduce the total overall design time by allowing the designer quick and easy access to the analysis tools.

A sample problem is described in section 6.0 of this paper. Details of the sample problem are shown in the appendixes; appendix A contains the input data and displayed output of the frequency-analysis evaluations, and appendix B contains similar contents for the time-analysis evaluations.

One objective of this development is to enable the user to assemble and analyze various control-system configurations using existing MATRIXx functions, user-defined functions (UDF), command files, and macros. A goal is to have, in a single software package, user-friendly control-system analysis tools that perform a variety of analyses. These analysis tools are easily integrated with control-law synthesis tools to form a complete environment. Within this paper, examples of flight control problems are used, but the approach is generic and should be useful to other control disciplines. All the control-system configurations assume positive feedback with the negative sign included within the control block. Features described are applicable to MATRIXx version 6.0, although upgraded versions will probably necessitate some changes in the software; the basic approach should remain applicable. All UDF's end with the notation "FUN", all command files end with the notation "CMD", and all plot command files end with the notation "PLT".

2.0. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A,B,C,D)</td>
<td>quadruple matrices for continuous system</td>
</tr>
<tr>
<td>ANSW</td>
<td>variable to differentiate whether same or different signals are used for all models</td>
</tr>
<tr>
<td>A&lt;sub&gt;s&lt;/sub&gt;</td>
<td>transfer-function matrix</td>
</tr>
<tr>
<td>B&lt;sub&gt;b&lt;/sub&gt;, B&lt;sub&gt;f&lt;/sub&gt;</td>
<td>control matrices (for feedback and feedforward signals, respectively) used in generic controller</td>
</tr>
<tr>
<td>C</td>
<td>state-space quadruple matrix of controller or state output matrix of a continuous system</td>
</tr>
<tr>
<td>C&lt;sub&gt;1&lt;/sub&gt;, C&lt;sub&gt;2&lt;/sub&gt;, C&lt;sub&gt;3&lt;/sub&gt;</td>
<td>matrices used in generic controller to describe different blocks</td>
</tr>
<tr>
<td>D&lt;sub&gt;b&lt;/sub&gt;, D&lt;sub&gt;f&lt;/sub&gt;</td>
<td>input-to-output transfer matrices (for feedback and feedforward signals, respectively) used in generic controller</td>
</tr>
<tr>
<td>D&lt;sub&gt;s&lt;/sub&gt;</td>
<td>diagonal scaling matrix</td>
</tr>
<tr>
<td>DBODE</td>
<td>MATRIXx discrete Bode response function</td>
</tr>
<tr>
<td>DELT</td>
<td>discrete sampling period, sec</td>
</tr>
<tr>
<td>DELTIME</td>
<td>time step for each element in output time vector, sec</td>
</tr>
<tr>
<td>DFP</td>
<td>Davidon-Fletcher-Powell</td>
</tr>
<tr>
<td>DIMS</td>
<td>row vector containing scalar input data</td>
</tr>
<tr>
<td>DLSIM</td>
<td>MATRIXx function for discrete time response to general inputs</td>
</tr>
<tr>
<td>E</td>
<td>input-to-output transfer matrix for discrete system</td>
</tr>
<tr>
<td>EVALCL</td>
<td>table containing vectors of closed-loop eigenvalues and damping ratios</td>
</tr>
<tr>
<td>(F,G,H,E)</td>
<td>quadruple matrices for a discrete system</td>
</tr>
<tr>
<td>F</td>
<td>state transition matrix of a discrete controller</td>
</tr>
<tr>
<td>FREQ</td>
<td>MATRIXx frequency-response function</td>
</tr>
<tr>
<td>G</td>
<td>control matrix for a discrete system</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>GAINM</td>
<td>gain margin, dB</td>
</tr>
<tr>
<td>GUSTAT</td>
<td>statistical data (standard deviation and mean) generated from time-simulation data with random input on first external disturbance path</td>
</tr>
<tr>
<td>GWSTAT</td>
<td>statistical data (standard deviation and mean) generated from time-simulation data with random input on second external disturbance path</td>
</tr>
<tr>
<td>H</td>
<td>state output matrix for a discrete system</td>
</tr>
<tr>
<td>I</td>
<td>identity matrix</td>
</tr>
<tr>
<td>inf</td>
<td>infimum (greatest lower bound)</td>
</tr>
<tr>
<td>j</td>
<td>( = \sqrt{-1} )</td>
</tr>
<tr>
<td>LBOUND, UBOUND</td>
<td>lower and upper bounds for actuator saturation</td>
</tr>
<tr>
<td>NMOD</td>
<td>number of models</td>
</tr>
<tr>
<td>NS</td>
<td>number of states for SDP or SCP</td>
</tr>
<tr>
<td>NSCP</td>
<td>number of states for SCP</td>
</tr>
<tr>
<td>NSC1, NSC2, NSC3</td>
<td>number of states for SC1, SC2, and SC3, respectively</td>
</tr>
<tr>
<td>NSDC</td>
<td>number of states for SDC</td>
</tr>
<tr>
<td>NSGDP</td>
<td>number of states for SGDP</td>
</tr>
<tr>
<td>NU</td>
<td>number of control inputs (used in Bode response)</td>
</tr>
<tr>
<td>P</td>
<td>state-space quadruple matrices of plant</td>
</tr>
<tr>
<td>PHASEM</td>
<td>phase margin, deg</td>
</tr>
<tr>
<td>Q</td>
<td>weighting matrix</td>
</tr>
<tr>
<td>RMS</td>
<td>root mean square</td>
</tr>
<tr>
<td>ROUND</td>
<td>MATRIXx function for computing an integer closest to input number</td>
</tr>
<tr>
<td>SC1, SC3</td>
<td>feedforward quadruples for generic controller</td>
</tr>
<tr>
<td>SC2</td>
<td>feedback quadruple for generic controller</td>
</tr>
<tr>
<td>SCP</td>
<td>quadruple for continuous plant model</td>
</tr>
<tr>
<td>SDC</td>
<td>quadruple for discrete controller model</td>
</tr>
<tr>
<td>SDP</td>
<td>quadruple for discrete plant model</td>
</tr>
<tr>
<td>SGDP</td>
<td>quadruple for a discrete model made up of a combined airplane and disturbance model</td>
</tr>
<tr>
<td>SIGI</td>
<td>minimum singular value of inverse return difference matrix at plant input</td>
</tr>
<tr>
<td>SIGO</td>
<td>minimum singular value of inverse return difference matrix at plant output</td>
</tr>
<tr>
<td>SISO</td>
<td>single-input, single-output</td>
</tr>
<tr>
<td>SVLT</td>
<td>singular-value loop transfer</td>
</tr>
<tr>
<td>SVLTI</td>
<td>singular value of loop transfer at plant input</td>
</tr>
<tr>
<td>SVLTO</td>
<td>singular value of loop transfer at plant output</td>
</tr>
<tr>
<td>T</td>
<td>discrete sampling period, sec</td>
</tr>
<tr>
<td>TMAX</td>
<td>maximum time of simulation run, sec</td>
</tr>
<tr>
<td>TS</td>
<td>output time step</td>
</tr>
<tr>
<td>U</td>
<td>input disturbance</td>
</tr>
<tr>
<td>UDF</td>
<td>user-defined function</td>
</tr>
<tr>
<td>X</td>
<td>steady-state covariance matrix</td>
</tr>
<tr>
<td>XURMS</td>
<td>RMS values for states with random disturbance on first external disturbance path</td>
</tr>
<tr>
<td>XWRMS</td>
<td>RMS values for states with random disturbance on second external disturbance path</td>
</tr>
<tr>
<td>Y</td>
<td>output</td>
</tr>
<tr>
<td>YURMS</td>
<td>RMS values for outputs with random disturbance on first external disturbance path</td>
</tr>
<tr>
<td>YVV</td>
<td>row vector that defines outputs for feedback to controller</td>
</tr>
<tr>
<td>YWRMS</td>
<td>RMS values for outputs with random disturbance on second external disturbance path</td>
</tr>
<tr>
<td>z</td>
<td>discrete frequency parameter</td>
</tr>
</tbody>
</table>
3.0. Frequency Analyses

The types of analyses described in this section are frequency responses, for several control-system configurations, and steady-state solutions. Each control-system configuration is composed of linear models that represent a discrete controller and a discrete plant; these models are assembled by using the programming capability within BUILD. All the analysis routines have been developed to handle several systems automatically in one computer run, which allows comparison of different control systems at different flight conditions. One restriction is that all systems being compared must have the same number of states, inputs, and outputs. However, models with different numbers of states can always be compared by adding dummy states with stable eigenvalues to the lower order models so that the order of each model is the same.

Continuous-system analyses can also be accomplished by using these control-system analysis tools. A continuous system can always be discretized at a high frequency compared with the bandwidth of interest and then analyzed as a discrete system. A Tustin transformation (ref. 3) is a good discretization approach, because it does not add the phase lag associated with a zero-order hold. TUSTIN.FUN has been developed to allow users to analyze continuous control systems. It is suggested that the sampling frequency be at least four times greater than the highest frequency of interest.

The structure for the frequency-analysis section is shown in figure 1. FREQANAL.CMD serves as an executive and calls other command files (FREQEVAL.CMD, PLOT.CMD, and USERFILE.CMD) that execute each of the main parts of the frequency-analysis software. All the configurations for the frequency-evaluation analyses are assembled and evaluated using FREQEVAL.CMD. Data that are generated can then be displayed using PLOT.CMD. USERFILE.CMD is an optional user-developed file that can be used to create all the data needed for the frequency-analysis and plotting command files. When USERFILE.CMD is selected via a menu option, the user is prompted to specify the name of the desired file. As an example, the first author has developed a command file (SYSTEM.CMD) that uses the input and output data from a control design program and generates discrete quadruples of the plant and controller. Whether a user file is used or not, any of the main command files can be run separately if the required data are available.

The following subsections describe the contents of each command file and associated UDF.

3.1. Frequency Evaluation

Five different frequency and steady-state analyses are presently available:

1. Singular-value robustness test
2. Singular-value loop transfer
3. Bode frequency response
4. Steady-state covariance analysis to external disturbance
5. Closed-loop eigenvalues

The command file FREQEVAL.CMD manages the various analyses. Figure 2, the frequency evaluation flowchart, shows the UDF's that have been developed and the flow structure. There are many menus in the software. A typical frequency-evaluation menu (fig. 3) is invoked in FREQEVAL.CMD to select any of the five analyses.

A menu option is also available to change the frequency range, since it is typical to perform some of the frequency-response tests over different frequency ranges. NOMEG.FUN, which is used for all the frequency-response tests, has been developed to create a vector of frequency data points that are linearly spaced on a logarithmic scale. The inputs to NOMEG.FUN allow multiple frequency ranges with different data-point densities for each range, specified as the number of points per decade. The vector
of frequency data points is then composed of sub-vectors, each with its specified density of frequency points. This feature is important when evaluating a control system with a lightly damped eigenvalue, because additional data points can be calculated in the frequency range where the control-response variable is varying relatively fast. In contrast, fewer data points are needed in frequency ranges with slowly varying control responses. This flexibility results in reduced time to perform the analysis and still allows for the important response characteristics.

The program BUILD is used to assemble the system configurations in all the evaluations described. The programming capability of BUILD allows an automated approach to the assembly of the system configuration, which speeds the evaluation process and helps minimize errors. The only data required are the discrete system quadruple matrices \((F,G,H,E)\) for both the controller and plant and the number of states in each quadruple. The only restrictions are that the number of plant outputs must equal the number of controller inputs and the number of controller outputs must equal the number of plant inputs. The program can calculate the number of inputs and outputs to each dynamic system and make the appropriate connections. There is no limit to the number of systems that can be evaluated in one computer run. The five types of analysis are described in more detail in the subsections that follow.

3.1.1. Singular-value robustness test. The option exists to calculate the stability margins of a closed-loop system for a multiplicative error at either the plant input or plant output. The results of this test give an indication of system stability when all loops are opened with simultaneous gain and phase changes in all loops. An example of the multiplicative error at the plant input is shown in figure 4, where a stable transfer matrix \(A\) is assumed to be acting simultaneously on all input paths. The closed-loop system is guaranteed to be stable if (ref. 4)

\[
\hat{\sigma}(\Delta) < \delta_{sm}
\]

(1)

where \(\hat{\sigma}(\Delta)\) is the maximum singular value and \(\delta_{sm}\) is the stability margin. The calculation for \(\delta_{sm}\) depends on whether \(\Delta\) is fully populated or diagonal (ref. 5) as follows:

\[
\delta_{sm} = \frac{1}{\hat{\sigma}(1 - CP)^{-1}CP} \quad \text{(for } \Delta \text{ fully populated)}
\]

(2)

\[
\delta_{sm} = \frac{1}{\mu(1 - CP)^{-1}CP} \quad \text{(for } \Delta \text{ diagonal)}
\]

(3)

The state-space quadruple matrices of the controller and plant are represented by \(C\) and \(P\), respectively, and \(\mu\) represents the structured singular value.

The expressions shown in equations (2) and (3) are for a closed-loop system with input disturbance \(U\) and output \(Y\). These expressions are implemented using the input-to-output map shown in figure 5. SV.FUN is supplied with quadruples of the discrete plant and discrete controller along with the sampling period and outputs closed-loop-system quadruple matrices that represent the transfer function for the configuration shown in figure 5. SV.FUN uses BUILD to create this configuration. After the first model is complete, the procedure is repeated for other models, except that BUILD is used only to update the quadruples. SSVIO.FUN is supplied with the quadruples generated in SV.FUN and outputs the singular values of the system being processed along with the minimum singular value of the inverse-return difference matrix. Within SSVIO.FUN, the closed-loop quadruple \((F,G,H,E)\) representing the transfer function is then split into components to compute the complex frequency response for \(A_s\) in the discrete domain (ref. 3) as follows:

\[
A_s = H(zI - F)^{-1}G + E
\]

(4)

\[
z = \exp(j\omega T)
\]

(5)

where \(\omega\) is the angular frequency, \(T\) is the discrete sampling period, and \(I\) is an identity matrix. A menu option is available to allow the solution of equation (4) directly for the unstructured singular value shown in equation (2) or to allow the computation of the structured singular value shown in equation (3). One shortcoming of the unstructured-singular-value test is the conservativeness of the results when considering multiloop-system stability margins (ref. 6). The singular value changes with system units (degrees, radians, etc.). This scaling problem contrasts with the classical Bode test for stability, which is invariant with changes in the system units. Reference 5 shows that a scaling matrix can be used to eliminate the scaling problem. The solution for \(\mu\) requires the calculation of a diagonal scaling matrix \(D_s\) at each frequency as follows:

\[
\mu(F, G, H, E) \leq \inf_{D_s} \hat{\sigma}(D_sA_sD_s^{-1})
\]

(6)
Unfortunately, $D_s$ must be solved iteratively\(^1\) as an optimization problem by minimizing $\bar{\sigma}$ with respect to the components of $D_s$. When the number of paths is three or less, relation (6) becomes an equality, but when the number of paths is greater than three, the upper bound for $\bar{\sigma}$ becomes a reasonable approximation to $\mu$. Two convergence algorithms are available: the default algorithm is a Davidon-Fletcher-Powell (DFP) algorithm (ref. 7), and the other is a modified Newton-Raphson algorithm. Based on the authors' experience, the DFP operates significantly faster and has been successfully used on several different problems. The second algorithm requires the calculation of a Hessian matrix, which takes a considerable amount of time. SVDFP.FUN and SVGNR.FUN have been developed to execute the DFP and modified Newton-Raphson algorithms, respectively.

A menu option allows selection of the iteration rate for calculating $D_s$. The most accurate choice is to compute $D_s$ at every frequency. To reduce calculation time, the user can compute $D_s$ less often. For example, if the default value of five is chosen for the $D_s$ computation, the same $D_s$ is used for five consecutive frequencies. The five frequencies include the two frequency points immediately preceding and immediately after the frequency at which the computation takes place.

**3.1.2. Singular-value loop transfer.** The singular-value loop transfer (SVLT) is an open-loop frequency-response test for which all loops are broken either at the plant input or the plant output. This test gives an indication of open-loop response, such as the crossover frequency and high-frequency attenuation. In a single-input, single-output (SISO) system, the SVLT is identical to the magnitude response in the Bode test. SVLT.FUN has been developed for the SVLT test. Within SVLT.FUN, BUILD is used to assemble two configurations (fig. 6), one for the loops broken at the plant input and the other for the loops broken at the plant output. Both the input and output SVLT responses are automatically generated over the frequency range of interest, since the calculation is relatively fast. The configuration for the plant input is assembled and linearized, and a singular-value frequency response is then computed using the MATRIXx function FREQ. The procedure is repeated for the output configuration. After the first model has been analyzed, the procedure is repeated for other models; BUILD is used only to update the plant and controller matrices.

\(^1\) Work done by Barton J. Bacon of Langley Research Center as part of a Ph.D. thesis entitled “Order Reduction for Closed-Loop System” for the School of Aeronautical and Astronautical Engineering, Purdue University.

**3.1.3. Bode frequency response.** The Bode frequency response is a classical stability test for SISO dynamic systems. BFR.FUN is supplied with system quadruples for a discrete plant and discrete controller, with a frequency vector, and with a discrete sampling time. The outputs of BFR.FUN are vectors of gain and phase and data for gain and phase margins. In BFR.FUN, BUILD is used to construct the various configurations; each plant is opened consecutively with all other plant inputs closed. This construction is illustrated in figure 7, where one loop is shown broken at the plant input and the other loops remain closed. The box with “-1” in figure 7 is included to create a 180° phase shift that normally occurs with negative-feedback control systems. In this paper, positive feedback is assumed.

A system quadruple is formed and a frequency response is computed using the MATRIXx function DBODE for each opened single-loop configuration. In the example of figure 7, NU frequency responses are computed for each model evaluated. After one model has been analyzed, the procedure is repeated for other models.

DBODE in MATRIXx version 6.0 allows the phase angle to exceed the principal angle range of 360° and results in decreased resolution on phase-angle plots. PRA.FUN has been developed to convert the phase-angle vector data to a principal angle between 0° and -360°. A second UDF, MMARGIN.FUN, accepts the principal phase angle from PRA.FUN, along with amplitude data from DBODE, and then calculates the gain and phase margins. Linear interpolation is used to calculate the amplitude crossover frequency and the corresponding phase angle, to which 180° is added to calculate phase margin. Linear interpolation is also used to calculate the frequency at which the phase angle becomes -180°. The gain margin is then calculated from the amplitude data at the corresponding frequency. A stable control system must have a positive phase margin and a negative gain margin (in dB).

**3.1.4. Steady-state covariance analysis.** Typically, a control engineer is interested in evaluating the response of a closed-loop control system to random external disturbances such as wind gusts. SSCOV.FUN calculates the state and output steady-state covariances for two random external disturbances. The data required for SSCOV.FUN are the sampling period and discrete-system quadruple matrices. These matrices represent the controller and a combined plant composed of the disturbance model and original plant. The outputs from SSCOV.FUN are the state and output RMS values that result from the two external-disturbance inputs.
A representative block diagram with a gust spectrum as the external-disturbance model is shown in figure 8.

Input signals to the combined plant are the controller feedback commands plus two signals for the random external disturbances; the output signals from the combined plant are the measurements going to the controller plus two outputs from the disturbance model. Input and output restrictions are that the two random external-disturbance signals must be the last two input paths going to the combined plant and that the outputs from the disturbance model must be the last two outputs from the combined plant. The reason for this restriction is that all connections, except the last two inputs and outputs, are made between the plant and controller. Furthermore, the disturbance-model outputs are later normalized to unity. If there is only one input disturbance of interest, the second disturbance path can always be used as a dummy input.

BUILD is used to construct the configuration of figure 8, which is then linearized to form the discrete-system quadruple \((F,G,H,E)\). A MATRIXx function is used as follows to solve the discrete Lyapunov equation:

\[
X = FXF^T + GQG^T
\]  

(7)

where \(F\) is the discrete state transition matrix, \(G\) is the discrete control matrix for the random inputs, \(X\) is the steady-state covariance matrix, and \(Q\) is a weighting matrix. The state vectors that are output from SSCOV.FUN are composed of the square root of the diagonal elements of \(X\). The output vectors are calculated by taking the square root of the diagonal elements of \(HXH^T\), where \(H\) is the output matrix and \(E\) is assumed to be zero. The steady-state covariance is calculated separately for each input, and the RMS data are then normalized by the RMS values of the external-disturbance-model outputs.

3.1.5. Closed-loop eigenvalues. A capability to calculate the closed-loop eigenvalues from the quadruple matrices of the discrete plant and discrete controller has been developed using CL.FUN. This function is used mainly as a check for the user, who typically calculates the closed-loop eigenvalues during the design process. Using BUILD, the closed-loop configuration is constructed and then linearized to form a quadruple matrix. The closed-loop discrete eigenvalues are calculated along with the equivalent continuous eigenvalues and damping ratios. This latter step is accomplished in ZTOS.FUN. Vectors representing the discrete eigenvalues, the corresponding continuous eigenvalues, and the damping ratios are displayed in a table.

3.2. Plotting

Four of the five analyses described in section 3.1 (Frequency Evaluation), with the closed-loop eigenvalue calculation being the exception, have plotting command files that can be executed. PLOT.CMD (fig. 1) serves as the executive for the specialized plot command files. Several menus are defined and invoked within PLOT.CMD. One menu is for selecting the specific plot command file to be executed. The choices are as follows:

1. SV.PLT
2. SVLT.PLT
3. BFR.PLT
4. SSCOV.PLT

These plotting files are used to view the results from the corresponding analysis (items 2 through 5, respectively, in fig. 3) selected in the frequency-evaluation menu. A second menu allows the selection of six different screen options. A third menu allows hard-copy plots to be made. The specific plot command files are described in detail below.

The minimum singular value of the inverse return difference matrix can be displayed on a log-log plot using SV.PLT. A menu option is invoked in SV.PLT to allow the user to display the singular values for either the plant input or the plant output. The available selections are determined by the type of analysis that has been performed. Multiple-model data can be put on the same plots when comparing various models.

SVLT.PLT allows the user to plot the singular values of the loop-transfer function. A menu option allows the user to display the singular values for either the plant input or the plant output. The user has the option of displaying multiple-model data on a common graph. The user also may choose the singular values that are to be displayed on this common graph. All the singular values are displayed on a log-log plot.

Bode frequency-response data are displayed on semilog plots by using BFR.PLT. A menu option allows the user to display either the amplitude in decibels, the phase angle (deg), or a Nyquist polar plot. The magnitude on the Nyquist plot is in actual units, as opposed to the desired units of dB. A dB scale is not used because MATRIXx polar plots do not allow negative numbers. Multiple models can be displayed on the same plot, and the desired open-loop data can be selected and displayed.

The steady-state covariance data generated in SSCOV.FUN are displayed by SSCOV.PLT by using bar charts. A menu option allows the user to select plots of either the states or output variables for one.
of the two external-disturbance paths. Two different prompts allow the user to enter, in vector form, the models to be used for the plot and to select the vector of elements (states or outputs) that will be displayed.

3.3. User File

One menu option in FREQANAL.CMD (fig. 1) is the choice of a user file, which is an optional file to be developed by the user. When this option is selected, the user is prompted to enter a command file name. The user-file option has been included as a convenience to allow the user a method of automatically preprocessing data for use in the frequency-evaluation file. This option does not have to be invoked if the user has already loaded the required data into MATRIXx prior to executing FREQANAL.CMD. (See appendix A and “Sample Problem,” section 6.0.)

SYSTEM.CMD is an example of how the user-file option can be utilized. SYSTEM.CMD was developed by the first author to create system quadruples of the discrete controller and discrete plant for FREQANAL.CMD. In general, the approach used is to combine feedback gains from a control-design program with other controller dynamics to form a state-space matrix that represents the controller transfer function. The plant is developed by combining individual continuous-system matrices \((A,B,C,D)\) that have been previously constructed. These matrices are then assembled into a system quadruple and discretized to form the discrete plant, which is used in most of the frequency-evaluation analyses. A second discrete plant, a combination of an airplane model with a Dryden wind-gust model, is also generated for use in the covariance analysis. All the various steps are accomplished by UDF's that have been assembled to form SYSTEM.CMD.

4.0. Time Analysis

The time-analysis command files give the user a capability to evaluate control-system time responses as a function of signal commands or random external disturbances. The approach taken is similar to that described in section 3.0. Figure 9 contains the time-analysis flowchart. TIMEANAL.CMD executes other command files via menu selection to either preprocess data, build and simulate the control system to obtain time histories, or plot the generated data. The controller is assumed to be discrete, but the plant can be either discrete or continuous in the time simulations used with signal commands. An explicit solution is used to evaluate the all-discrete simulations, whereas a combination of a discrete solution and numerical integration is used for the hybrid time simulations. The time response to random external disturbances assumes a discrete plant that includes the disturbance model.

BUILD is used to assemble the closed-loop control-system configurations, which consist of linear models of the plant and controller. An option exists to incorporate actuator saturation. The controller structure is general and allows for both feedback and feedforward compensation, which is also assembled in BUILD.

The command files that can be selected via the time-analysis menu are as follows:

1. TEVAL.CMD—computes time response to a signal-generator command
2. TGUST.CMD—time response to a random input
3. TPLOT.CMD—used for displaying time histories that are generated in both TEVAL.CMD and TGUST.CMD
4. USERFILE—an optional file developed by the user

It is possible for the general user to execute any of these files, as long as the appropriate variables are available. The subsections that follow describe the contents of each command file and associated UDF in more detail.

4.1. Time Evaluation

All the time responses, except those due to a random external disturbance, are computed using TEVAL.CMD. This file serves as an executive for both discrete and hybrid closed-loop-system simulations. The hybrid closed-loop system has a discrete controller and a continuous plant. Menu selection allows the user to choose whether a limiter is to be used for actuator position saturation. If the choice is to use a limiter, then it is assumed that the variables for the upper and lower saturation bounds have been previously loaded into MATRIXx.

TEVAL.CMD executes either DC1.CMD (if a discrete plant is to be used) or DC2.CMD (if a continuous plant is to be used), as shown in figure 9. Both DC1.CMD and DC2.CMD build the closed-loop control system and connect the system to a signal generator. After the total system is constructed, DC1.CMD or DC2.CMD runs the simulation. The output of these command files is a matrix with column vectors that represent the output time histories. DC1.CMD and DC2.CMD are nearly identical. They both call GENC.FUN, which assembles the discrete generic controller, and they both execute SIGNAL.CMD for the signal generator. The difference is that DC1.CMD calls DISCRETE.FUN, which assembles the discrete generic controller, and they both execute SIGNAL.CMD for the signal generator. The difference is that DC1.CMD calls DISCRETE.FUN, which assembles the discrete generic controller, and they both execute SIGNAL.CMD for the signal generator.
hybrid control system. These files are each discussed in subsequent subsections.

4.1.1. Generic-controller structure. A generic-controller configuration is available for use within GENC.FUN. As shown within the dotted lines in figure 10, the structure allows the inclusion of a discrete feedback control C2 and a discrete feed-forward control, C1 or C3. BUILD is used to assemble and connect the blocks. Logic within GENC.FUN checks the quadruples SC1 and SC3 to determine whether the corresponding C1 or C3 is to be connected. For example, if \( ||SC1||_2 \) is zero, then C1 is not used and C3 is evaluated. If \( ||SC3||_2 \) is also zero, then an error message is displayed, which indicates that there is no controller, and the program aborts. The feedback control quadruple that corresponds to C2 must be in the form

\[
SC2 = \begin{bmatrix}
A & B_b & B_f \\
C & D_b & D_f
\end{bmatrix}
\]

(8)

where \( B_b \) and \( D_b \) are the input matrices for the feedback measurements, and \( B_f \) and \( D_f \) are the input matrices for the feedforward signals from C3. If C1 is used instead of C3, then matrices \( B_f \) and \( D_f \) are not needed. If C1 is used as the feedforward control, then the output vector from C1 must be the same size as the output vector from C2, which is also the number of control signals going to the plant \( P \).

As shown in figure 11, various controller configurations are possible with this structure. If C1 is a null matrix and C3 an identity matrix, then the configuration in figure 11(a) is obtained by making \( B_b \) and \( B_f \) equal and making \( D_b \) and \( D_f \) equal. A negative sign at the summing junction can be obtained by making \( B_f \) and \( D_f \) the negative of \( B_b \) and \( D_b \), respectively. The configuration in figure 11(b) is obtained by making C1 a null matrix and letting C2 have the general form of equation (8). Finally, the configuration shown in figure 11(c) is obtained by using C1 and C2, with \( B_f \) and \( D_f \) not included in C2, and with C3 equal to zero. If C2 is a null matrix and C1 is an identity matrix, the command signal is input directly to the actuators, and the open-loop plant time response can be obtained.

4.1.2. Signal generator. SIGNAL.CMD is an executive command file for selecting a specific type of signal to be used for the simulation. A maximum of six signal generators of the same type can be assembled using BUILD. The simulation output time vector is also calculated in this file. First, the user is prompted for the maximum time TMAX of the simulation run. Next, a prompt appears for the user to decide whether the output time vector is to be limited to approximately 100 data points. If the response is “yes,” a second prompt then appears and requests the user to input the time step DELTIME between each element in the output time vector. The output time step \( T_S \) is then calculated to be some multiple of DELTIME by evaluating the equation

\[
T_S = \text{DELTIME} \times \text{ROUND}[(\text{TMAX/DELTIME})/100]
\]

(9)

where \( \text{ROUND} \) is a \( \text{MATRIX}x \) function used to compute the integer closest to the calculated value. The software does not allow this integer to be less than one.

Equation (9) is used for both discrete and continuous systems, but for a discrete system, the user should input the discrete sampling period (DELT) so that the output time step will be an even multiple of DELT. For a hybrid system, the user can input any number and \( T_S \) will be computed as shown in equation (9). If the user decides not to limit the output time vector to approximately 100 data points (an option available only for hybrid systems), the number of data points is determined directly by dividing TMAX by DELTIME. A very large number of data points can be calculated if the user is not cautious.

SIGNAL.CMD can handle multiple model cases. A menu option allows the user to select whether the same signal is to be used for all models or if different signals are to be used. The variable ANSW from this menu option is saved for use in the command files that build the specific signal generators. Another menu appears for the user to select the specific signal to be used. Two signal generators are currently available; one signal generator is for a pulse command (PULSE.CMD) and the second signal generator is for a ramp command (RAMP.CMD). Two of the signal sources available within BUILD, “pulse train” and “ramp,” are basic building blocks for PULSE.CMD and RAMP.CMD.

PULSE.CMD can be used to create several different types of pulse signals, such as one pulse of variable width and amplitude, a step command signal, or a pulse train. A series of prompts allow the user to select the signal command paths for each run and to select the signal pulse width and the signal pulse amplitude. If different signals are to be used for the other models to be evaluated, these three prompts are repeated for each model. The start time is initially defaulted to zero for all signals, but a menu option is available to change this time. Currently, all signals must be set to the same starting time, but this restriction may be removed in the future.

There are three variables that the user inputs via prompts in RAMP.CMD. The user is required to select the signal-path number for each run, enter the
puts plus two additional inputs for the random disturbance. The plant must equal the number of controller outputs for the controls can be either commands at a ramp with saturation. Outputs from both the plant and signal generator are used for display.

One current restriction is that the vector of plant outputs must be composed of outputs from the vehicle, followed by outputs from the controls. The outputs for the controls can be either commands at the control inputs or control-surface positions if actuator dynamics are included as part of the plant model. This restriction may be removed in a future modification.

4.2. Time Response to External Disturbance

The second option in the time analysis menu is TGUST.CMD (fig. 9), which evaluates the time response due to a random disturbance. Typically, the random disturbance considered by flight-control engineers is from wind gusts, but the computer code is general and allows any type of external-disturbance model. The user must supply a discrete plant system quadruple (SGDP) that is a combination of the original plant model and a disturbance model. There are certain input and output restrictions for SGDP that must be met. First, the number of inputs to the plant must equal the number of controller outputs plus two additional inputs for the random disturbance, and the two disturbance inputs must be the last two input paths. Second, the last two outputs from the plant must be from the disturbance model, since these outputs are later normalized to unity. If there is only one input disturbance of interest, the second disturbance path can always be used as a dummy input. The plant can have as many outputs as desired. Connections from the plant to the controller are defined in the variable YVV.

A typical example that a user might consider is a combined plant composed of a longitudinal airplane model and a Dryden wind-gust model with two outputs, one output for the longitudinal axis and one output for the vertical axis of the airplane. Airplane time responses, including control-surface positions, can then be evaluated as a function of each separate input disturbance. Therefore, this computer code allows both axes to be evaluated during the same set of runs.

As shown in figure 9, TGUST.CMD calls GUSTR.FUN. A menu allows selection of the input disturbance path, with only one input being selectable at a time for a given simulation run. GUSTR.FUN uses BUILD to construct the closed-loop control system. The only two inputs to this closed-loop system are the random disturbances. A white-noise disturbance of zero mean and normal distribution is input to the plant. A menu within GUSTR.FUN allows the user to select the default number of 1000 data points or to insert a different number of data points. Each data point is equivalent to one discrete sampling period, which is the conversion factor for the output time vector. The MATRIXx function DLSIM is used to generate the simulation data. The standard deviation of the disturbance-model output (either of the last two outputs from the combined plant) is calculated first; all vectors in the output matrix are then normalized by this standard deviation. The output vectors are available for plotting, and the standard deviation and mean of all outputs are available in matrix form. (See appendix B.) As with the other analysis files, multiple models can be processed in the same run.

4.3. Plotting

The data generated in TEVAL.CMD and TGUST.CMD can be displayed by selecting TPLCT.CMD from the main time-analysis menu. There is one menu in TPLCT.CMD, and it offers a choice between two plot files named TEVAL.PLT and GUSTS.PLT. TEVAL.PLT is used to plot the time-history data generated in TEVAL.CMD, and GUSTS.PLT is used to plot the time-history data generated in TGUST.CMD.
When TEVAL.PLT is selected, a display is shown with information such as the total number of data columns, number of runs, number of models used, number of plant outputs, number of controls, number of command model outputs, and number of signal commands. Some of the displayed data are input to TEVAL.PLT, and some of the data are calculated within this command file. There is also a prompt for the user to enter a vector of variables desired for plotting. The user must know the order of data columns in terms of corresponding variables, and then must enter the column numbers to be plotted. However, the order of data columns is consistent with the order of outputs from the simulation model. The display is a helpful reminder as to the number of variables involved. The user can insert a display title by entering the desired characters within apostrophes.

TGUST.PLT plots time-history data generated by TGUST.CMD. When this command file is selected, a display is shown with information similar to that described for TEVAL.PLT. The value for the disturbance-model output represents the two disturbance paths that are used to generate the data. The user is prompted to enter a vector of variables to be plotted. Again, the user must know the order of data columns in terms of corresponding variables, and then must enter the column numbers to be plotted. The display serves as a useful reminder.

4.4. User File

An optional user command file is available for the time analysis (fig. 9) that is similar to the user file described in section 3.3 for the frequency analysis. When this option is selected, a prompt appears for the user to enter a command-file name (within apostrophes). The user-file option has been included mainly as a convenience to allow the user a method of automatically processing data and generating the required inputs for use in TEVAL.CMD and TGUST.CMD. This option does not have to be invoked if the user desires to load the needed data into MATRIXx prior to executing TIMEANAL.CMD. A second use for this file could be to post-process data and save the data within a selected file.

5.0. Integration With Control-Design Tools

A goal of this integrated approach is to reduce the total overall design time by allowing the designer to quickly and accurately evaluate the control system by using several types of analysis. The idea is to combine separate control-design tools and these analysis tools in a manner that allows easy access to and between the design and analysis tools. The user-friendly control-system analysis tools described in this paper minimize the risk of human errors, since most of the analysis configurations are automatically assembled.

An example of the integrated approach used by the authors is illustrated in figure 14. A user input data file is first loaded into MATRIXx for processing prior to executing a control-design program. Internal to the block labeled “PROCESS INPUT DATA” are multiple UDF’s that generate all data needed by the control-design program. As shown in figure 15, the UDF’s are combined to form a command file, and each UDF performs a specific task. The first task selects from a data base the plant model(s) to be used in the control design. All UDF’s have been developed to process multiple models automatically in the same run. The second task generates a reduced model by selecting the states and controls that are of interest. The third task selects the vehicle outputs that are to be used for control feedback. Other UDF’s have been developed that combine the vehicle and actuators to form a new plant, discretize the plant according to the selected sampling period, scale selected variables, convert state and control weightings to equivalent discrete weightings, and add both process and sensor noise variances.

The control-design program presently being used is written in FORTRAN, but could be in any language. Since existing design programs are available, it seems reasonable that the newly developed tools interface with these programs. The data generated by “PROCESS INPUT DATA” are saved on an ASCII file. These data can then be read by other programs.

Data generated by a control-design program, typically feedback gains (but could be any data such as eigenvectors and eigenvalues), are saved on a file which is then loaded back into MATRIXx. These data are then used by the analysis tools that have been discussed. The blocks shown within the dotted lines in figure 14 represent the external process performed by either the designer or some optimization process. After reviewing data generated by the analysis files, a decision is made to either accept the design or change some input design parameters. If the latter decision is made, the cycle is repeated. If the design is acceptable, a typical next step is to evaluate the control law in a nonlinear simulation.

6.0. Sample Problem

This section contains descriptions and results of sample problems that are illustrated in detail in appendixes A and B. To make the sample problems easier to follow, the organization of appendixes A and B correspond to the organization of sections 6.1 and 6.2, respectively.
6.1. Description of Frequency-Analysis Sample

Appendix A contains the input data and the resulting screen output for the frequency analysis. Appendix B contains similar information for the time analysis. The sample plant is an airplane model with nine states (four vehicle states and five actuator states) and one control.

6.1.1. Input data. In appendix A, a discrete-plant quadruple (SDP) is shown with nine states (NS), one control, and three outputs, and a discrete-controller quadruple (SDC) is shown with two states (NSDC), three controls, and one output. Recall that the number of controller outputs must equal the number of plant inputs, and the number of plant outputs must equal the number of controller inputs. The sampling period (DELT) is 0.031 sec (to be consistent with the corresponding real-time simulation model at Langley Research Center), and only one model is being evaluated. When more than one model is to be evaluated, SDP and SDC will be stacked by columns with the first model in the first set of columns. The quadruples (SDP and SDC) are used in the evaluations of singular-value robustness, singular-value loop transfer, Bode frequency response, and closed-loop eigenvalues. The steady-state covariance analysis requires a plant with an external-disturbance model. A third-order Dryden wind-gust model is combined with the plant (SDP) to evaluate the system response to external wind gusts in both the longitudinal and vertical directions. This new combined discrete-plant quadruple (SGDP) has 12 states (NSGDP), 3 controls, and 7 outputs. Two of the additional inputs and outputs are for the wind-gust model, and the other two additional outputs are for the actuator positions. The controller and sampling period remain the same.

6.1.2. Singular-value robustness. The singular-value multiplicative errors at the plant input and at the plant output are evaluated first. The frequency range selected is from 0.01 to 10 rad/sec with a density of 20 data points per decade. Since there is only one input, the structured and unstructured singular values are identical for this case. The structured singular value is selected for the outputs and new scale factors are calculated every fifth frequency point. The Davidon-Fletcher-Powell convergence algorithm is also selected. Outputs from the first two convergence passes are shown in appendix A as displayed on the computer terminal. The data point for the first convergence pass is calculated twice to get good starting values. The data shown are the minimum singular value (SVAL), final step size (KAPPA), gradient of the two scale factors (GRADI), and the two scale factors (DOUT). The first path always has a scale factor of one; therefore, the number of scale factors is always one less than the number of paths. Figure 16 contains log-log plots of the minimum singular value at the plant input (SIGI) and the minimum singular value at the plant output (SIGO).

6.1.3. Singular-value loop transfer and Bode frequency response. Both the singular-value loop transfer and Bode frequency response are calculated over a revised frequency range of 0.1 to 100 rad/sec with a density of 20 points per decade. Figure 17 contains log-log plots of the maximum singular value at the plant input (SVLTI) and the maximum singular value at the plant output (SVLTO). There are two plots that illustrate the Bode frequency response. Figure 18 contains semilog frequency-response plots of gain in decibels and phase angle. A second illustration is shown in figure 19, which is a Nyquist polar plot of amplitude versus phase angle. The amplitude scale for the Nyquist plot is dimensionless, as opposed to dB, since the radius of the MATRIXx pole plot must be positive.

6.1.4. Steady-state covariance. Typical results from the steady-state covariance analysis are illustrated in the bar chart in figure 20. Selections 1 (XURMS) and 2 (YURMS) are for the state and output RMS values, respectively, with a normalized wind-gust disturbance in the first disturbance path. States 4, 5, and 6 are plotted in the top portion of figure 20, and outputs 3, 4, and 5 are plotted in the bottom portion of the figure.

6.1.5. Closed-loop eigenvalues. The final evaluation is the closed-loop eigenvalue test. The tabular eigenvalue data (EVALCL) are calculated and remain on file for observation after exiting the FREQANAL.CMD menu but prior to exiting MATRIXx. The table is set up with the complex eigenvalue data (EVALCL) are calculated over a revised frequency range of 0.1 to 1000 rad/sec with a density of 20 points per decade.

6.1.6. Additional output data. Other output data (in addition to EVALCL) include the phase margin (PHASEM) and gain margin (GAINM). The four columns of PHASEM represent the phase margin in degrees, frequency in rad/sec, model number, and control loop number. The first column of GAINM is the gain margin in dB; the remaining three columns represent the same quantities as in PHASEM. The RMS data (XURMS, YURMS, XWRMS, YWRMS) for the two external-disturbance...
paths are also shown. XURMS and XWRMS have 14 elements corresponding to the 12 states in SGDP plus 2 states in SDC; YURMS and YWRMS have 7 elements corresponding to the 7 outputs from SGDP.

The example described is for one model. When multiple models are evaluated, the output vectors are stacked by columns; the first set of vectors represents the first model. As an example, for the case of $q$ models and $p$ outputs, the first $p$ columns of the output matrix will be from the first model, the second $p$ columns will be from the second model, and the total number of columns in the output matrix will be the product of $p$ and $q$. This same data stacking approach is used in the time analysis.

6.2. Description of Time-Analysis Sample

The input data and resulting screen output for the time-analysis sample described in this subsection are shown in appendix B. The sample plant is an airplane model with 9 states (4 vehicle states and 5 actuator states) and 2 controls.

6.2.1. Input data. Input data for the time analysis shown in appendix B are slightly more complicated than those needed for the frequency analysis. The generic controller (SC1, SC2, SC3) has dynamic feedback (SC2) with two states (NSC2), two outputs, and six inputs. The six inputs are partitioned into three inputs for the feedback signals from the plant and three identical inputs of opposite sign for the control inputs from the signal generator (columns 6 to 8 of SC2 are the negative of columns 3 to 5). This set of inputs is equivalent to a summing junction between the feedback signals and the command signals, as shown in figure 11(a). The feedforward control is just a unity gain matrix (SC3). Note that SC1 and NSC1 must be set to zero if they are not used.

The plant selected for the sample problem is continuous (SCP) with nine states (NSCP), two controls, and seven outputs. The seven outputs include five outputs from the vehicle and two outputs for the control positions. Three of the vehicle outputs are fed back to SC2, and the other outputs are used only for display. Three quantities that must be defined are located in variable DIMS. DIMS(9) contains the sampling period of 0.031 sec, DIMS(10) contains the number of vehicle outputs, and DIMS(11) is the starting point for a row vector that defines the three vehicle outputs that are fed back. Three other additional inputs required are the number of models used (NMOD) and row vectors that contain the lower and upper saturation values of the actuators (LBOUND, UBOUND). These last two variables are required; if a limiter is not being used, the variables can be set to the scalar zero.

The input data for the time response to external disturbances (TGUST.CMD) consist of a discrete quadruple for the combined plant (SGDP) with 12 states (NSGDP), 4 inputs, and 9 outputs. The inputs to SGDP consist of two controls and two random white-noise signals with normal distribution. The discrete controller (SDC) has two states (NSDC), two outputs that go to the plant, and three measurement inputs from the plant (YVV).

6.2.2. Time evaluation. In the first example, TEVAL.CMD is selected. A continuous plant with actuator saturation is being used for the 10-sec simulation with approximately 100 data points of output at time intervals which are a multiple of DELT. A ramp signal generator with a slope of 0.2 and a saturation level of 1 is the command to the first input path; the other two nonselected input paths have default input commands of zero.

After this simulation run is completed, a second run is selected for the same maximum time and number of data points. The command to the first path is a step with an amplitude of 100; implementation of this step is accomplished by selecting a pulse and setting the pulse width equal to the maximum time of the simulation run (10 sec). Defaults include setting the start time of the pulse to zero, setting the amplitude of the nonselected signal generators to zero, and setting the pulse frequency to a small number.

An option exists to save or delete the files within BUILD. It is generally advisable to delete these files, since they will remain with other data that the user may desire to save. If the BUILD files are kept, it is then possible to edit these files, after finishing with TIMEANAL.CMD, and to run special cases.

The plotting option is selected with the option of a hard-copy output on file MATLASEAR.DAT. A split screen is selected with four variables for the upper plot and three variables for the lower plot. Prior to the selection of the variable, a display shows the user that there are 22 columns of data for the two runs and that the data for each run are distributed as 5 columns for the plant (airplane) outputs, 2 columns for the control outputs from the plant, 3 columns for the model outputs, and 1 column for the signal-generator output. The simulation title REGULATOR is attached to the plot as shown in figure 21. Variable names cannot be printed within a key, but the line style of the curves always appears in a known order.

6.2.3. Time response to external disturbance. TGUST.CMD is selected as another example. First, the random disturbance for the first input path (channel 1) is selected with the default value of 1000 data points. After this run is complete, the
one variable is shown in the lower part of the figure.

6.2.4. Additional output data. After quitting TIMEANAL.CMD, the generated data are available in memory. Data that are needed must be saved prior to exiting MATRIXx, or they will be lost. Examples of statistical data for the external-disturbance input paths (channels 1 and 2) are illustrated by GUSTAT and GWSTAT, respectively. The first row of each variable represents the standard deviation of the time-history data for each displayed output (defined by columns), and the second row represents the mean of the time-history data. All data are for a normalized input disturbance of 1.

7.0. Concluding Remarks

One objective of this work is to enable the user to assemble and analyze various control-system configurations within the MATRIXx software package. Implementation of a high-level system approach to control-law analysis has been accomplished by combining basic MATRIXx functions with user-defined functions and command files. Programming features within MATRIXx and its BUILD module allow various control-system analysis configurations to be assembled automatically after the user selects the analysis to be made. Embedded within these tools are menu-driven options that require minimal input data and interaction by the user. This approach minimizes the mistakes by the designer and the time required to perform the various analyses. Key attributes of these control analysis tools include simplicity, user-friendliness, flexibility, and the ability to trade off between time and accuracy.

Frequency-response analysis tools and time-response analysis tools are both available. All these analysis routines have been developed to handle multiple models automatically in one computer run. This feature allows, in minimum time, comparison of different control systems at different flight conditions.

The only inputs required for the frequency evaluations are discrete state-space quadruples for a plant and a controller, the corresponding number of states for each quadruple, and the discrete sampling period. Using these inputs, five different evaluations can be performed. The input data are minimized for simplicity.

A vector of frequency data points, based upon user inputs which define the frequency range of interest and the density of data points within that range, is automatically created by the program. This frequency vector can be easily changed for each analysis. The frequency vector can be composed of several subvectors, each with a different density of data points. This feature is important when evaluating a control system with a lightly damped eigenvalue, because additional data points can be calculated in the frequency range where the control-response variable changes rapidly. In contrast, fewer data points are needed in frequency ranges with slowly varying control responses. This flexibility results in reduced time to perform the analysis, but still allows the important response features to be accurately analyzed.

The frequency evaluations are applicable to both single input-output systems and multiple input-output systems. One frequency-evaluation test developed in recent years for multiple input-output systems is the structured singular value for a multiplicative perturbation at either the plant input or the plant output. Theoretically, a new scaling matrix must be computed at each new frequency. Since the scaling matrix must be solved iteratively, the process is time-consuming. The user has the option of reducing the number of frequencies at which the scaling matrix is calculated; this option decreases the computation time with the penalty of a slight decrease in accuracy. It is also possible to compute the unstructured singular-value frequency response.

A generic controller has been developed for use in simulations that use signal-generator commands. The structure allows the incorporation of a dynamic feedback controller and two options for a dynamic feedforward controller. Several different control-system configurations can be assembled and evaluated, including configurations that allow signal commands to be input directly to the actuators.

The generic controller must be in discrete form, but the plant can be either discrete (for an all-discrete simulation) or continuous (for a hybrid simulation). The all-discrete simulation is valid only at the sampling intervals, but it is more efficient than the hybrid simulation because it takes less computer time. The hybrid system requires numerical integration for the plant equations of motion, but outputs can be observed between sampling times. With either type of simulation, the user may choose whether a limiter is to be used for actuator position saturation.

The time-analysis command files give the user the capability to evaluate control-system time responses as a function of signal commands or random external disturbances. Presently, either ramp
or pulse signal generators are available for command inputs, but others may be added in the future. The pulse command can be either a single pulse, a step, or a pulse train, and the ramp may have saturation included. Time responses and statistical data (mean and standard deviation) are generated when an external-disturbance input is used. The external disturbance is white noise with zero mean and normal distribution.

Options exist for the inclusion of user-developed command files. The user-file selection has been included mainly as a convenience to allow the user a method of automatically preprocessing data. This option does not have to be invoked if the user has already loaded the required data into MATRIXx prior to executing the selected command file.

Plot command files are available for both frequency and time evaluations. Singular-value frequency-response data are displayed on log-log plots, Bode frequency-response data are displayed on semilog plots, and steady-state root-mean-square data due to an external disturbance are displayed in bar charts. Time-history outputs are displayed on linear scaled plots.

A plan has been demonstrated for integration of the analysis tools with control-design tools; this plan allows easy access to and between the two processes. This integrated approach allows the designer to easily change design parameters, run the control-synthesis program, and then perform various analyses. Total time for this complete cycle is significantly reduced relative to an approach which has few integrated tools.

References


NASA Langley Research Center
Hampton, VA 23665-5225
January 11, 1989
APPENDIX A

DATA FOR FREQUENCY-ANALYSIS SAMPLE

The "Sample Problem" section (6.0) describes the input data needed for the frequency analysis. These data and the output as displayed on the screen are shown below. For clarity, the titles, section numbers, and user responses to prompts are in boldface print. Also, explanations are located on the right side of the page.

A.1 INPUT DATA

SDP =

COLUMNS 1 THRU 6
6.6066D-03 2.8002D+00 9.7528D-02 2.8682D-13 0.0000D+00 0.0000D+00

COLUMNS 7 THRU 10
-1.9398D-02 -2.3460D-02 -1.1230D-02 4.1398D-03
-3.5455D-04 -3.4715D-04 -2.1587D-04 5.3172D-05
-5.5898D-03 -6.7448D-03 -5.1480D-03 1.2374D-03
-1.6246D-04 -1.1376D-04 -9.2150D-05 1.1557D-05
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
-2.6515D-01 0.0000D+00 5.7684D-02 0.0000D+00

NS =
9.

SDC =
8.0880D-01 -7.8095D-01 -1.0667D+00 -2.1447D+00 1.2979D-01
0.0000D+00 1.0000D+00 3.1000D-02 3.1000D-02 0.0000D+00
1.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
| NSDC | =
|------|---
| 2.   |   
| DELT | =
| 3.1000D-02 |   
| SGDP | =

**Columns 1 Thru 6**

<table>
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<tr>
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<td>-3.7840D-04</td>
<td>0.0000D+00</td>
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<td>0.0000D+00</td>
<td>0.0000D+00</td>
<td>0.0000D+00</td>
</tr>
</tbody>
</table>
| COLUMNS 7 Thru 12
| -1.9398D-02 | -2.3460D-02 | -1.1230D-02 | 1.4204D-03  | 1.6756D-03  | 4.4128D-08  |
| -5.5898D-03 | -6.7448D-03 | -5.1480D-03 | 3.0294D-06  | 8.1752D-05  | 2.1547D-09  |
| -1.6246D-04 | -1.1376D-04 | -9.2150D-05 | 4.8484D-08  | 1.2716D-06  | 2.2316D-11  |
| 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| -1.2748D-01 | 7.0635D-02   | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| -7.0635D-02 | -1.2748D-01 | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| 0.0000D+00  | 0.0000D+00  | 3.9455D-01  | 9.9475D-01  | 9.9475D-01  | 5.2335D-05  |
| 0.0000D+00  | 0.0000D+00  | 9.9475D-01  | 9.9475D-01  | 9.9475D-01  | 5.2335D-05  |
| 0.0000D+00  | 0.0000D+00  | 9.9475D-01  | 9.9475D-01  | 9.9475D-01  | 5.2335D-05  |
| 2.6515D-01  | 5.7684D-02   | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| 2.5144D-01  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| 0.0000D+00  | 0.0000D+00  | 3.0000D+01  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 1.0000D+00  | 0.0000D+00  | 0.0000D+00  |
| 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 0.0000D+00  | 3.3670D-03  | 0.0000D+00  |
A.2 SINGULAR-VALUE ROBUSTNESS

<> EXEC ('MUGDIR:FREQANAL.CMD')

SELECT COMMAND FILE
1 EXEC FREQEVAL.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 1

ANALYSIS:
1 CHANGE OMEG
2 S.V. MULT ERR
3 S.V. LOOP TRAN.
4 BODE FREQ RESP
5 SSCOV
6 CLOSED LOOP EIG
7 QUIT

SELECT> 1
OMEG MENU:
1 CHANGE OMEG
2 REVIEW OMEG
3 QUIT

SELECT> 1

NEW OMEG FREQ RANGES<MIN,MAX1,***,MAXN> [.01 10]

N =
1.

PTS/DECADE FOR EACH RANGE(NOM=2u)<1,2,***,N> [20]

OMEG MENU:
1 CHANGE OMEG
2 REVIEW OMEG
3 QUIT

SELECT> 3

ANALYSIS:
1 CHANGE OMEG
2 S.V. MULT ERR
3 S.V. LOOP TRAN.
4 BODE FREQ RESP
5 SSCOV
6 CLOSED LOOP EIG
7 QUIT

SELECT> 2

OPTION 2. (SINGULAR VALUES – MULTIPLICATIVE ERROR)

<OMEGA,SIG,SVAL,DSCALED,GRA>=SSVIO(S,NS,OMEG,DELT)
<SimB,NSimB,SOm,NSom>=SV(Sdp,NS,SDC,NSDC,DELT)

PLEASE CHOOSE:
1 INPUTS ONLY
2 OUTPUTS ONLY
3 BOTH

SELECT> 3

NCOUNT SELECTION
1 KEEP DEFAULT (5)
2 CHANGE NCOUNT
3 SELECT NCOUNT=0

SELECT> 1
FUNCTION SELECTION
1 DEFAULT (SVDFP)
2 SELECT SVGNR
SELECT> 1

COMPLETED EXAMPLE FOR 1 INPUT

NCOUNT SELECTION
1 KEEP DEFAULT (5)
2 CHANGE NCOUNT
3 SELECT NCOUNT=0
SELECT> 1

FUNCTION SELECTION
1 DEFAULT (SVDFP)
2 SELECT SVGNR
SELECT> 1

SVALI =
1.0090
KAPPA =
1.
GRADI =
-0.0012
-0.0157
DOUT =
9.0403
0.0896

COMPLETED FIRST POINT, INITIAL CALCULATION

SVALI =
1.0089
KAPPA =
1.
GRADI =
-0.0013
-0.0004
DOUT =  
9.0614
0.1473

I =
1.

COMPLETED FIRST POINT, FINAL CALCULATION

SVALI =
1.0166

KAPPA =
1.

GRADI =
1.0D-03*

-0.8142
-0.6040

DOUT =
9.0721
0.1379

I =
2.

COMPLETED SECOND POINT

SELECT COMMAND FILE

1 EXEC FREQEVAL.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 2

AMP = AMPL(DB)
PICTURE TAKING:
1 EVERYTHING
2 SOMETHINGS
3 NOTHING
SELECT> 2

Available devices are:
1 VT220
2 VT240/Monochrome
3 Tektronix 4014
4 MATLN03
SELECT> 4

ENTER GRAPHICS FILENAME [MATLASER.DAT]: CARRIAGE RETURN

PLOT SELECT:
1 SV
2 SVLT
3 BFR
4 GUST
5 QUIT
SELECT> 1

PLOT MENU
1 WHOLE
2 1/2 UP/LOW
3 1/2 UP. 1/4 LOW.
4 1/4 UP. 1/2 LOW.
5 1/4 QUAD
6 BODE DEFAULT
7 QUIT
SELECT> 2

MEASURED OUTPUTS

NY = 3.

CONTROL COMMANDS

NBU = 1.
SELECT FOR UPPER

CHOOSE:

1 SIGI
2 SIGO
3 QUIT

SELECT> 1

ENTER VECTOR OF MODELS DESIRED [1] TOP HALF PLOTTED (FIG. 16)

SELECT FOR LOWER

CHOOSE:

1 SIGI
2 SIGO
3 QUIT

SELECT> 2

ENTER VECTOR OF MODELS DESIRED [1] LOWER HALF PLOTTED (FIG. 16)

TAKE PICTURE NOW:

1 YES
2 NO

SELECT> 1

PLOT SELECT:

1 SV
2 SVLT
3 BFR
4 GUST
5 QUIT

SELECT> 5

A.3 SINGULAR-VALUE LOOP TRANSFER AND BODE FREQUENCY RESPONSE

SELECT COMMAND FILE

1 EXEC FREQUEVAL.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 1
ANALYSIS:
1 CHANGE OMEG
2 S.V. MULT ERR
3 S.V. LOOP TRAN.
4 BODE FREQ RESP
5 SSCOV
6 CLOSED LOOP EIG
7 QUIT
SELECT> 1

OMEG MENU:
1 CHANGE OMEG
2 REVIEW OMEG
3 QUIT
SELECT> 1

NEW OMEG FREQ RANGES<MIN,MAX1,***,MAXN> [.1 100]
N  =
1.

PTS/DECADE FOR EACH RANGE(NOM=20)<1,2,***,N> [20]

OMEG MENU:
1 CHANGE OMEG
2 REVIEW OMEG
3 QUIT
SELECT> 3

A.3.1 SINGULAR-VALUE LOOP TRANSFER

ANALYSIS:
1 CHANGE OMEG
2 S.V. MULT ERR
3 S.V. LOOP TRAN.
4 BODE FREQ RESP
5 SSCOV
6 CLOSED LOOP EIG
7 QUIT
SELECT> 3

COMPLETED SINGULAR VALUE LOOP TRANSFER CALCULATION
SELECT COMMAND FILE

1 EXEC FREQEVAL.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 2

AMP = AMPL(DB)

PICTURE TAKING:

1 EVERYTHING
2 SOME THINGS
3 NOTHING

SELECT> 2

PLOT SELECT:

1 SV
2 SVLT
3 BFR
4 GUST
5 QUIT

SELECT> 2

PLOT MENU

1 WHOLE
2 1/2 UP/LOW
3 1/2 UP, 1/4 LOW.
4 1/4 UP, 1/2 LOW.
5 1/4 QUAD
6 BODE DEFAULT
7 QUIT

SELECT> 2

MEASURED OUTPUTS

NY = 3.

CONTROL COMMANDS

NBU = 1.
SELECT FOR UPPER

CHOOSE:

1 SVLTI
2 SVLTO
3 QUIT

SELECT> 1

ENTER VECTOR OF MODELS DESIRED <ON A SINGLE PLOT> [1]

ENTER SV(S) TO BE PLOTTED [1] TOP HALF PLOTTED (FIG. 17)

SELECT FOR LOWER

CHOOSE:

1 SVLTI
2 SVLTO
3 QUIT

SELECT> 2

ENTER VECTOR OF MODELS DESIRED <ON A SINGLE PLOT> [1]

ENTER SV(S) TO BE PLOTTED [1] LOWER HALF PLOTTED (FIG. 17)

TAKE PICTURE NOW:

1 YES
2 NO

SELECT> 1

PLOT SELECT:

1 SV
2 SVLT
3 BFR
4 GUST
5 QUIT

SELECT> 5
A.3.2 BODE FREQUENCY RESPONSE

SELECT COMMAND FILE

1 EXEC FREQEVAL.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 1

ANALYSIS:

1 CHANGE OMEG
2 S.V. MULT ERR
3 S.V. LOOP TRAN.
4 BODE FREQ RESP
5 SSCOV
6 CLOSED LOOP EIG
7 QUIT

SELECT> 4

COMPLETED BODE FREQUENCY RESPONSE CALCULATION

SELECT COMMAND FILE

1 EXEC FREQEVAL.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 2

AMP =AMPL(DB)

PICTURE TAKING:

1 EVERYTHING
2 SOMETHINGS
3 NOTHING

SELECT> 2

PLOT SELECT:

1 SV
2 SVLT
3 BFR
4 GUST
5 QUIT

SELECT> 3
PLOT MENU
1 WHOLE
2 1/2 UP/LOW
3 1/2 UP, 1/4 LOW.
4 1/4 UP, 1/2 LOW.
5 1/4 QUAD
6 BODE DEFAULT
7 QUIT

SELECT> 2

CONTROL COMMANDS

NBU =

SELECT FOR UPPER

CHOOSE:

1 DB
2 PHASE
3 NYQUIST
4 QUIT

SELECT> 1

ENTER VECTOR OF MODELS DESIRED <ON A SINGLE PLOT> [1]

ENTER THE CHANNEL TO BE GRAPHED [1] TOP HALF PLOTTED (FIG. 18)

SELECT FOR LOWER

CHOOSE:

1 DB
2 PHASE
3 NYQUIST
4 QUIT

SELECT> 2

ENTER VECTOR OF MODELS DESIRED <ON A SINGLE PLOT> [1]

ENTER THE CHANNEL TO BE GRAPHED [1] LOWER HALF PLOTTED (FIG. 18)

TAKE PICTURE NOW:

1 YES
2 NO

SELECT> 1
PLOT SELECT:
1  SV
2  SVLT
3  BFR
4  GUST
5  QUIT

SELECT> 3

PLOT MENU
1  WHOLE
2  1/2 UP/LOW
3  1/2 UP. 1/4 LOW.
4  1/4 UP. 1/2 LOW.
5  1/4 QUAD
6  BODE DEFAULT
7  QUIT

SELECT> 1

CONTROL COMMANDS

NBU  =  1.

FOR SINGLE PLOT

CHOOSE:
1  DB
2  PHASE
3  NYQUIST
4  QUIT

SELECT> 3

ENTER VECTOR OF MODELS DESIRED <ON A SINGLE PLOT> [1]

ENTER THE CHANNEL TO BE GRAPHED [1]

CALULATING THE AMPLITUDE AT THIS TIME.

NYQUIST POLAR PLOT COMPLETED (FIG. 19)

TAKE PICTURE NOW:
1  YES
2  NO

SELECT> 1
PLOT SELECT:

1  SV
2  SVLT
3  BFR
4  GUST
5  QUIT

SELECT> 5

A.4 STEADY-STATE COVARIANCE

SELECT COMMAND FILE

1  EXEC FREQEVAL.CMD
2  EXEC PLOT.CMD
3  EXEC USERFILE
4  EXEC AJO USERFILE
5  QUIT

SELECT> 1

ANALYSIS:

1  CHANGE OMEG
2  S.V. MULT ERR
3  S.V. LOOP TRAN.
4  BODE FREQ RESP
5  SSCOV
6  CLOSED LOOP EIG
7  QUIT

SELECT> 5

COMPLETED STEADY-STATE COVARIANCE CALCULATION

SELECT COMMAND FILE

1  EXEC FREQEVAL.CMD
2  EXEC PLOT.CMD
3  EXEC USERFILE
4  EXEC AJO USERFILE
5  QUIT

SELECT> 2

AMP = AMPL(DB)

PICTURE TAKING:

1  EVERYTHING
2  SOME THINGS
3  NOTHING

SELECT> 2
PLOT SELECT:

1  SV
2  SVLT
3  BFR
4  GUST
5  QUIT

SELECT> 4

PLOT MENU

1  WHOLE
2  1/2 UP/LOW
3  1/2 UP. 1/4 LOW.
4  1/4 UP. 1/2 LOW.
5  1/4 QUAD
6  BODE DEFAULT
7  QUIT

SELECT> 2

MEASURED OUTPUTS

NY  =

3.

GUSTS INPUTS = 2

SELECT FOR UPPER

CHOOSE:

1  XURMS
2  YURMS
3  XWRMS
4  YWRMS
5  QUIT

SELECT> 1

ENTER VECTOR OF MODELS DESIRED [1]

SELECT VECTOR OF ELEMENTS DESIRED [4 5 6] TOP HALF PLOTTED (FIG. 20)

SELECT FOR LOWER

CHOOSE:

1  XURMS
2  YURMS
3  XWRMS
4  YWRMS
5  QUIT

SELECT> 2
ENTER VECTOR OF MODELS DESIRED [1]

SELECT VECTOR OF ELEMENTS DESIRED [3 4 5]  LOWER HALF PLOTTED (FIG. 20)

TAKE PICTURE NOW:

1  YES
2  NO

SELECT> 1

PLOT SELECT:

1  SV
2  SVLT
3  BFR
4  GUST
5  QUIT

SELECT> 5

A.5 CLOSED-LOOP EIGENVALUES

SELECT COMMAND FILE

1  EXEC FREQEVAL.CMD
2  EXEC PLOT.CMD
3  EXEC USERFILE
4  EXEC AJO USERFILE
5  QUIT

SELECT> 1

ANALYSIS:

1  CHANGE OMEG
2  S.V. MULT ERR
3  S.V. LOOP TRAN.
4  BODE FREQ RESP
5  SSCOV
6  CLOSED LOOP EIG
7  QUIT

SELECT> 6

COMPLETED CLOSED-LOOP EIGENVALUES CALCULATION
SELECT COMMAND FILE

1 EXEC FREQeval.CMD
2 EXEC PLOT.CMD
3 EXEC USERFILE
4 EXEC AJO USERFILE
5 QUIT

SELECT> 5

A.6 ADDITIONAL OUTPUT DATA

EVALCL =

-1.2610D-01 -7.0068D-02i -6.2456D+01 -8.4981D+01i 5.9220D-01 +0.0000D+00i
-1.2610D-01 +7.0068D-02i -6.2456D+01 +8.4981D+01i 5.9220D-01 +0.0000D+00i
3.9387D-01 -8.1817D-18i -3.0056D+01 -6.7009D-16i 1.0000D+00 +0.0000D+00i
3.2015D-01 +5.3010D-01i -1.5458D+01 +3.3145D+01i 4.2268D-01 +0.0000D+00i
3.2015D-01 -5.3010D-01i -1.5458D+01 -3.3145D+01i 4.2268D-01 +0.0000D+00i
9.1988D-01 -9.4606D-02i -2.5242D+00 -3.3060D+00i 6.0686D-01 +0.0000D+00i
9.1988D-01 +9.4606D-02i -2.5242D+00 +3.3060D+00i 6.0686D-01 +0.0000D+00i
9.7906D-01 -1.0570D-02i -6.8090D-01 -3.4825D-01i 8.9031D-01 +0.0000D+00i
9.7906D-01 +1.0570D-02i -6.8090D-01 +3.4825D-01i 8.9031D-01 +0.0000D+00i
9.9960D-01 -4.0888D-03i -1.2555D-02 -1.3195D-01i 9.4722D-02 +0.0000D+00i
9.9960D-01 +4.0888D-03i -1.2555D-02 +1.3195D-01i 9.4722D-02 +0.0000D+00i

PHASEM =

5.0008D+01 3.0826D+00 1.0000D+00 1.0000D+00

GAINM =

-1.6040D+01 1.1110D+01 1.0000D+00 1.0000D+00

XURMS =

1.6452D+00
9.9085D-04
9.1346D-04
6.2951D-03
2.6258D-02
1.4032D-02
5.1980D-04
3.7986D-04
7.4870D-05
1.0000D+00
1.2548D-15
2.1641D-15
1.5925D-03
9.6856D-04

33
YURMS =
9.9085D-04
9.1346D-04
9.0288D-03
1.3070D-02
2.2461D-03
1.0000D+00
4.2248D-18

XWRMS =
5.9138D+02
4.8425D-01
4.0372D-01
2.4170D+00
4.0363D+01
2.1609D+01
7.9894D-01
5.8395D-01
1.1496D-01
6.5833D-12
2.9700D+02
1.5378D+04
2.4466D+00
8.9393D-01

YWRMS =
4.8425D-01
4.0372D-01
3.2727D+00
2.0088D+01
3.4487D+00
6.5833D-12
1.0000D+00
APPENDIX B

DATA FOR TIME-ANALYSIS SAMPLE

The "Sample Problem" section (6.0) describes the input data needed for the time analysis. These data and the output as displayed on the screen are shown below. For clarity, the titles, section numbers, and user responses to prompts are in boldface print. Also, explanations are located on the right side of the page.

B.1 INPUT DATA

SC1 =
0.

NSC1 =
0.

SC2 =

COLUMN 1 THRU 6
8.088D-01 -7.8095D-01 -1.0667D+00 -2.1447D+00 1.2979D-01 1.0667D+00
0.0000D+00 1.0000D+00 3.1000D-02 3.1000D-02 0.0000D+00 -3.1000D-02
-8.2059D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
-1.4107D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMN 7 THRU 8
2.1447D+00 -1.2979D-01
-3.1000D-02 0.0000D+00
0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00

NSC2 =
2.

SC3 =
1. 0. 0.
0. 1. 0.
0. 0. 1.

NSC3 =
0.
SCP =

COLUMNS 1 THRU 6
-4.6153D-02 -1.6189D+01 2.8067D-13 -3.2163D+01 0.0000D+00 0.0000D+00
-7.1404D-04 -3.0264D-01 9.8946D-01 -2.2643D+01 0.0000D+00 0.0000D+00
-1.0716D-04 -7.9189D-01 -3.9554D-13 -3.1000D-14 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

COLUMNS 7 THRU 11
-2.6605D+00 0.0000D+00 -5.6114D-01 0.0000D+00 0.0000D+00
-2.6658D-02 0.0000D+00 -6.2345D-01 0.0000D+00 0.0000D+00
-8.2124D-01 0.0000D+00 -2.5625D-01 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00
0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00 0.0000D+00

NSCP =

9.

DIIMS(9) =
3.1000D-02

DIIMS(10) =
5.

DIIMS(11:13) =
2. 3. 5.

NMOD =
1.
\[
\text{SGDP} = -3.0376 \times 10^{-6}
\]
\[
\text{UBOUND} = \begin{bmatrix}
-2.2064 \times 10^{-5} \\
9.9029 \\
3.0419 \times 10^{-2} \\
1.1012 \times 10^{-1} \\
-1.9398 \times 10^{-2} \\
-2.3460 \times 10^{-2} \\
\end{bmatrix}
\]
\[
\text{LBOUND} = \begin{bmatrix}
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
0.0000 \\
\end{bmatrix}
\]
\[
2.5144 \times 10^{1}
\]
\[
9.9858 \times 10^{-1} \\
-4.9898 \times 10^{-1} \\
-2.3061 \times 10^{-9} \\
0.0000 \\
0.0000 \\
0.0000 \\
\]
\[
\text{COLUMNS 1 THRU 6}
\]
\[
\begin{bmatrix}
9.9858 \times 10^{-1} \\
-4.9898 \times 10^{-1} \\
-2.3061 \times 10^{-9} \\
0.0000 \\
0.0000 \\
0.0000 \\
\end{bmatrix}
\]
\[
9.9858 \times 10^{-1} \\
-4.9898 \times 10^{-1} \\
-2.3061 \times 10^{-9} \\
0.0000 \\
0.0000 \\
0.0000 \\
\]
COLUMNS 13 THRU 16

| -4.6986E-04 | -2.0145E-04 | 1.2858E-05 | 1.8572E-05 |
| -5.9214E-06 | -3.2479E-06 | 1.9912E-07 | 3.5655E-07 |
| -1.3495E-04 | -9.2150E-05 | 2.8235E-08 | 9.0683E-07 |
| -1.2327E-06 | -1.0219E-06 | 2.9631E-10 | 9.3896E-09 |
| 7.7808E-01  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 1.8085E+00  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 1.2418E-02  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 1.5299E-02  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 0.0000E+00  | 2.0182E-02  | 0.0000E+00 | 0.0000E+00 |
| 1.8013E-02  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 0.0000E+00  | 0.0000E+00  | 0.0000E+00 | 2.2037E-02 |
| 0.0000E+00  | 0.0000E+00  | 0.0000E+00 | -9.3255E-01 |
| 0.0000E+00  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 0.0000E+00  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 0.0000E+00  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |
| 0.0000E+00  | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 |

NSGDP =

12.

SDC =

| 8.0880E-01 | -7.8095E-01 | -1.0667E00 | -2.1447E00 | 1.2979E01 |
| 0.0000E+00 | 1.0000E00  | 3.1000E-02 | 3.1000E-02 | 0.0000E+00 |
| -8.2059E00 | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |
| -1.4107E00 | 0.0000E+00  | 0.0000E+00 | 0.0000E+00 | 0.0000E+00 |

NSDC =

2.

YVW =

2. 3. 5.

B.2 TIME EVALUATION

<> EXEC ('MUGDIR:TIMEANAL.CMD')

SELECT COMMANDFILE

1 TEVAL.CMD
2 TGUST.CMD
3 TPLOT.CMD
4 USERFILE
5 AJO USERFILE
6 QUIT

SELECT> 1
CHOOSE PLANT

1 CONTINUOUS PLANT
2 DISCRETE PLANT

SELECT> 1

IS LIMITER USED

1 NO
2 YES

SELECT> 2

SAME SIGNALS FOR ALL MODELS

SELECT

1 YES
2 NO

SELECT> 1

PLEASE ENTER TIME MAXIMUM [10]

1 100 DATA PT LIMIT OK

1 YES
2 NO

SELECT> 1

ENTER TIME STEP WHICH IS APPROX. TMAX/100 [DELT]

SELECT SIGNAL

1 PULSE
2 RAMP

SELECT> 2

NUMBER OF SIGNAL COMMANDS

NSIG =

3.

SELECT CHANNEL(S) FOR SIGNAL COMMANDS <I1,I2,ETC> [1]

ENTER RAMP SLOPE(S) <I1,I2,ETC> [.2]

ENTER RAMP SATURATION <I1,I2,ETC> [1]

TIME EVALUATION FOR FIRST RUN COMPLETED
CHANGE CHANNELS
1 YES
2 NO
SELECT> 1
SAME SIGNALS FOR ALL MODELS
SELECT
1 YES
2 NO
SELECT> 1
PLEASE ENTER TIME MAXIMUM [10]
100 DATA PT LIMIT OK
1 YES
2 NO
SELECT> 1
ENTER TIME STEP WHICH IS APPROX. TMAX/100 [DELT]
SELECT SIGNAL
1 PULSE
2 RAMP
SELECT> 1
DEFAULT MENU:
1 KEEP DEFAULTS
2 REVIEW VALUES
3 START TIME
4 NON-SELECTED PULSE
5 PULSE FREQUENCY
6 SAVE CHANGES
SELECT> 1
NUMBER OF SIGNAL COMMANDS

\[ \text{NSIG} = 3. \]

SELECT CHANNEL(S) FOR SIGNAL COMMAND <l1,l2,ETC> [1]
SELECT PULSE WIDTHS FOR SIGNALS SELECTED <l1,l2,ETC> [10]
SELECTED PULSE MAGNITUDE(S) <l1,l2,ETC> [100]

TIME EVALUATION FOR SECOND RUN COMPLETED
CHANGE CHANNELS

1 YES
2 NO

SELECT> 2

DELETE SYSTEM BUILD

1 YES
2 NO

SELECT> 1

END OF TEVAL.CMD

SELECT COMMANDFILE

1 TEVAL.CMD
2 TGUST.CMD
3 TPILOT.CMD
4 USERFILE
5 AJO USERFILE
6 QUIT

SELECT> 3

PICTURE TAKING:

1 EVERYTHING
2 SOMETHINGS
3 NOTHING

SELECT> 2

Available devices are:

1 VT220
2 VT240/Monochrome
3 Tektronix 4014
4 MATLN03

SELECT> 4

ENTER GRAPHICS FILENAME [MATLASER.DAT]: CARRIAGE RETURN

SELECT ANALYSIS:

1 TIME EVALUATION
2 GUSTS
3 QUIT

SELECT> 1
PLOT MENU
1 WHOLE
2 1/2 UP/LOW
3 QUIT

SELECT> 2

ENTER SIMULATION NAME(MAX 22 CHAR) FOR PLOT TITLE (NAME SHOULD HAVE AN ASTERISK AT THE BEGINNING AND BE SURROUNDED BY APOSTROPHES) "REGULATOR"

NO.COLUMNS,NO.RUNS,NMOD,PLANT OUTPUTS,CONTROLS,MODEL,SIGNAL COMMANDS

T =
22. 2. 1. 5. 2. 3. 1.

SELECT FOR UPPER
ENTER VECTOR OF VARIABLES DESIRED [2 3 4 11]

SELECT FOR LOWER
ENTER VECTOR OF VARIABLES DESIRED [5 6 7]

TAKE PICTURE NOW:
1 YES
2 NO

SELECT> 1

SELECT ANALYSIS:
1 TIME EVALUATION
2 GUSTS
3 QUIT

SELECT> 3

B.3 TIME RESPONSE TO EXTERNAL DISTURBANCE

SELECT COMMANDFILE
1 TEVAL.CMD
2 TGUST.CMD
3 TPLT.CMD
4 USERFILE
5 AJO USERFILE
6 QUIT

SELECT> 2
\[ <Y,T,ST> = GUSTR(SGDP, NSGD, SDC, NSDC, DELT, YV, COUNTER) \]

**CHOOSE CHANNELS**

1  CHANNEL1  
2  CHANNEL2  
3  QUIT  

**SELECT > 1**

**NO. OF DATA PTS.**

1  KEEP DEFAULT (1000)  
2  CHANGE  

**SELECT > 1**

TIME RESPONSE FOR FIRST CHANNEL COMPLETED

**CHOOSE CHANNELS**

1  CHANNEL1  
2  CHANNEL2  
3  QUIT  

**SELECT > 2**

**NO. OF DATA PTS.**

1  KEEP DEFAULT (1000)  
2  CHANGE  

**SELECT > 2**

ENTER NO. OF DATA PTS [2000]  

TIME RESPONSE FOR SECOND CHANNEL COMPLETED

**CHOOSE CHANNELS**

1  CHANNEL1  
2  CHANNEL2  
3  QUIT  

**SELECT > 3**

END OF TGUST.CMD

**SELECT COMMANDFILE**

1  TEVAL.CMD  
2  TGUST.CMD  
3  TPLLOT.CMD  
4  USERFILE  
5  AJO USERFILE  
6  QUIT  

**SELECT > 3**
PICTURE TAKING:

1 EVERYTHING
2 SOME THINGS
3 NOTHING

SELECT> 2

SELECT ANALYSIS:

1 TIME EVALUATION
2 GUSTS
3 QUIT

SELECT> 2

PLOT MENU

1 WHOLE
2 1/2 UP/LOW
3 QUIT

SELECT> 2

SIGNAL INPUTS:

1 CHANNEL 1
2 CHANNEL 2

SELECT> 2

NCY = 2

NO. COLUMNS, NO. MODELS, TOTAL OUTPUTS, PLANT OUTPUTS, CONTROLS, GUST

V = 9. 1. 9. 5. 2. 2.

SELECT FOR UPPER
ENTER VECTOR OF VARIABLES DESIRED [2 9] TOP HALF PLOTTED (FIG. 22)

SELECT FOR LOWER

SIGNAL INPUTS:

1 CHANNEL 1
2 CHANNEL 2

SELECT> 2
NO.COLUMNS,NO.MODELS,TOTAL OUTPUTS,PLANT OUTPUTS,CONTROLS,GUST

V =

9. 1. 9. 5. 2. 2.

ENTER VECTOR OF VARIABLES DESIRED [6]

LOWER HALF PLOTTED (FIG. 22)

TAKE PICTURE NOW:

1 YES
2 NO

SELECT> 1

SELECT ANALYSIS:

1 TIME EVALUATION
2 GUSTS
3 QUIT

SELECT> 3

SELECT COMMANDFILE

1 TEVAL.CMD
2 TGUST.CMD
3 TPLOT.CMD
4 USERFILE
5 AJO USERFILE
6 QUIT

SELECT> 6

B.4 ADDITIONAL OUTPUT DATA

GUSTAT =

COLUMNS 1 THRU 6
5.2292E-01 6.3624E-04 5.0874E-04 3.2339E-03 3.2607E-03 1.0395E-02
-2.9174E-01 2.2937E-04 -2.0502E-04 -6.7829E-04 -1.3392E-03 -3.1300E-03

COLUMNS 7 THRU 9
1.7837E-03 1.0000E+00 0.0000E+00
-5.3791E-04 5.9155E-02 0.0000E+00

GWSTAT =

COLUMNS 1 THRU 6
3.3245E+02 3.8928E-01 2.8748E-01 1.0632E+00 2.3758E+00 2.0152E+01
2.3273E+01 -2.2649E-02 7.8752E-04 3.0784E-01 1.0629E-01 1.4603E+00

COLUMNS 7 THRU 9
3.4581E+00 0.0000E+00 1.0000E+00
2.5132E-01 0.0000E+00 4.3018E-02

45
Figure 1. Frequency-analysis flowchart.
Figure 2. Frequency-evaluation flowchart.
ANALYSIS

1. CHANGE OMEG
2. S.V. ERROR
3. S.V. LOOP TRANSFER
4. BODE FREQ RESPONSE
5. SSCOV
6. CLOSED LOOP EIG
7. QUIT

Figure 3. Frequency-evaluation menu.

Figure 4. Multiplicative error at plant input.

Figure 5. Input-to-output map.
Figure 6. Singular-value loop transfer.

Figure 7. Bode configuration.
Figure 8. Configuration for steady-state covariance.

Figure 9. Time-analysis flowchart.
Figure 10. Generic control configuration.

(a) \( C_1 = 0; \ C_3 = 1; \ B_f = B_b; \ D_f = D_b. \)

(b) \( C_1 = 0; \ B_f \) and \( D_f \) general.

(c) \( C_3 = 0, \ B_f \) and \( D_f \) not included.

Figure 11. Control configurations.
GENC LIMITER PLANT
OUTPUTS DEFINED BY YVV

(a) Closed-loop control system.

SIGNAL
CGENC or DGENC
SYSLIMITERTPLANT

(b) Signal generator and closed-loop control system.

Figure 12. Time-evaluation simulation.

Figure 13. Hybrid simulation.
Figure 14. Integrated control-design and analysis approach.

Figure 15. Command file for process input data.
Figure 16. Computer-generated output for singular-value multiplicative error.

Figure 17. Computer-generated output for singular-value loop transfer.
Figure 18. Computer-generated output for Bode frequency response.

Figure 19. Computer-generated output for Nyquist plot.
Figure 20. Computer-generated output for steady-state covariance plot. (State and output numbers added for clarity.)

Figure 21. Computer-generated output for regulator time response. (Numbers on plots represent outputs and are added for clarity.)
Figure 22. Computer-generated output for time response of combined plant to random disturbance. (Numbers on plots represent outputs and are added for clarity.)
The basic functions embedded within a user-friendly software package (MATRIXx) are used to provide a high-level systems approach to the analysis of linear control systems. Various control-system analysis configurations are assembled automatically to minimize the amount of work by the user. Interactive decision making is incorporated via menu options and at selected points, such as in the plotting sections, by inputting data. One section in the paper describes the frequency and steady-state analysis. There are five evaluations—the singular-value robustness test, the singular-value loop-transfer frequency response, the Bode frequency response, the steady-state covariance analysis, and the closed-loop eigenvalue calculations. Another section describes time-response simulations. These simulations, which are linear except for actuator saturation, may have either a continuous plant or a discrete plant and include a generic controller structure for both feedback and feedforward dynamics. A time response for a random white-noise disturbance is available. This paper describes the configurations and key equations used for each type of analysis, the restrictions that apply, the type of data required, and a sample problem. One approach to integrating the design and analysis tools is also presented.